Metal Structures

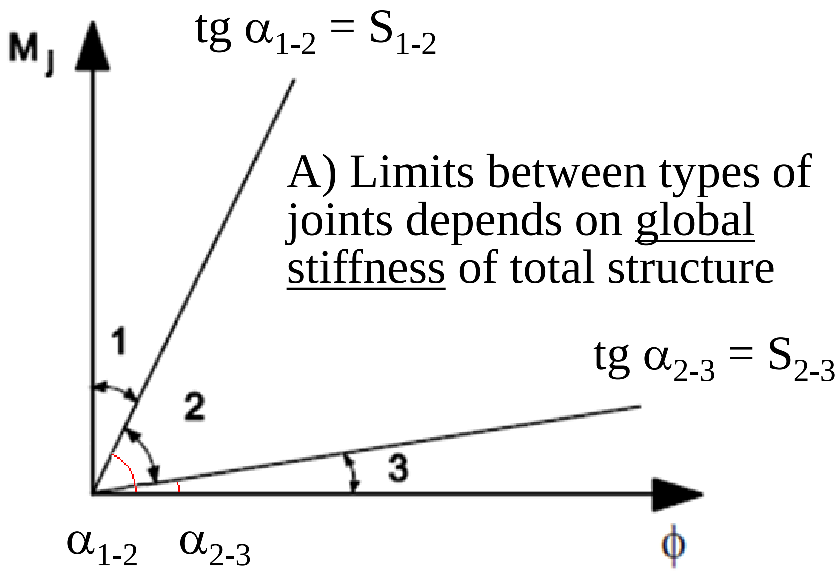
Lecture XV

Stiffness of joints

Contents

Local stiffness of joint

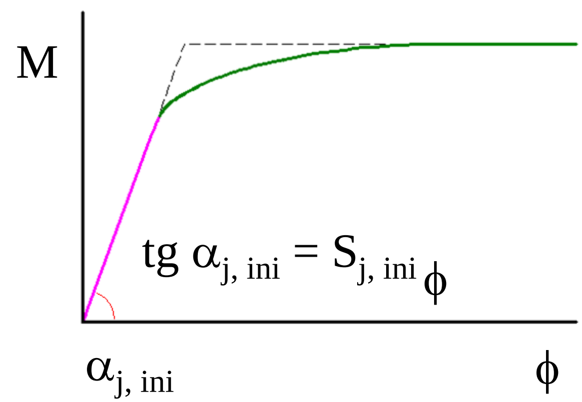
According to information presented in previous lecture: type of node depends on ratio between global stiffness (for total structure) and local stiffness of node elements (plates, bolts). Local stiffness of joint is defined by many dimensions and factors, concern various phenomena.



A) Limits between types of joints depends on <u>global stiffness</u> of total structure

Photo: EN 1993-1-8 fig 5.4

Photo: Author



B) Characteristic of joint depends on <u>local stiffness</u> of sub-parts of joint

Eurocode procedures for calculating joint stiffness focus on beam-column tension joints.

Recommendation for this type of joint, come from experiecne:

1. It’s recommended to take into consideration only three rows of bolt in tensed part of joint (the furthest from compressed flange of beam: 1, 2, 3).

2. Although it, we should applied rows through whole high of beam for bolted joint category E.

3. One row of bolt = 2 bolts only; we are not sure, if for 4 bolts in row formulas are the same

4. Shear force is applied to the lowest row of bolts only

5. Eurocodes not presented cleary information about influence of vertical rib over flange of beam on stifeness and resistance of joint. Recomendation: square rib (axa) of thickness the same as web of beam. Rib should reach ≈ 200 mm over the highest row of bolts. Solution is not very popular – rib is in collision with concrete plate.

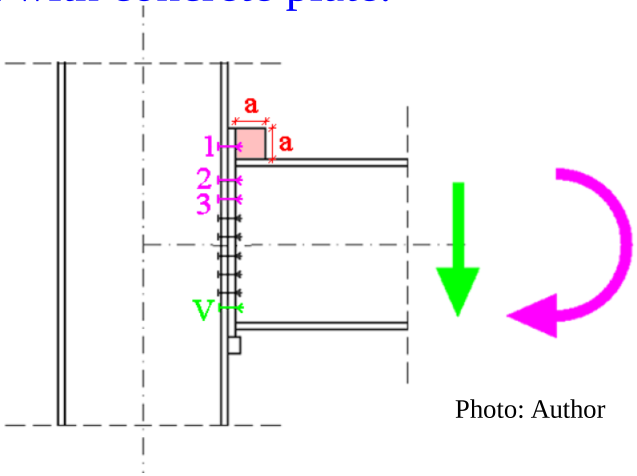


Photo: Author

Dimensions

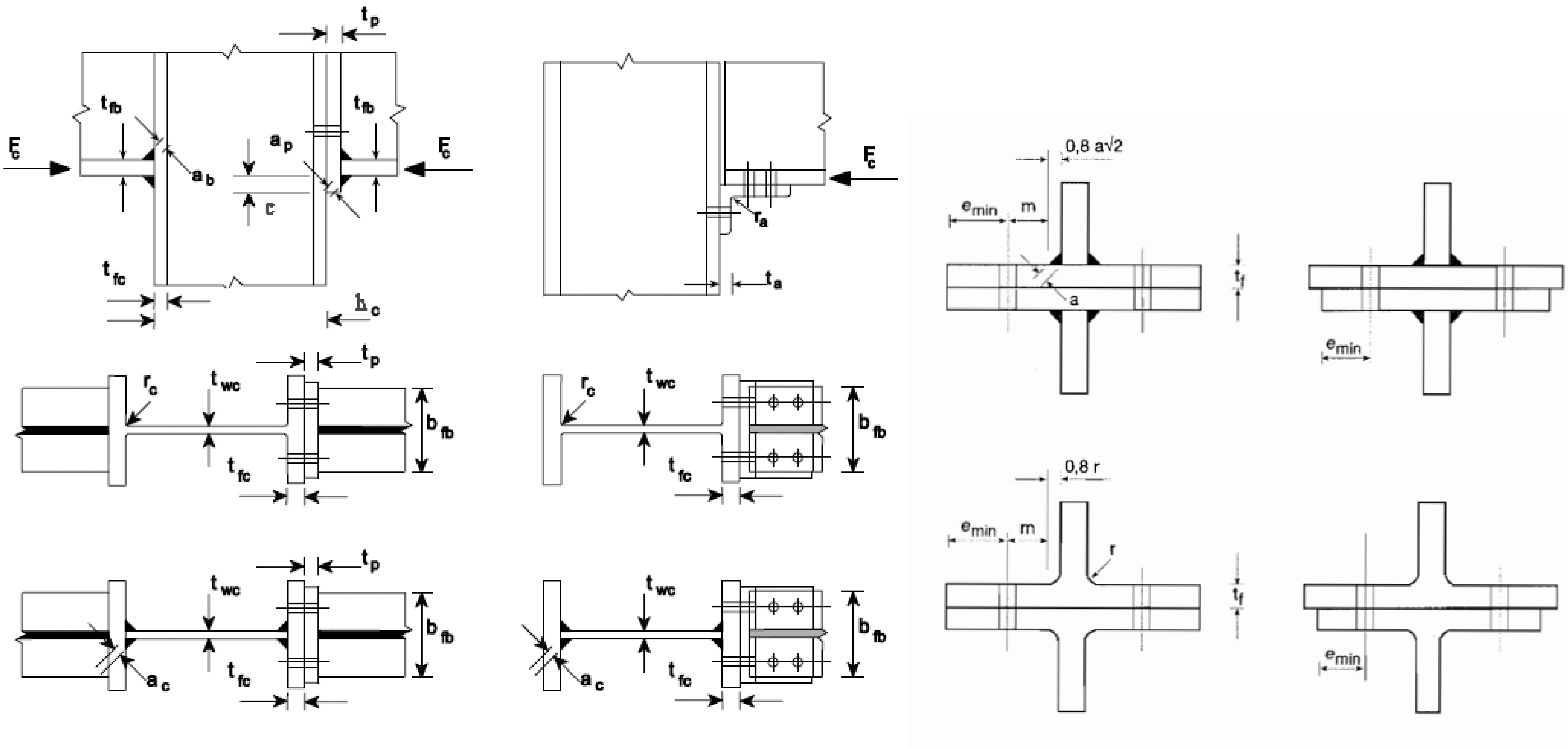
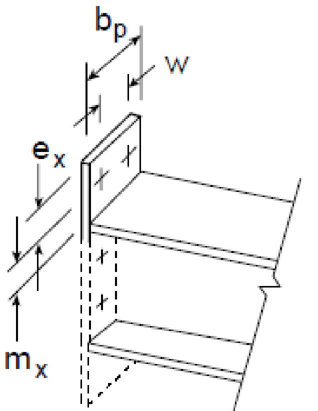
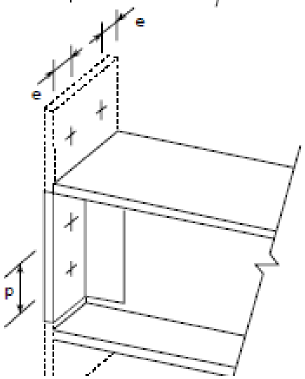
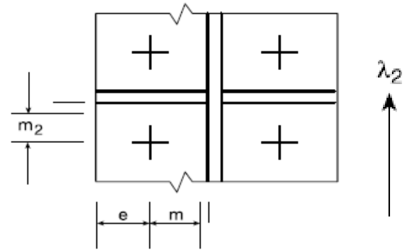


Photo: EN 1993-1-8 fig. 6.2 , fig. 6.6

Dimensions

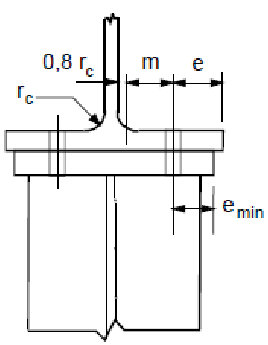


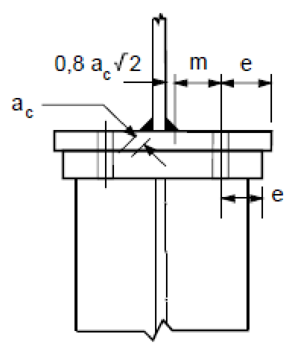




$$\lambda_1 = \frac{m}{m + e}$$

$$\lambda_2 = \frac{m_2}{m + e}$$





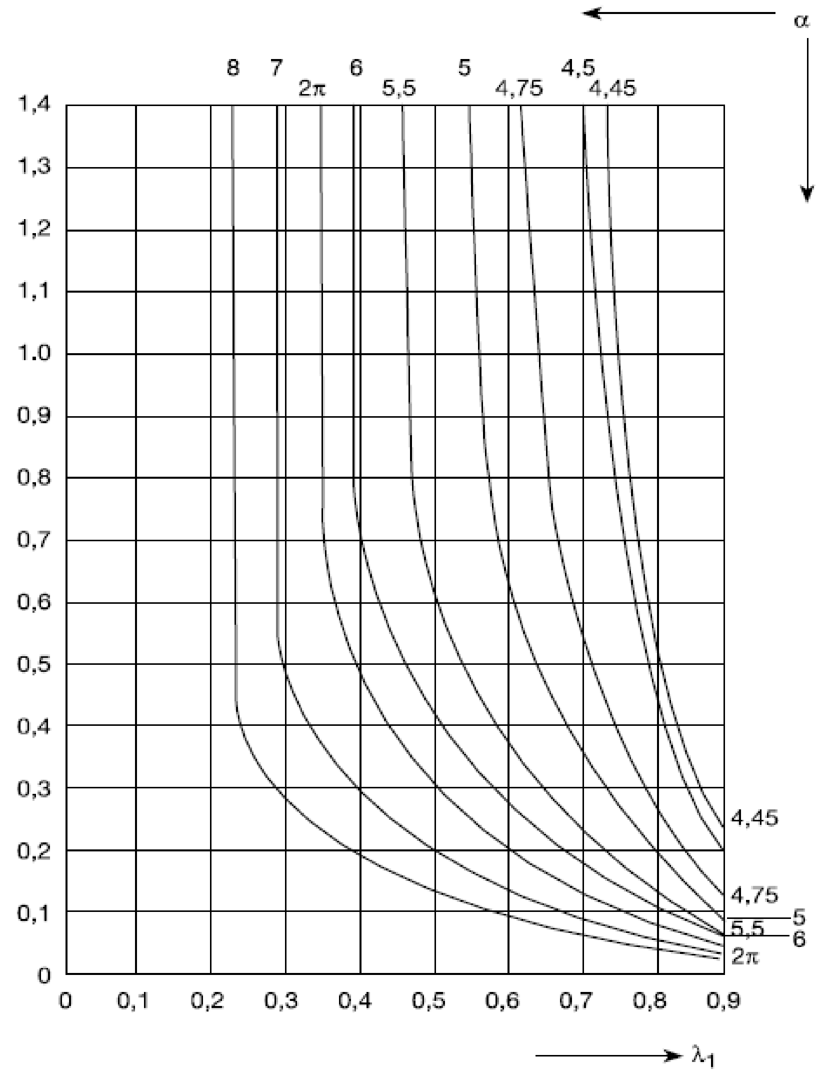


Photo: EN 1993-1-8 fig 6.8, 6.10. 6.11

Dimensions

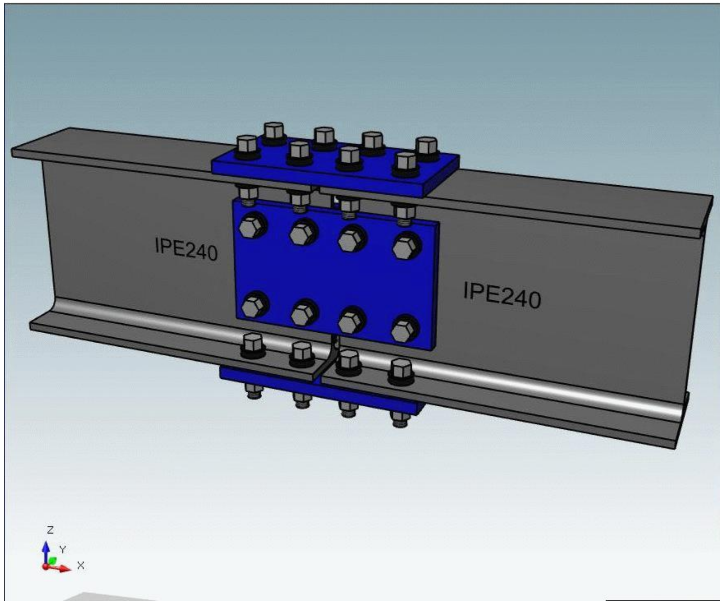


Photo: gsi-eng.eu

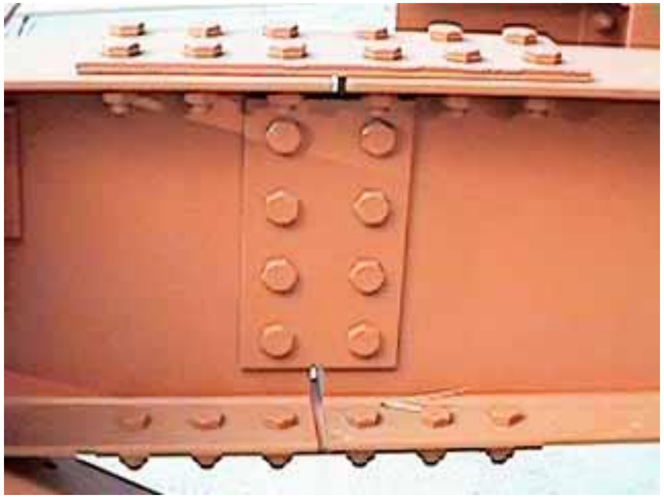


Photo: amsd.co.uk

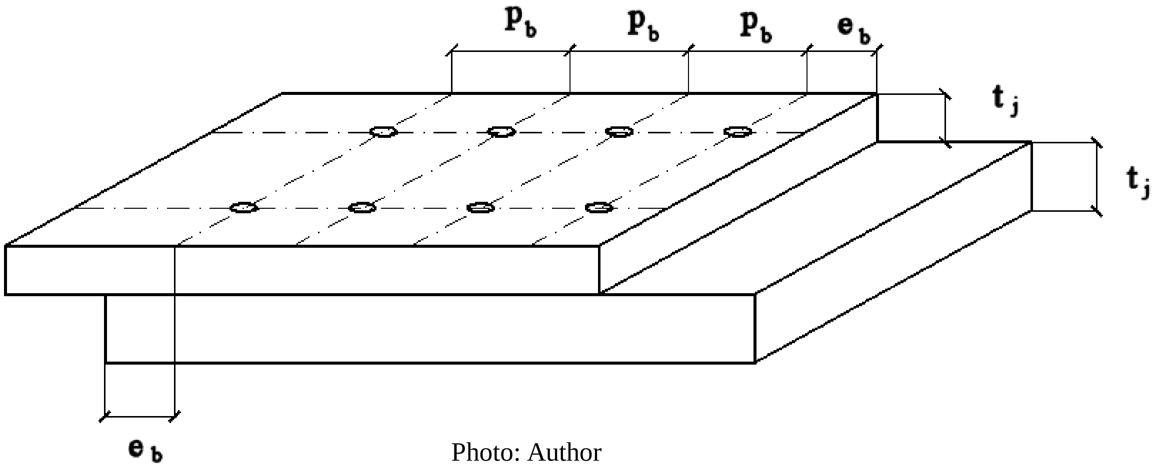


Photo: Author

Dimensions

Arms of actions:

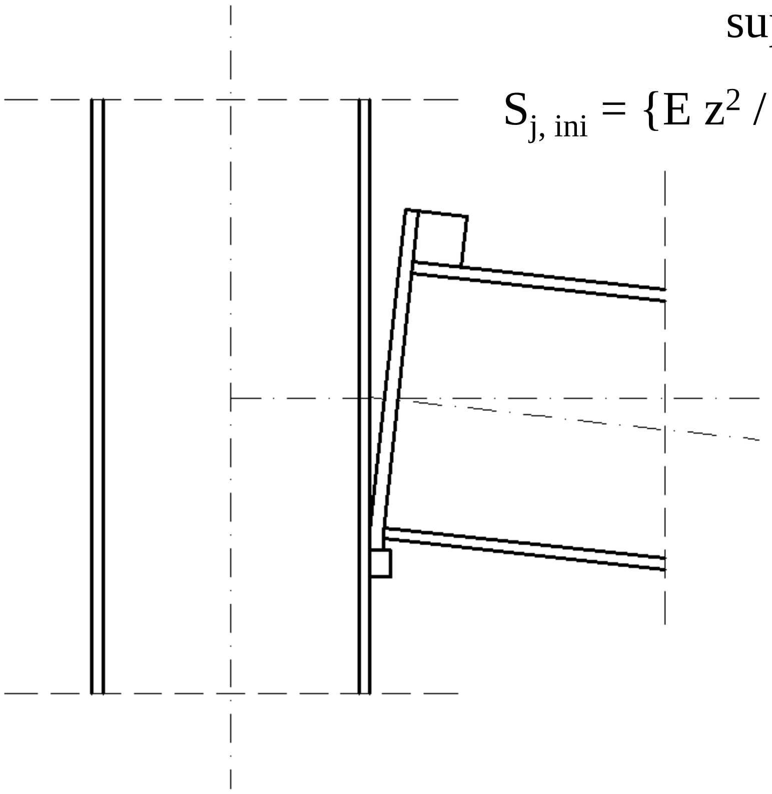
"frame" joints:

$$S_{j,\ ini} = E\ z^2 / [\ \Sigma\ (1 / k_i)]$$

Photo: Author

support joints:

$$S_{j,\ ini} = \{E\ z^2 / [\ \Sigma\ (1 / k_i)]\}\ e / (e + e_k)$$



Theoretical situation: no bolts in joint – rotation of beam around bottom part of compressed zone. Theoretical axis of rotation: centre of gravity (CoG) of compressed beam's flange.

Arm of action z is defined as distance between CoG of bottom flange to CoG of top flange or axis of bolt (AoB):

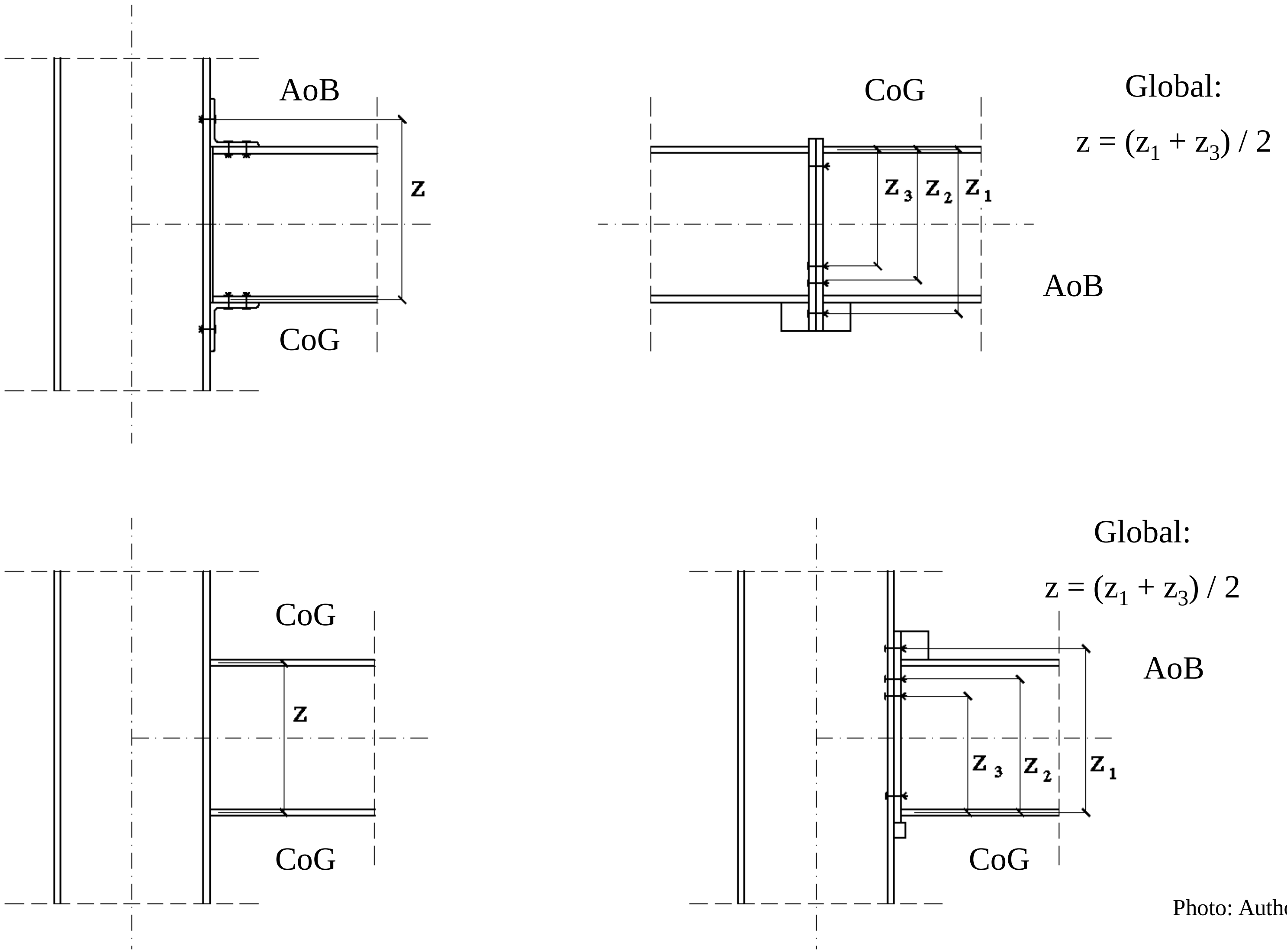
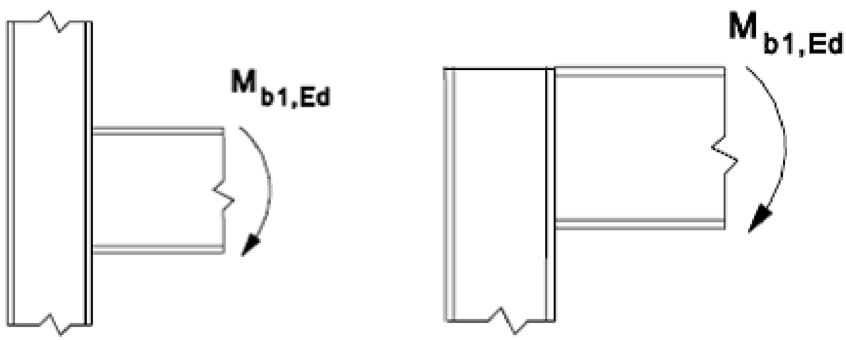
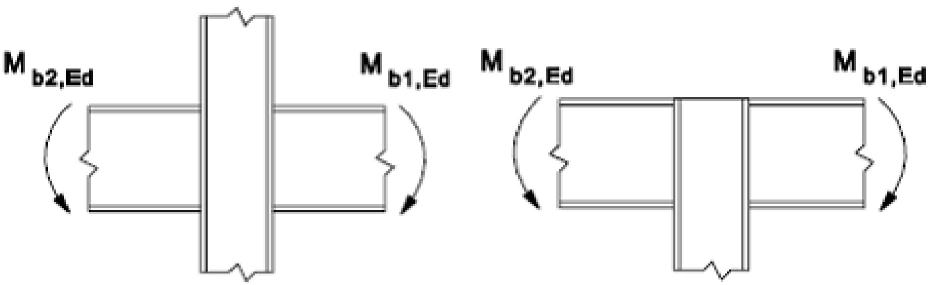


Photo: Author

Beam-column geometry

EN 1993-1-8 tab. 5.4

Type of joint configuration	Action	Ttransformation parameter β
	$M_{b1,\ Ed}$	$\beta \approx 1$
	$M_{b1,\ Ed} = M_{b2,\ Ed}$	$\beta = 0$ *)
	$M_{b1,\ Ed} = M_{b2,\ Ed} > 0,0$	$\beta \approx 1$
	$M_{b1,\ Ed} = M_{b2,\ Ed} < 0,0$	$\beta \approx 2$
	$M_{b1,\ Ed} + M_{b2,\ Ed} = 0,0$	
*) in this cane the value of β is the exact value rater than an approximation		

EN 1993-1-8 tab. 6.3

β	ω
$0{,}0 \le \beta \le 0{,}5$	$\omega = 1{,}0$
$0{,}5 \le \beta < 1{,}0$	$\omega = \omega_1 + 2(1 - \beta)\,(1 - \omega_1)$
$\beta = 1{,}0$	$\omega = \omega_1$
$1{,}0 < \beta < 2{,}0$	$\omega = \omega_1 + 2(1 - \beta)\,(\omega_2 - \omega_1)$
$\beta = 2{,}0$	$\omega = \omega_2$

$$\omega_1 = 1 / \sqrt{[1 + 1{,}3(b_{eff,\,c,\,wc}\ t_{wc} / A_{vc})^2]}$$

$$\omega_2 = 1 / \sqrt{[1 + 5{,}2(b_{eff,\,c,\,wc}\ t_{wc} / A_{vc})^2]}$$

$$A_{vc} = A_{vc,\,column}\ \ (\approx h_w\ t_w)$$

β , ω - two ways to take into account shape of join (one beam to column / two beams to column…)

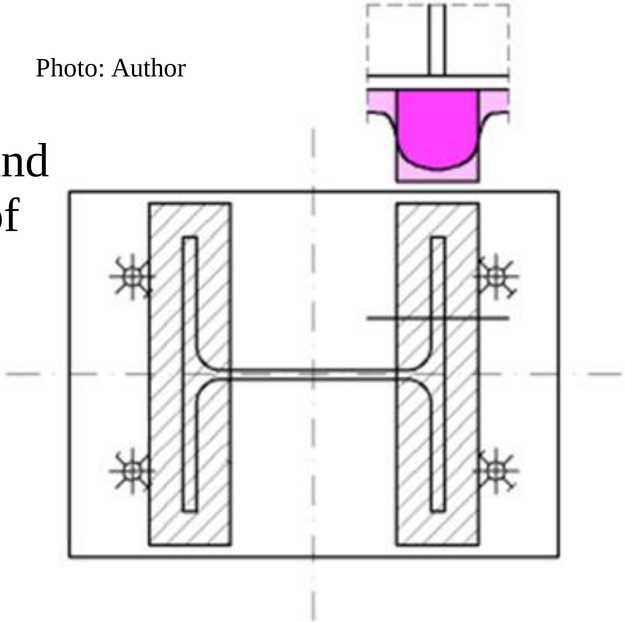
Column base

Photo: diy.stackexchange.com

Photo: osha.gov

1. Concrete (f_{cd}) decides about resistance of base: it is the weakest element, which can be destroyed as first. Resistance of base depends on effective area of contact between steel and concrete.

Photo: Author



2. Effective area is the area of the cross-section of the column and its the nearest neighborhood. This is the result of linearisation of stress under base plate.

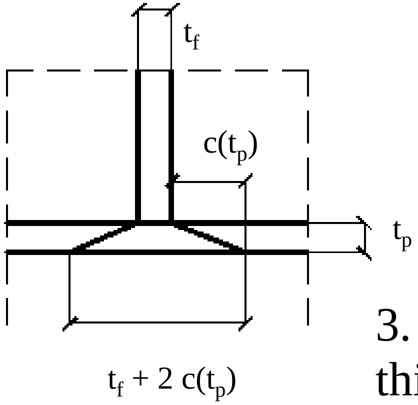
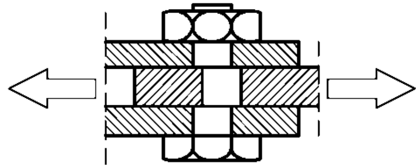
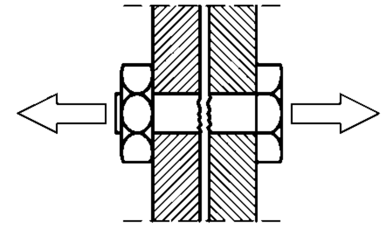


Photo: Author

3. Range of the nearest neighborhood c is proportional to the thickness of base plate.

Categories of bolted joints and loads

					
Categories of bolted joint	A	B	C	D	E
Types of loads	Static without changing the direction of the bending moments; aerodynamic	Static with changing the direction of the bending moments; aerodynamic	Dynamic	Static; aerodynamic	Dynamic
Types of bolts	„normal”	preloaded		„normal”	preloaded

Changing the direction of the bending moment:
various combinations of loads

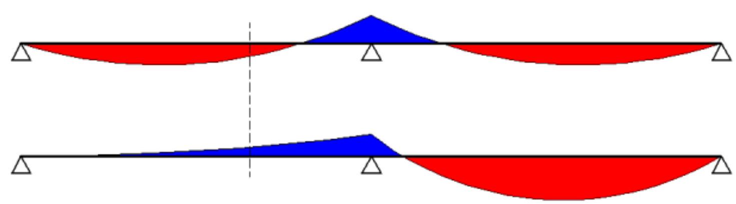


Photo: Author

Shear of bolt’s shank and bearing (deformation of plates as a effect of contact with shank) – two very important phenonena occur in case of shear bolted joint.

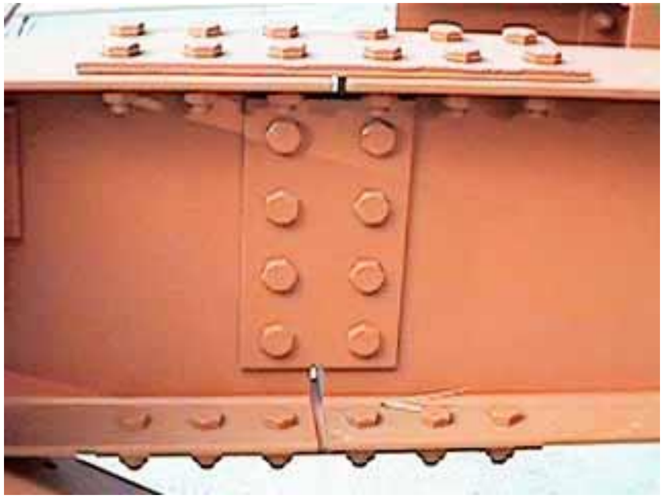


Photo: amsd.co.uk

Photo: ascelibrary.org

Photo: ceprofs.civil.tamu.edu

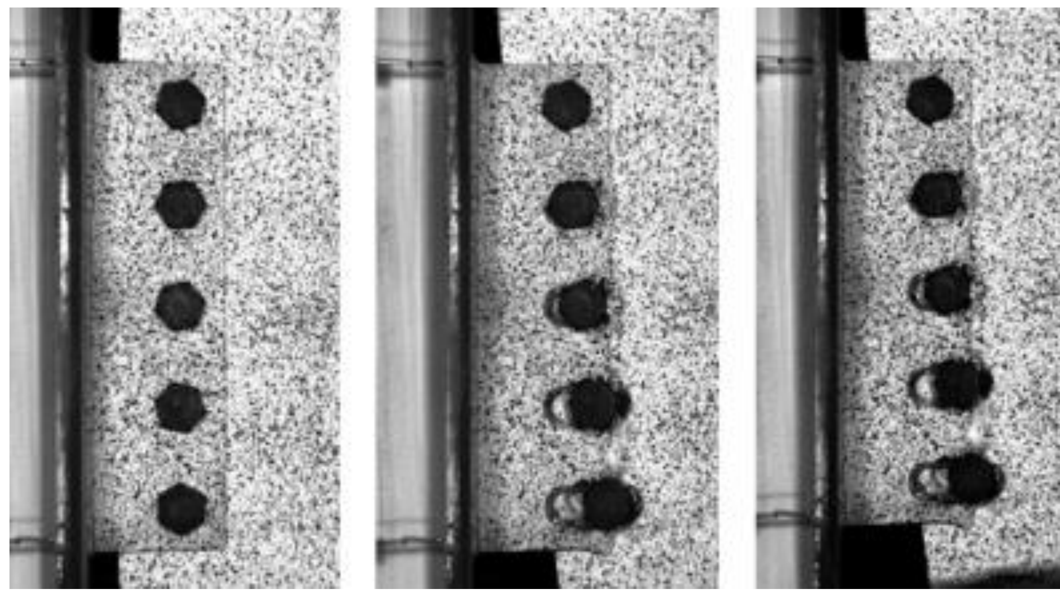


Photo: ascelibrary.org

Stresses in bolted tensile joint

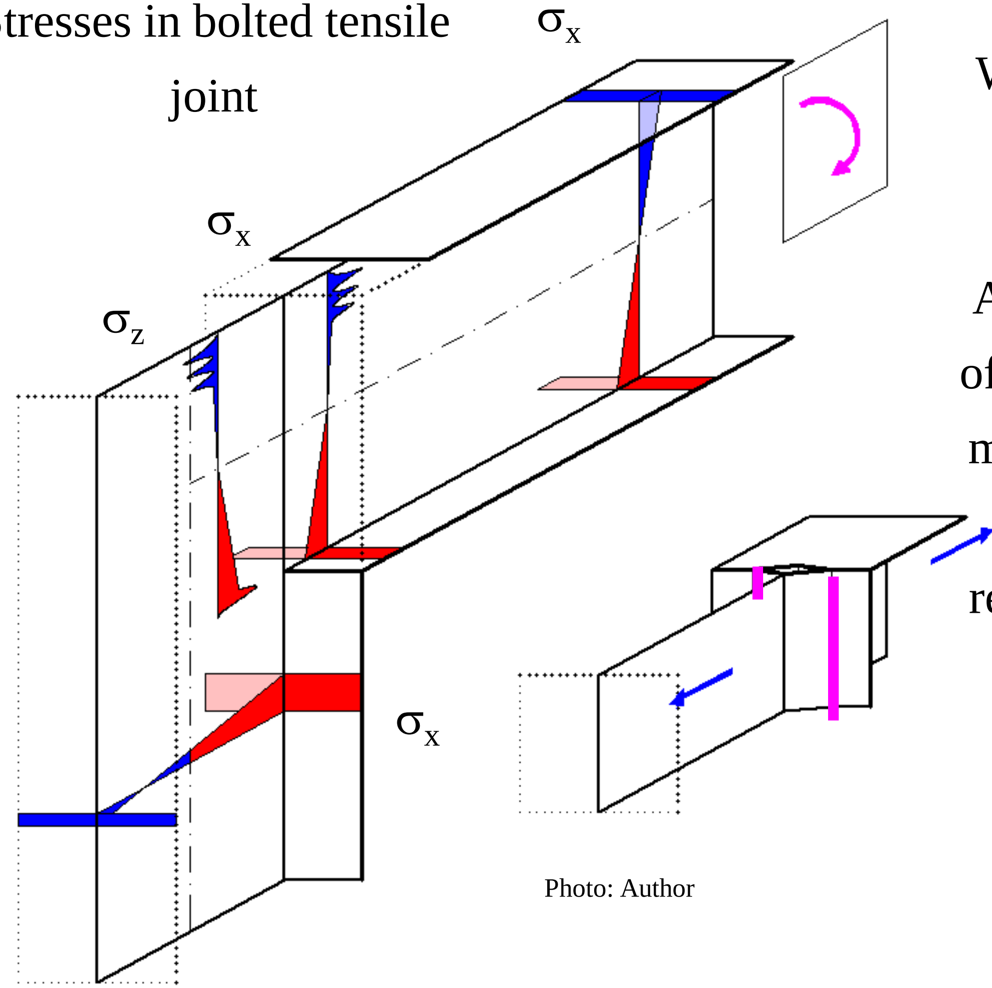
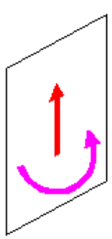


Photo: Author

We have a full set of cross-sectional forces in beam: M_{Ed} , N_{Ed} , V_{Ed} .

Axial force is taken into account only in form of stipulation in EN 1993-1-8 6.2.7.1 (2) that it must not exceed 5% of beam's resistance; it is then neglected in calculations. This requirement is another condition to be met for beam.

Shear force is traditionally applied by calculation to compressed zone only.



Shear force in column don't affected on joint.

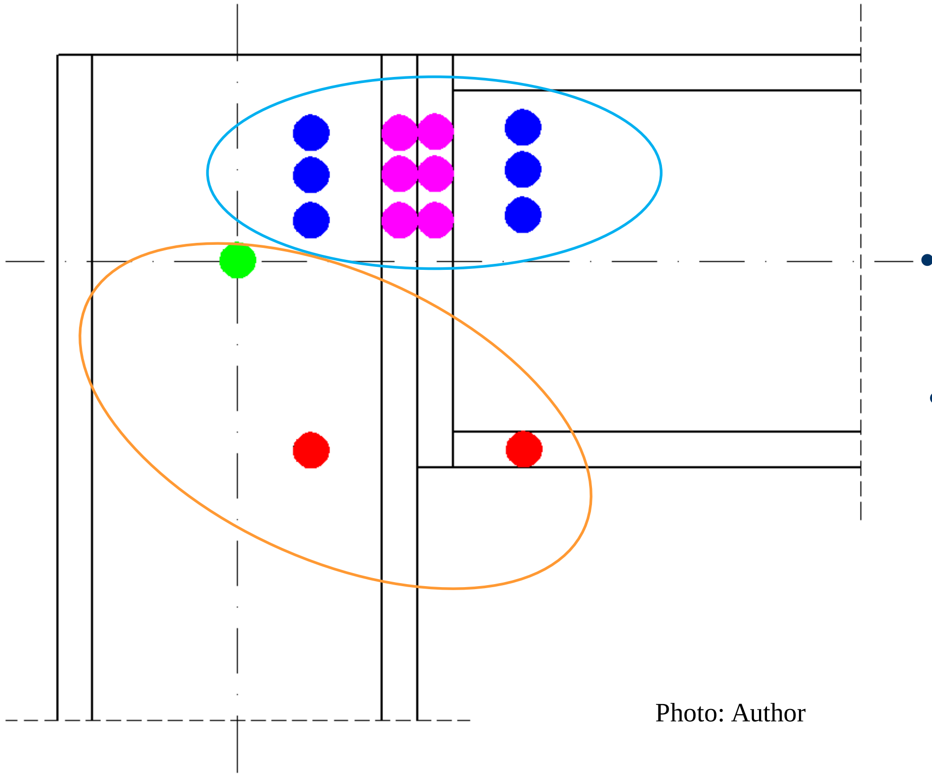


Photo: Author

Tensed zone

Shear zone is included in compressed zone.

Various resistances for various rows of bolts:

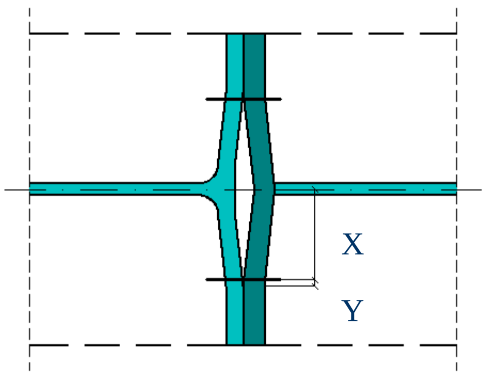
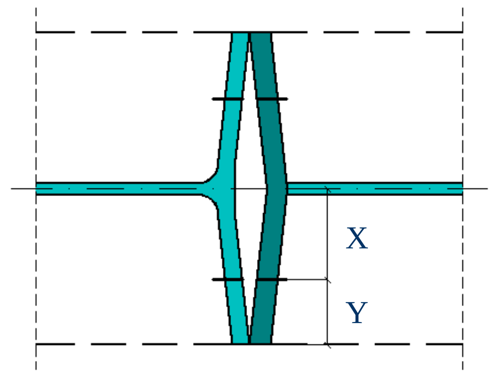
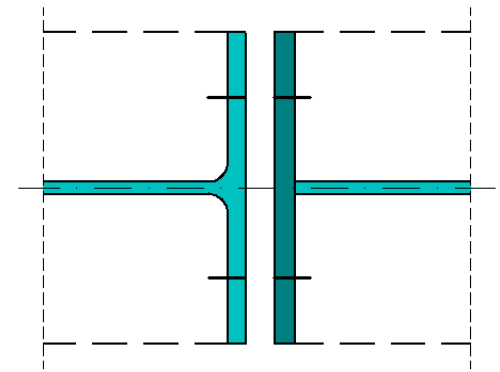
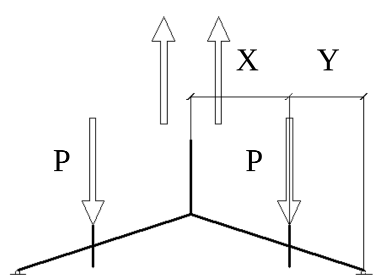
- Local transverse tension of column's web around I, II, III row of bolts;
- Local bending of column's flange around I, II, III row of bolts;
- Local bending of end plate around I, II, III row of bolts;
- Local longitudinal tension of beam's web around I, II, III row of bolts;

The same resistance for each row of bolts:

- Local transverse compression of beam's flange;
- Local transverse compression of column's web under global longitudinal compression;
- Local shearing of column's web between "scissors" of compressed and tensed zones;

Photo: Author

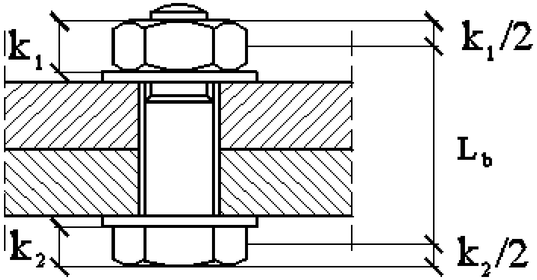
3 models of destruction must be analysed in bended zone.

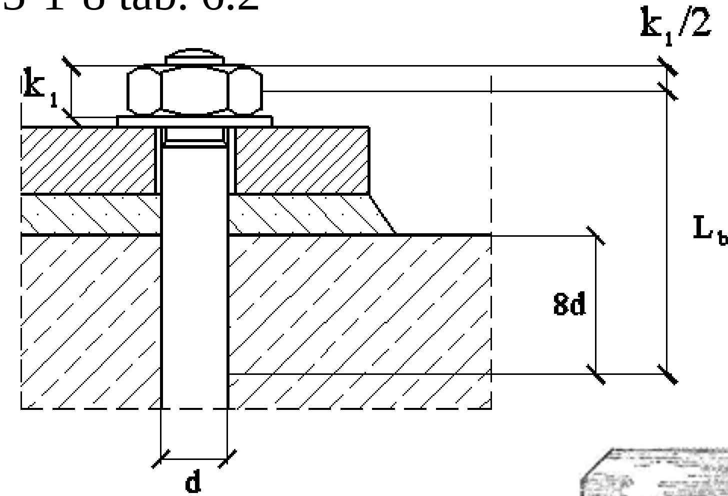
Model 1 – complete yielding of flange / end plate	Model 2 – bolt failure with (partial) yelding of flange / eng plate	Model 3 – bolt failure
Thin column’s flange / end plate – prying action		Fat column’s flange / end plate
		
Force in bolt = P Total force in tensed zone = F $P\,Y = (F/2)\,(X+Y)$ $P = (F/2)\;(X+Y)/Y$ $(X+Y)/Y > 2 \;\rightarrow\; P > F$ (!!!)		Force in bolt = P Total force in tensed zone = F $P = F/2$

Prying action - when it can occur?

Photo: Author

EN 1993-1-8 tab. 6.2





$L_b \leq L_b^*$ → Prying forces

$L_b > L_b^*$ → No prying forces

$L_b^* = 8{,}8\, m^3 A_s / (\Sigma l_{eff}\ \ t_f^3)$

A_s – area of bolt cross-section in threated portion

t_f – the thickness of the thinnest plate

m → #t / 6

Σl_{eff} → #t / 23 – 25, #t / 27

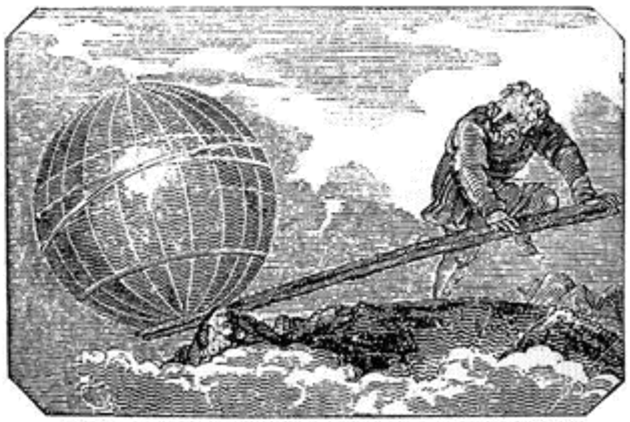


Photo: physics.weber.edu

Give me the place to stand, and I shall move the Earth

Calculation model: effective area of stress concentration - effective length

Flange, plate → l_{eff}

Web → b_{eff} (other symbol, but value the same as for flange / plate l_{eff})

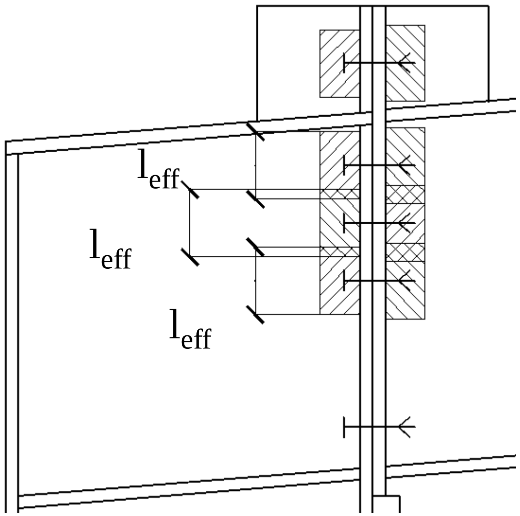


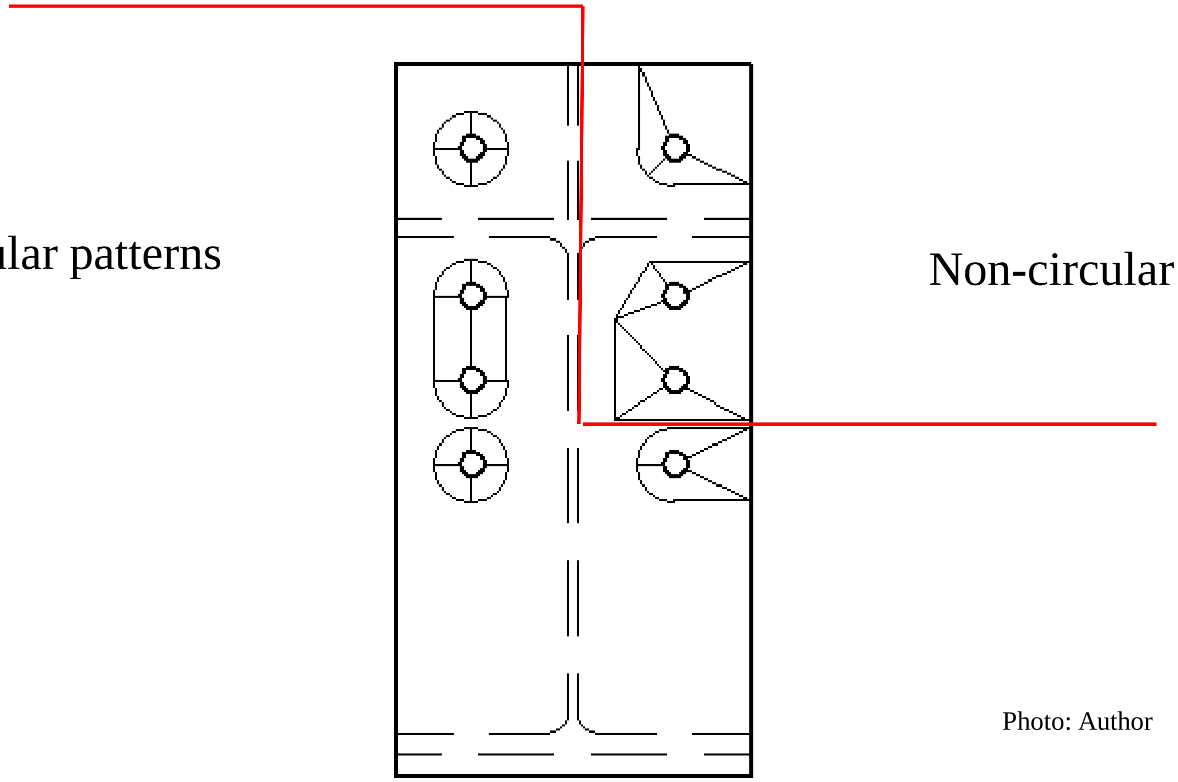
Photo: Author

There is possible, that effective areas from two row of bolts would be common. In this situation we must analysed group of bolts Σl_{eff}, not separate bolts l_{ef} .

Σl_{eff} is important for resistance only, l_{ef} is important for resistance and stiffness.

Generally, breakage of plate / flange is possible by two ways:

Circular patterns



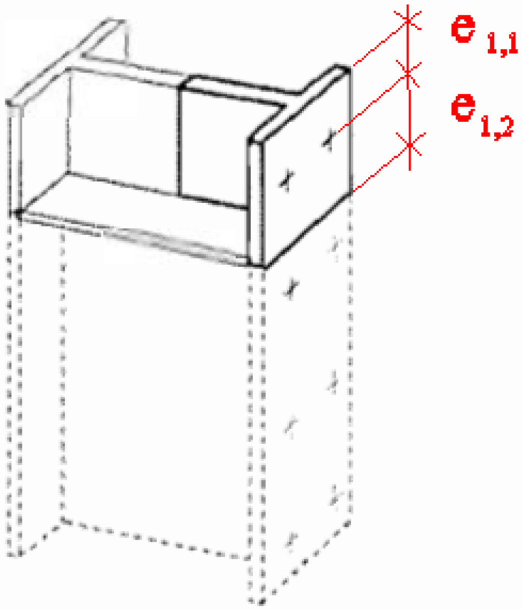
Non-circular patterns

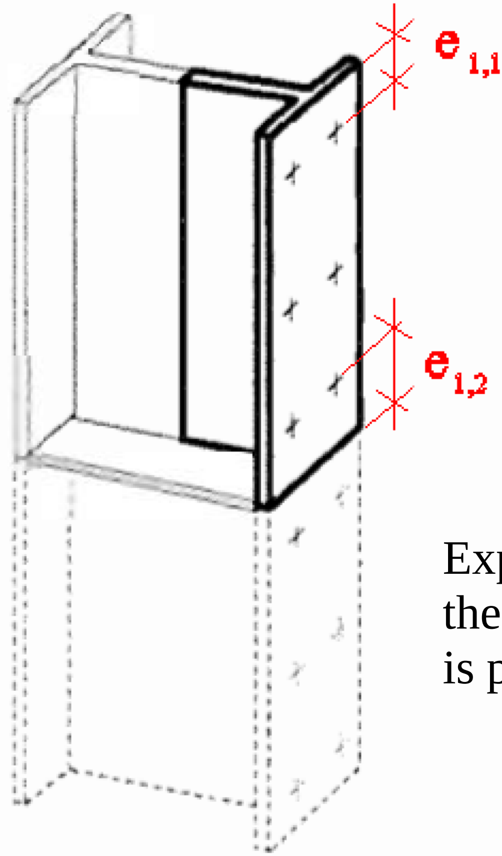
Photo: Author

There are different values of l_{eff} for both. We must calculate l_{eff} for both and take into following consideration less of them.

$e_{1,1}$ – distance from bolt to end of column's flange

Photo: Author





Explanation is not given in the Polish version of EN; is presented in amendment

$e_{1,2}$ – the least distance from bolt to the nearest stiffener

The information in the Eurocode (row name) ↔ (location) is not completely clear; other interpretations can also be found in the literature

Analysis of literature and Eurocode indicates existence of 4 differently working rows of bolts. Descriptions from EN 1993-1-8 will be given for these four series.

One over flange of beam / column stiffener
First below flange of beam / column stiffener
Next below flange of beam / column stiffener
Last below flange of beam / column stiffener

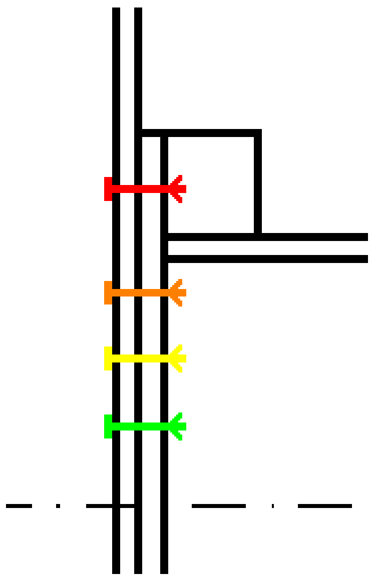
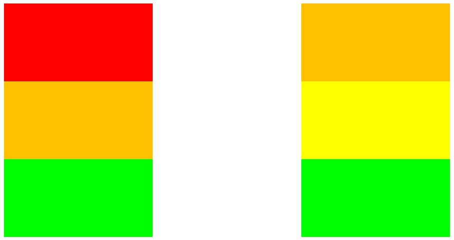


Photo: Author

The most often, one of two situations is applicated:

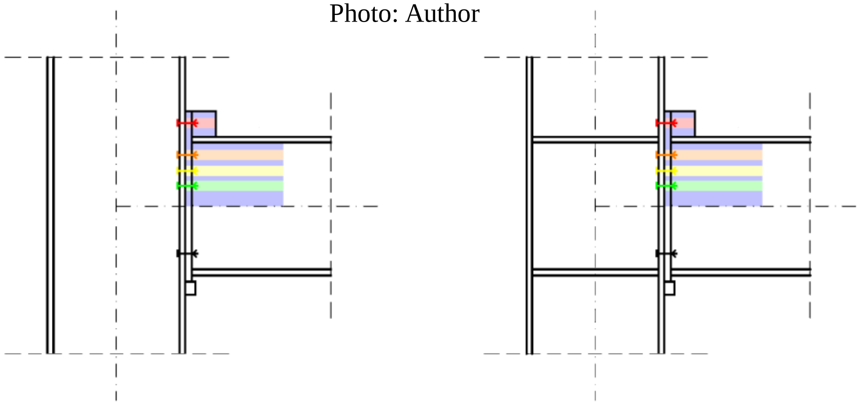


End-plate / beam web

(Web stiffeners doesn't matter)

EN 1993-1-8 tab. 6.6

Photo: Author

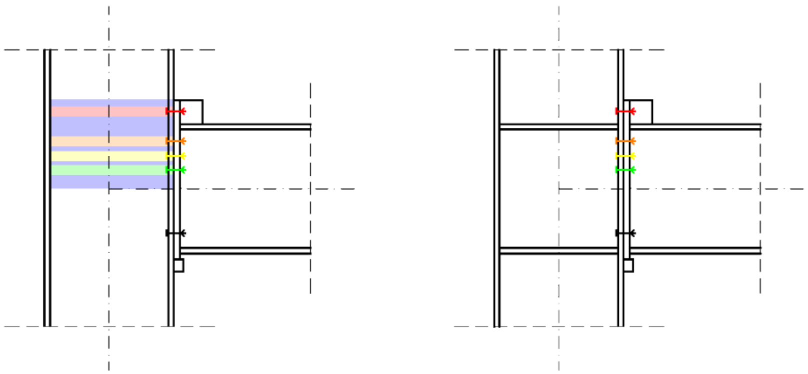


Bolt-row location	Bolt-row considered individually		As part of a group of bolt-rows	
	Circular $l_{eff,\,cp}$	Non-circular $l_{eff,\,nc}$	Circular $\Sigma l_{eff,\,cp}$	Non-circular $\Sigma l_{eff,\,nc}$
Bolt-row outside tension flange of beam	min ($2\pi m_x$; $\pi m_x + w$; $\pi m_x + 2e$)	min ($4m_x + 1{,}25e_x$; $e + 2m_x + 0{,}625e_x$; $0{,}5b_p$; $0{,}5w + 2m_x + 0{,}625e_x$)	-	-
First bolt-row below tension flange of beam	$2\pi m$	αm	$\pi m + p$	$0{,}5p + \alpha m - 2m - 0{,}625e$
Other inner bolt-row	$2\pi m$	$4m + 1{,}25e$	$2p$	p
Other end row-bolt	$2\pi m$	$4m + 1{,}25e$	$\pi m + p$	$2m + 0{,}625e + 0{,}5p$

Unstiffened column flange / unstiffened column web

EN 1993-1-8 tab. 6.4

Photo: Author



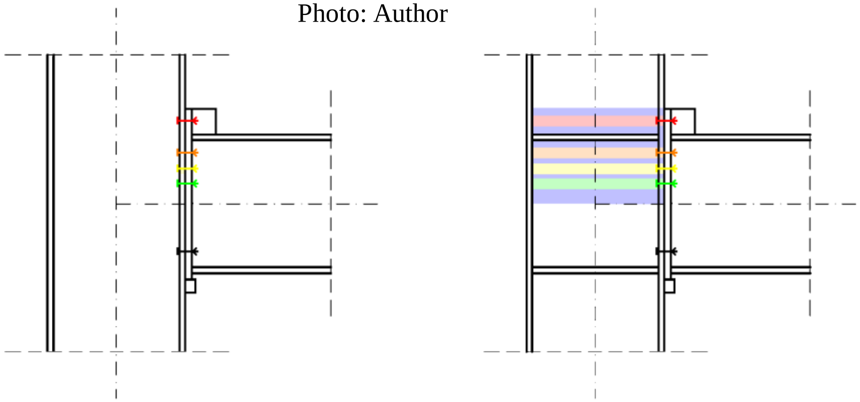
Bolt-row location	Bolt-row considered individually		As part of a group of bolt-rows	
	Circular $l_{eff,\ cp}$	Non-circular $l_{eff,\ nc}$	Circular $\Sigma l_{eff,\ cp}$	Non-circular $\Sigma l_{eff,\ nc}$
(Top) end row bolt	min ($2\pi m$; $\pi m + 2e_{1,1}$)	min ($4m + 1{,}25e$; $2m + 0{,}625e + 2e_{1,1}$)	min ($\pi m + p$; $2e_{1,1} + p$)	min ($2m + 0{,}625e + 0{,}5p$; $e_{1,1} + 0{,}5p$)
Inner bolt-row	$2\pi m$	$4m + 1{,}25e$	$2p$	p
(Bottom) end bolt row	min ($2\pi m$; $\pi m + 2e_{1,b}$)	min ($4m + 1{,}25e$; $2m + 0{,}625e + 2e_{1,b}$)	min ($\pi m + p$; $2e_{1,b} + p$)	min ($2m + 0{,}625e + 0{,}5p$; $e_{1,b} + 0{,}5p$)

Part not completely clear in Eurocode; $e_{1,1}$ according to #t / 21 ; $e_{1,b}$ could be problably taken as distance to bottom end of column or to joint in lower lewel of frame

Stiffened column flange / stiffened column web

EN 1993-1-8 tab. 6.5

Photo: Author



Bolt-row location	Bolt-row considered individually		As part of a group of bolt-rows	
	Circular $l_{eff,\,cp}$	Non-circular $l_{eff,\,nc}$	Circular $\Sigma l_{eff,\,cp}$	Non-circular $\Sigma l_{eff,\,nc}$
End bolt-row adjacent to a stiffener	min ($2\pi m$; $\pi m + 2e_{1,1}$)	min ($e_{1,1} + \alpha m - 2m - 0{,}625e$; αm)	-	-
Bolt-row adjacent to a stiffener	$2\pi m$	αm	$\pi m + p$	$0{,}5p + \alpha m - 2m - 0{,}625e$
Other inner bolt-row	$2\pi m$	$4m + 1{,}25e$	$2p$	p
Other end bolt-row	min ($2\pi m$; $\pi m + 2e_{1,2}$)	min ($4m + 1{,}25e$; $2m + 0{,}625e + 2e_{1,2}$)	min ($\pi m + p$; $2e_{1,2} + p$)	min ($2m + 0{,}625e + 0{,}5p$; $e_{1,2} + 0{,}5p$)

Recommendation in literature

<u>Informal</u> remarks on the effect of the reinforcing rib above the beam flange

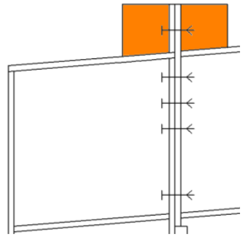
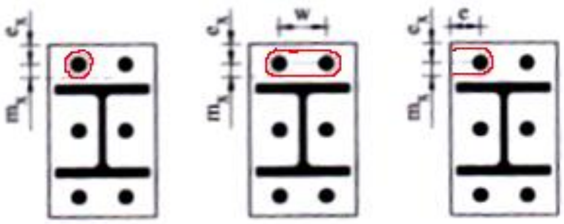


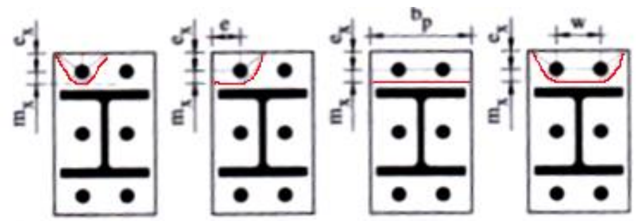
Photo: Author

Circular patterns



1 2 3

Non-circular patterns



1 2 3 4

Photo: J. Goczek, Ł. Supeł, M. Gajdzicki, Przykłady obliczeń konstrukcji stalowych, Politechnika Łódzka 2011

Formulas for effective lengths given, in the table, apply to specific failure mechanisms. Reinforcement of plate over beam flange by a vertical rib (→ #t / 4) will make **second** mechanism in circular ($\pi m_x + w$) and **third** + **fourth** mechanisms in non-circular ones ($e + 2m_x + 0{,}625e_x$; $0{,}5b_p$) impossible (rib will prevent collapse involving both bolts at the same time). These formulas can be omitted from calculations.

Flange cleat

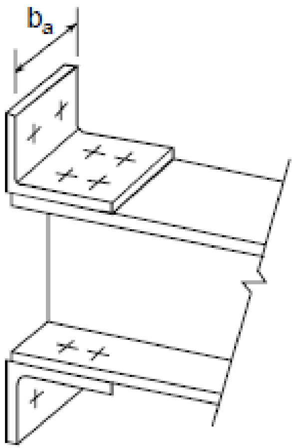


Photo: EN 1993-1-8 fig. 6.12

$$l_{eff} = b_a / 2$$

Photo: Behaviour of stiffened flange cleat joints, D. SkejicD. Dujmovic, D. Beg

Effective areas in compressed part of column web

EN 1993-1-8 6.2.6.2

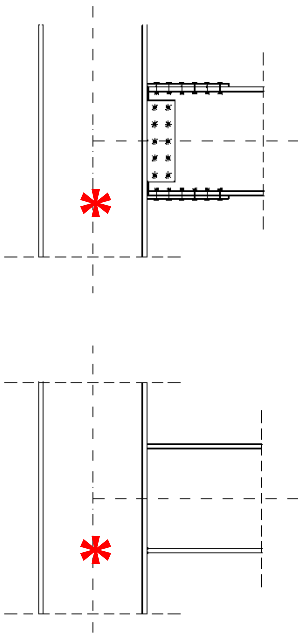
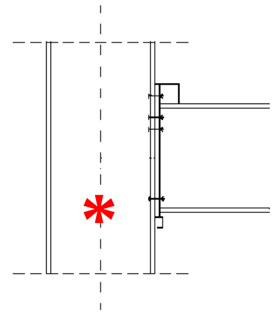
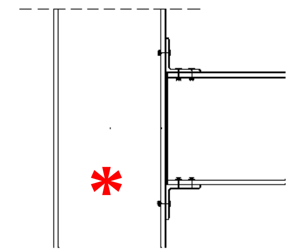
			
$b_{eff,\,c,\,wc}$	$t_{fb} + 2\sqrt{2}\, a_b + 5(t_{fc} + s)$	$t_{fb} + 2\sqrt{2}\, a_p + 5(t_{fc} + s) + s_p$	$2t_a + 0{,}6\, r_a + 5(t_{fc} + s)$

Photo: Author

Effective areas in compressed part

Column:	s_p	s	d_{wc}
Welded I-beam	$\min(t_p + c\ ;\ 2t_p)$	$\sqrt{2}\, a_c$	$h_c - 2(t_{fc} + \sqrt{2}\, a_c)$
Hot rolled I-beam		r_c	$h_c - 2(t_{fc} + r_c)$

c – length of end-plate out off bottom flange of beam

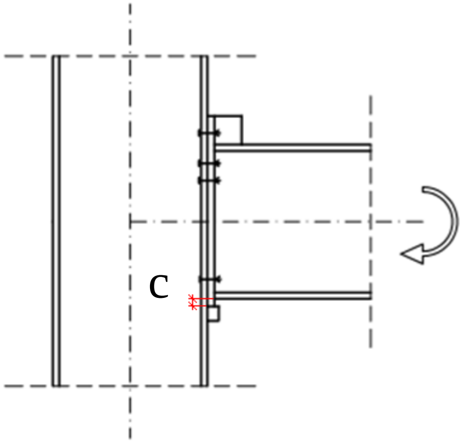


Photo: Author

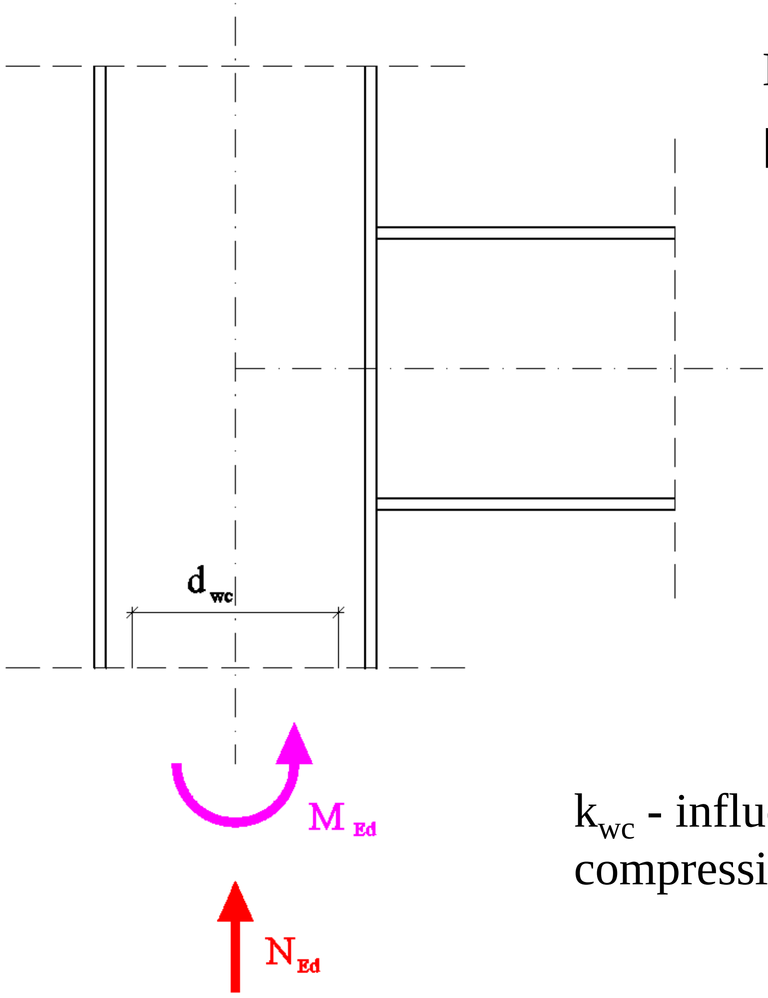
EN 1993-1-8 6.2.6.2

$\bar{\lambda}_p$	ρ
$\leq 0{,}72$	1,0
$> 0{,}72$	$(\bar{\lambda}_p - 0{,}2) / (\bar{\lambda}_p)^2$

$$\bar{\lambda}_p = 0{,}932 \sqrt{[(b_{eff,\,c,\,wc}\, d_{wc}\, f_{y,\,wc}) / (E\, t_{wc}^2)]}$$

ρ - simplified calculation of instability in compressed part of web - without full calculation of instability factor (χ), only reduction factor.

Effective areas in compressed part



Max compression for plane part of web (d_{dwc}):

$[\ \sigma\ (N_{ed} + M_{Ed})\ ]_{dwc} = \sigma_{com,\ Ed}$

$\sigma_{com,\ Ed}$ / $f_{y,\ wc}$	k_{wc}
$\leq 0{,}7$	1,0
$> 0{,}7$	$1{,}7 - \sigma_{com,\ Ed} / f_{y,\ wc}$

EN 1993-1-8 6.2.6.2

k_{wc} - influence of double - transverse and longitudinal - compression of the column flange

Photo: Author

Component method: resistance and stiffness of joint is effect of resistance and stiffness its components.

→ #14 / 17

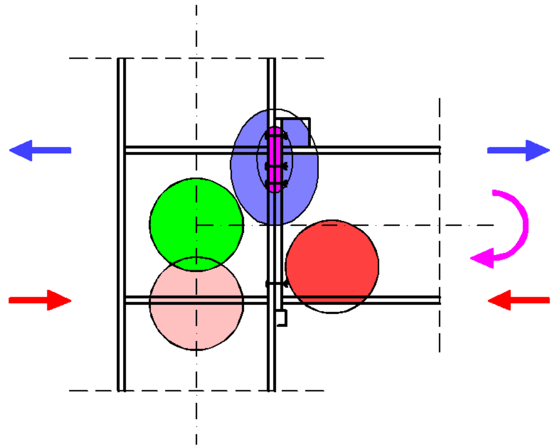


Photo: Author

For stifness, joint is analysed as a complex of springs.

For resistance, the most important is the weaknest component (the weaknest link).

Photo: dynamicbusiness.com.au

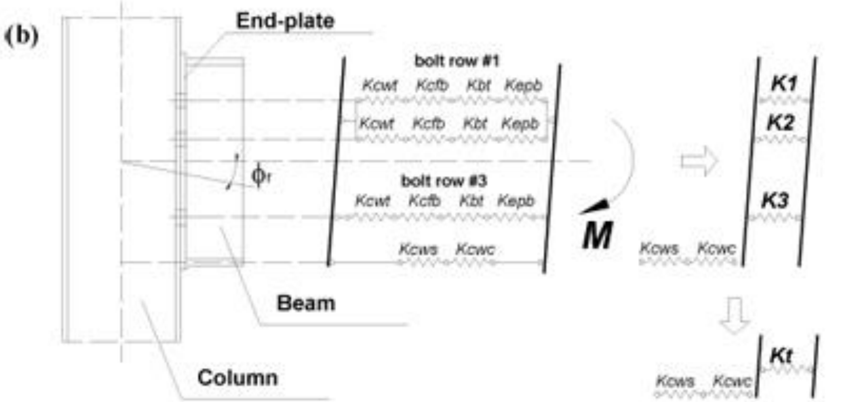


Figure 2. (a) Active components for bolted beam-to-column end-plate connections and (b) joint rotational stiffness according to EC3-1.8 (2003).

Photo: scielo.br

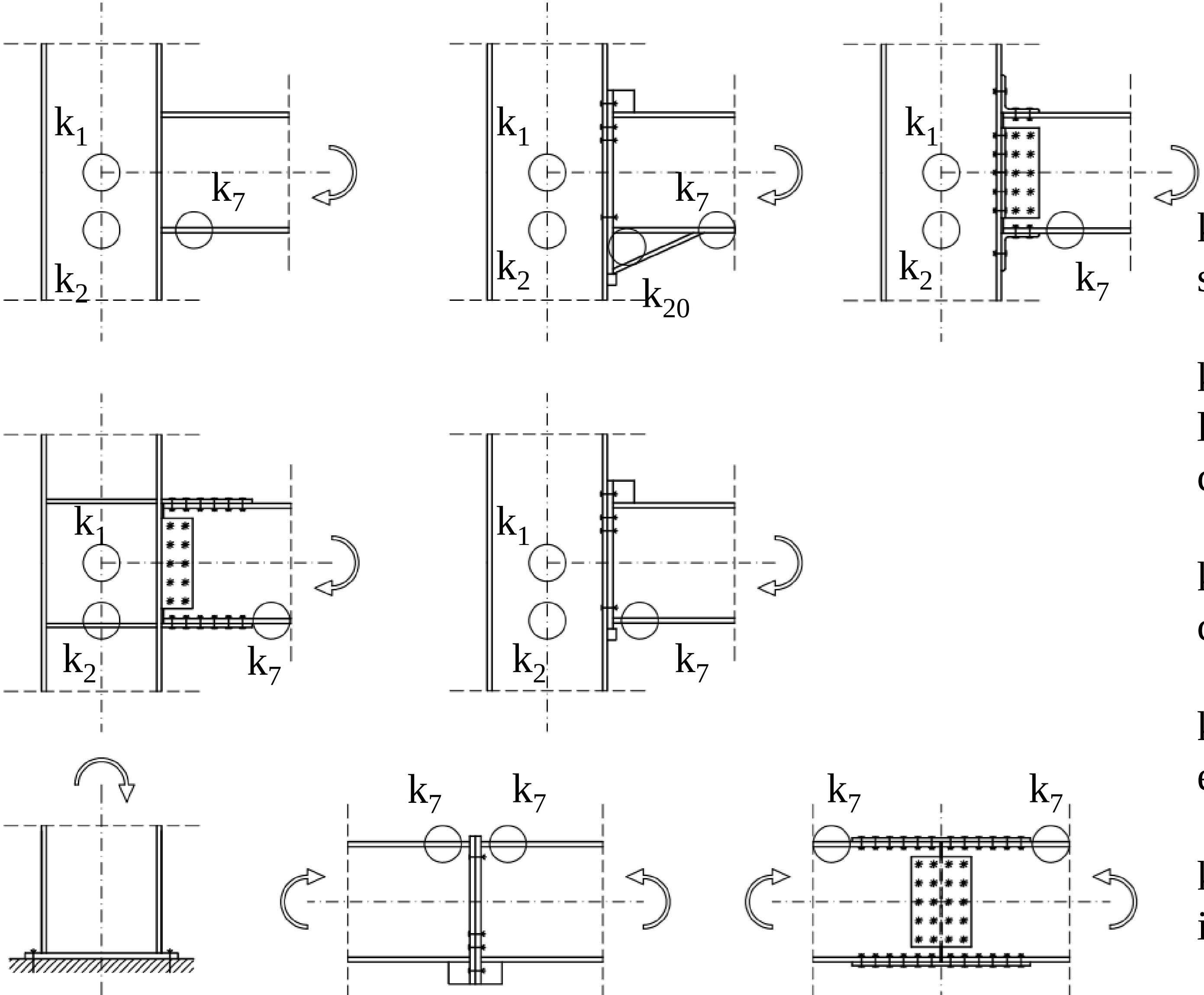


Photo: Author

k_1 – column web in shear;

k_2 – column web in local transversal compression;

k_7 – flange of beam in compression;

k_{19} – welds (each on each positions);

k_{20} – haunched beam in compression;

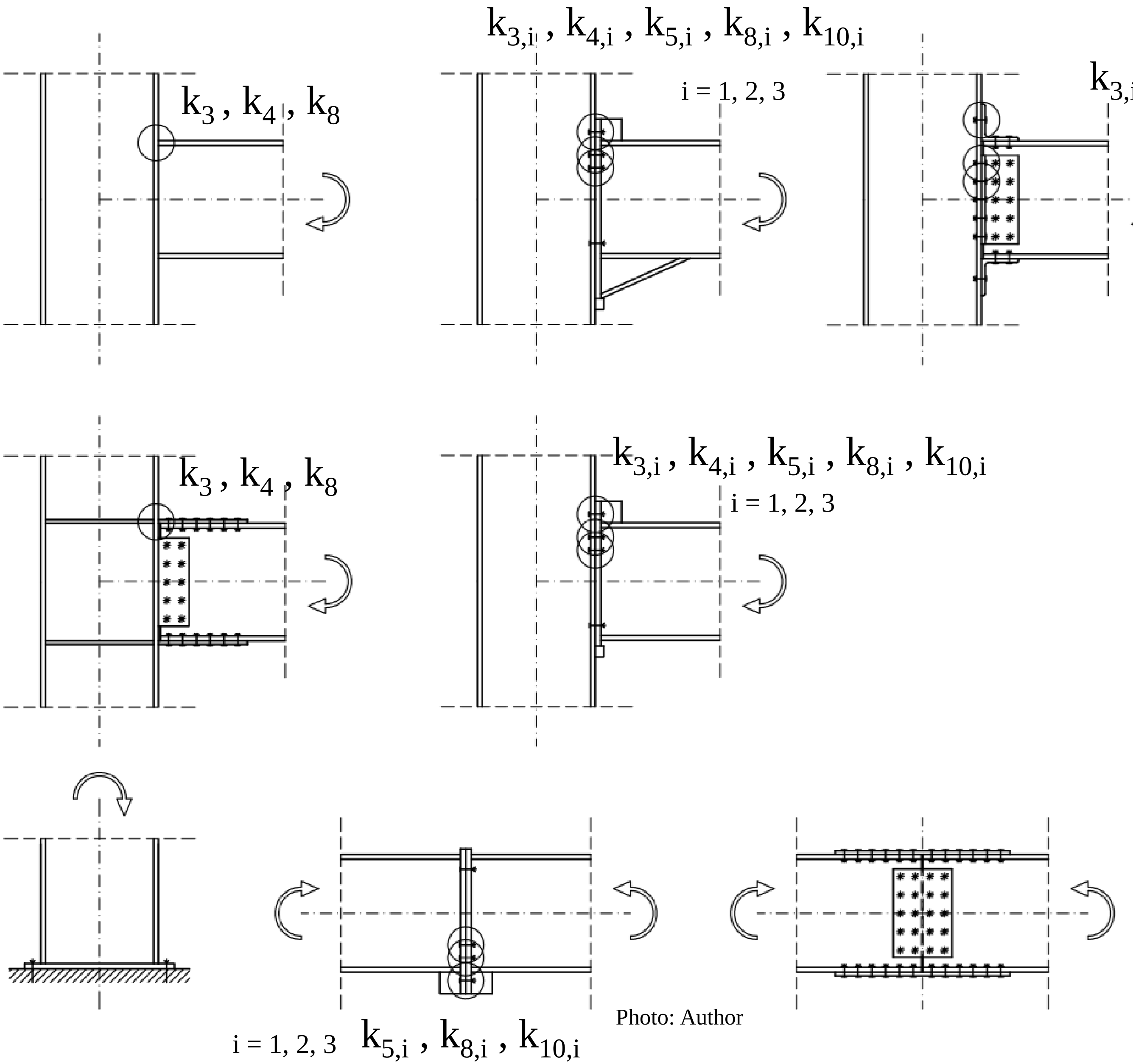


Photo: Author

k_3 – column web in local transversal tension;

k_4 – column flange in bending;

k_5 – end plate in bending;

k_6 – flange cleat in bending;

k_8 – beam web in tesnion;

k_{10} – bolt in tension;

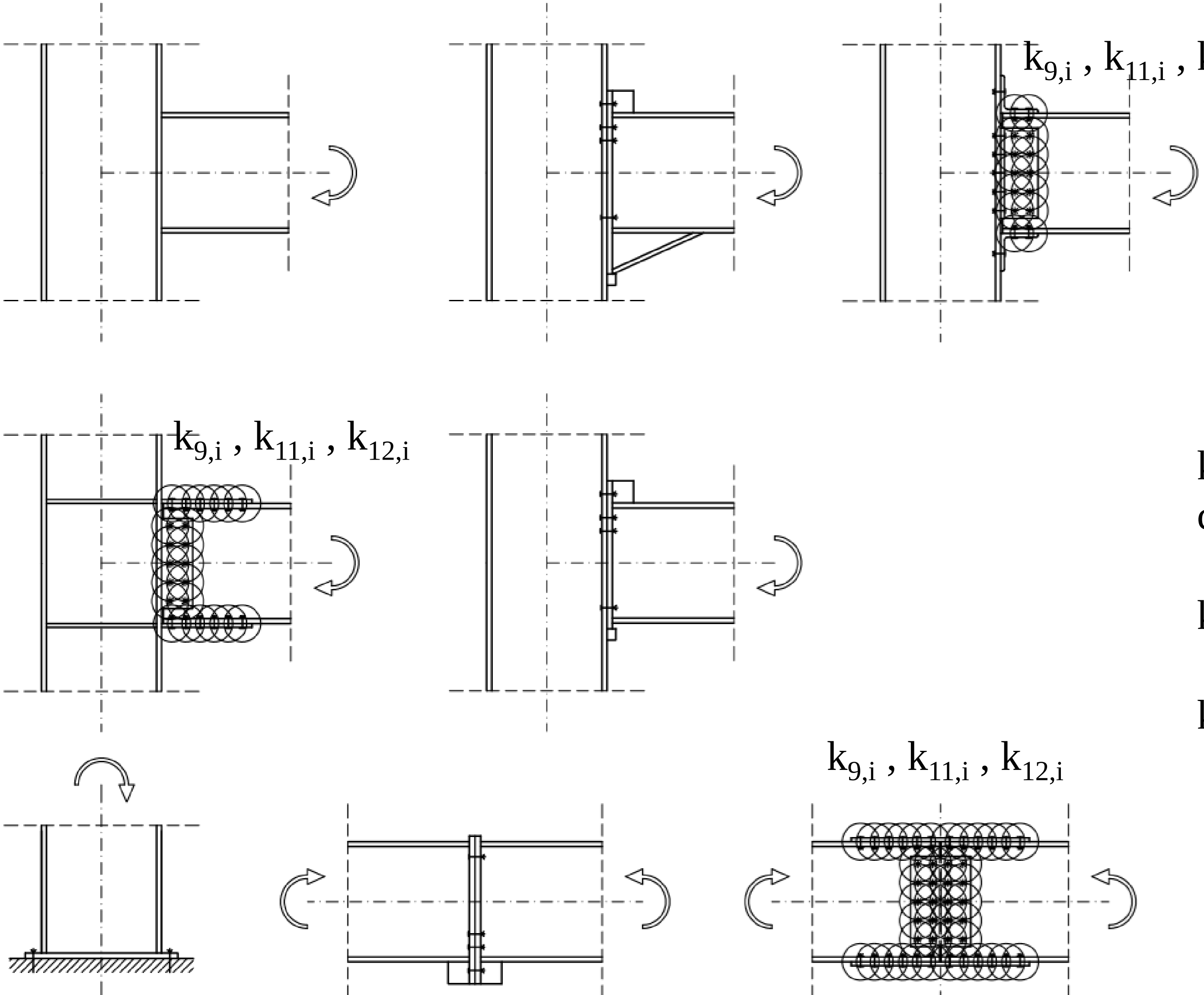
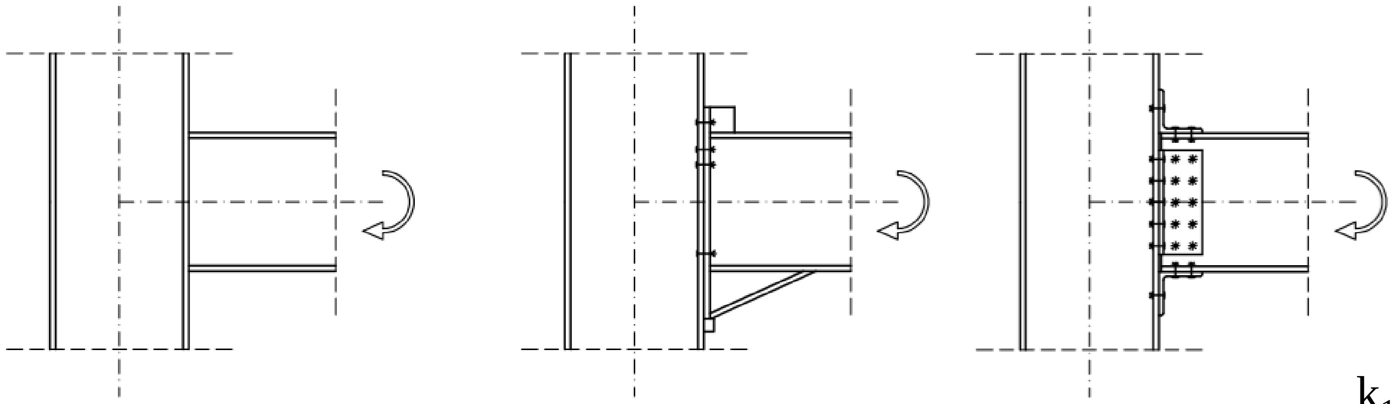


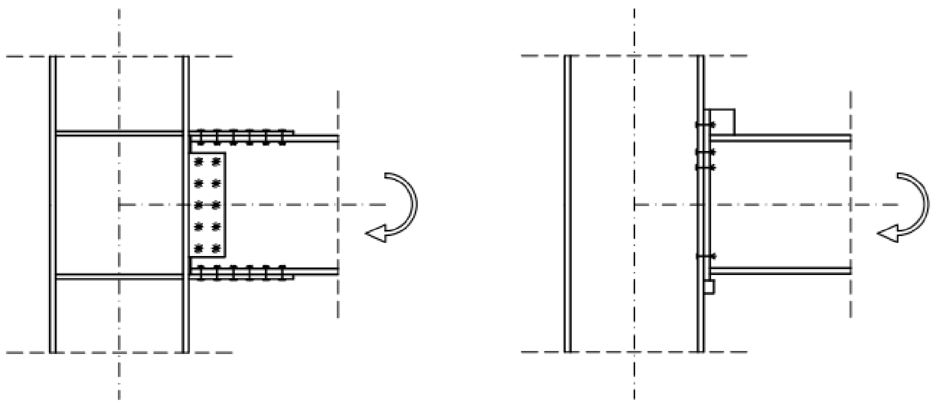
Photo: Author

k_9 – plate in tension or compression;

k_{11} – bolt in shear;

k_{12} – bolt in bearing;





k_{13} – concrete in compression;

k_{14} – base plate in bending under compression;

k_{15} – base plate in bending under tension;

k_{16} – anchor bolt in tesnion;

k_{17} – anchor bolt in shear;

k_{18} – anchor bolt in bearing;

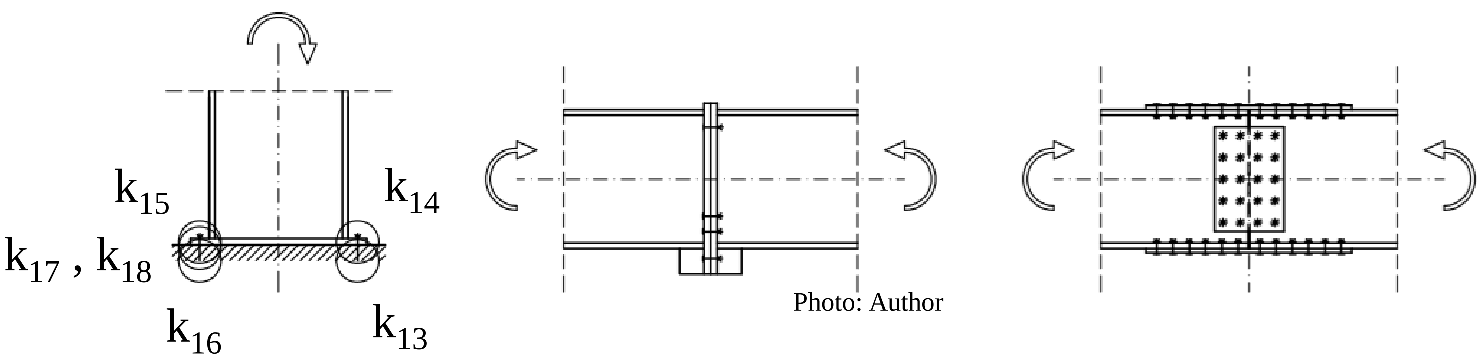


Photo: Author

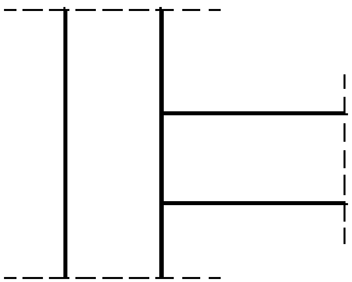
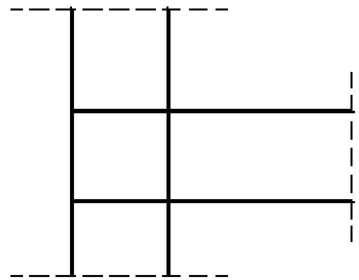
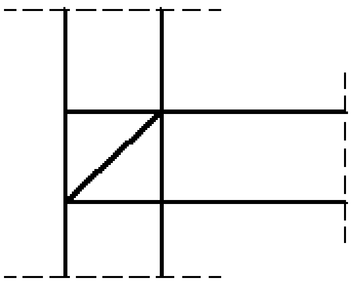
Values of k_i

k_1

EN 1993-1-8 tab. 6.11

Column web in shear

Photo: Author

 	
$0{,}38\, A_{vc} / \beta\, z$	$\rightarrow \infty$

$A_{vc} \rightarrow$ active area of web $\approx h_w\, t_w$

$z \rightarrow$ #t / 8, 9

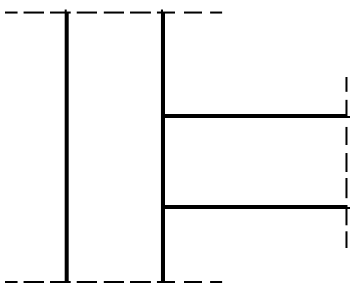
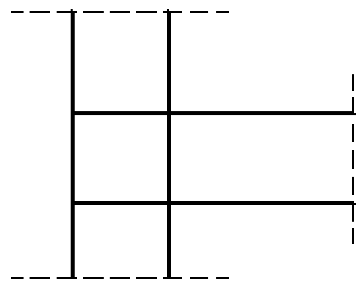
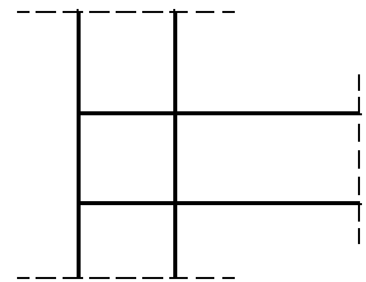
$\beta \rightarrow$ #t / 10

k_2

Column web in transversal compression

k_3

Column web in transversal tension

	Welded, bolted	Bolted	Welded
			
k_2	$0{,}7\, b_{eff,\,c,\,wc}\, t_{wc} / d_c$	$\rightarrow \infty$	
k_3	$0{,}7\, b_{eff,\,t,\,wc}\, t_{wc} / d_c$		$\rightarrow \infty$

$b_{eff,\,t,\,wc} \rightarrow$ #t / 23 – 25, #t / 27

$b_{eff,\,c,\,wc} \rightarrow$ #t / 28

$d_c = h_c - 2\, t_{fc}$

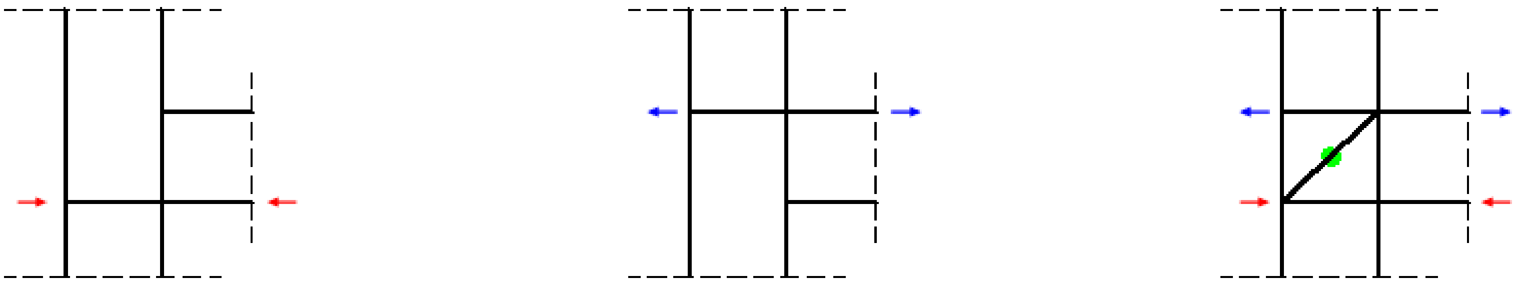
Photo: Author

EN 1993-1-8 tab. 6.11

Attention

There is information in Eurocode, that infinitive value of k_1, k_2 and k_3 can be taken into consideration for stiffned sub-part of joint. But there is no information, which type of stiffeners should be applied for different sub-parts.

According to literature, rule is that stiffeners are applied conforming to type of load:



$k_1 \to \infty$ $k_2 \to \infty$ $k_3 \to \infty$

Photo: Author

EN 1993-1-8 tab. 6.11

$\mathbf{k_4}$

Column flange in bending

$0{,}9\, l_{eff}\, t_{fc}^{\,3} / m^3$

$\mathbf{k_5}$

End-plate in bending

$0{,}9\, l_{eff}\, t_p^{\,3} / m^3$

$\mathbf{k_6}$

Flange cleat in bending

$0{,}9\, l_{eff}\, t_a^{\,3} / m^3$

$m \rightarrow$ #t / 6

$l_{eff} \rightarrow$ #t / 23 – 25, #t / 28

$t_x \rightarrow$ thickness of column’s flange / end plate / flange cleat

k_7

Beam flange and beam web in compression

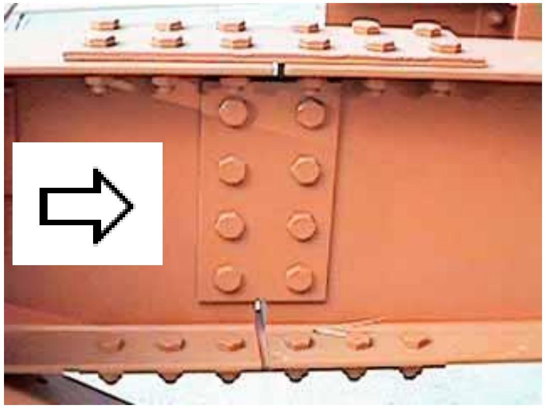
k_8

Beam web in tension

k_9

Web / flange plate in tension or compression

Photo: amsd.co.uk



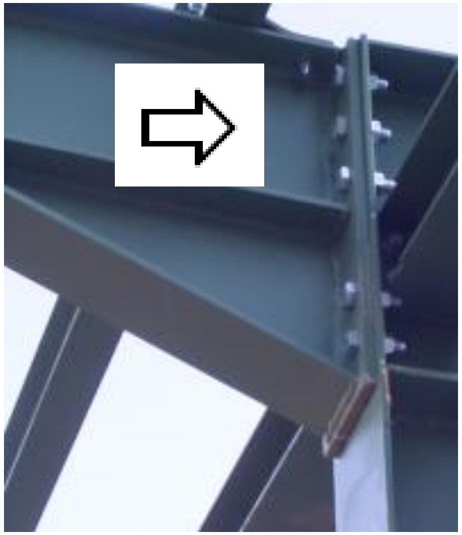


Photo: resources.scia.net

EN 1993-1-8 tab. 6.11 – no information about value. Parts important for resistance of joint only, not for stiffness. Stiffness of web, flange on plate in their planes is very big. This means, local stiffness can be taken into consideration as

$$\rightarrow \infty$$

Stiffness of total join is in proportion to Σ (1 / k_i)]. When k_i tends to infinitive, its reversion tends to 0. This parts have no effect on total stiffness of joins.

k_{10} $1{,}6\ A_s / L_b$ EN 1993-1-8 tab. 6.11

Bolts in tension $L_b , A_s \rightarrow$ #t / 18

k_{11}

Bolts in shear

k_{12}

Bolts in bearing

Categories of bolted joint	A	B, C
k_{11}	$16\ n_b\ d^2\ f_{ub} / (E\ d_{M16})$	$\rightarrow \infty$
k_{12}	$24\ n_b\ k_b\ k_t\ d\ f_u / E$	$\rightarrow \infty$

d – diamete of bolt

$d_{M16} = 16$ mm

n_b = number of bolt-rows

$k_b = \min (1{,}25\ ;\ 0{,}25\ e_b / d + 0{,}5\ ;\ 0{,}25\ p_b / d + 0{,}375)$

$k_t = \min (2{,}5\ ;\ 1{,}5\ t_j / d_{M16})$

$e_b , p_b \rightarrow$ #t / 7

k_{13}

Concrete in compression (including grout)

EN 1993-1-8 tab. 6.11

$E_c \sqrt{(b_{eff}\, l_{eff})}\ / (1{,}275\ E)$

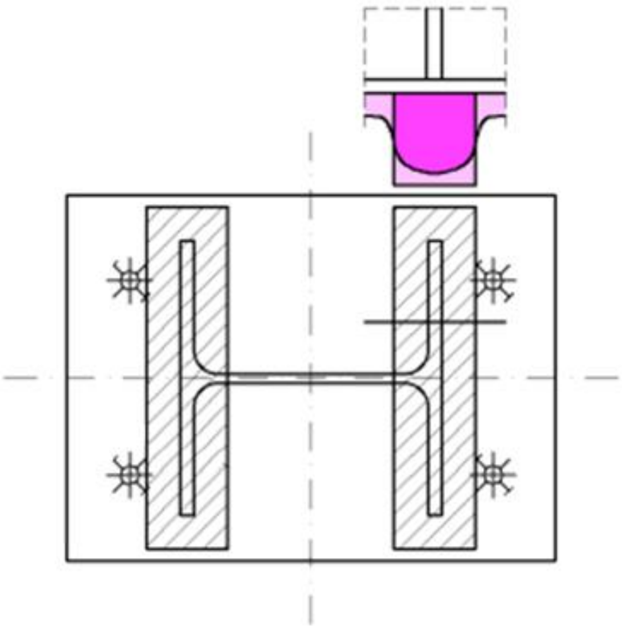


Photo: Author

$b_{eff}\ , l_{eff} = b_{eff}\ , l_{eff}\,(c = 1{,}25\ t_p)$

$c \rightarrow$ #t / 12

k_{14}

Plate in bending under compression

EN 1993-1-8 tab. 6.11 – already taken into consideration in k_{13} .
Part important for resistance only, not for stiffness.

k_{15}

Base plate in bending under tension

k_{16}

Anchor bolts in tension

	With prying forces	Without prying forces
k_{15}	$0{,}85\, l_{eff}\, t_p^{\,3} / m^3$	$0{,}425\, l_{eff}\, t_p^{\,3} / m^3$
k_{16}	$1{,}6\, A_s / L_b$	$2{,}0\, A_s / L_b$

EN 1993-1-8 tab. 6.11

$m \rightarrow$ #t / 6

Prying forces $\rightarrow$ #t / 15 - 18

L_b , $A_s \rightarrow$ #t / 18

$l_{eff} \rightarrow$ #t / 42

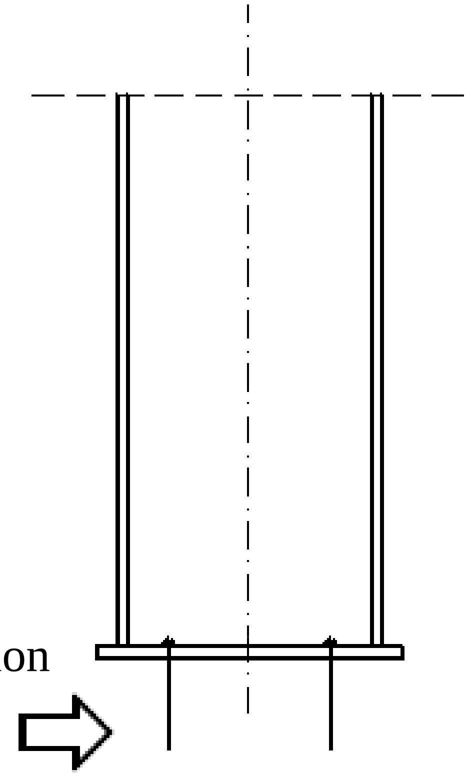
k_{17}

Anchor bolts in shear

k_{18}

Anchor bolts in bearing

EN 1993-1-8 tab. 6.11 – no information about value. Parts important for resistance only, not for stiffness.



Sear and bearing of anchor bolt in analysed in horizontal direction. For stifness of support important is vertical direction These two phenomena have no influence on stiffness of support.

k_{19}

Welds

k_{20}

Haunched beams

EN 1993-1-8 tab. 6.11 – no information about value. Parts important for resistance only, not for stiffness.

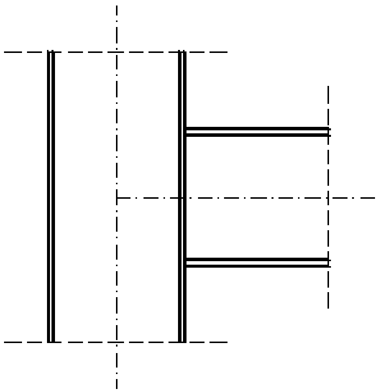
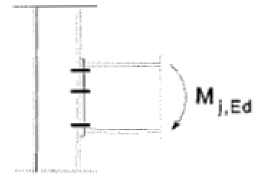
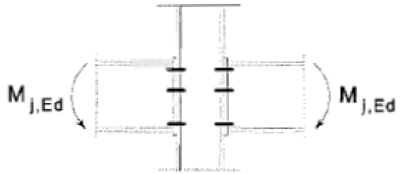
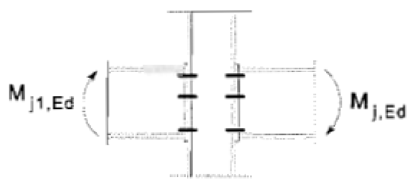
Photo: resources.scia.net

Welds are too tiny elements to have influence on stiffness.

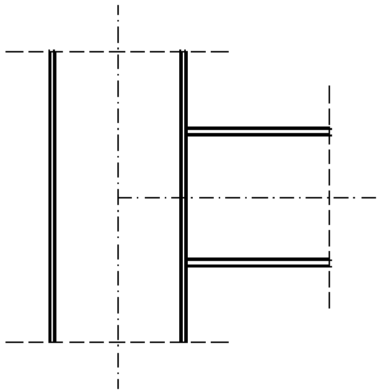
Haunched beam is rather massive element of big stiffness in its plane:

$\rightarrow \infty$

Spring models

 Photo: Author		k_1 ; k_2 ; k_3
	 $M_{j,\ Ed,\ l} = - M_{j,\ Ed,\ r}$	k_2 ; k_3
	 $M_{j,\ Ed,\ l} \neq - M_{j,\ Ed,\ r}$	k_1 ; k_2 ; k_3

EN 1993-1-8 tab. 6.9 tab. 6.10



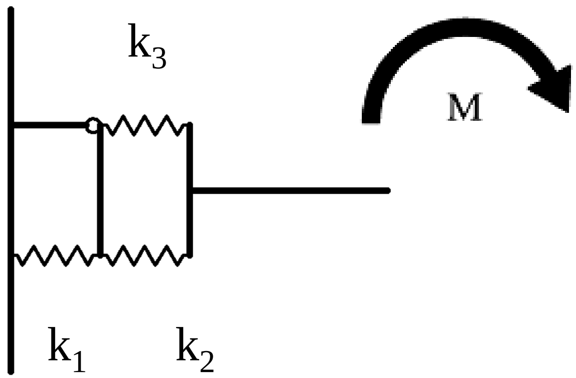
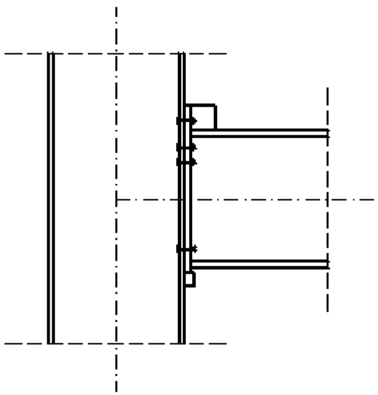
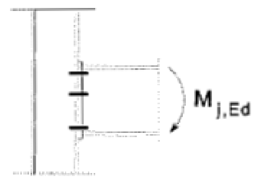
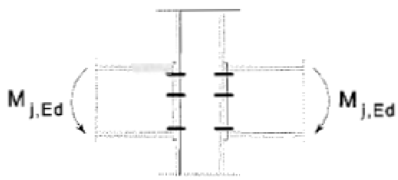
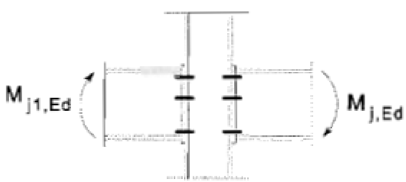
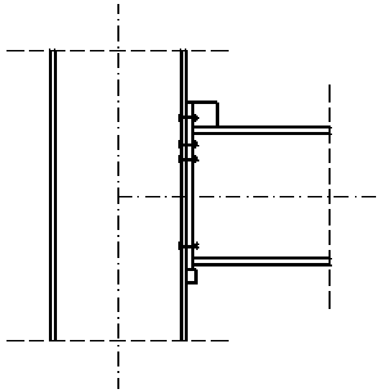


Photo: Author

 Photo: Author		One bolt-row in tension	k_1 ; k_2 ; k_3 ; k_4 ; k_5 ; k_{10}
		Two or more bolt-row in tension	k_1 ; k_2 ; k_{eq}
	 $M_{j,\,Ed,\,l} = - M_{j,\,Ed,\,r}$	One bolt-row in tension	k_2 ; k_3 ; k_4 ; k_5 ; k_{10}
		Two or more bolt-row in tension	k_2 ; k_{eq}
	 $M_{j,\,Ed,\,l} \neq - M_{j,\,Ed,\,r}$	One bolt-row in tension	k_1 ; k_2 ; k_3 ; k_4 ; k_5 ; k_{10}
		Two or more bolt-row in tension	k_1 ; k_2 ; k_{eq}

EN 1993-1-8 tab. 6.9 tab. 6.10



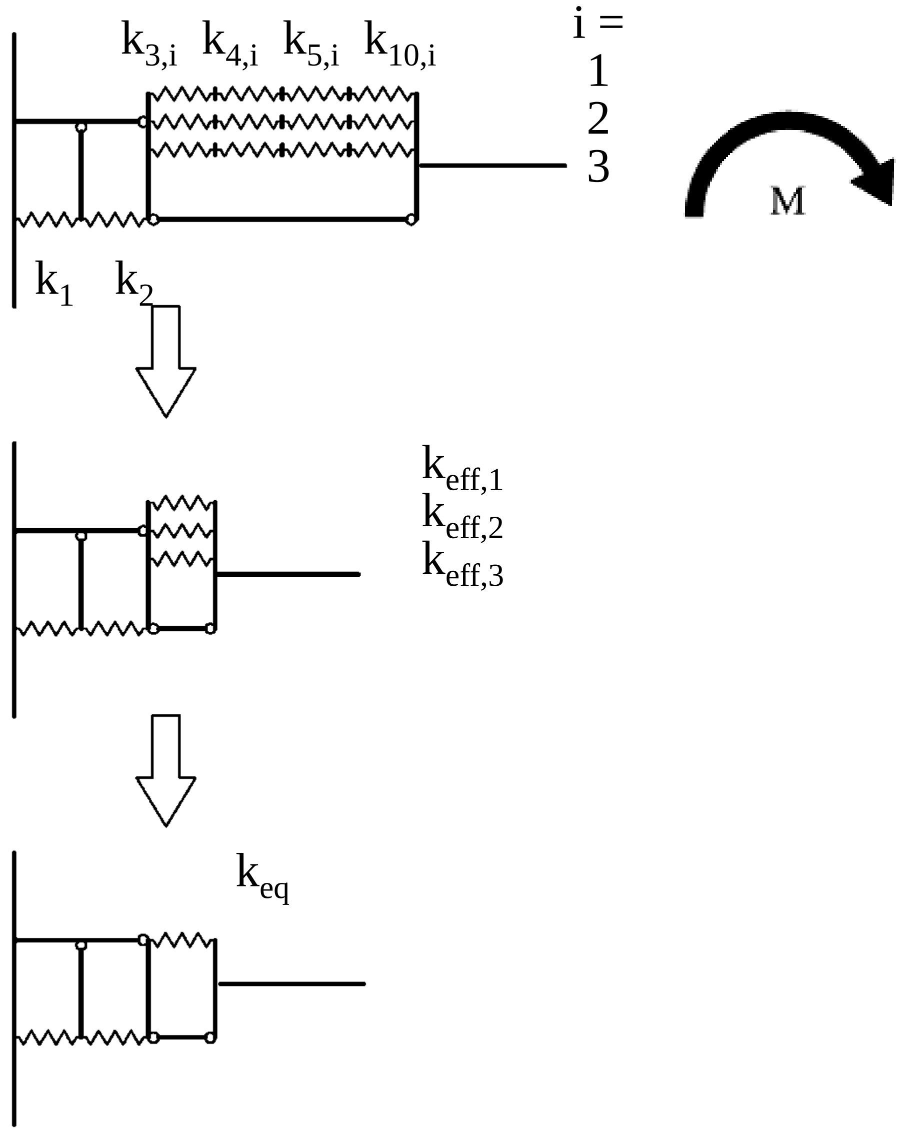


Photo: Author

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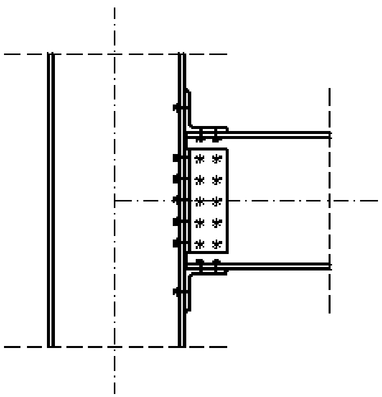
<td rowspan="3">
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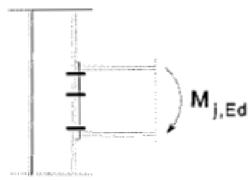
<td></td>
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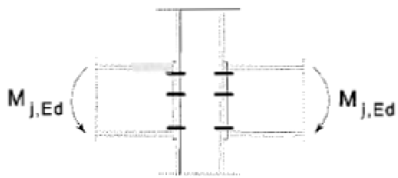
<td>
$M_{j,\,Ed,\,l} = - M_{j,\,Ed,\,r}$</td>
<td>k_2 ; k_3 ; k_4 ; k_6 ; k_{10} ; k_{11}^{tf} ; k_{11}^{bf} ;
k_{12}^{tf} ; k_{12}^{bf} ; k_{12}^{tL} ; k_{12}^{bL}</td>
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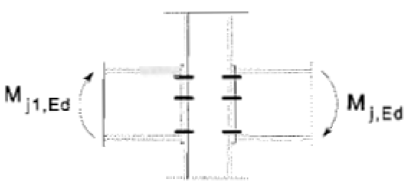
<td>
$M_{j,\,Ed,\,l} \neq - M_{j,\,Ed,\,r}$</td>
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k_{11}^{bf} ; k_{12}^{tf} ; k_{12}^{bf} ; k_{12}^{tL} ; k_{12}^{bL}</td>
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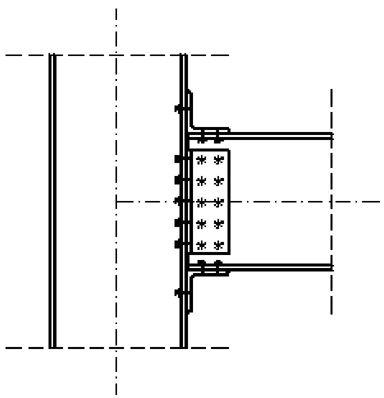
EN 1993-1-8 tab. 6.9 tab. 6.10



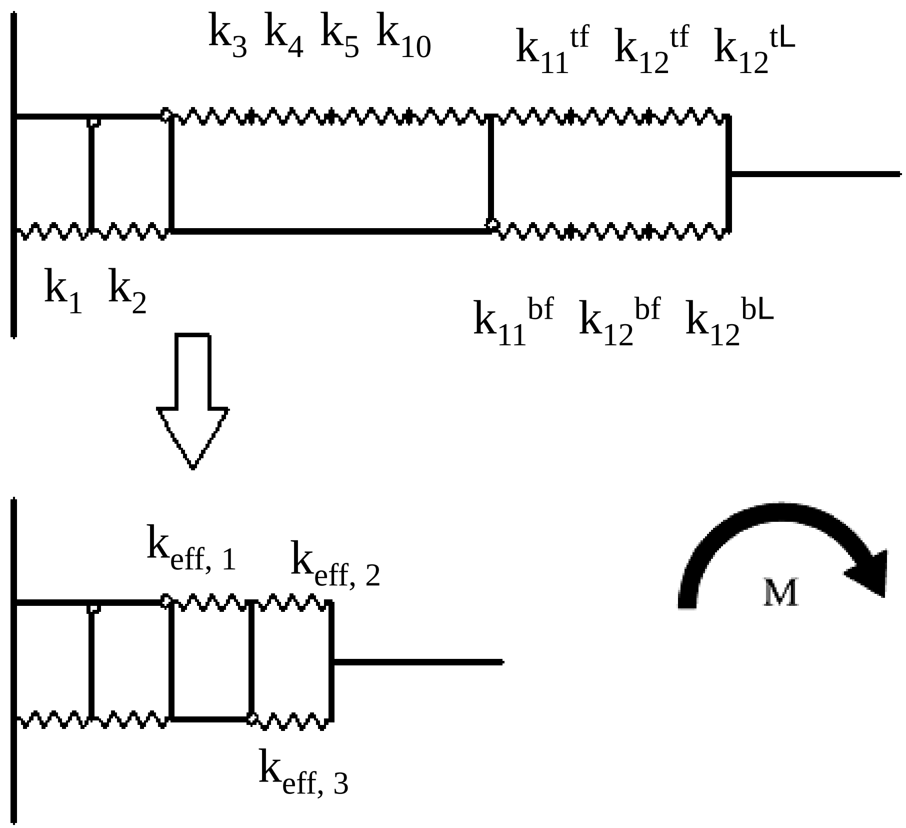
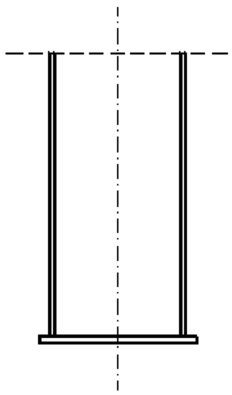


Photo: Author

 Photo: Author	One bolt-row in tension	k_{13} ; k_{15} ; k_{16}
	Two or more bolt-row in tension	k_{13}^{1} ; k_{15}^{1} ; k_{16}^{1} ; k_{13}^{2} ; k_{15}^{2} ; k_{16}^{2} ; k_{13}^{3} ; k_{15}^{3} ; k_{16}^{3} ...

EN 1993-1-8 tab. 6.9 tab. 6.10

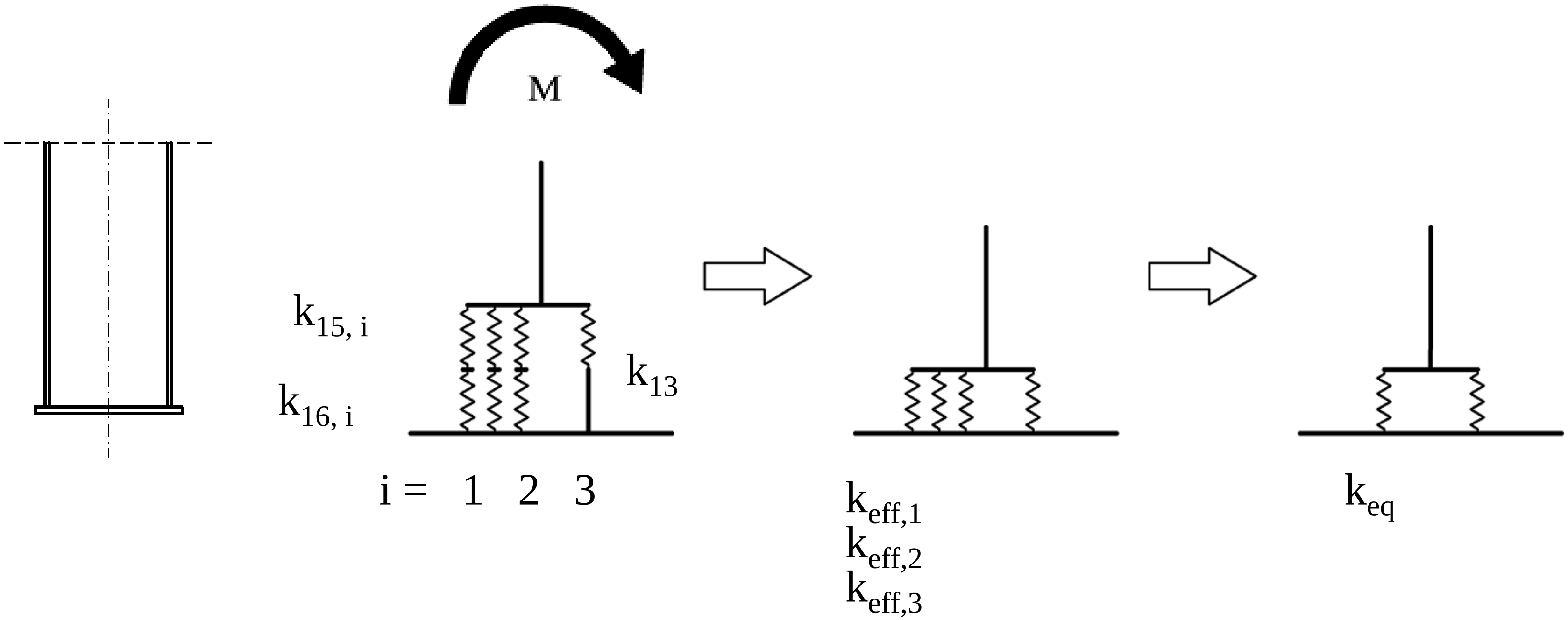


Photo: Author

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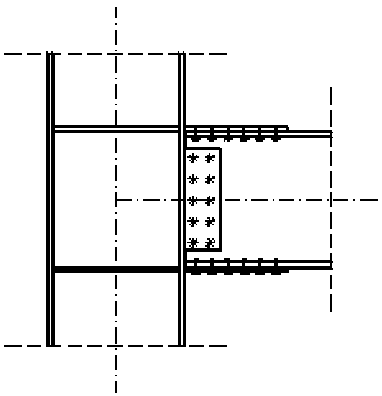
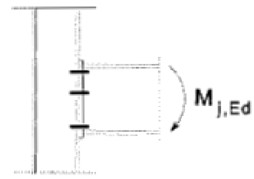
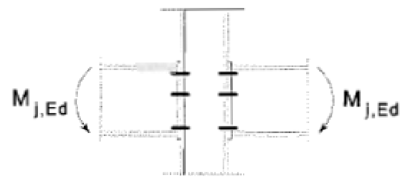
<td rowspan="3">
Photo: Author</td>
<td>
</td>
<td>k_1 ; k_2 ; k_3 ;
k_{11}^{tf} ; k_{11}^{bf} ; k_{12}^{tf} ; k_{12}^{bf} ; k_{12}^{tp} ; k_{12}^{bp}</td>
</tr>
<tr>
<td>

$M_{j,\, Ed,\, l} = - M_{j,\, Ed,\, r}$</td>
<td>k_2 ; k_3 ;
k_{11}^{tf} ; k_{11}^{bf} ; k_{12}^{tf} ; k_{12}^{bf} ; k_{12}^{tp} ; k_{12}^{bp}</td>
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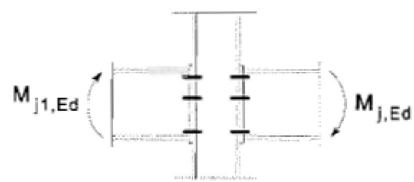
<td>

$M_{j,\, Ed,\, l} \neq - M_{j,\, Ed,\, r}$</td>
<td>k_1 ; k_2 ; k_3 ;
k_{11}^{tf} ; k_{11}^{bf} ; k_{12}^{tf} ; k_{12}^{bf} ; k_{12}^{tp} ; k_{12}^{bp}</td>
</tr>
</table>

In analogy to EN 1993-1-8 tab. 6.9 tab. 6.10

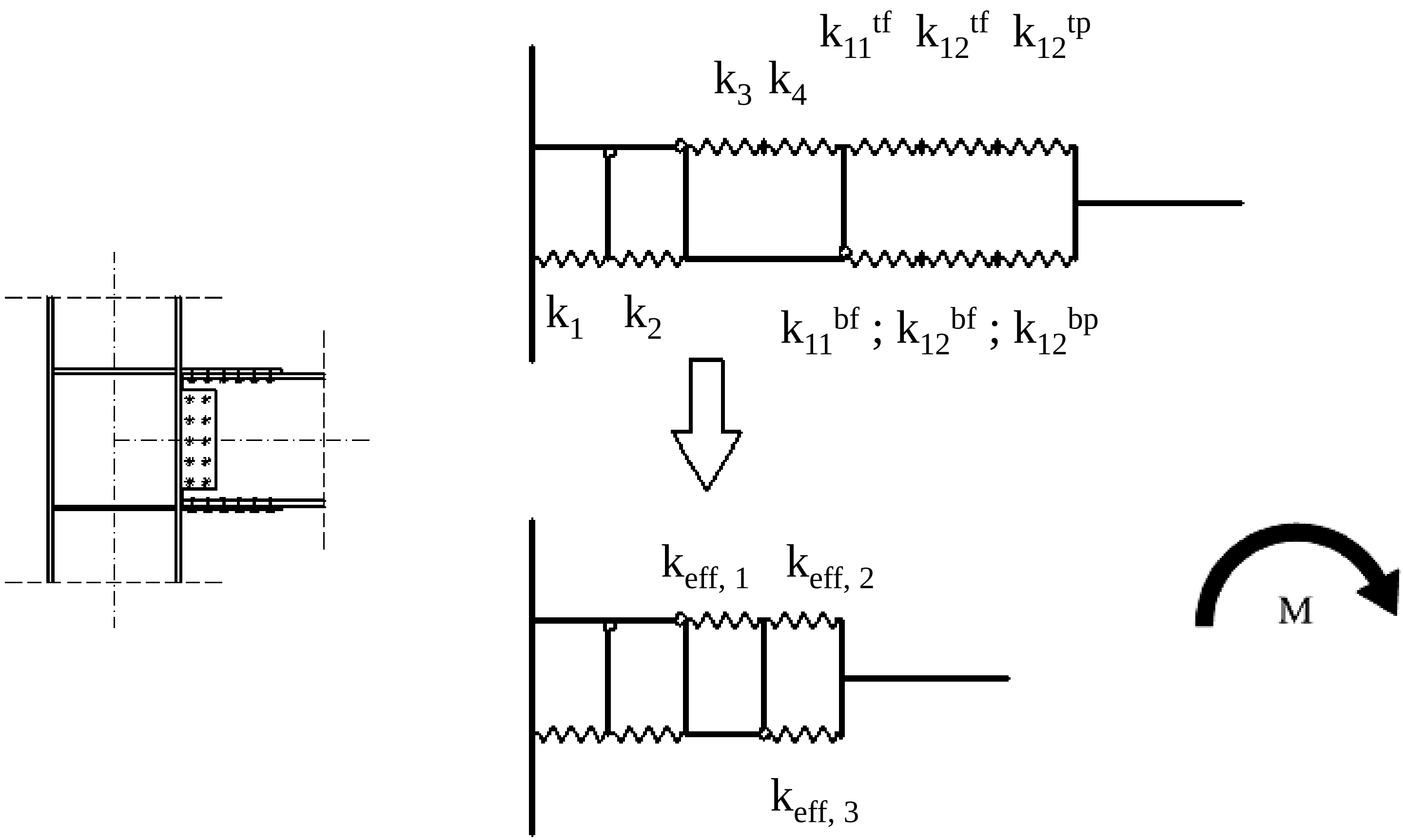
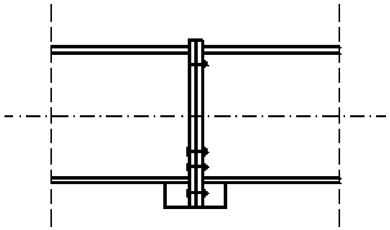


Photo: Author

 Photo: Author	One bolt-row in tension	k_5^l ; k_5^r ; k_{10}
	Two or more bolt-row in tension	k_{eq}

In analogy to EN 1993-1-8 tab. 6.9 tab. 6.10

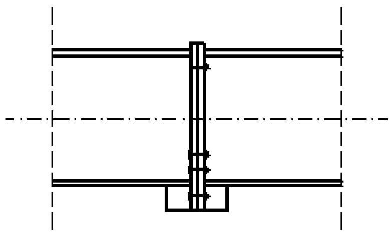
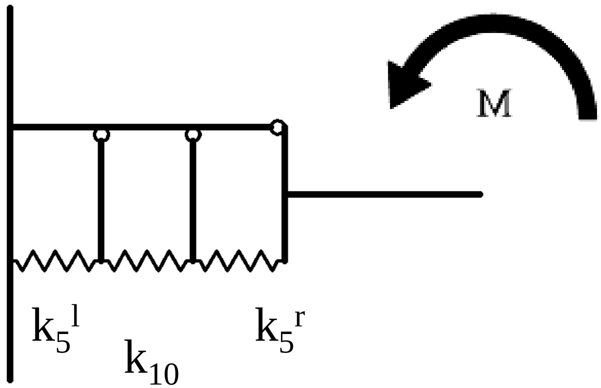
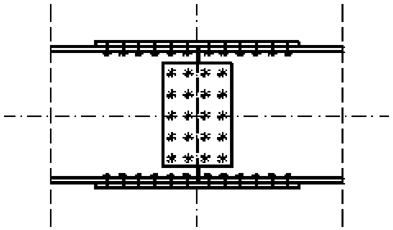
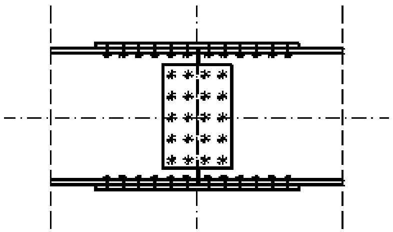


Photo: Author



 Photo: Author	k_{11}^{tfl} ; k_{11}^{bfl} ; k_{12}^{tfl} ; k_{12}^{bfl} ; k_{12}^{tpl} ; k_{12}^{bpl} ; k_{11}^{tfr} ; k_{11}^{bfr} ; k_{12}^{tfr} ; k_{12}^{bfr} ; k_{12}^{tpr} ; k_{12}^{bpr}

In analogy to EN 1993-1-8 tab. 6.9 tab. 6.10



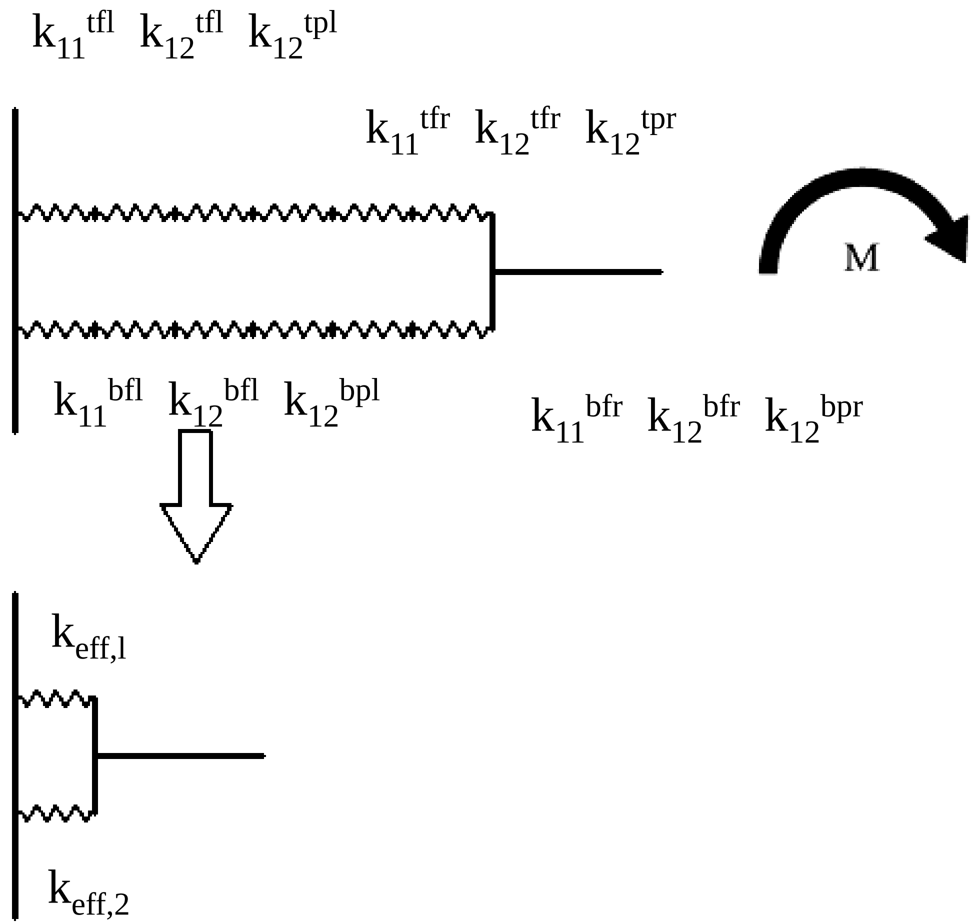


Photo: Author

Equivalent and effective springs

$$k_{eq} = \Sigma \,(k_{eff,\, r}\, h_r) \,/\, z_{eq}$$

$$k_{eq}$$

EN 1993-1-8 (6.29)

$$k_{eff,\, r} = 1 \,/\, \Sigma \,(1 \,/\, k_{i,\, r} \,)$$

$$z_{eq} = \Sigma \,(k_{eff,\, r}\, h_r^{\,2}) \,/\, \Sigma \,(k_{eff,\, r}\, h_{ri})$$

$$k_{i,\, r} = k_{3,\, r} \,,\, k_{4,\, r} \,,\, k_{5,\, r} \,,\, k_{10,\, r}$$

h_r - arm of action; the same as for resistance

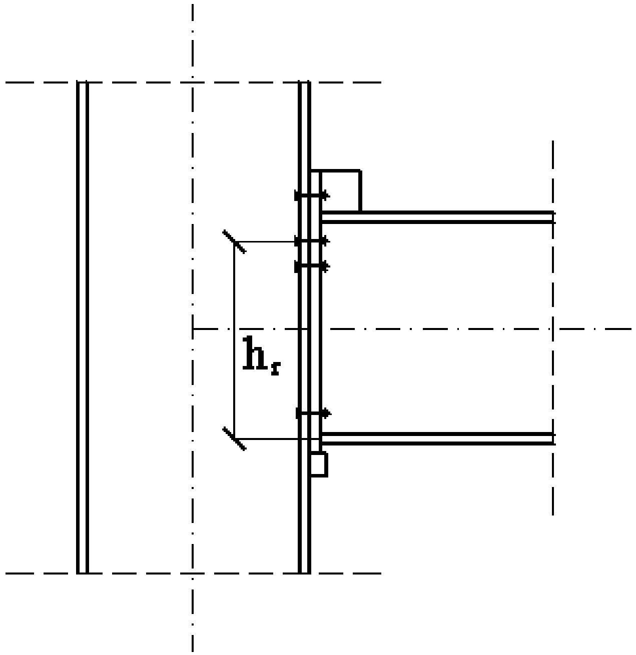
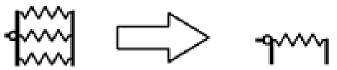


Photo: Author

$k_{i,\, r}$ k_{eff}

$k_{eff,\, r}$



k_{eq}

Photo: Author

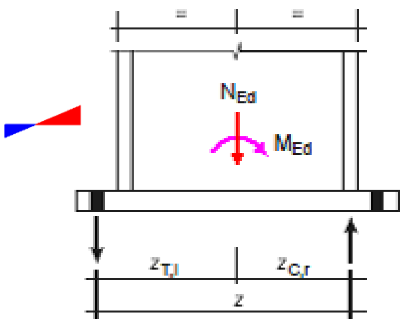
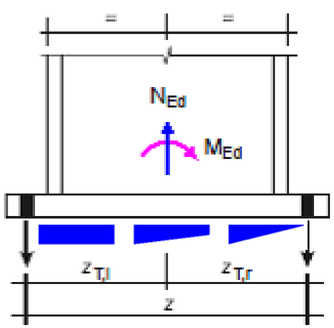
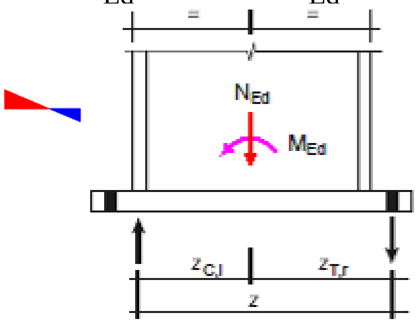
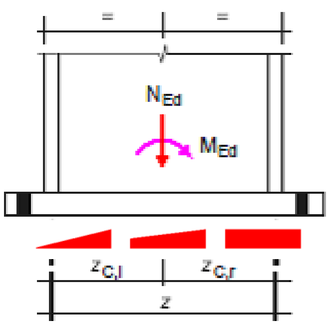
Loading	Lever arms	**Rotation stiffness $S_{j,\,ini}$**	
Left-T, Right-C, exampe: $M_{Ed} > 0$; $N_{Ed} < 0$ 	$z = z_{T,\,l} + z_{C,\,r}$ $e = M_{Ed} / N_{ed}$	$N_{Ed} > 0 \quad e > z_{T,\,l}$	$N_{Ed} \leq 0 \quad e \leq -z_{C,\,r}$
		$\mathbf{E\, z^2\, e / \{\mu\, (e + e_k)\, [(1 / k_{T,\,l}) + (1 / k_{C,\,r})]\}}$ $e_k = (z_{C,\,r}\, k_{C,\,r} - z_{T,\,l}\, k_{T,\,l}) / (k_{T,\,l} + k_{C,\,r})$	
Left-T, Right-T, exampe: $M_{Ed} > 0$; $N_{Ed} > 0$ 	$z = z_{T,\,l} + z_{T,\,r}$ $e = M_{Ed} / N_{Ed}$	$N_{Ed} > 0 \quad 0 < e < z_{T,\,l}$	$N_{Ed} > 0 \quad -z_{T,\,r} < e \leq 0$
		$\mathbf{E\, z^2\, e / \{\mu\, (e + e_k)\, [(1 / k_{T,\,l}) + (1 / k_{T,\,r})]\}}$ $e_k = (z_{T,\,r}\, k_{T,\,r} - z_{T,\,l}\, k_{T,\,l}) / (k_{T,\,l} + k_{T,\,r})$	

Photo: Author

EN 1993-1-8 tab. 6.12, first part

<table>
<tr><th>Loading</th><th>Lever arms</th><th colspan="2">Rotation stiffness $S_{j, ini}$</th></tr>
<tr><td rowspan="2">Left-C, Right-T, exampe:
$M_{Ed} < 0$; $N_{Ed} < 0$

</td><td rowspan="2">$z = z_{C, l} + z_{T, r}$

$e = M_{Ed} / N_{ed}$</td><td>$N_{Ed} > 0 \quad e \leq -z_{T, r}$</td><td>$N_{Ed} \leq 0 \quad e > z_{C, l}$</td></tr>
<tr><td colspan="2">$\mathbf{E\, z^2\, e / \{\mu\, (e + e_k)\, [(1 / k_{C, l}) + (1 / k_{T, r})]\}}$

$e_k = (z_{T, r}\, k_{T, r} - z_{C, l}\, k_{C, l}) / (k_{C, l} + k_{T, r})$</td></tr>
<tr><td rowspan="2">Left-C, Right-C, exampe:
$M_{Ed} > 0$; $N_{Ed} < 0$

</td><td rowspan="2">$z = z_{C, l} + z_{C, r}$

$e = M_{Ed} / N_{Ed}$</td><td>$N_{Ed} \leq 0 \quad 0 < e < z_{C, l}$</td><td>$N_{Ed} \leq 0 \quad -z_{C, r} < e \leq 0$</td></tr>
<tr><td colspan="2">$\mathbf{E\, z^2\, e / \{\mu\, (e + e_k)\, [(1 / k_{C, l}) + (1 / k_{C, r})]\}}$

$e_k = (z_{C, r}\, k_{C, r} - z_{C, l}\, k_{C, l}) / (k_{C, l} + k_{C, r})$</td></tr>
</table>

Photo: Author

EN 1993-1-8 tab. 6.12, second part

$$k_{T,l} = k_{15,l} + k_{16,l}$$

$$k_{T,r} = k_{15,r} + k_{16,r}$$

$$k_{C,l} = k_{13,l}$$

$$k_{C,r} = k_{13,r}$$

$\mu \rightarrow$ #14 / 77 - 79

EN 1993-1-8 6.3.4 (1)

Examples of stiffness joint calculation

Photo: amsd.co.uk

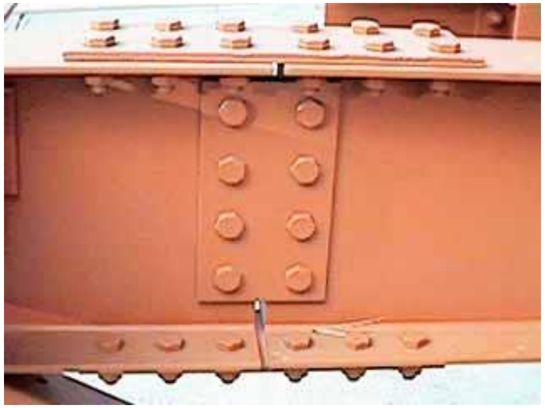


Photo: microstran.com.au

Photo: j-p.com.ua

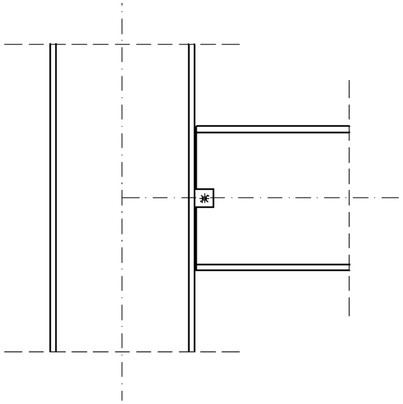
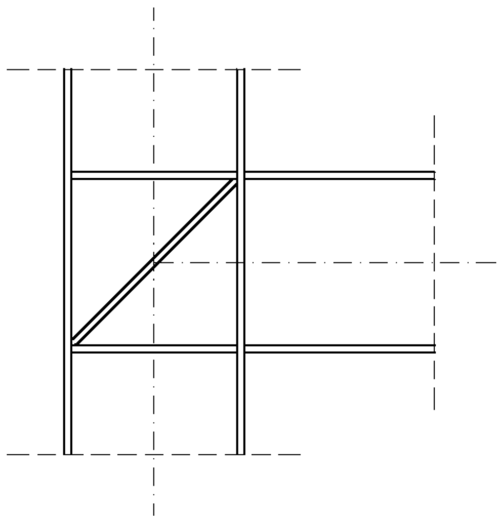
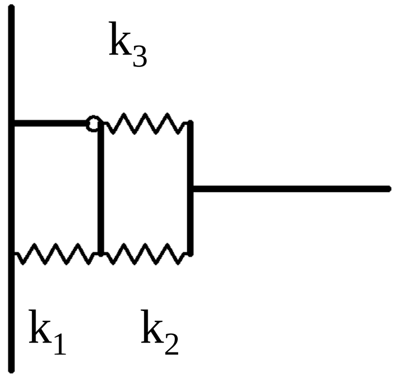


Photo: Author



k_1 , k_2 , k_3

Photo: microstran.com.au



Welded joint, horizontal
and diagonal stiffeners.

Photo: Author

Welded joint, horizontal and diagonal stiffeners.

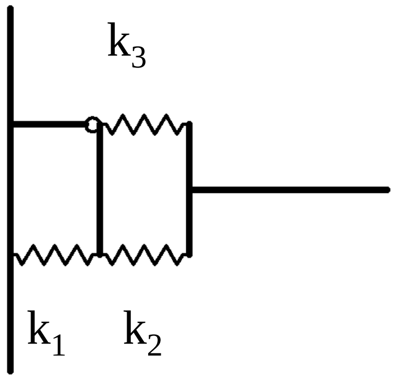


Photo: Author

$k_1 \rightarrow \infty$ (#t / 36)

$k_2 \rightarrow \infty$ (#t / 37)

$k_3 \rightarrow \infty$ (#t / 37)

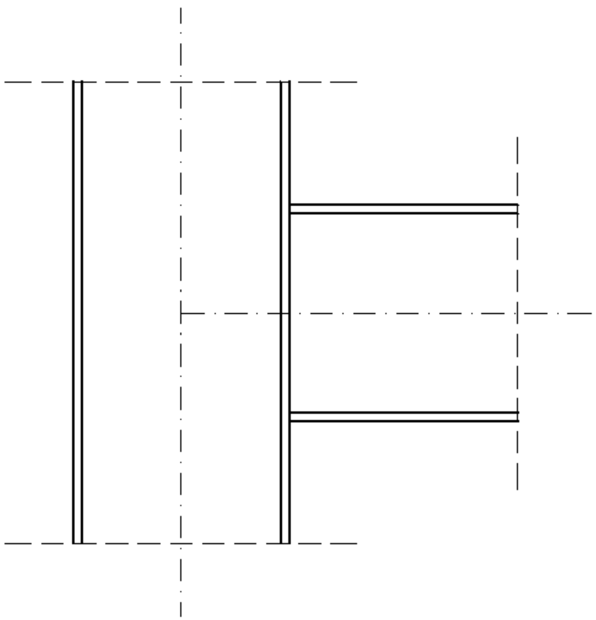
$$S_{j,\,ini} = E\,z^2 / [\,\Sigma\,(1 / k_i)] =$$

$$= E\,z^2 / [(1/\infty) + (1/\infty) + (1/\infty)] \rightarrow$$

$$\rightarrow E\,z^2 / (0 + 0 + 0) \rightarrow \infty$$

$S_{j,\,ini} \rightarrow \infty$ → rigid joint

k_1 , k_2 , k_3



Welded joint, no stiffeners.

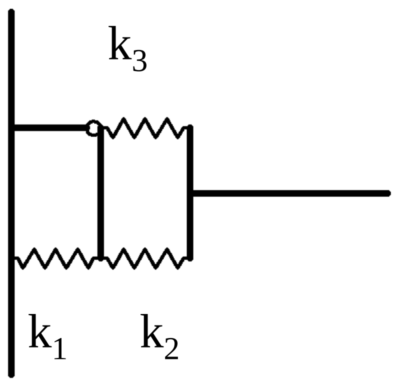


Photo: Author

Welded joint, no stiffeners.

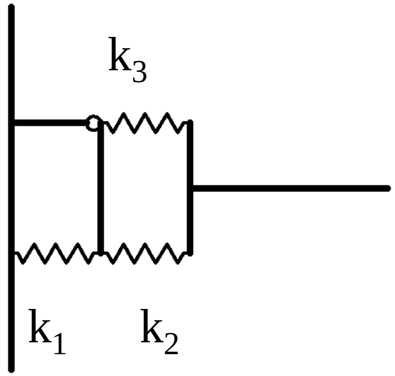


Photo: Author

$k_1 << \infty$ (#t / 36)

$k_2 << \infty$ (#t / 37)

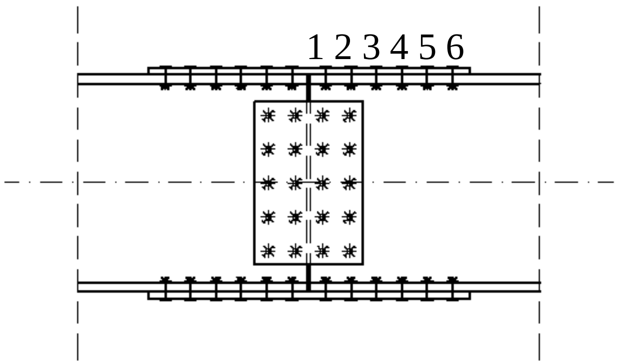
$k_3 << \infty$ (#t / 37)

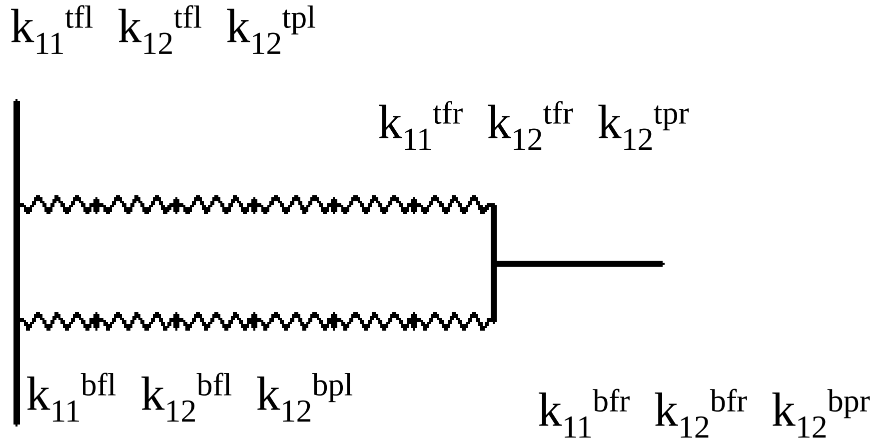
$$S_{j, ini} = E z^2 / [\Sigma (1 / k_i)] << \infty$$

$S_{j, ini} << \infty \rightarrow$ rigid, semi-rigid or hinged joint

Bolted joint, bolts category B or C

Photo: Author





There are 3 coefficients for each row of bolts: ultimately there are 36 springs for to and 36 for bottom in this case.

$$k_{11}^{tfl}\ ;\ k_{11}^{bfl}\ ;\ k_{12}^{tfl}\ ;\ k_{12}^{bfl}\ ;\ k_{12}^{tpl}\ ;\ k_{12}^{bpl}\ ;$$
$$k_{11}^{tfr}\ ;\ k_{11}^{bfr}\ ;\ k_{12}^{tfr}\ ;\ k_{12}^{bfr}\ ;\ k_{12}^{tpr}\ ;\ k_{12}^{bpr} \rightarrow k_{eff}$$

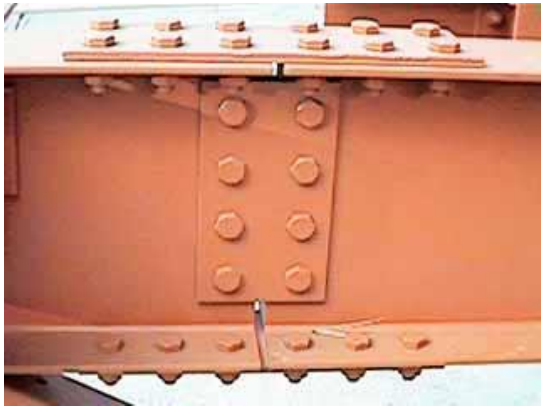


Photo: amsd.co.uk

Bolted joint, bolts category B or C

$k_{11}^{i} \to \infty$ (#t / 41)

$k_{12}^{i} \to \infty$ (#t / 41)

$$k_{eff} = 1 / \Sigma (1 / k_i) = 1 / \Sigma (1 / \infty) \to 1 / 0 \to \infty$$

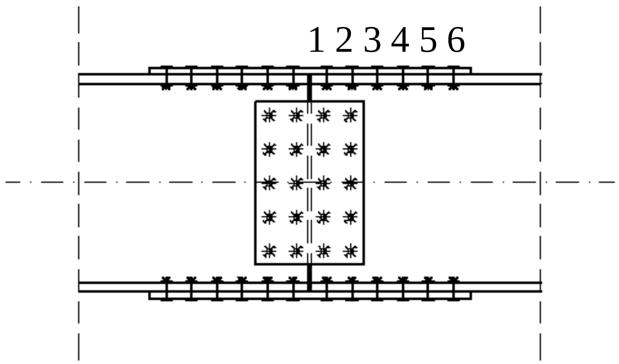
$$S_{j,\, ini} = E\, z^2 / (1 / k_{eff}^{\,top} + 1 / k_{eff}^{\,bottom}) =$$

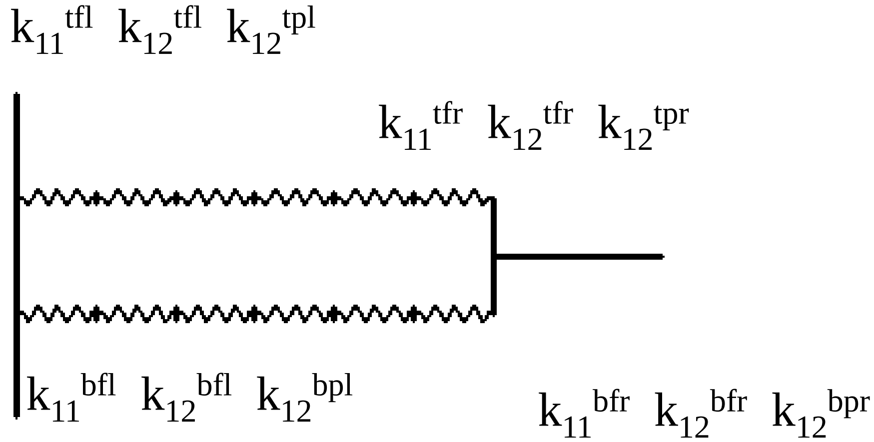
$$= E\, z^2 / (1 / \infty + 1 / \infty)] \to$$

$$\to E\, z^2 / (0 + 0 + 0 + 0) \to \infty$$

$S_{j,\, ini} \to \infty \to$ rigid joint

Bolted joint, bolts category A





There are 3 coefficients for each row of bolts: ultimately there are 36 springs for to and 36 for bottom in this case.

$$k_{11}^{tfl} ; k_{11}^{bfl} ; k_{12}^{tfl} ; k_{12}^{bfl} ; k_{12}^{tpl} ; k_{12}^{bpl} ;$$
$$k_{11}^{tfr} ; k_{11}^{bfr} ; k_{12}^{tfr} ; k_{12}^{bfr} ; k_{12}^{tpr} ; k_{12}^{bpr} \rightarrow k_{eff}$$

Photo: Author

Bolted joint, bolts category A

$k_{11}^{i} << \infty$ (#t / 41)

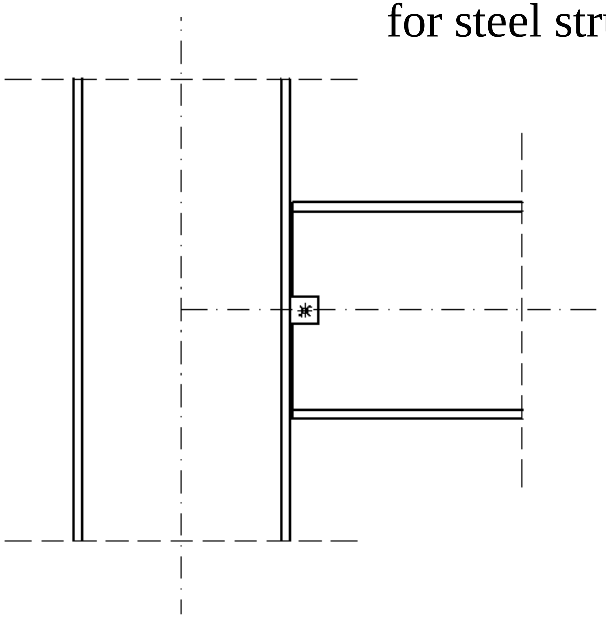
$k_{12}^{i} << \infty$ (#t / 41)

$k_{eff} = 1 / \Sigma (1 / k_i) << \infty$

$S_{j,\,ini} = E\, z^2 / (1 / k_{eff}^{\,top} + 1 / k_{eff}^{\,bottom}) << \infty$

$S_{j,\,ini} << \infty \rightarrow$ rigid, semi-rigid or hinged joint

This joint can be treatred as ideal hinged, but resistance for one bolt in very small. This type of joint is no recomended for steel structures (excep electro-energetic towers).



Bolted joint, bolts category A

Photo: Author

Photo: inzynierbudownictwa.pl

Photo: galeria.budownictwopolskie.pl

Photo: mlelectric.eu

Bolted joint, bolts category A

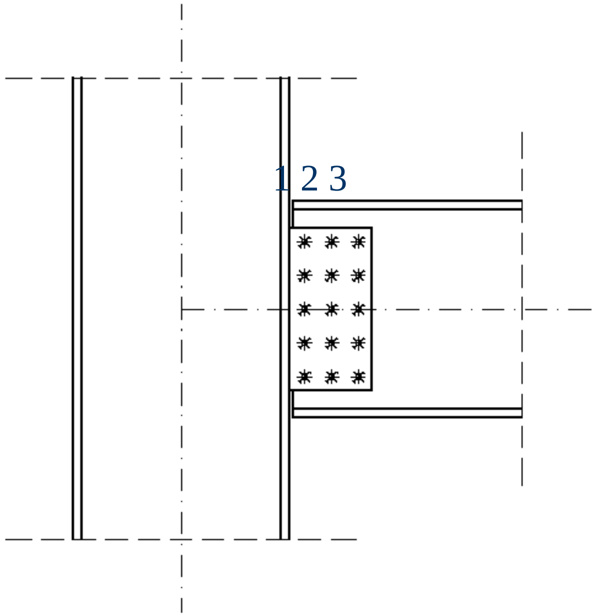


Photo: Author

There is no immobilisation of flanges, and, because of this, there is no clear way to build spring model of joint.

Rough approximation: there is possible, that for web plate, the farthest bolt rows (most top and most bottom rows) can be assumed as immobilisation of flanges.

Ultimately, k_i should be multiplied by additional factor θ:

$$\theta = J_y / J_{I, y}$$

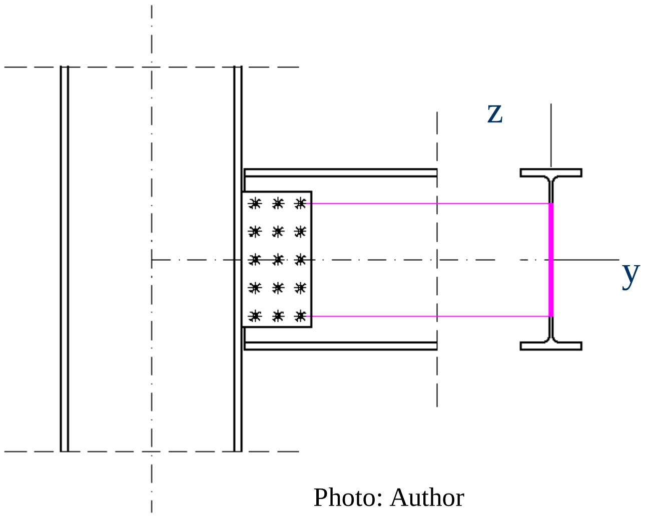


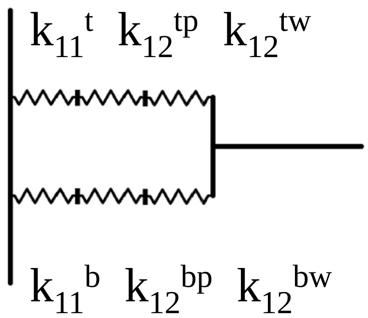
Photo: Author

No bending moment acts from beam on column – no k_1, k_2 and k_3.

Web plate - k_9, but value of k_9 tends to infinitive and can be omitted.

Only k_{11} and k_{12} are taken into consideration.

Photo: Author



There are 3 coefficients for each row of bolts: ultimately there are 9 springs for top and 9 for bottom in this case.

Bolted joint, bolts category A

$k_{11} << \infty$ (#t / 41)

$k_{12} << \infty$ (#t / 41)

$S_{j,\,ini} = E\, z^2 / \{ \Sigma\, [1 / (\theta\, k_i)] \} << \infty$

$S_{j,\,ini} << \infty \rightarrow$ rigid, semi-rigid or hinged joint

According to results of experiments, we can assume, that there are always pinned joints, if:

- web only is supported;
- for bolts are applied slotted holes.

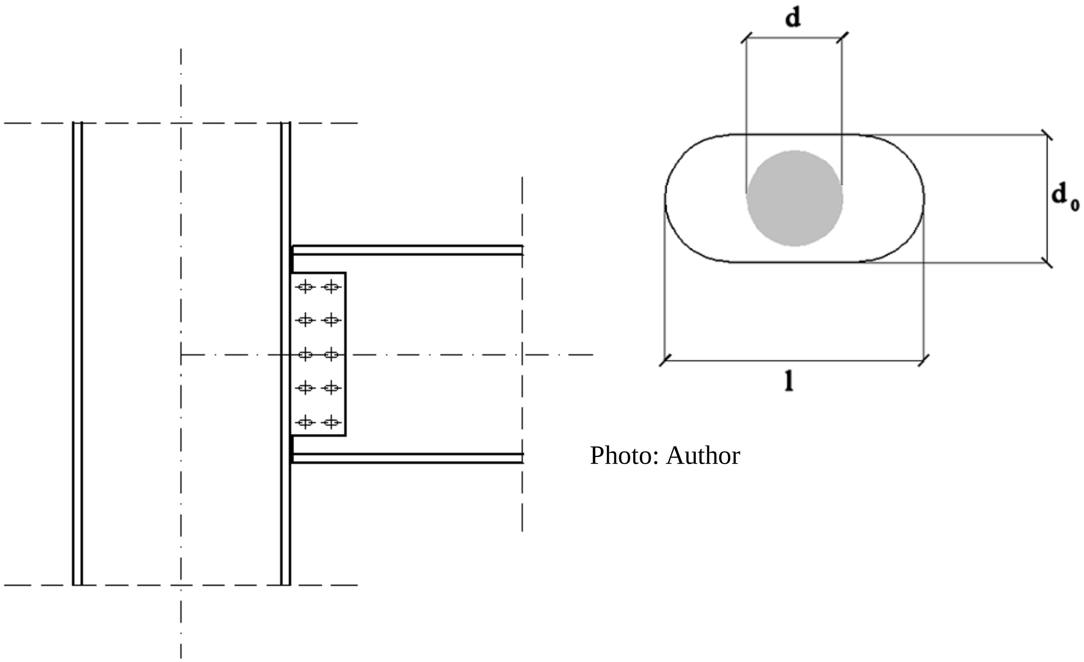
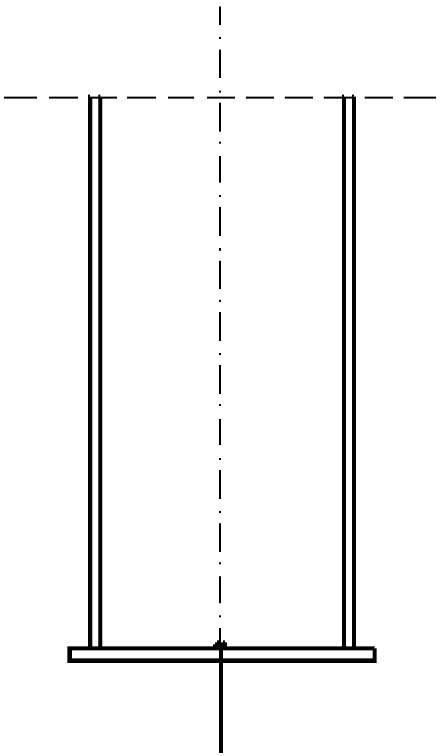


Photo: Author

Photo: tekla-detailed-structural-fabrication.com

Photo: Author



k_{13} ; k_{15} ; k_{16}

Photo: j-p.com.ua

$k_{13} << \infty$ (# / 42)

$k_{15} << \infty$ (# / 43)

$k_{16} << \infty$ (# / 43)

$z = 0$

$e = 0$

$$S_{j,\ ini} = \{E\ z^2 / [\ \Sigma\ (1 / k_i)]\}\ [e / (e + e_k)] \rightarrow$$

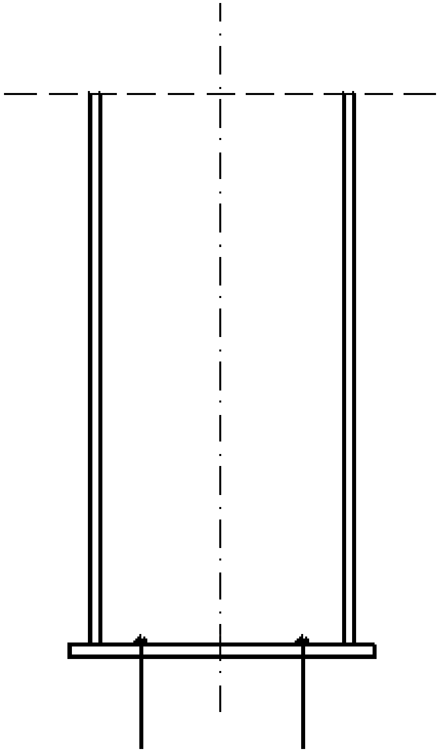
$$E\ 0^2 / [(1 / k_{13}) + (1 / k_{15}) + (1 / k_{16})\ ]\}\ [0 / (0 + e_k)] \rightarrow$$

$$\rightarrow \{0 / [\text{something}]\}\ [0 / (\text{something})] = 0$$

Column base, axial force only,
one row of anchors.

$S_{j,\ ini} = 0 \rightarrow$ pinned joint

Photo: Author



k_{13} ; k_{15} ; k_{16}

$k_{13} << \infty$ (# / 42)
$k_{15} << \infty$ (# / 43)
$k_{16} << \infty$ (# / 43)
$z \neq 0$
$e \neq 0$

$$S_{j,\,ini} = \{E\, z^2 / [\, \Sigma\, (1 / k_i)]\}\ [e / (e + e_k)] =$$

$$= \{E\, z^2 / [(1 / k_{13}) + (1 / k_{15}) + (1 / k_{16})\ ]\}\ [e / (e + e_k)] \neq 0$$

$S_{j,\,ini} \rightarrow$ rigid, semi-rigid or hinged joint

Column base, axial force only, more than one row of anchors.

Semi-rigid joints

What to do in this situation?

Photo: Author

The result of analisys:

Initial assumption	Could be in real structure	What it means for calculations of joint?
Hinged	Hinged	
	Semi-rigid	
Rigid	Rigid	

To avoid problem of differene between theoretical (fictitious) distribution of cross-sectional forces and real (unknown) one, which arises due to the flexibility of the joints, a **computational verification of stiffness** is necessary.

For each semi-rigid joints → modification of static scheme (of course, hinged and rigid joints are not modified).

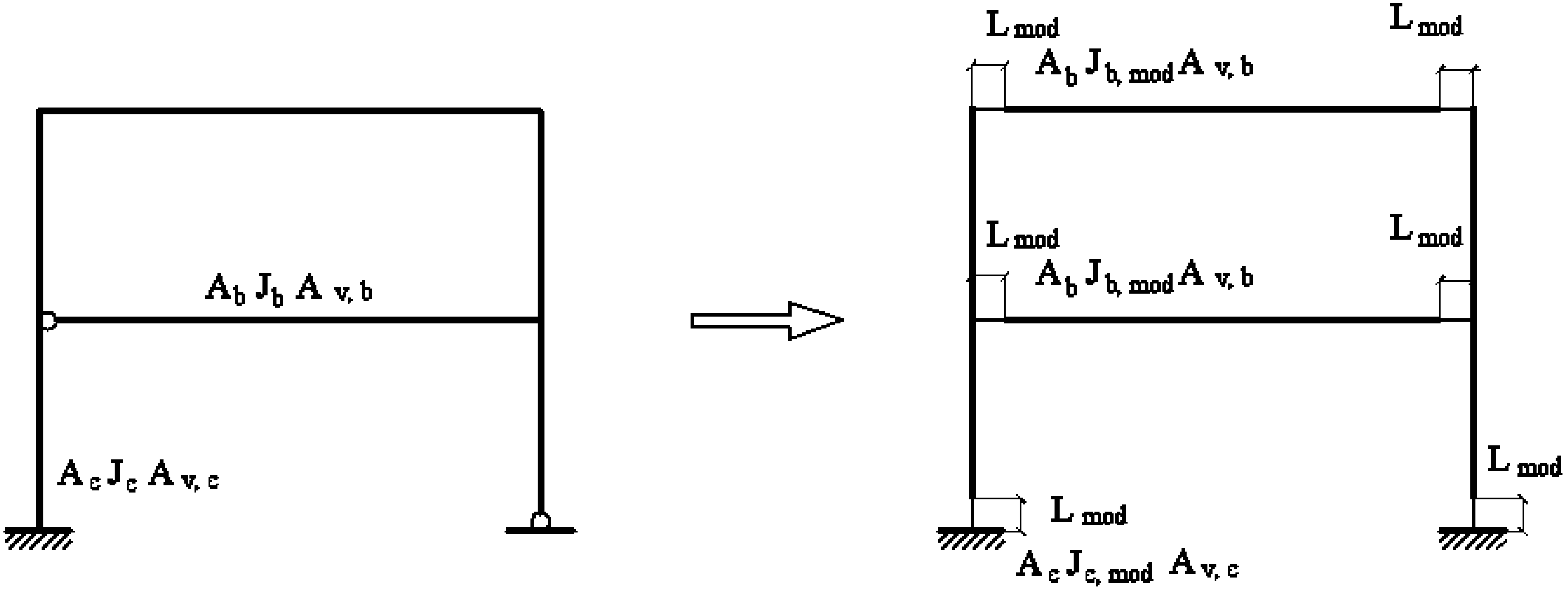


Photo: Author

$$J_{mod} = \min (J \quad ; \quad S_{j,\,ini}\, L_{mod} / E)$$

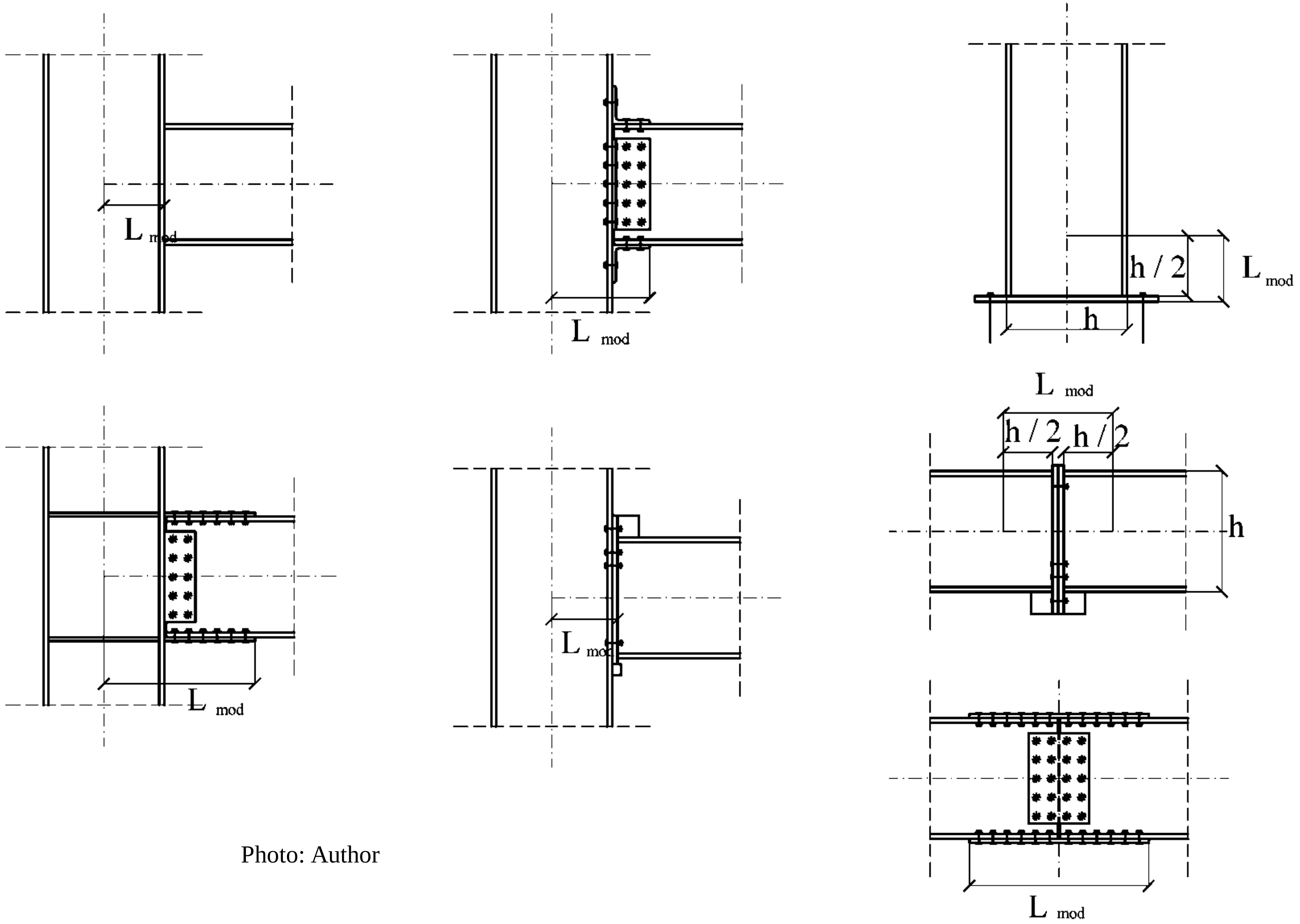


Photo: Author

Example of modification

Rigid joints beam-column

$J_c = J_b$

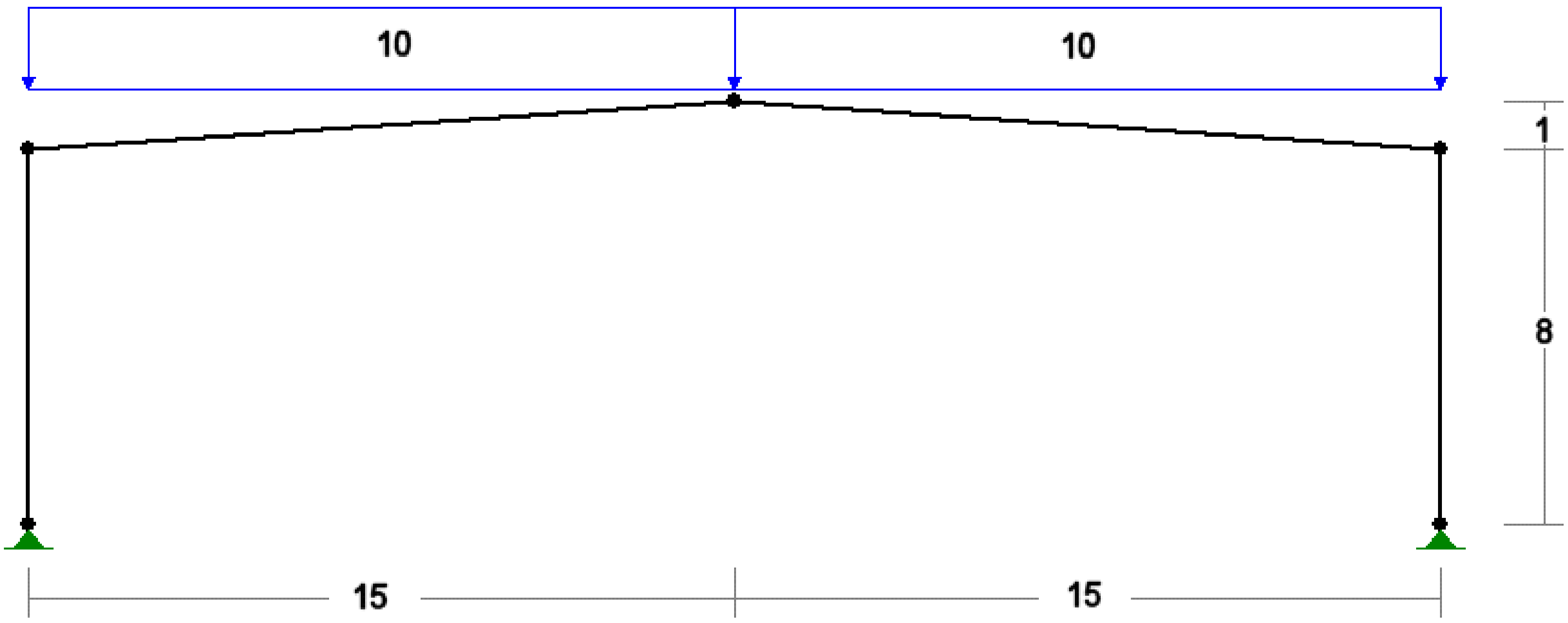


Photo: Author

M [kNm]

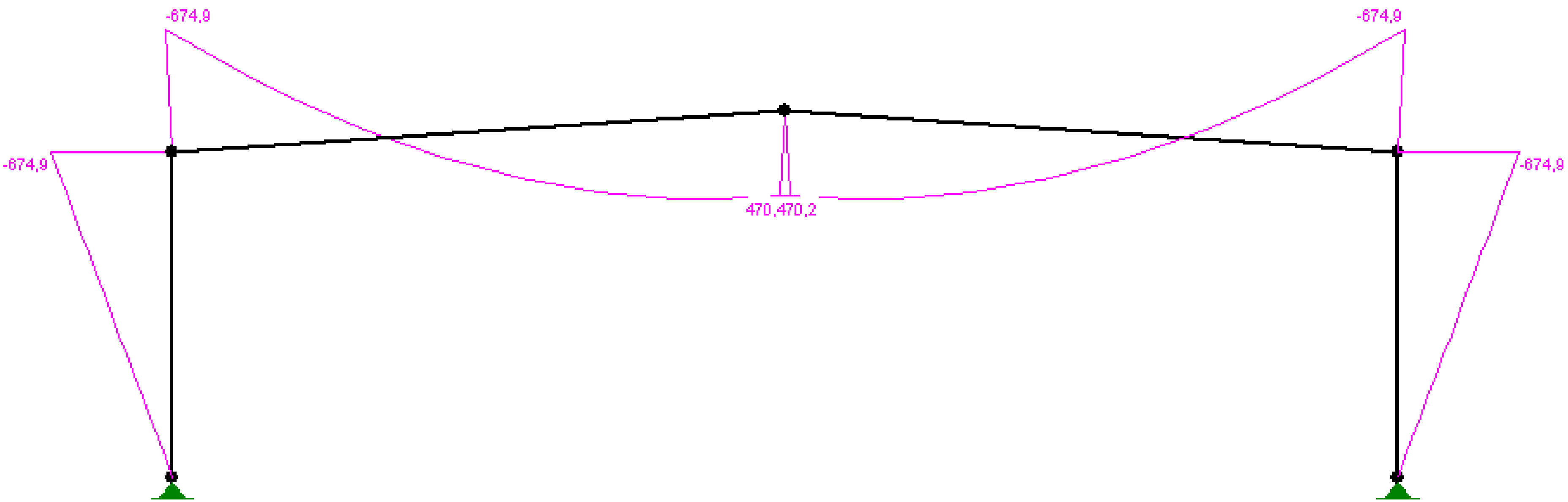


Photo: Author

Semi-rigid joints (limits):

$$0{,}5\ E\ J_b\ /\ L_b < S_{j,\ ini} < 25\ E\ J_b\ /\ L_b$$

Analysis of 5 cases, what’s happened, when

hinged joint (0,5) < $S_{j,\ ini}$ < rigid joint (25)

$$S_{j,\ ini,\ 1} = 5\ E\ J_b\ /\ L_b$$

$$S_{j,\ ini,\ 2} = 10\ E\ J_b\ /\ L_b$$

$$S_{j,\ ini,\ 3} = 15\ E\ J_b\ /\ L_b$$

$$S_{j,\ ini,\ 4} = 20\ E\ J_b\ /\ L_b$$

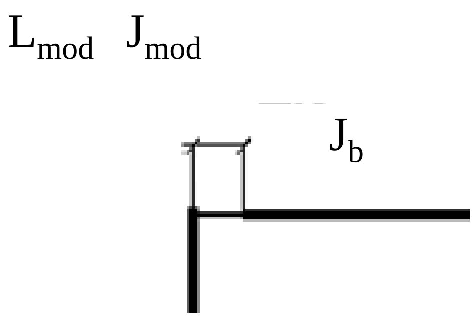


Photo: Author

$$L_{mod} = 0{,}4\ m$$

$$J_{mod} = S_{j,\ ini}\ L_{mod}\ /\ E$$

Joint	M_{eaves} [kNm]	M_{ridge} [kNm]	M_{eaves} [%]	M_{ridge} [%]
Pinned (theoretically)	0,0	1225,0	-100	+160,5
$S_{j, ini, 1}$	574,9	581,2	-14,8	+23,6
$S_{j, ini, 2}$	626,2	524,7	-7,2	+11,6
$S_{j, ini, 3}$	645,6	503,1	-4,3	+7,0
$S_{j, ini, 4}$	655,7	491,7	-2,8	+4,6
Rigid	674,9	470,2	+/- 0,0	+/- 0,0

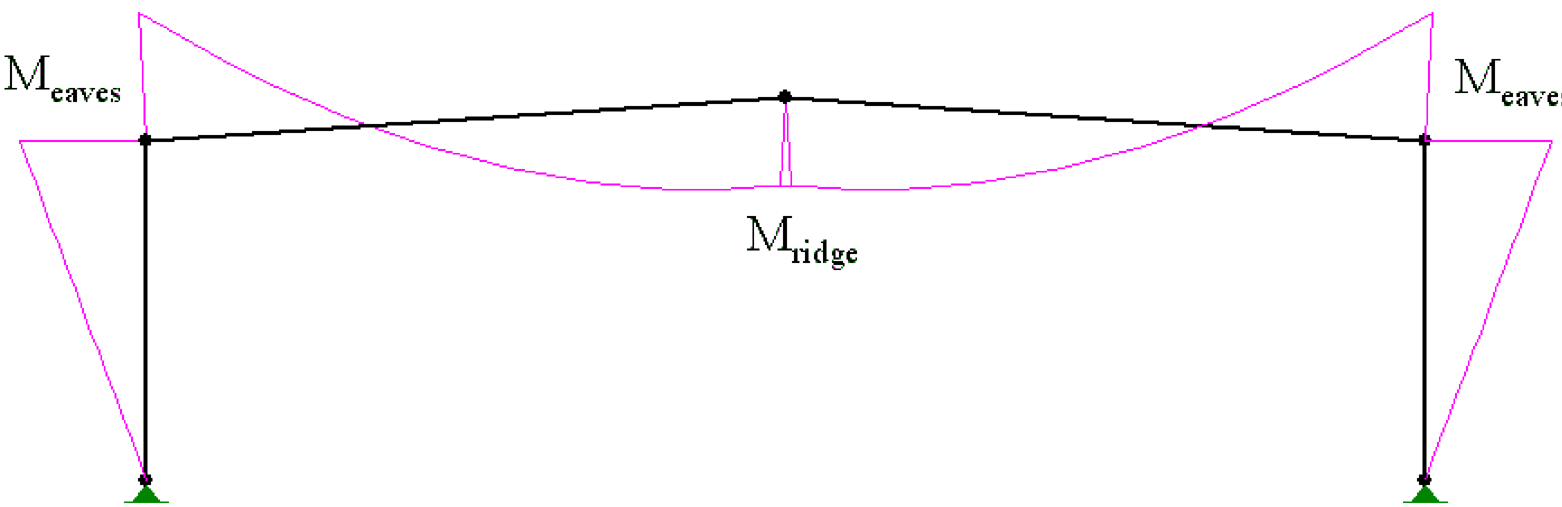


Photo: Author

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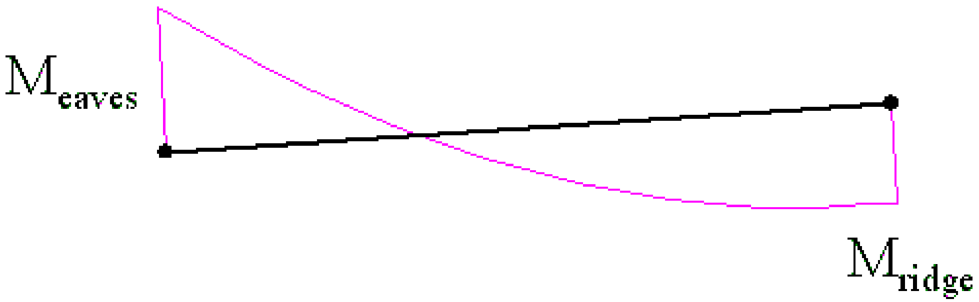
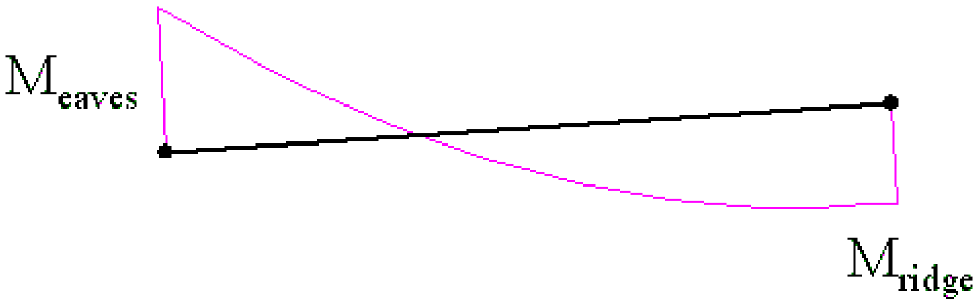


Joint	M_{eaves} [kNm]	M_{ridge} [kNm]	M_{eaves} / M_{ridge}	Remarks
$S_{j, ini, 1}$	574,9	581,2	0,99	Uniform effort at both points
$S_{j, ini, 2}$	626,2	524,7	1,19	
$S_{j, ini, 3}$	645,6	503,1	1,20	
$S_{j, ini, 4}$	655,7	491,7	1,34	
Rigid	674,9	470,2	1,44	Uneven effort at both points

Photo: Author



Joint	M_{eaves} [kNm]	M_{ridge} [kNm]	I	eaves	ridge	Dead weight [kg / m]
$S_{j, ini, 1}$	574,9	581,2	IPE 550 A	0,99	1,00	93,8
Rigid	674,9	470,2	IPE 600 A	0,91	0,64	109,6

There is possible, that for semi-rigid joints we can make lighter structure.

For comparison - example of frame with different stiffness of girded and columns

Rigid joints beam-column

A) $2\ J_c = J_b$

B) $J_c = 2\ J_b$

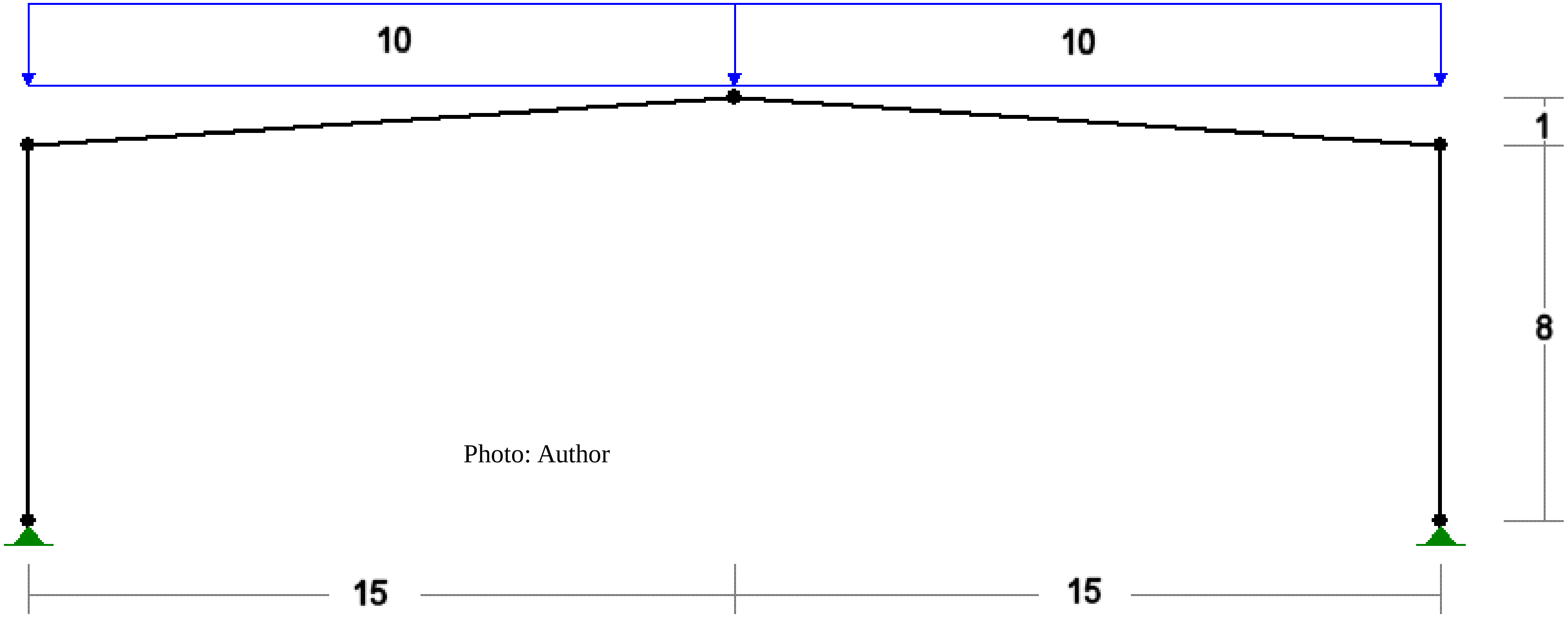


Photo: Author

Joint	M_{eaves} [kNm]	M_{ridge} [kNm]
Semi-rigid, $S_{j, ini, 1}$	574,9	581,2
Rigid, $J_c = J_b$	674,9	470,2
Rigid A ($2 J_c = J_b$)	594,2	559,5
Rigid B ($J_c = 2 J_b$)	724,0	415,1

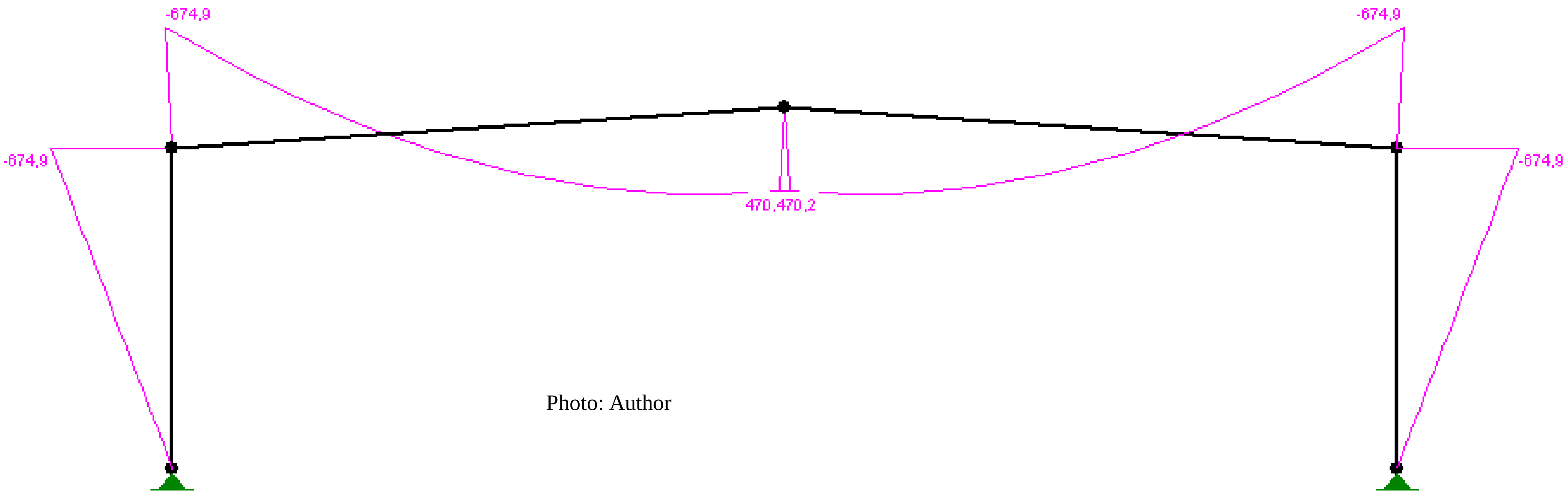
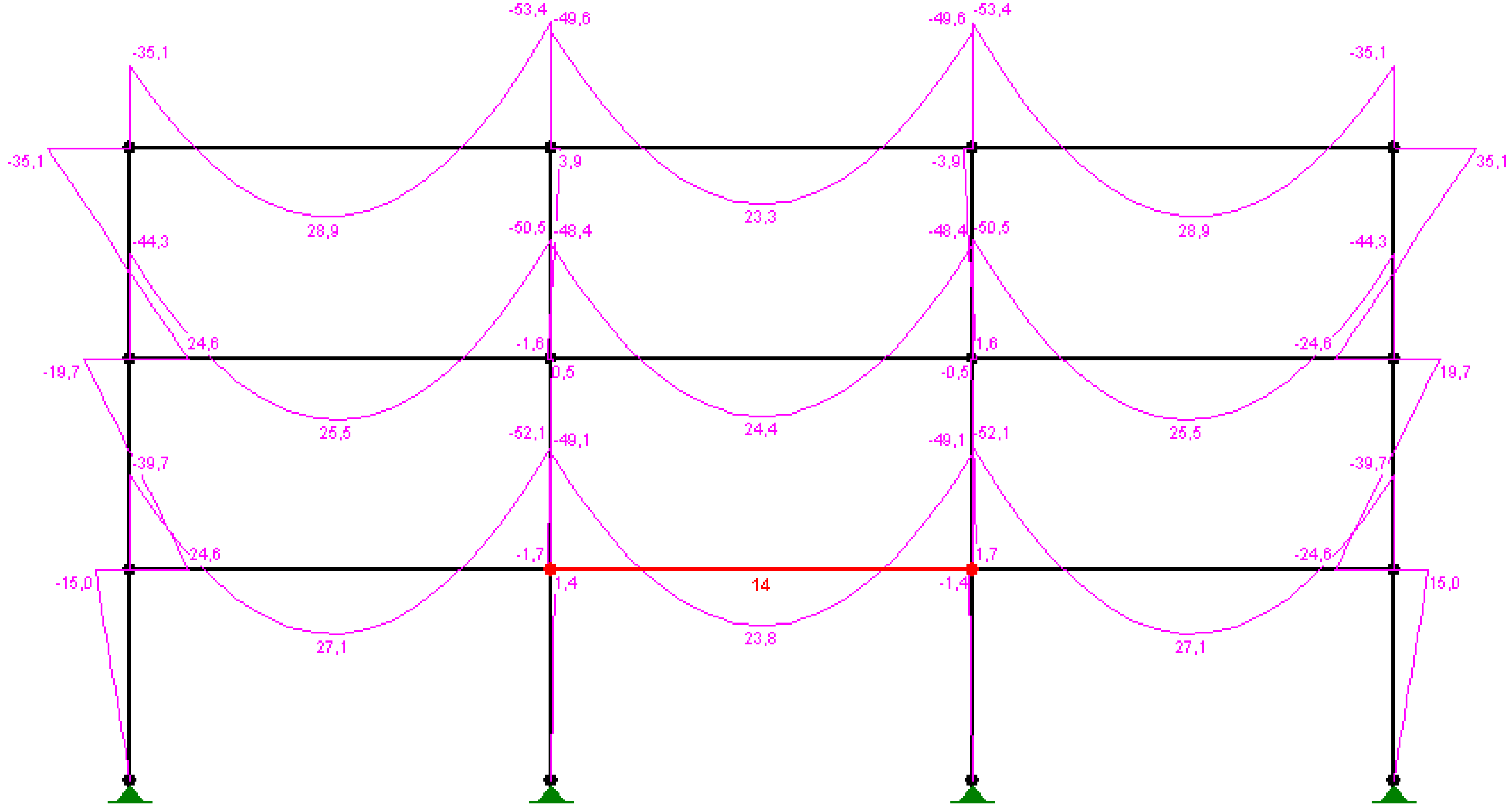


Photo: Author

But, for one-bay frame: max moment for girder = max moment for column; because of economic, there will be rather the same cross-section for column and girder (except a high column susceptible to buckling, which must have a larger cross-section). Additionally: column should be much more massive than roof girder, because of instability of column.

Generally, there is no sense to correct envelope of bending moments by change of the stiffness of elements in single-bay one-storey frame.

Photo: Author



Multi-bay multi-storey: max moment for girder in not the same as max moment for column; for this situation we can try to correct envelope of bending moments by change of J for columns and girders.

Idea to deliberately design semi-rigid joints to correction envelope of bending moments, did not catch on. During its life, structure works under various loads and actions. As a result, increasing of backlash and deformation gradually reduce the stiffness of the joints.

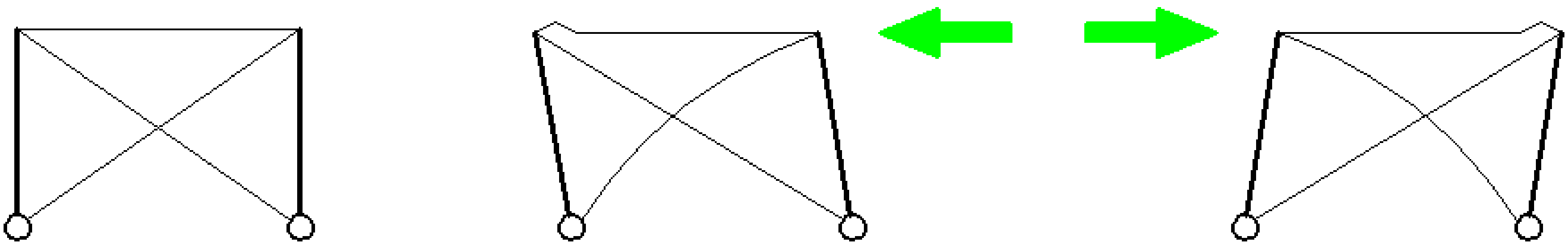


Photo: Author

$S_{j,\ ini}$ change values during time of exploatation, especially for semi-rigid joints. Because of this, it's not good idea to correct envelop of bending moments by semi-rigid joints.

Better way: correct by change of the stiffness of elements.

Photo: steltech.co.nz

There are used non uniform members for the best fit resistance and envelope of bending moments.

Photo: quora.com

Examination issues

Parts of tension joint important for stiffness

Effective and equivalent springs in springs model

Pinned joint - węzeł przegubowy
Semi-rigid joint - węzeł podatny
Flange cleat - nakładka z kątownika
End-plate - blacha czołowa
Grip length - grubość skleszczenia
Grout - podlewka
Prying force - siły przy efekcie dźwigni
Anchor bolt - kotew

Thank you for attention

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