

Metal Structures

Lecture XIV

Joints and connections in steel structure

Contents

Introduction → #t / 3

Rigid, semi-rigid and hinged joints → #t / 19

Technical solutions → #t / 26

Verification of stiffness → #t / 64

Calculations according to Eurocode → #t / 73

Examination issues → #t / 97

Introduction

Lectures Ist – VIIIth are introduction to design of metal structures. Information from IXth – XIIIth lectures allows to calculate the resistance, stability and deformation of the structural **members**. But without properly shaped of **joints**, we will only have a storehouse of elements, not a structure.



Photo: wikipedia



Photo: southweststeels.co.uk



Photo: wistal.pl

Calculation of joints is the most complicated and the most labor-intensive part of each project of steel structure. Many various local phenomena must be taken into consideration. It is possible, that it takes 60-70% of total time of calculations. Example: 5 lectures (9-13) about elements, 8 lectures (14-21) about joints.



Photo: j-p.com.ua



Photo: microstran.com.au



Photo: resources.scia.net

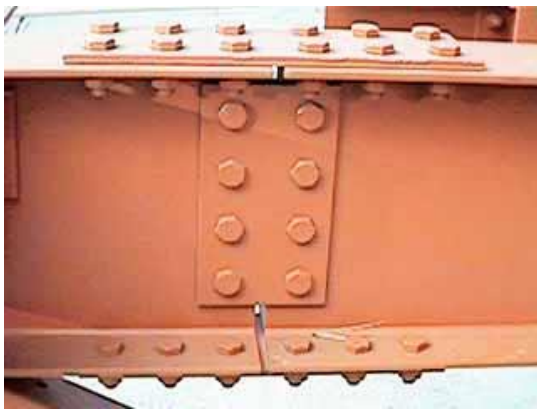


Photo: zs4-sanok.pl

Photo: amsd.co.uk



Photo: palatinepaints.co.uk

Joints and connections

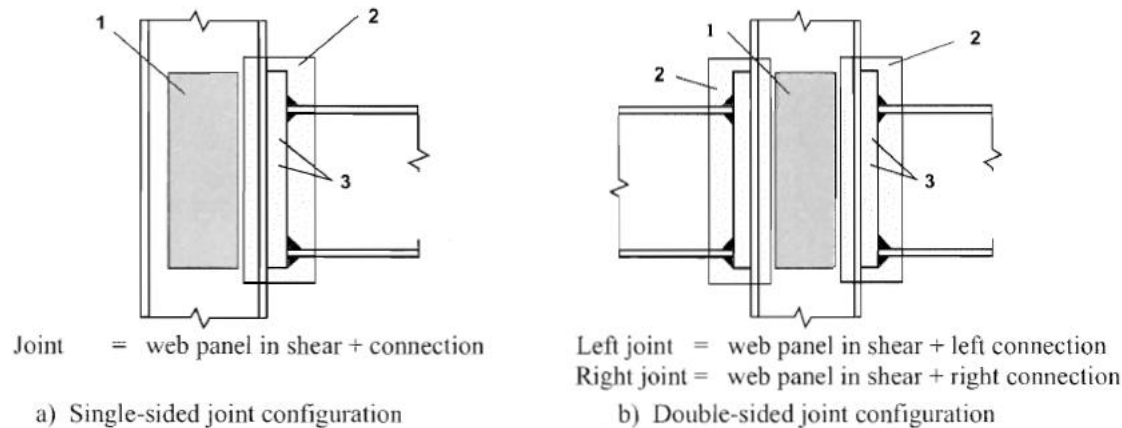
1.4.2 connection

Location at which two or more elements meet. For design purposes it is the assembly of the basic components required to represent the behaviour during the transfer of the relevant internal forces and moments at the connection.

EN 1993-1-8 1.4.2,
1.4.4

1.4.4 joint

Zone where two or more members are interconnected. For design purposes it is the assembly of all the basic components required to represent the behaviour during the transfer of the relevant internal forces and moments between the connected members. A beam-to-column joint consists of a web panel and either one connection (single sided joint configuration) or two connections (double sided joint configuration), see Figure 1.1.



1 web panel in shear
2 connection
3 components (e.g. bolts, endplate)

Photo: EN 1993-1-8 Fig. 1.1

Connection: Location... ...the assembly of the basic components...

Joint: Zonethe assembly of **all** the basic components...

Nodes on steel structure always consist on bolts or welds and additional plates (stiffeners, gusset plates). Local concentration of stresses and forces reveals on each node.

Because of it local concentration, short parts of connecting members (short part cooperated with subelements of node) must be taken into consideration. Specific local phenomena, resulting form concentration and cooperation, must be detaily analysed.

In simplification: the most important parts of node are weld and shank of bolts. Because of this, they coul be named „connections” – in opposite to „rest part of node”, i.e. stiffeners, gusset plates and cooperated parts of members.

Parts of structure

→ #3 / 77



Photo: Author

Each steel structure can be divided into three parts:

- members
- connections
- joints

Members

Bars, beams, purlins, rafters, girders, columns, bracings - calculations according to level of cross-section and level of element.



Photo: Author



Photo: civildigital.com

Example from 1st design project: resistance and stability of truss bars.

(~ 40% of calculation's conditions)

→ #3 / 78

Connections

Welds and shank of bolts - calculation according to level of point (for welds) or cross-section (shearing resistance or tension resistance of shank of bolts)

Example from 1st design project: resistance for welds.

→ #3 / 79



Photo: Author



Photo: ceprocs.civil.tamu.edu



Photo: researchgate.net

(~ 10% of calculation's conditions)

Joints

Small parts of members, where are contact between two or more members. There are many specific phenomenons on these short part of beams, columns, etc. Calculation according to level of cross-section and level of element.

Example from Ist design project: resistance of truss joints.

→ #3 / 80

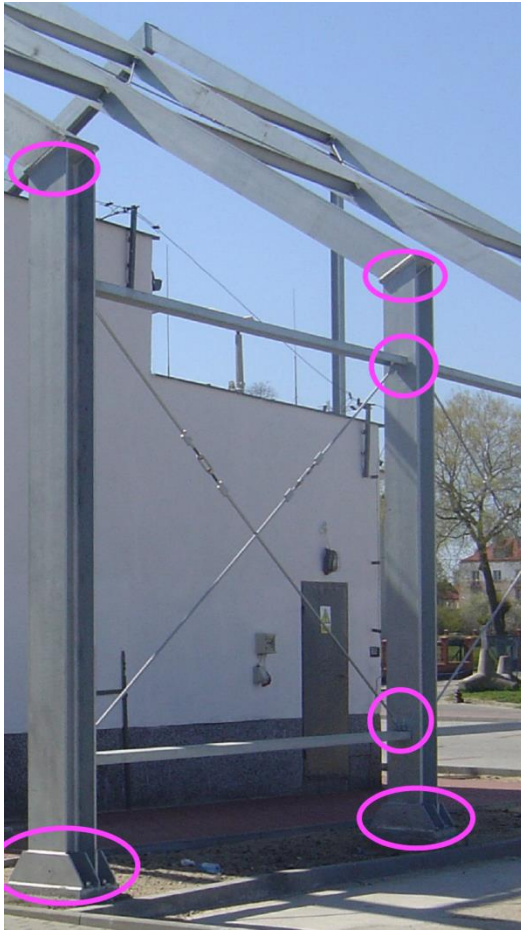


Photo: Author



Photo: sciELO.br



Photo: ascelibrary.org



Photo: osha.gov

(~ 60% of calculation's conditions)

Algorithm

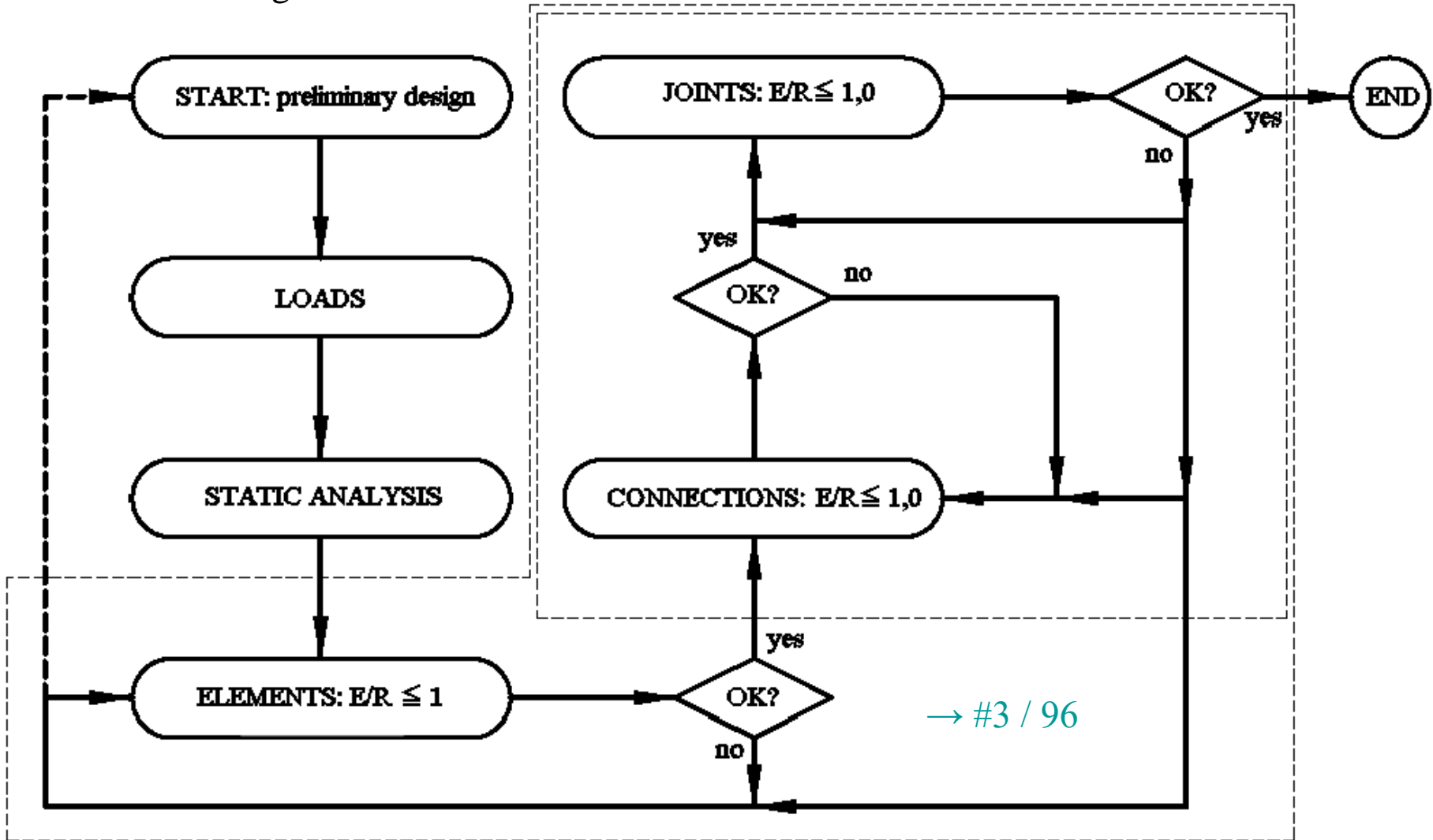


Photo: Author

Resistance of **connections** are resistance of welds and resistance of shank of bolts:



Photo: researchgate.net

- exceeding stresses in the weld;
- shearing of the bolt;
- tearing of the bolt;



Photo: ceprofs.civil.tamu.edu

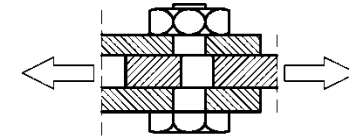


Photo: Author

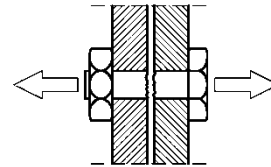


Photo: Author



Photo: forgemag.com

Behaviour of **joint** could be divided into two groups:

- interaction between plates/elements and connections;
- interaction between plates/elements each other



Photo: ascelibrary.org

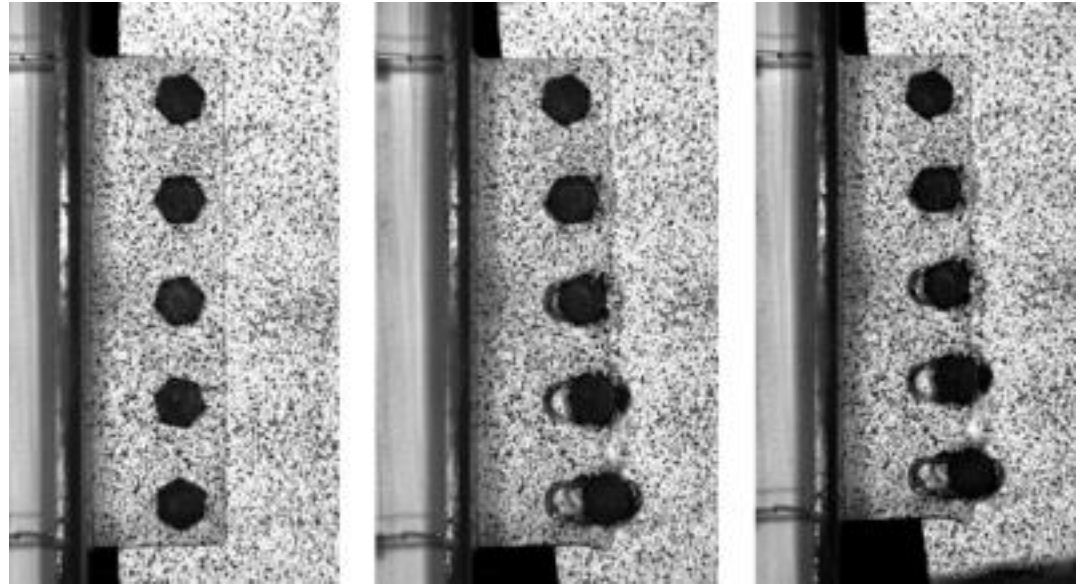


Photo: ascelibrary.org

Example for interaction plate-shank: bearing resistance: deformation or destruction of plate as the effect of interaction between shank of bolt and plate.

Other examples of destruction as the effect of interaction and contact between shank of bolt and plates:

- punching;
- local bending of plate around bolts under tension.

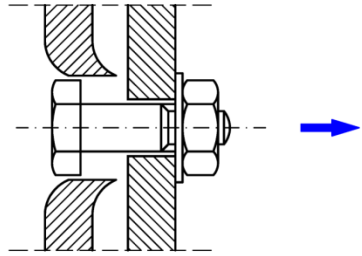


Photo: Author



Photo: knkmpk.blogspot.com



Photo: Author

This type of destruction occurs only as effect of interaction/contact between bolt and plate/element, not between weld and plate/element.

Selected examples of destruction as the effect of interaction/contact between plates/elements each other or local behaviour of plates/elements:

- netto area;
- block tearing;
- local instability in truss node;
- destruction of concrete under column.



Photo: quora.com



Photo: quora.com



Photo: sciELO.br



Photo: osha.gov

Stiffness of joint in second, besides resistance, parameter important for joints. According to Eurocode, we must check, if our assumption about joint - rigid or hinged - is true for real geometry and real cross-sections of members. Sometimes, because of big difference between assumption and real behaviour of joint, we must make static calculations once again.

Part of structure	Condition	
	Resistance	Stiffness
Member	ULS	SLS (deflection)
Joint	ULS	Qualification for different type of joint (hinged, semi-rigid, rigid)

Analysis of stiffness and resistance is made based on components method. (more information will be presented on lecture #14,

→ Des #1 / 73

Component method: resistance and stiffness of joint is effect of resistance and stiffness its components.

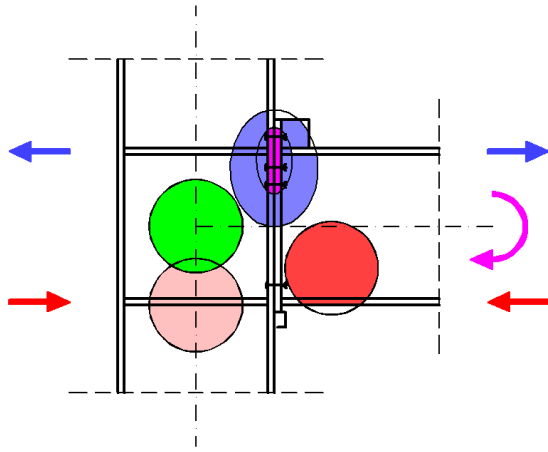


Photo: Author

For resistance, the most important is the weakest component (the weakest link).



Photo: dynamicbusiness.com.au

For stiffness, joint is analysed as a complex of springs.

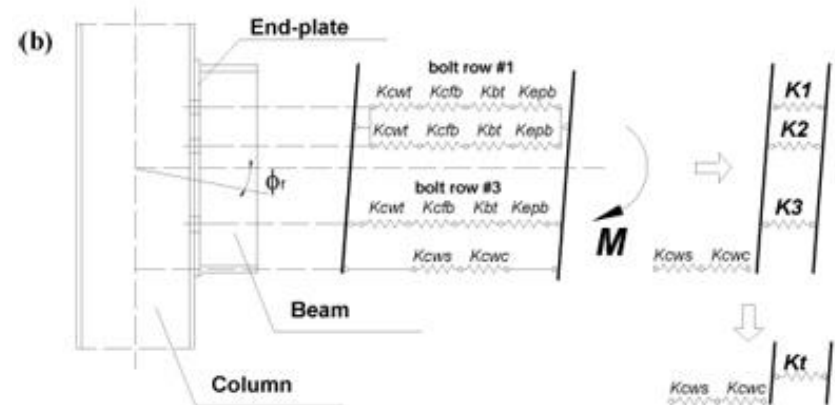


Figure 2. (a) Active components for bolted beam-to-column end-plate connections and (b) joint rotational stiffness according to EC3-1.8 (2003).

Photo: sciELO.br

Type	Connection	Joint		
		Stiffness	Resistance	
			Contact / interaction between connections (shank of bolt) and other elements	Contact / interaction between other elements
Welded	EN 1993-1-8 chapter 4 (welds) <u>Lecture #16, 17</u>	EN 1993-1-8 chapter 5 and 6 <u>Lecture #14,15</u>	No phenomenons	EN 1992-1-1 chapter 6 EN 1993-1-5 chapter 5, chapter 9 EN 1993-1-8 chapter 4, chapter 6, chapter 7 EN 1995-1-5 chapter 9 <u>Lecture #19, 20, 21</u>
Bolted	EN 1993-1-8 chapter 3 (shanks) <u>Lecture #18</u>		EN 1993-1-8 chapter 3, chapter 6 <u>Lecture #18, 19, 21</u>	EN 1993-1-8 chapter 4, chapter 6 <u>Lecture #18, 19, 20, 21</u>

Rigid, semi-rigid and hinged joints

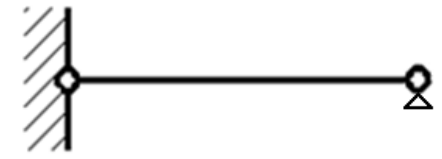
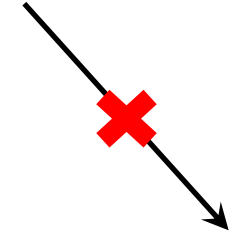
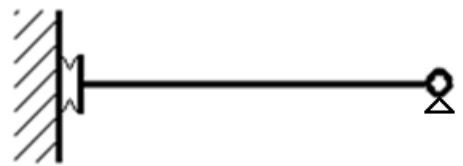
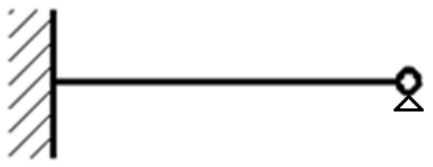
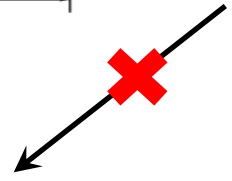
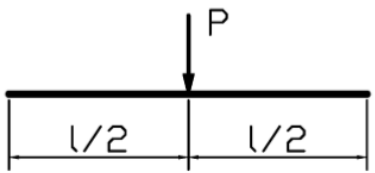


Photo: Author

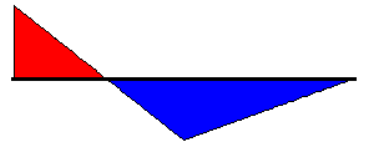
Generally, two types of joints - hinged and rigid – are used to modelling of structures. Both types based on various technical solutions: position of bolts, welds, and additional plates. In theory:

- **ideal hinge** joint: no bending moment, rotation depends on loads and cooperation with rest part of structure;
- **ideal rigid** joint: no rotation, bending moment depends on loads and cooperation with rest part of structure.

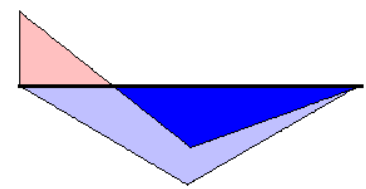
In reality, there are no ideal rigid (**infinite stiffness**) or ideal hinged (**zero stiffness**) nodes. There are only nodes with lesser or greater flexibility (semi-rigid).



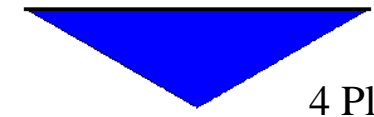
$3 P l / 16$



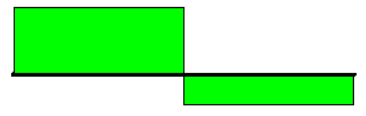
$2,5 P l / 16$



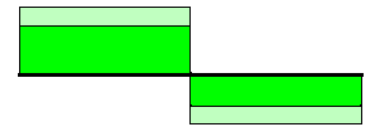
$4 P l / 16$



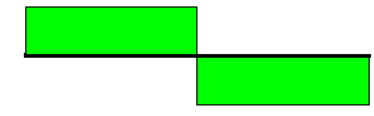
$11 P / 16$



$5 P / 16$



$8 P / 16$



$8 P / 16$

Photo: Author

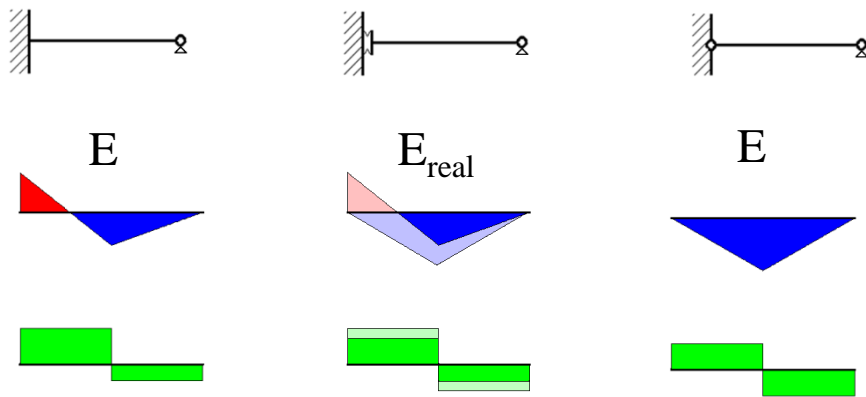
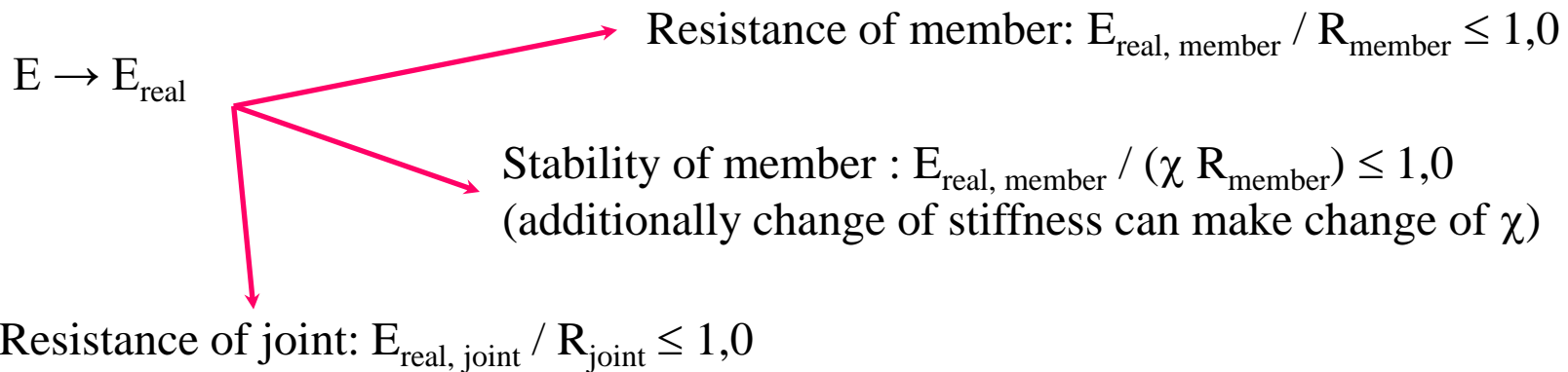


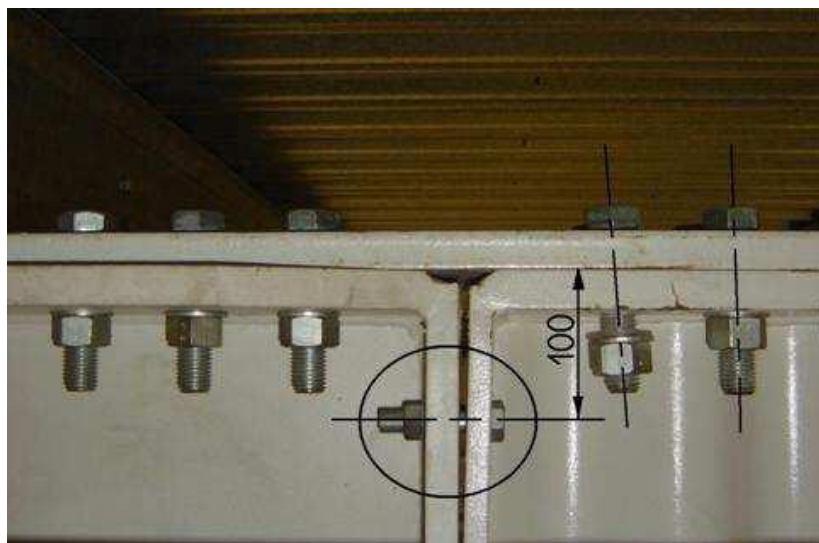
Photo: Author

Difference between theoretical (E) and real (E_{real}) distribution of cross-sectional forces has significant implications for calculations:



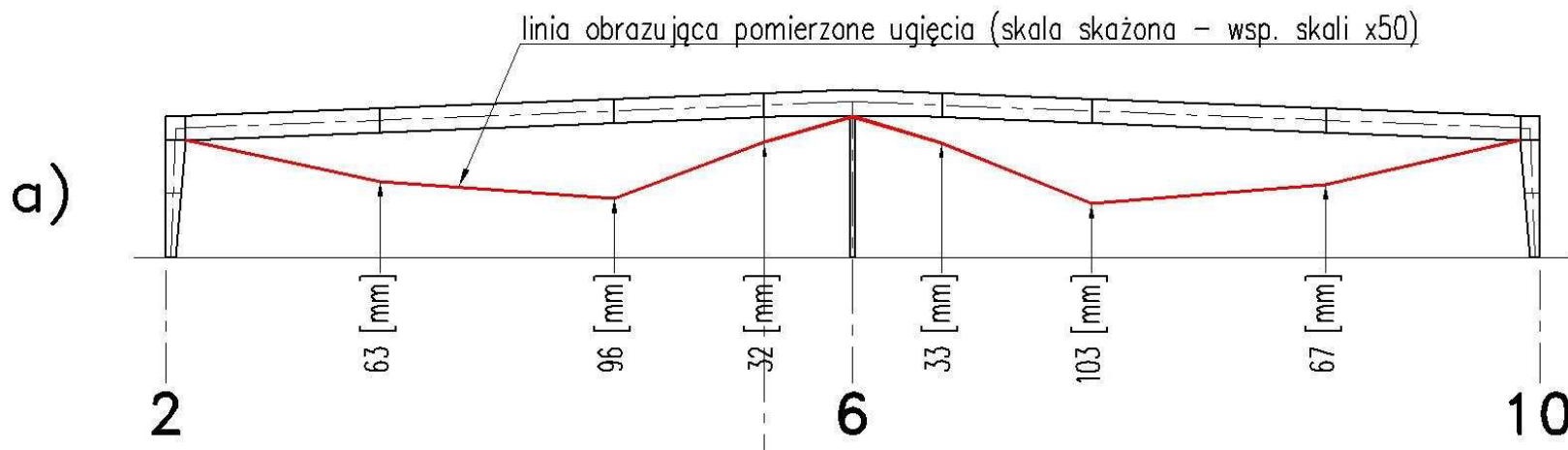
Additionally, change of joint stiffness makes change of deformation of structure (SLS)

Incorrectly designed or incorrectly executed bolted joint, example of effects: initial assumptions – rigid, real behaviour – semirigid.






Fortunately, it ended with exceeding SLS and not with collapse of structure.

Photo: Z. Kowal, Ł. Tkaczyk, Uszkodzenia doczołowo-stycznych połączeń dźwigarów z falistym środkiem, Awarie Budowlane 2007



The result of analysis:

Initial assumption	Could be in real structure	What it means for calculations of joint?
Hinged	Hinged	
	Semi-rigid	
Rigid	Rigid	

To avoid problem of difference between theoretical (fictitious) distribution of cross-sectional forces and real (unknown) one, which arises due to the flexibility of the joints, a **computational verification of stiffness** is necessary.

Initial assumption: rigid joints

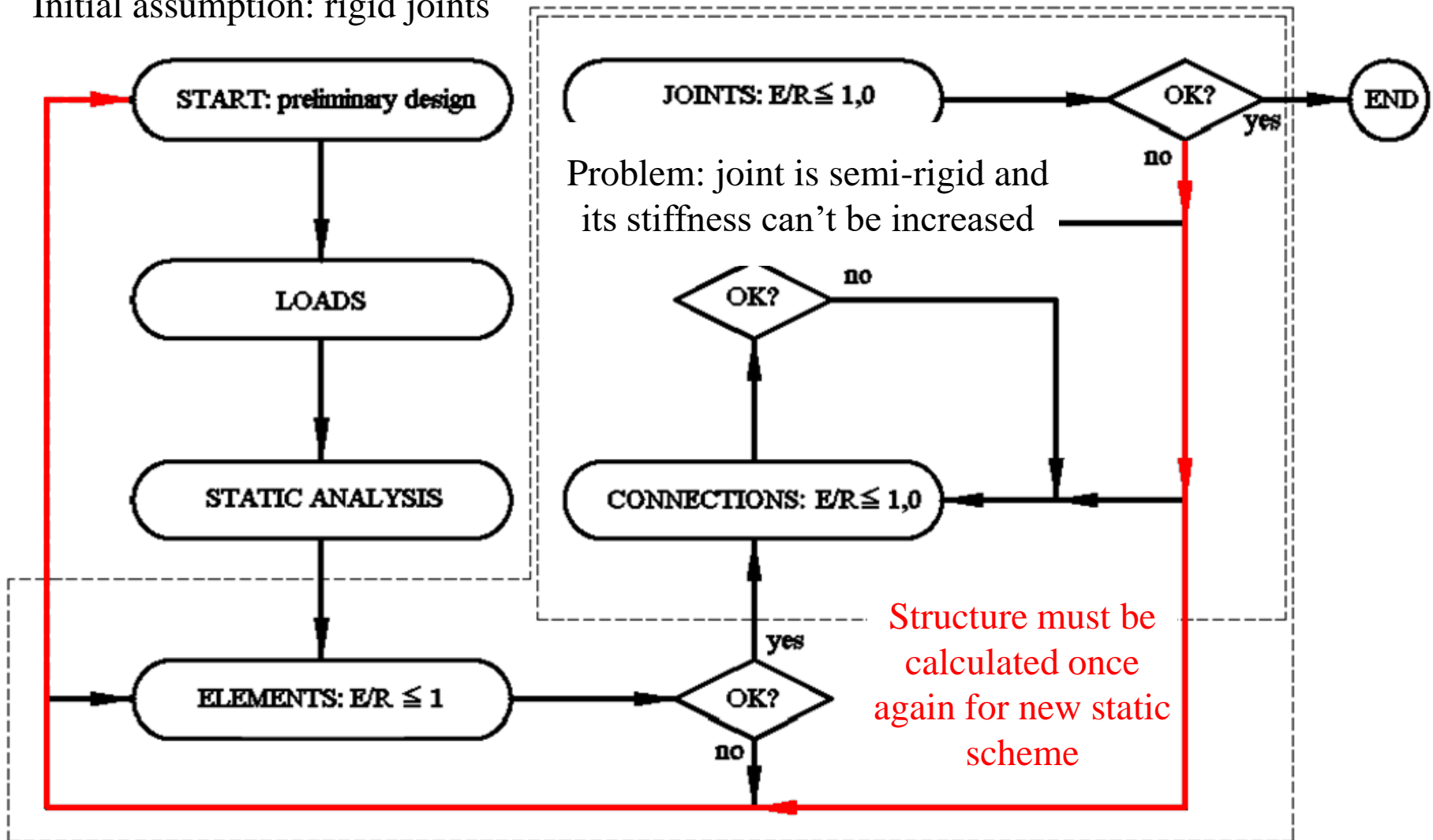
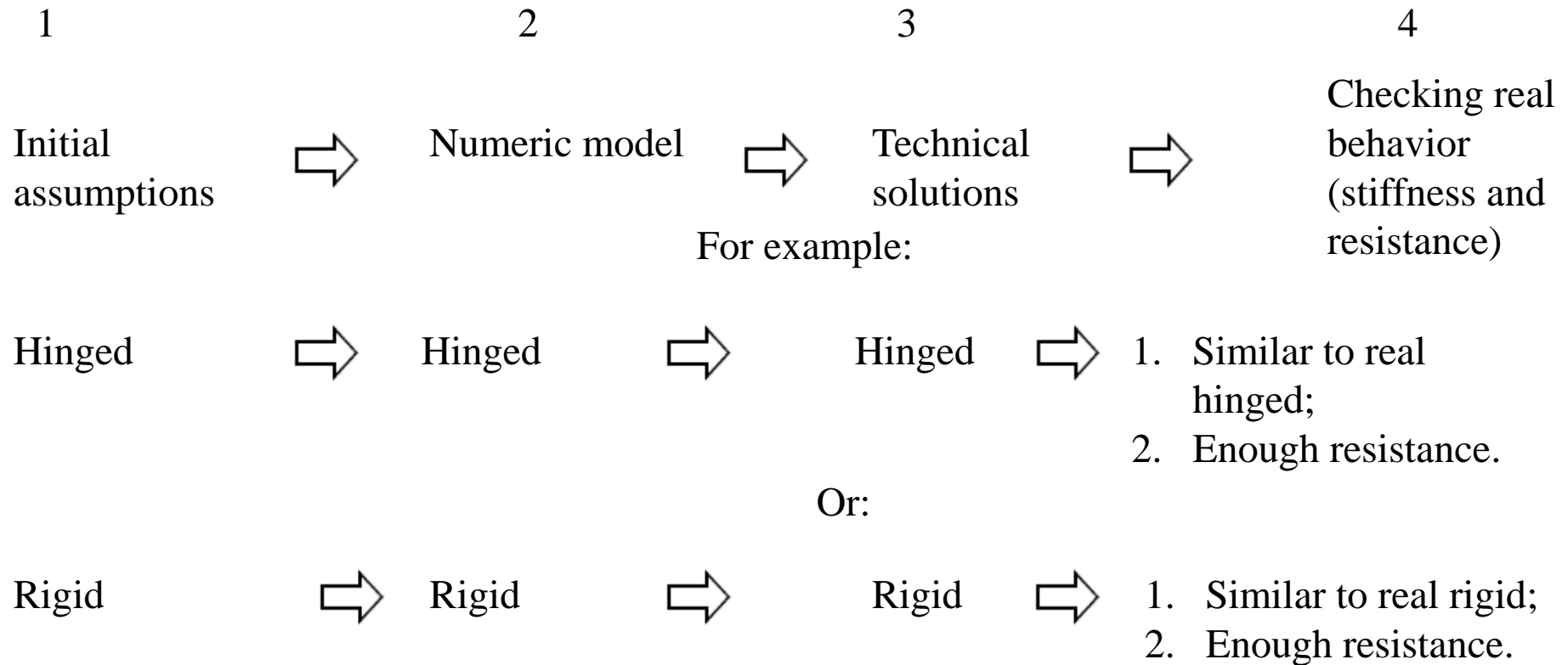


Photo: Author

Analysis of joints in steel structures involves four steps:

arrangement of rigid and hinged joints in structure - FEM numerical model - technical solution (position of bolts and welds, dimensions of gusset plates) - verification of stiffness and load-bearing capacity. **Stiffness in step 1, 2 and 4 must be the same.**



Technical solutions

Initial assumptions



Numeric model



Technical solutions

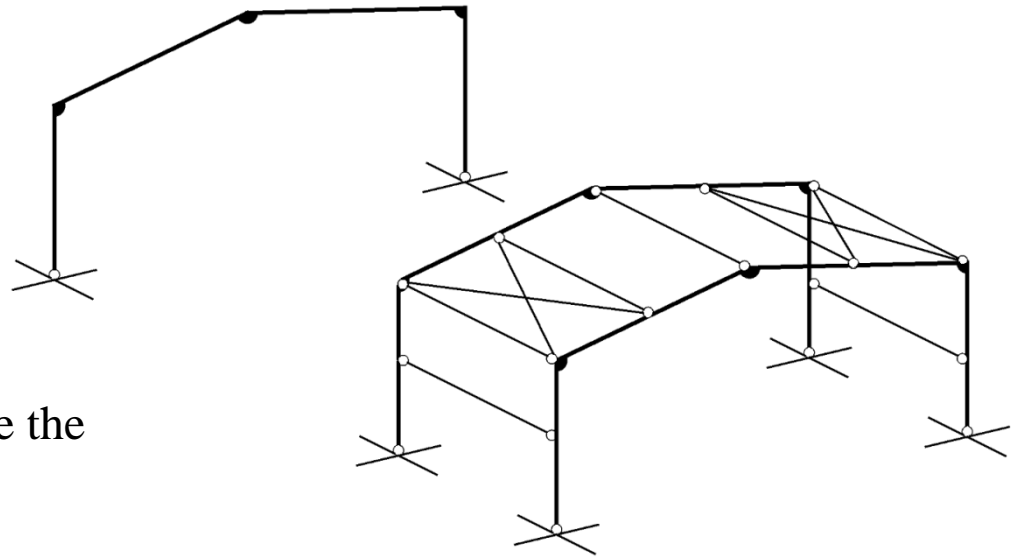


Real behavior

First and second point are easy. Initial assumptions based on experience. The most often used type of structure is:

- flat frame;
- rigid joints in plane of frame;
- hinged supports;
- hinged joints out of plane (between frame and purlins, girts and bracings).

Photo: Author



In numeric model types of joints must be the same as in initial assumptions.

Very complicated part of issue are supports of steel structure, especially in case of contact with other type of material (masonry, concrete). There are three types of supports according to initial assumptions:



rigid,



pinned,



roller.

Photo: Author

The most complicated and often disregarded problem is to provide steel structures with sufficient freedom of rotation and translation in the last two types of supports.

Support	Examples of supports
Rigid	Column;
Pinned	Column; Truss or beam on masonry, concrete or steel structure;
Roller	Truss or beam on masonry, concrete or steel structure;

For massive structures or structures with big loads (bridges), two various technical solutions are applied for pinned and roller supports.

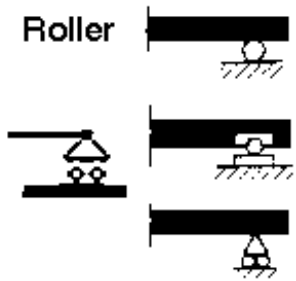


Photo: web.mit.edu



Photo: . fbcdn-photos-g-a.akamaihd.net



Photo: .texasescapes.com



Photo: web.mit.edu



Photo: wikipedia



Photo: .tatasteelconstruction.com

For „normal” truss and beams important factor is proportion between stiffness of Girder (truss or I-beam) and Support (column, wall...):

$$S_G / S_S$$

$$S_G = E_G J_G / L_G$$

$$S_S = E_S J_S / h_S$$

Stiffness of truss could be approximated by:

$$S_{\text{truss}} \approx 3,5 q L_G^2 \quad ; \quad q = (\Sigma F_i) / L_G$$

or (\rightarrow #9 / 11):

$$J_{\text{truss}} \approx 0,35 h_{1, \text{truss}}^2 A_{\text{chord}}$$

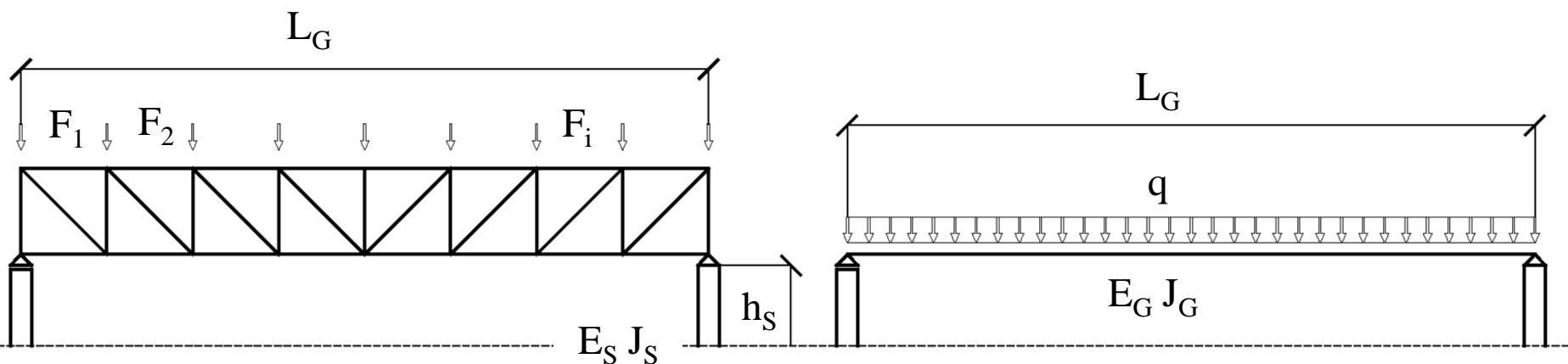


Photo: Author

Recommended technical solutions (based on experience):

(Big)	Various proportion S_G / S_S		(Small)
No rocker	Flat rocker	Round rocker	Elastomeric rocker

Elastomeric rocker is used, first of all, for bridges. For other type of structures is used very rare.



Photo: anthony-johnson-engineering.co.uk

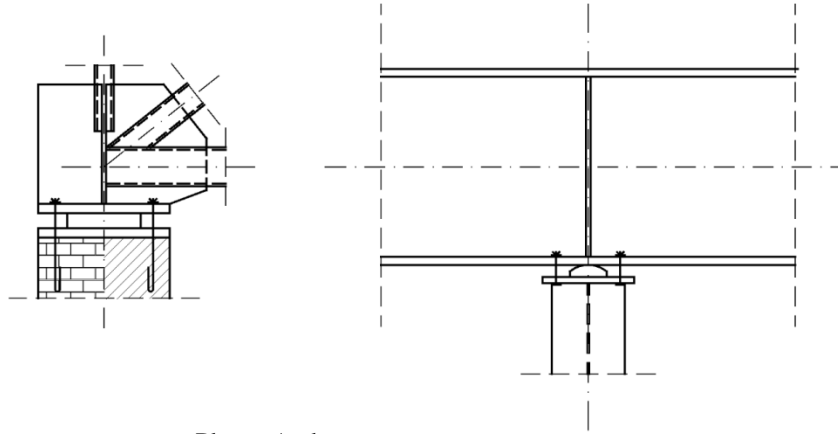


Photo: Author

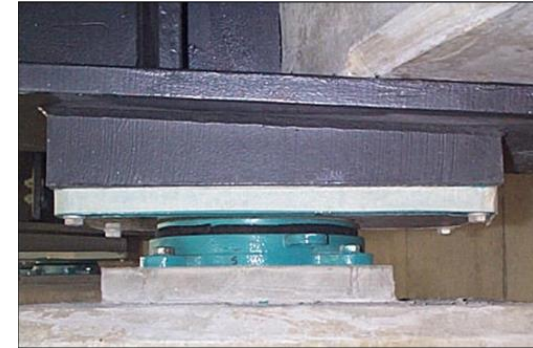


Photo: steelconstruction.info

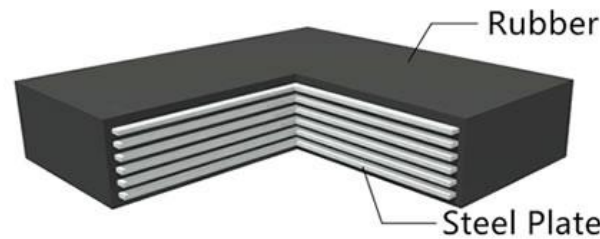


Photo: bridgebearing.org

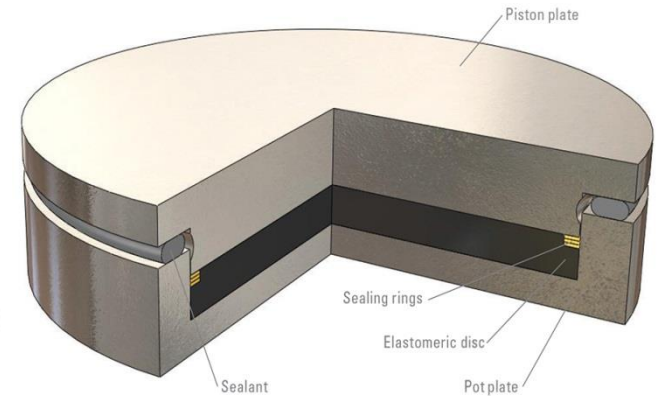


Photo: canambridges.com

Elastomer bearing consists of layers of steel and rubber. Provides good vertical load transfer. Work in horizontal direction and rotation is very close to ideal roller support.

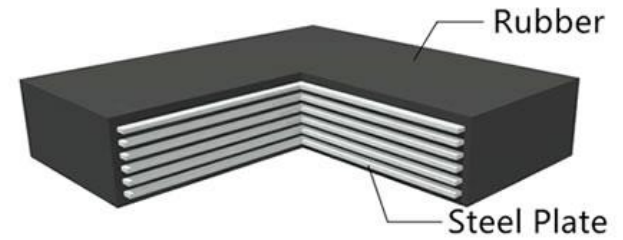


Photo: bridgebearing.org

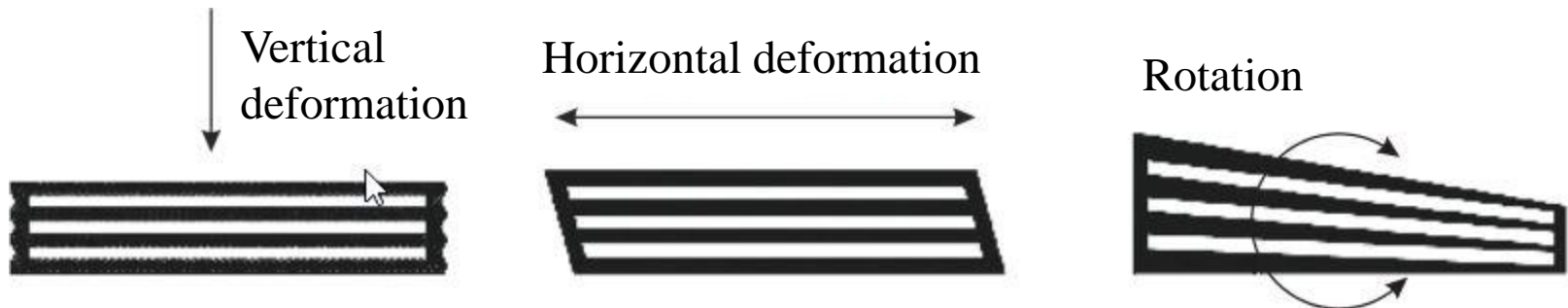


Photo: forbuild.eu

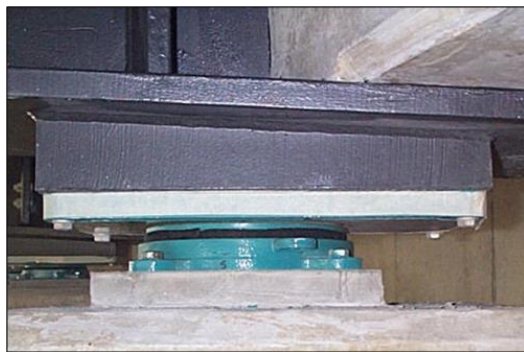


Photo: steelconstruction.info

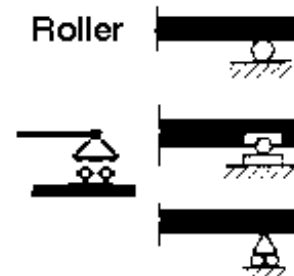


Photo: web.mit.edu

An important feature of structure CC3 is large safety margin. Real behavior of supports must be as close as possible to theoretical one, to avoid secondary effects come from supports different from the assumed one. They are usually impressive structures with large spans (and large reactions in the supports). Large reactions with supports other than ideal may cause significant secondary effects, dangerous for structure. That is why for CC3 it is so important to shape supports as close as possible to ideal.



Photo: alcoxsteel.com

Photo: cnxzl.com



Photo: wikipedia

On lecture will present recommendations applicable in civil engineering for structures of CC3. In case of CC2 and CC1 designs, it is possible to depart from strict compliance with recommendations. Recommendations are based only on engineering experience and are not mandatory. However, compliance with them allows you to avoid potential problems during exploitation of the structure.

Recommendations for CC3 are not necessarily valid for lower Consequence Classes. Structure should be as cheap as possible. Detailed design of supports can be very important for CC3. For lower classes, with a smaller assumed safety margin, it may not be necessary: too expensive in relation to safety of structure. Small differences between ideal and real behavior of structure can be accepted. Carrying out / resigning from thorough analysis should result from experience of designer.

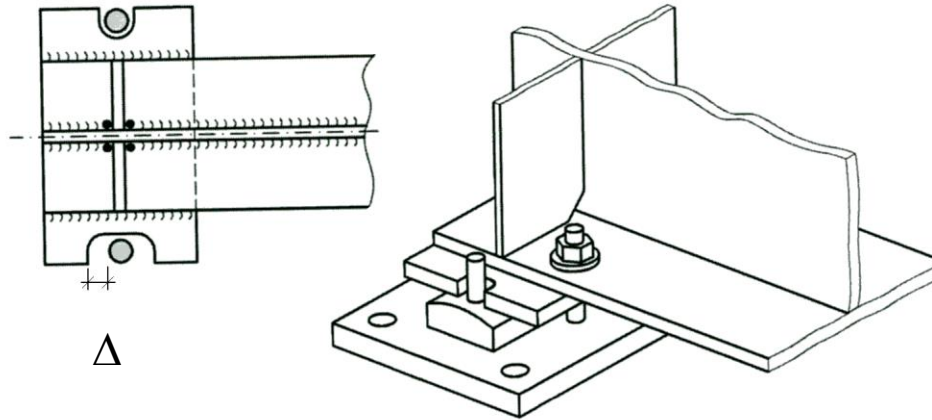
CC	$S_G / S_S > 20$	$20 \geq S_G / S_S > 10$	$10 \geq S_G / S_S > 5$	$5 \geq S_G / S_S$
3	No rocker	Flat rocker	Round rocker	Elastomeric rocker
2	No rocker	No rocker / flat rocker	Flat rocker / round rocker	Round rocker
1	There is usually no detailed analysis			

Translational degree of freedom for elastomeric rocker is achieved by specific shaping the bearing details. It is often the manufacturer's secret.

Translational degree of freedom for rest types of supports is achieved by slotted holes for anchor bolts.

Photo: K. Rykaluk, Konstrukcje metalowe cz I., DWS, Warszawa2016

Pinned support



Roller support

Δ

$$\Delta = q L^3 / (12 E W_{pl}) + \Delta T L \alpha_T$$

$\Delta T = |$ difference between 8°C and working temperature of structure $|$

For truss:

$$W_{pl} \approx J_{truss} / h_{1, truss}$$

Technical solution for support of column bases on other philosophy. Important is position of anchor bolts. Bending moment M_{Ed} is transferred to base by anchors on eccentricity e and by stresses in concrete.

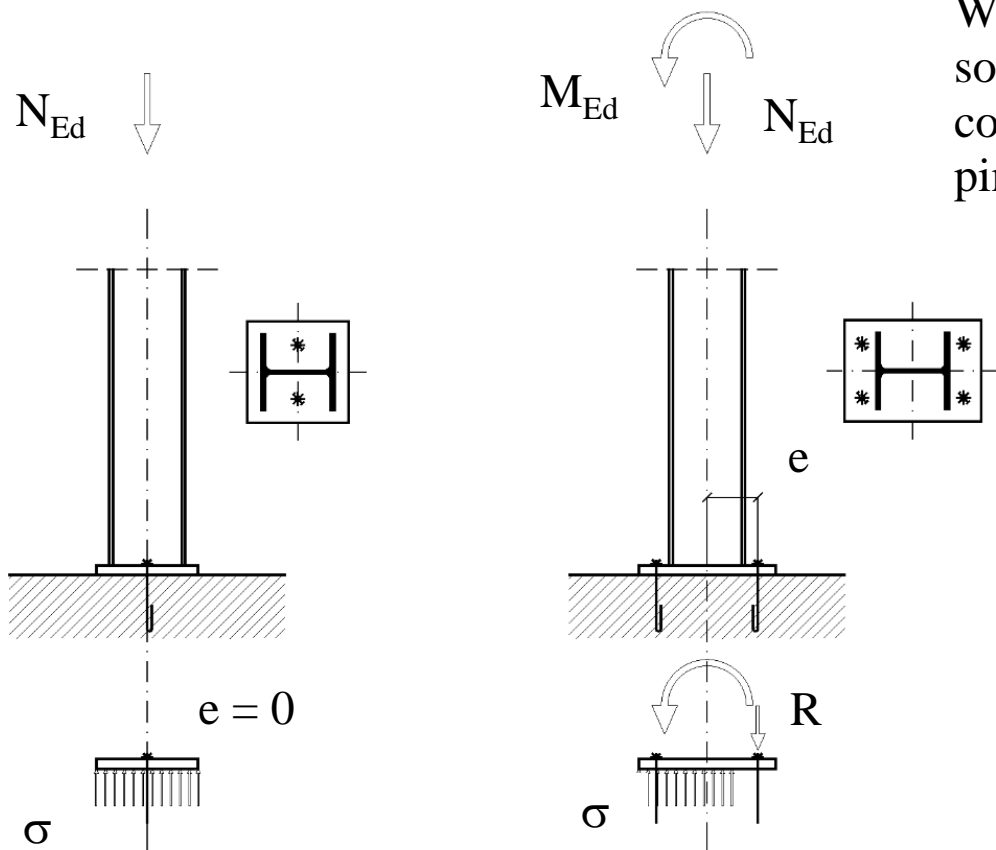


Photo: Author

When $e = 0$, this solution is taken into consideration as pinned support.

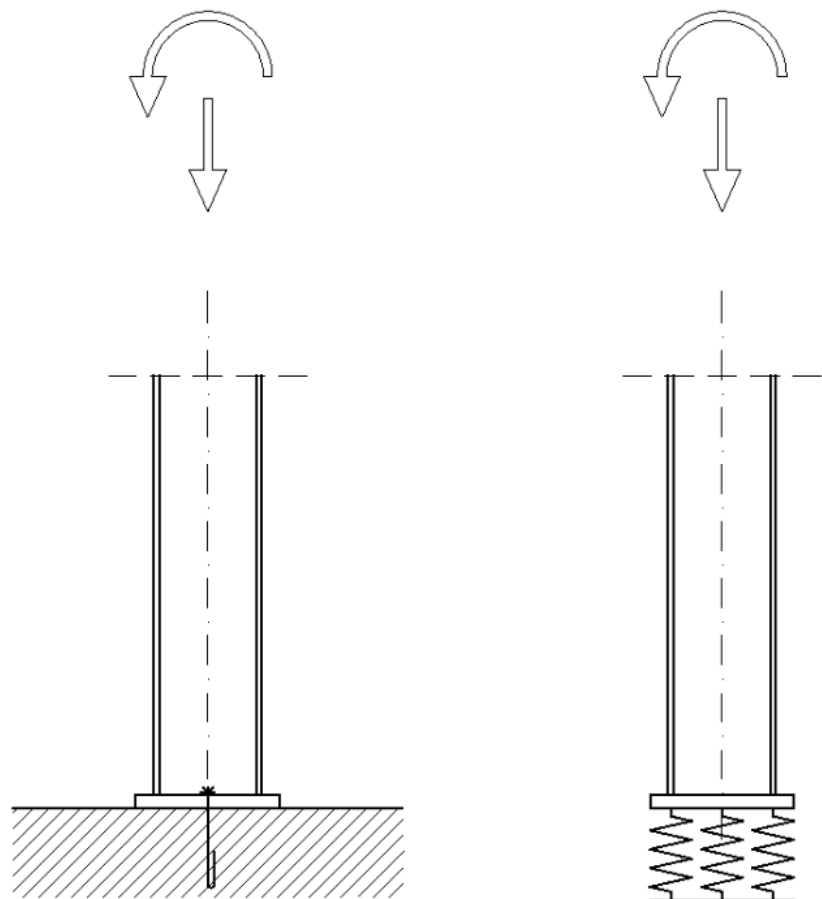


Photo: j-p.com.ua



Photo: srt251fpaler.blogspot.com

Of course, in very accurate model contact steel column – concrete base should be modelled as complex of springs.



According to this model, non-zero value of bending moment exists in support even in situation $e = 0$. But, according to tests and experiments, value of this moment is enough small to take this support into consideration as pinned.

Photo: Author



In case of contact between steel and other materials, it is not only necessary to analyze steel structure. Concrete and masonry are weaker than steel and are usually destroyed earlier than steel structure. For concrete and masonry, stress propagation inside the material is analyzed. Measure here is to compare area of direct contact with steel A_0 and the effective area A_{eff} inside the material. More information will be presented on Lec #20.

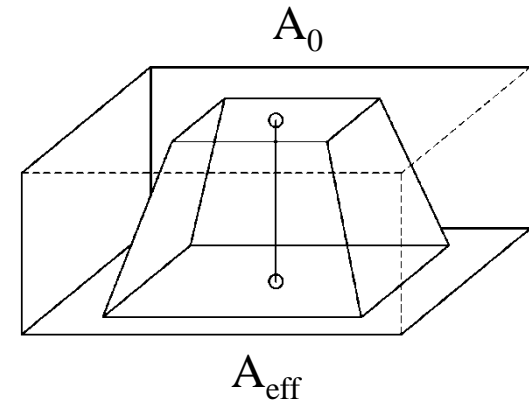
Photo: dailymail.co.uk



Photo: osha.gov

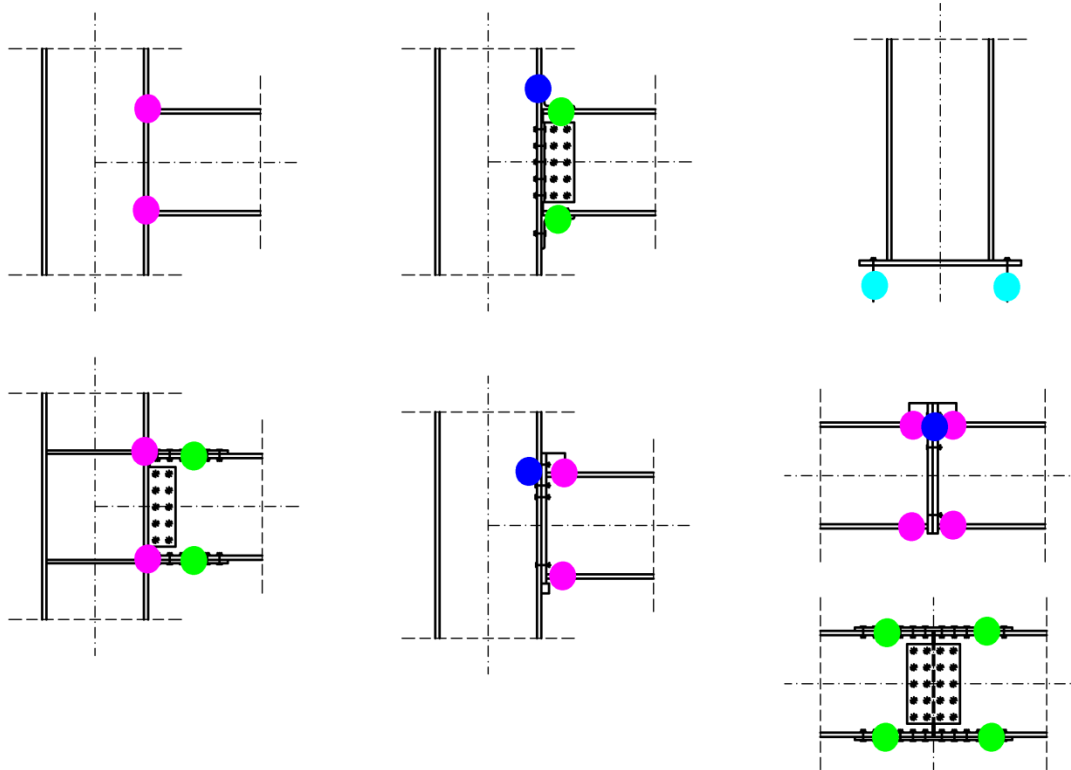


Photo: diy.stackexchange.com



General rules:

immobilization of flanges (and of web) → on the assumption: rigid joint.



Welds: between column and beam;
beam and end-plate; column and flange
plate

Bolts in tension

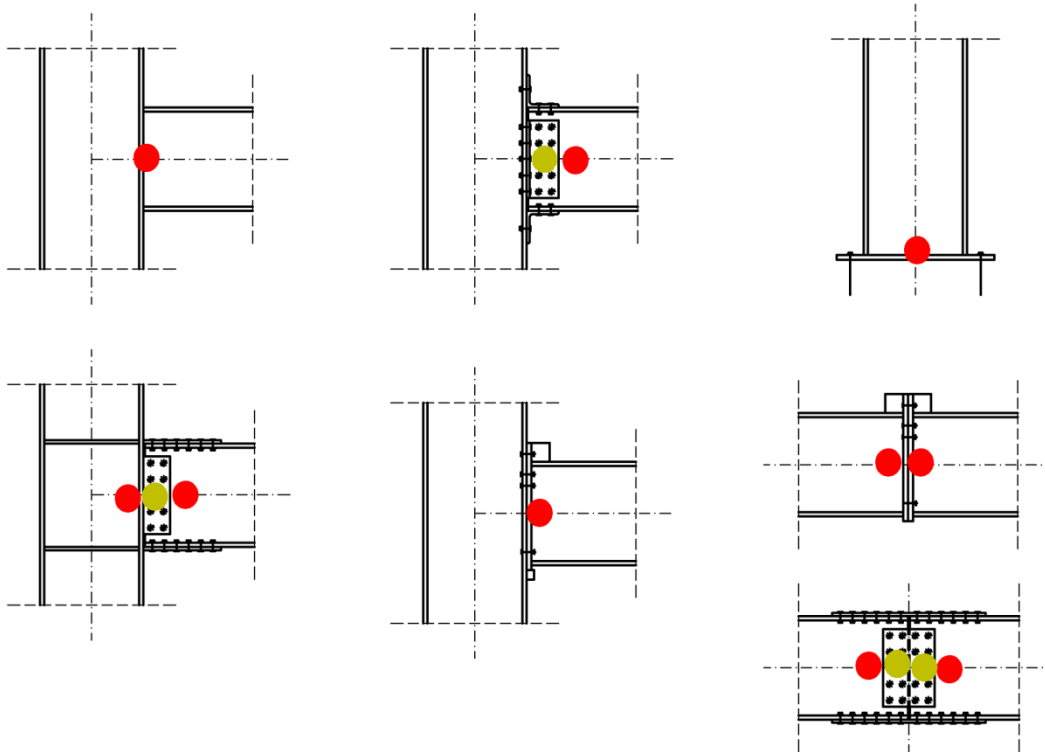
Anchor bolts in tension

Bolts in shear

Photo: Author

General rules:

immobilization of web only → on the assumption: hinged joint.



Welds: between column and beam;
beam and end-plate; column and web
plate; column and base plate

Bolts in web plate

Photo: Author

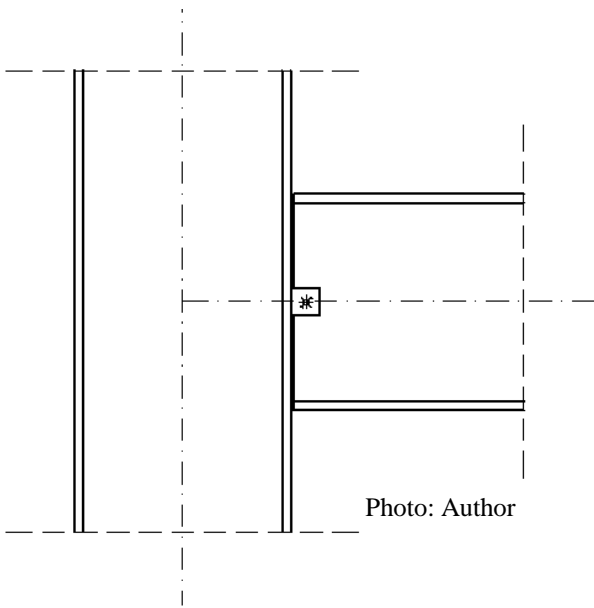


Photo: Author

This joint can be treated as nearly ideal hinge, but this type of joint is not recommended for steel structures (except electro-energetic towers).



Photo: galeria.budownictwopolskie.pl



Photo: inzynierbudownictwa.pl



Photo: mlectric.eu

According to results of experiments, we can assume, that there are always pinned joints, if:

- web only is supported;
- for bolts are applied slotted holes.

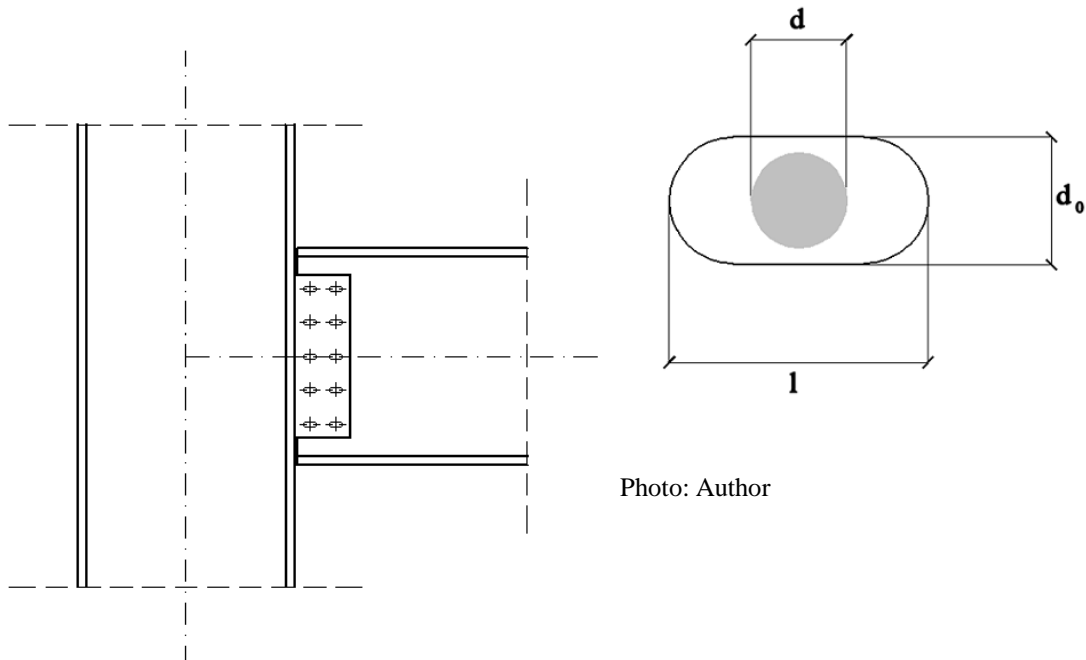


Photo: Author



Photo: tekla-detailed-structural-fabrication.com

Of course, slotted holes can be used for compensation of imperfection during erection of structures. The technical solutions used at that time are heading towards the development of a rigid joint.

But other solutions allow to get an pinned joint with them.

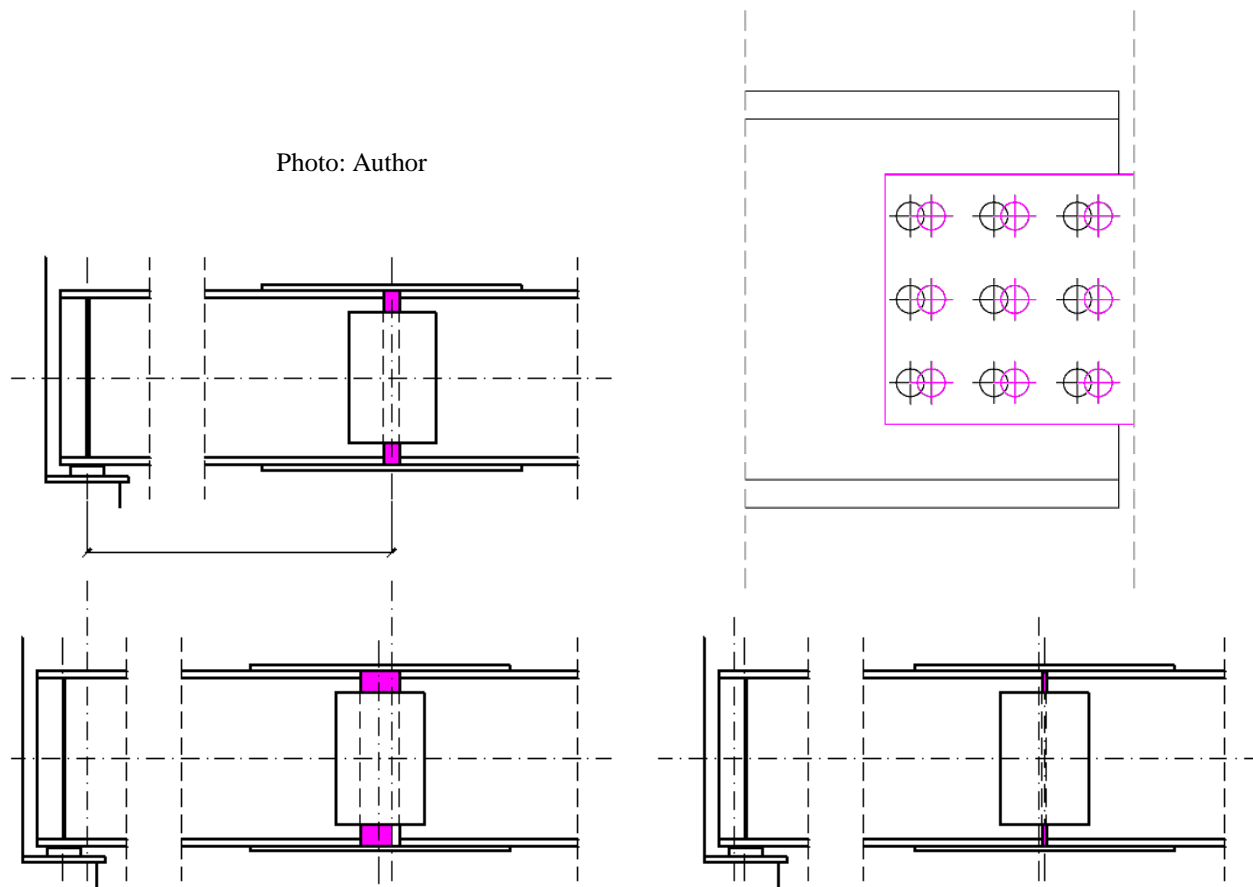


Photo: tekla-detailed-structural-fabrication.com

Photo: resources.scia.net



In plane of frame:

- Rigid joint;
- Bolted joint between girder and column;
- Welded joints between I-beam and end plate;
- Bolts in tensed part: tensile force (from bending moment);
- Bolts in compressed part: shear force;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates and stiffeners: Lec # 18, 19, 21;

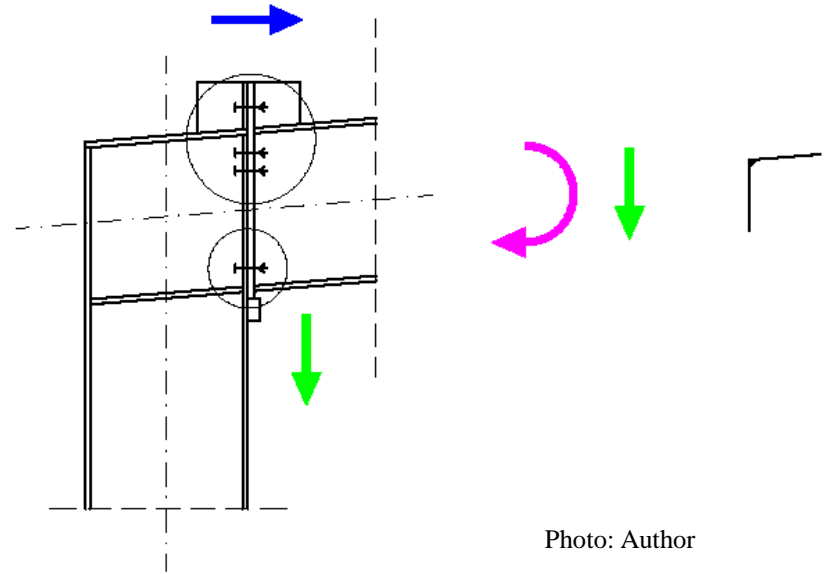


Photo: Author

In plane of frame:

- Rigid joint;
- Welded joint between plates and column;
- Bolted joints between I-beam and plates;
- Only shear forces act on bolts;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates and stiffeners: Lec # 18, 19, 21

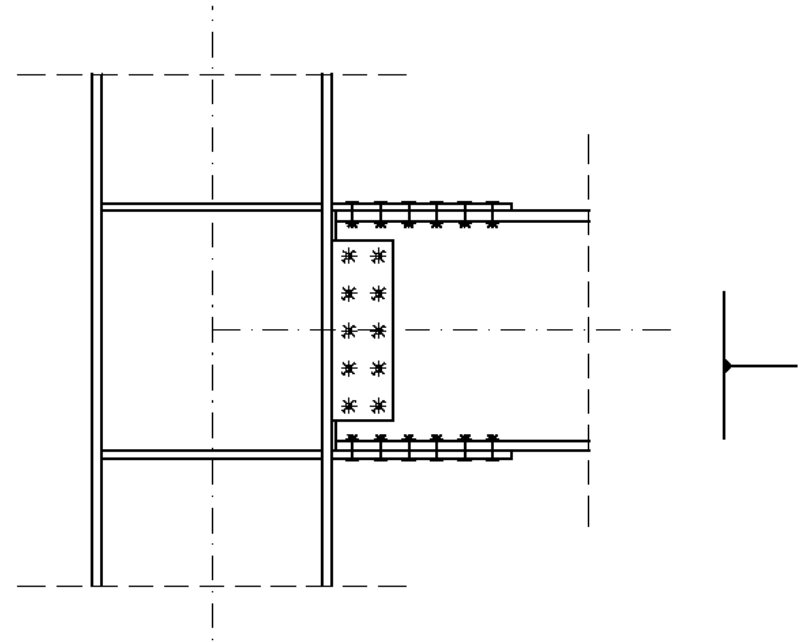
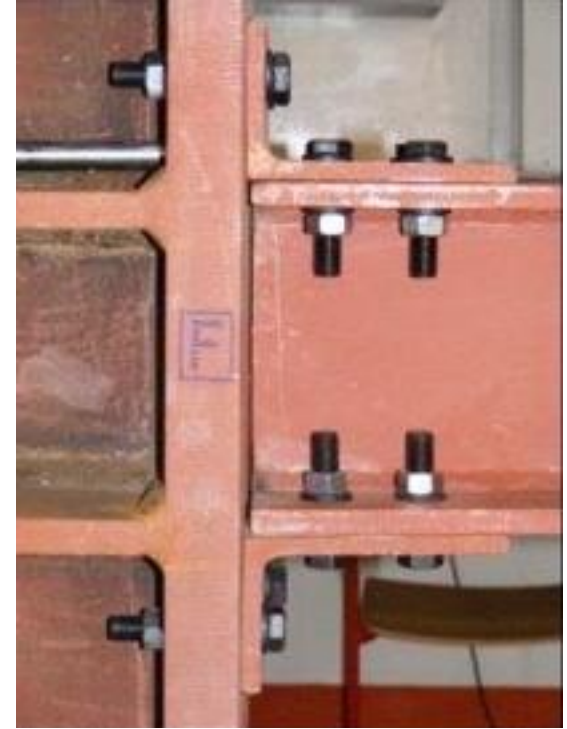


Photo: Author

Photo: Behaviour of stiffened flange cleat joints, D. Skejic, D. Dujmovic, D. Beg



In plane of frame:

- Rigid joint;
- Welded joint between L and column;
- Bolted joints between I-beam and L;
- Shear forces and axial forces act on bolts;
- Bolts: Lec # 18 , 19;
- L : Lec # 18, 19;

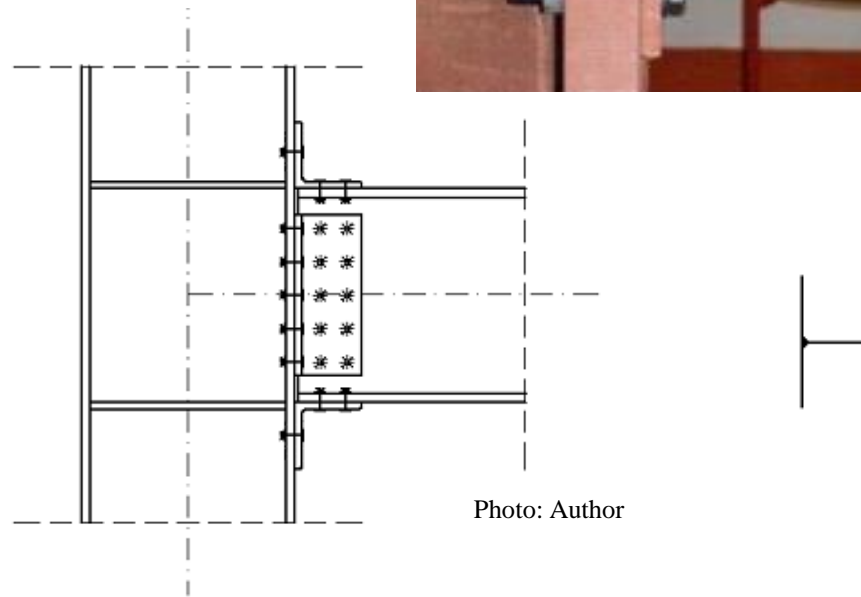


Photo: Author

In plane of frame:

- Fixed joint;
- Shear joint;
- Bolted joint between two parts of beam;
- Only shear forces act on bolts;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 19;

Photo: amsd.co.uk

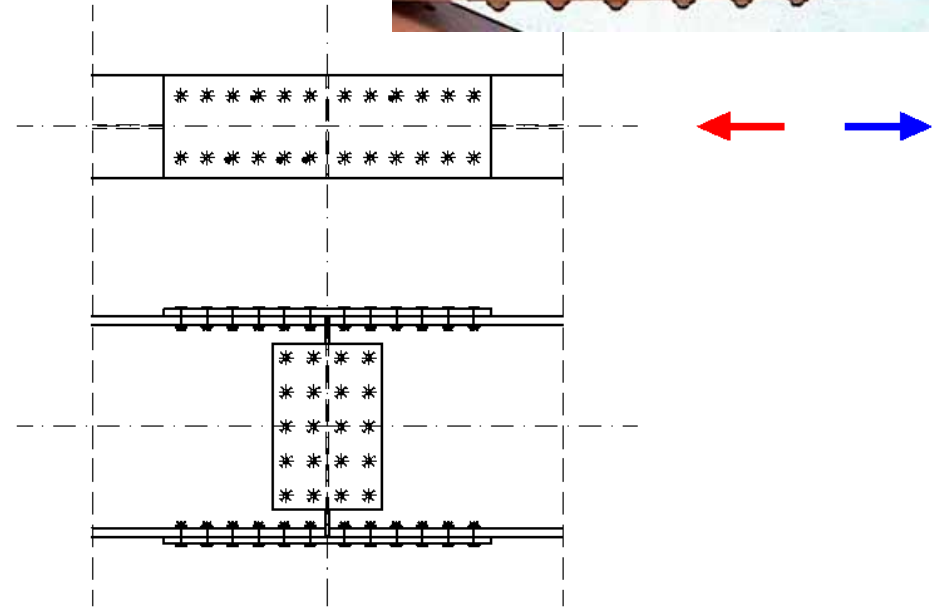
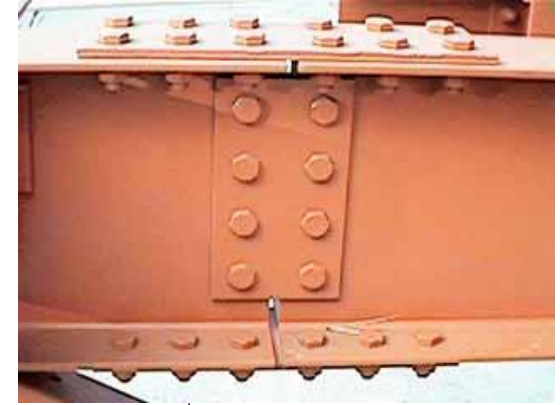
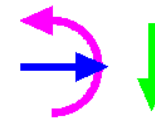


Photo: Author



In plane of frame:

- Fixed joint;
- Shear joint;
- Welded joint beam-column;
- Bolted off-set joint beam-beam;
- Only shear forces act on bolts;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 19;

Photo: moellerengineering.com

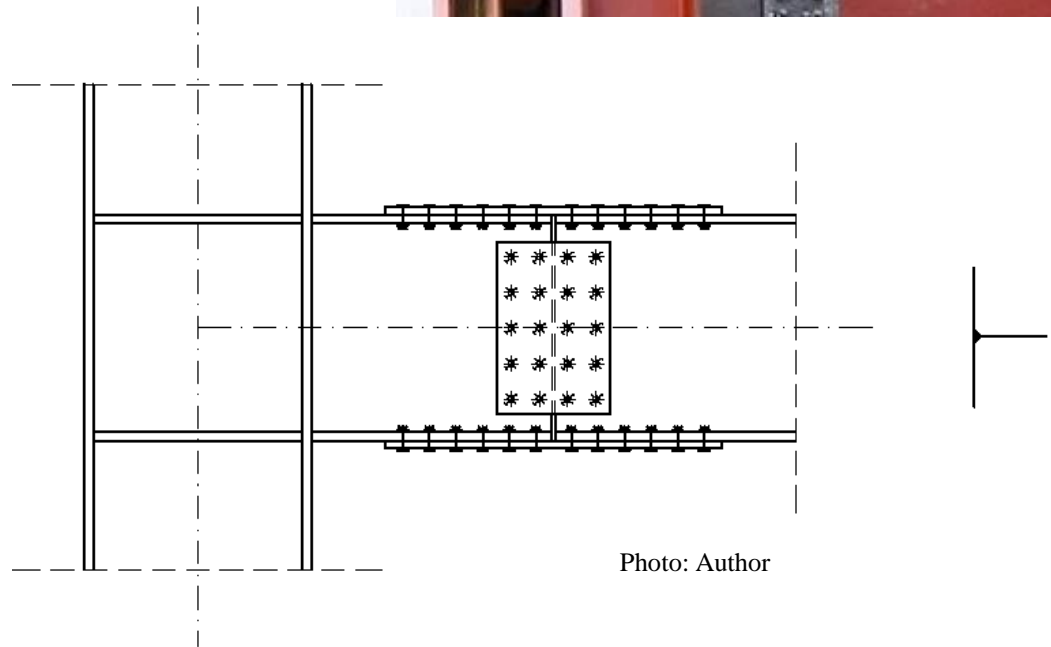


Photo: Author

In plane of frame:

- Fixed joint;
- Shear and tension joint;
- Joint between beams;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 19;

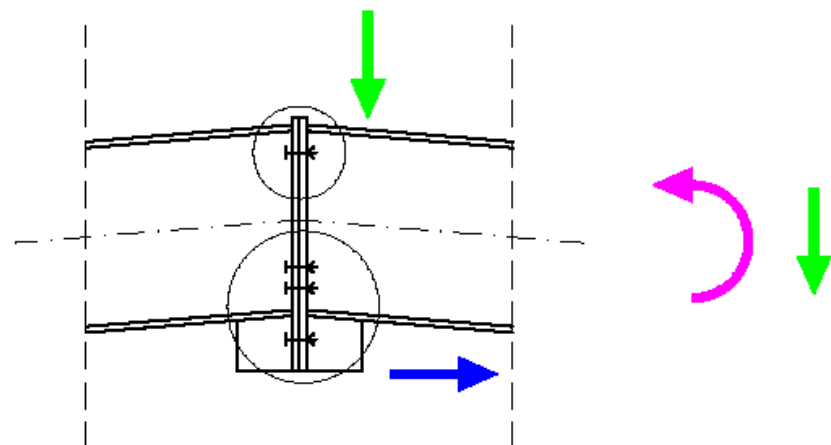


Photo: Author



Photo: farmbuildings.info

Stiffeners:

- Increase stiffness of web;
- Enable connection with secondary beams;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Stiffeners: Lec # 19, 21;

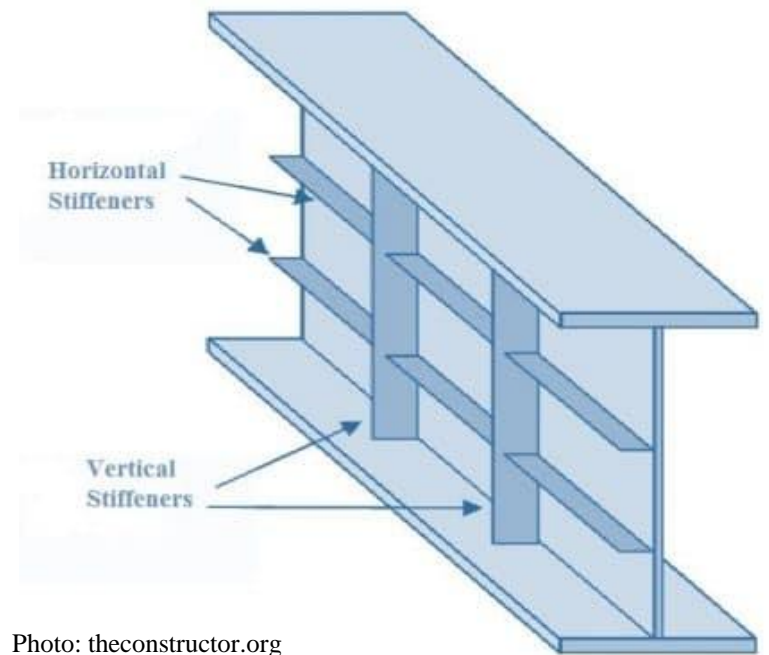


Photo: theconstructor.org



Photo: hebsteelstructure.en



Photo: mscsteel.com

Stiffeners in corners of frame: possible solutions (no stiffeners, horizontal only, horizontal and diagonal). No stiffeners means cheaper structure (smaller number of plates and welds, lighter transport members). But increasing number of stiffeners means increasing stiffness and resistance of joint.

Lec # 21;



Photo: sciencedirect.com



Photo: microstran.com.au

Photo: kodiaksteelhomes.com

In plane of frame:

- Hinged joint;
- Shear joint;
- Joint between beam and column;
- Bending moment = shear force · eccentricity;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 21;



Photo: ecs.umass.edu

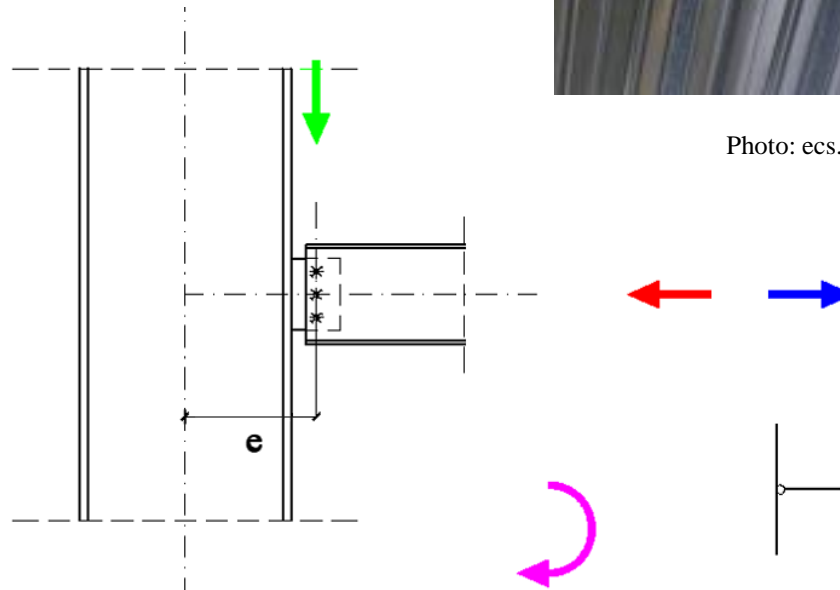


Photo: Author

In plane of frame:

- Hinged joint;
- Shear joint;
- Joint between beam / column and tie-beam;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec #18, 21;



Photo: kobexstal.pl

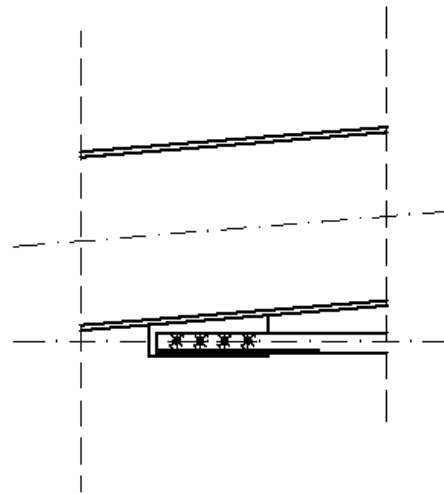


Photo: Author



Out of plane of frame:

- Hinged joint;
- Shear joint;
- Joint between secondary and primary beams or between frame and transversal beam;
- Bending moment = shear force · eccentricity;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Stiffeners: Lec # 18, 21;



Photo: mscsteel.com

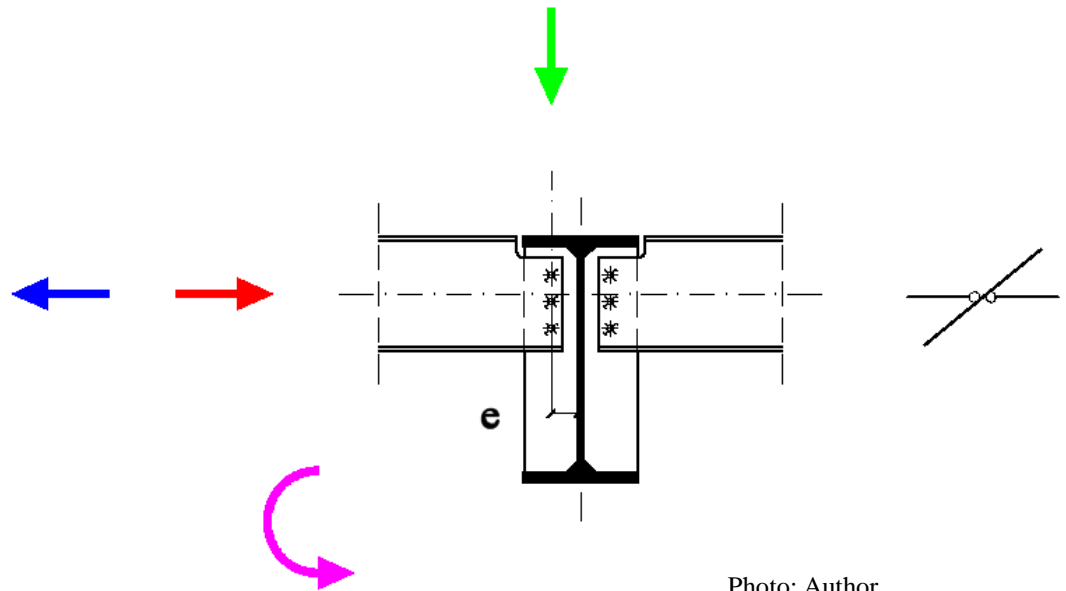


Photo: Author

Support for purlins:

- Various solutions for hot-rolled and cold-formed purlins;
- Various solutions, bolted or welded and bolted;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 21;



Photo: Author

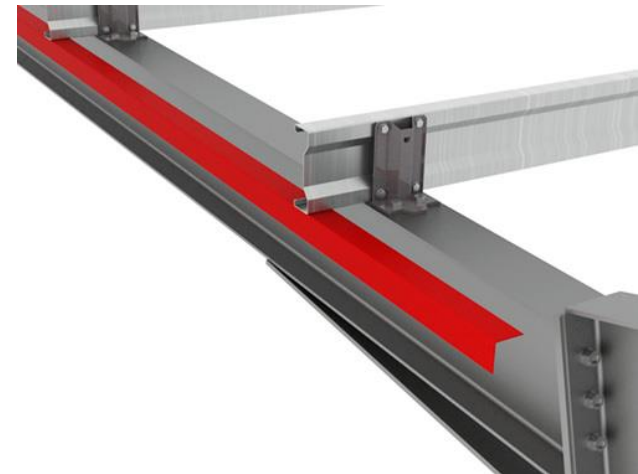


Photo: kingspan.com



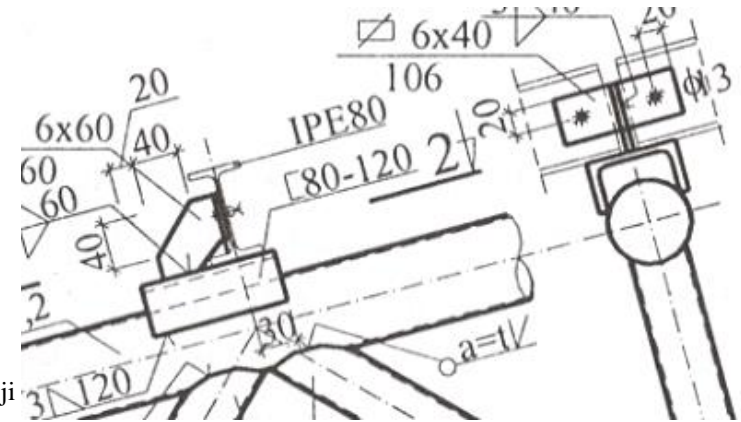
Photo: Author



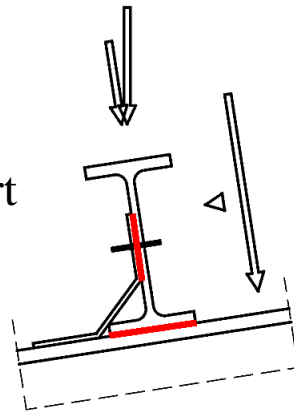
Photo: M. Gwóźdz, M. Maślak, Przykłady projektowania wybranych stalowych konstrukcji prętowych, Politechnika Krakowska 2003

Support for purlins

Hot-rolled

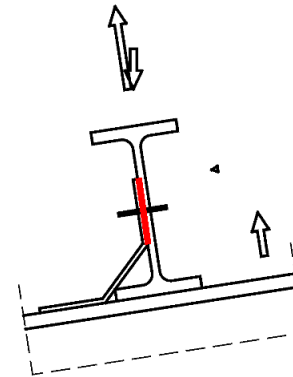


Snow + dead-weight +
wind pressure:
by contact between
bottom flange and
girder; web and support
plate.



→ #8 / 39

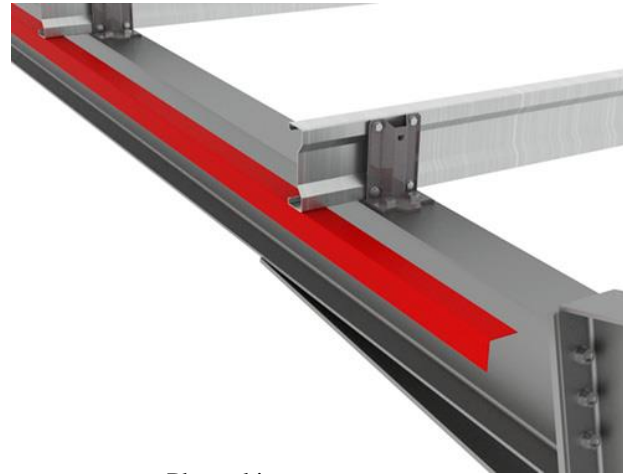
Photo: Author



Wind suction bigger than
dead-weight:
by contact between web
and support plate; shear
of bolt.



Photo: steel.ie

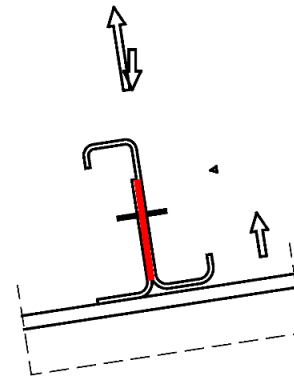
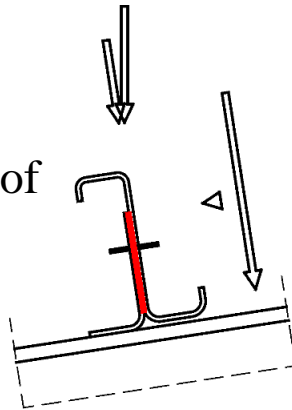


Support for purlins

Cold-formed

Photo: kingspan.com

Snow + dead-weight + wind pressure:
by contact between web and bolt; web and support plate.
No contact between bottom flange and roof girder.



Wind suction bigger than dead-weight:
by contact between web and bolt; web and support plate.

Photo: Author

→ #8 / 41

Support for wall girts:

- Various solutions, bolted or welded and bolted;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 21;



PhotoAuthor



Photo: devisdepro.com



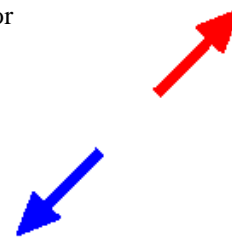
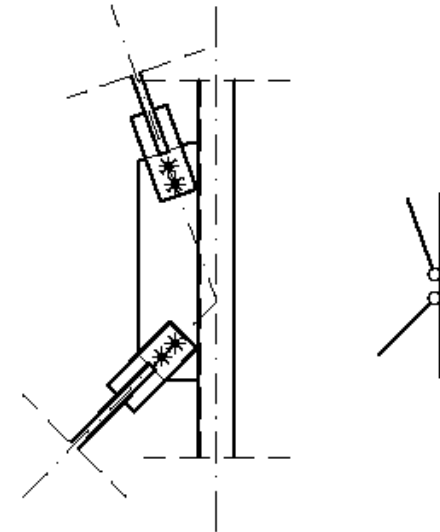
PhotoAuthor

Bracings:

- Hinged joint;
- Shear joint;
- Welds: Lec # 17;
- Bolts: Lec # 18, 19;
- Plates: Lec # 18, 21;



Photo: Author

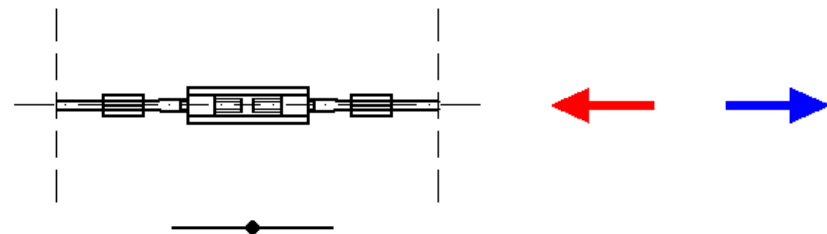


Bracings, tie beams, hangers:

- Rigging screw;
- Fixed joint;
- Tension joint;
- Bolts: Lec # 18;



Photo: Author



Nodes in trusses are a specific issue. Welded joints between truss bars can be treated as rigid or hinged. In the latter case, additional conditions must be met. More information will be presented on Lec # 21.

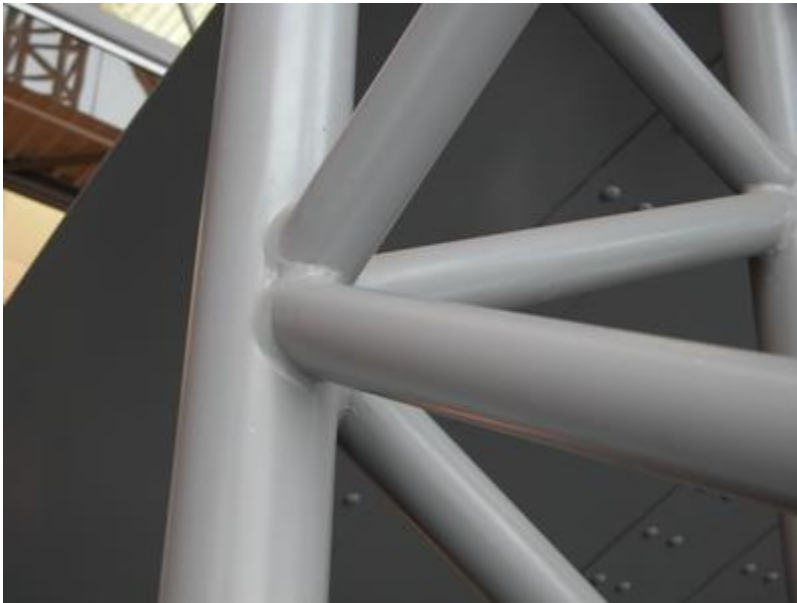


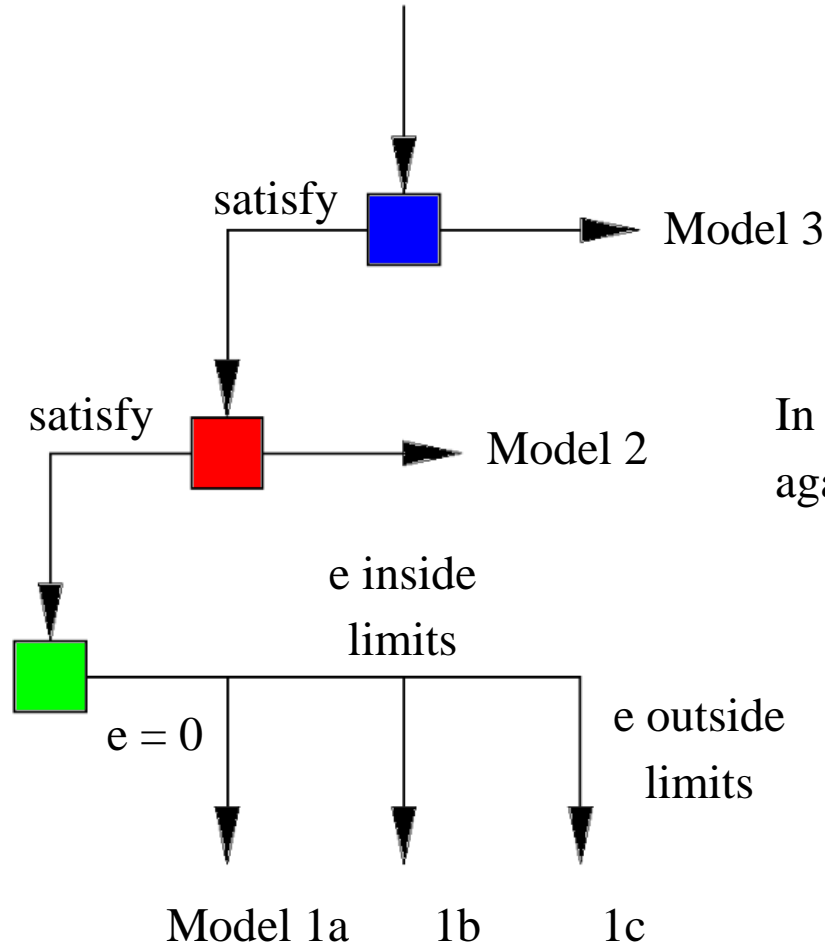
Photo: tboake.com



Photo: freimans.com

Truss

Photo: Author



In case 2 or 3, truss must be calculated once again according to static model 2 or 3.

→ Des #1/ 76



Photo: zs4-sanok.pl



Photo: encrypted-tbn0.gstatic.com

Specific problem are splice joints (between transport members) in trusses. Axial forces only existed in truss bars. Hollow sections are used first of all in modern truss.

There is no information in Eurocode about calculation of stiffness and resistances for joints between hollow sections.

Because of this, it is assumed that – if resistance of splice joint is enough – splice joints in truss no change static scheme of structure. Truss member with splice joint is treated as one continuous member without splice joint. More information will be presented on Lec # 21.

Verification of stiffness

Initial
assumptions



Numeric model



Technical
solutions



Real behavior

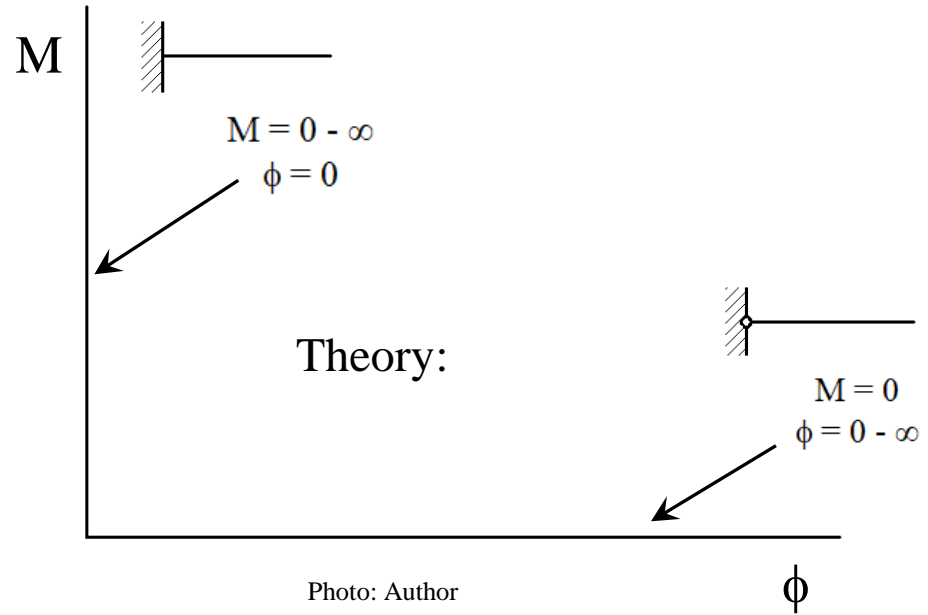
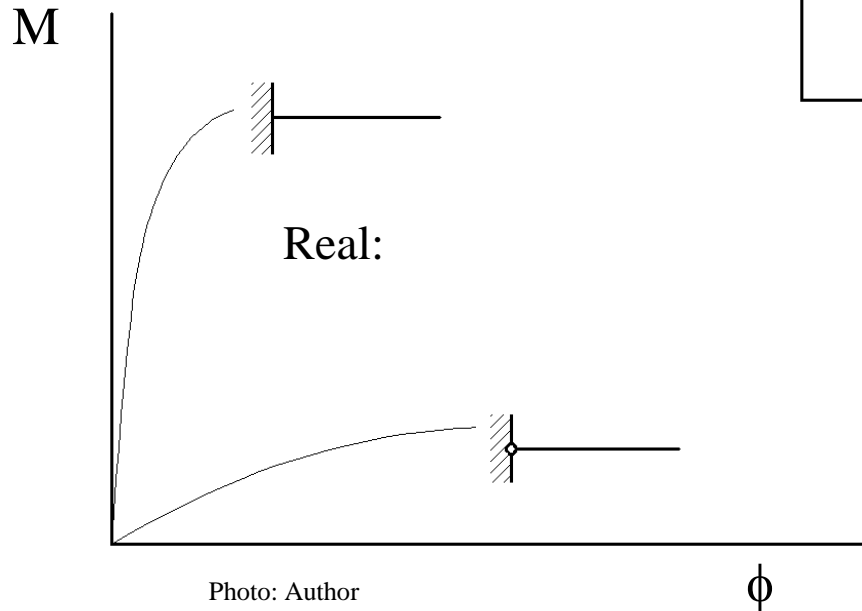
Real behavior of joint – based on geometry of joint (depth of cross-section of beam and column, thickness of plates, type of bolts...) – must be tested and compared with initial assumptions. Both points: initial assumptions and real behavior – must be the same. In case of difference, joint must be designed once again.

Verification of stiffness should be done before checking resistance. Formulas of resistance are dedicated for specific types of joint (specific types of behavior) and are not true for other types.

Joints

For joints important is their stiffness (relationship $M-\phi$)

→ #8 / 56



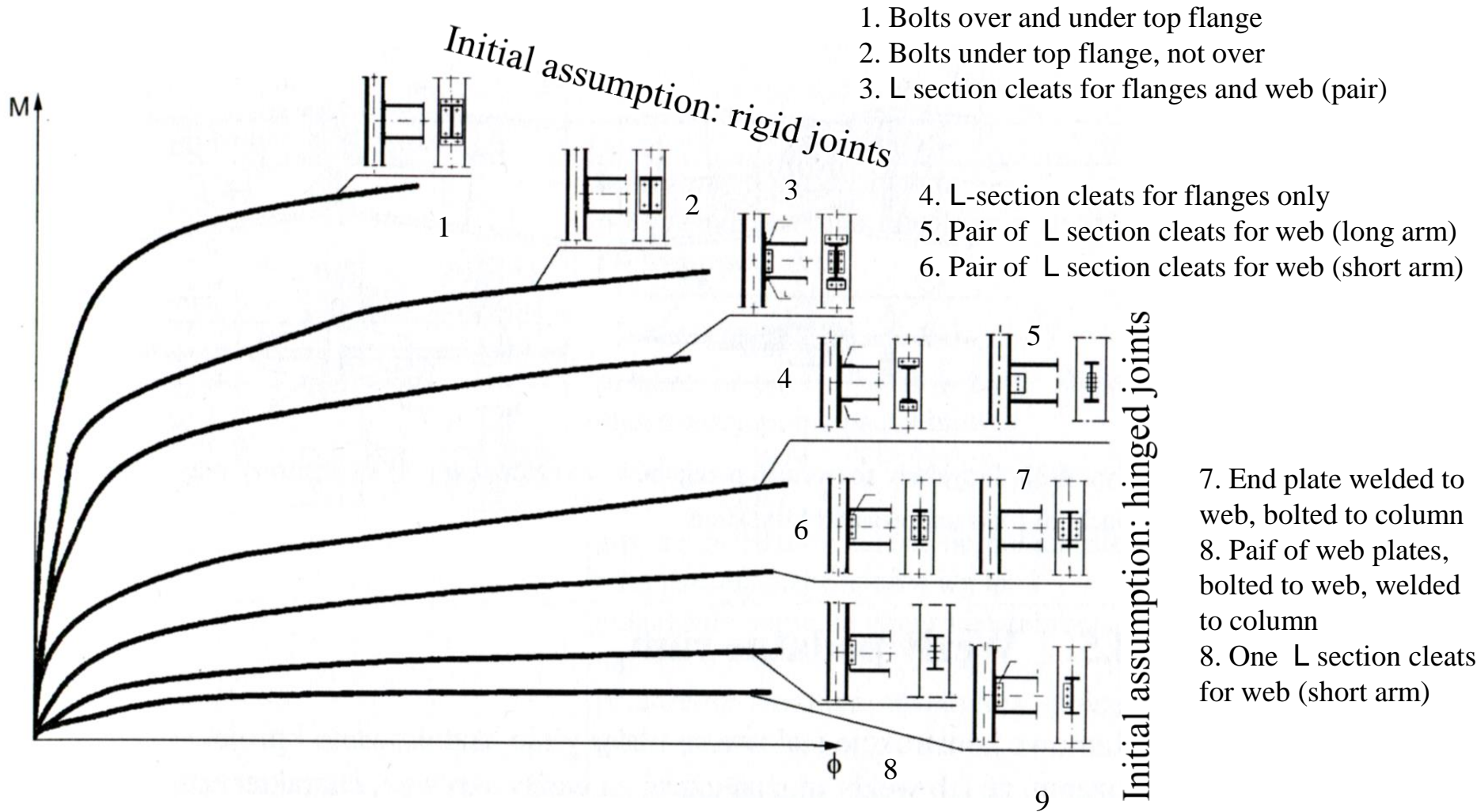


Photo: Łubiński M, Filipowicz A, Żółtowski W, "Konstrukcje metalowe", Arkady 2000

Result of tests and experiments

→ #8 / 57

Calculations (→ Lec # 14, 15):

What are limits 1-2 and 2-3?

What is stiffness of analysed joint?

What about comparison between limits and stiffness?

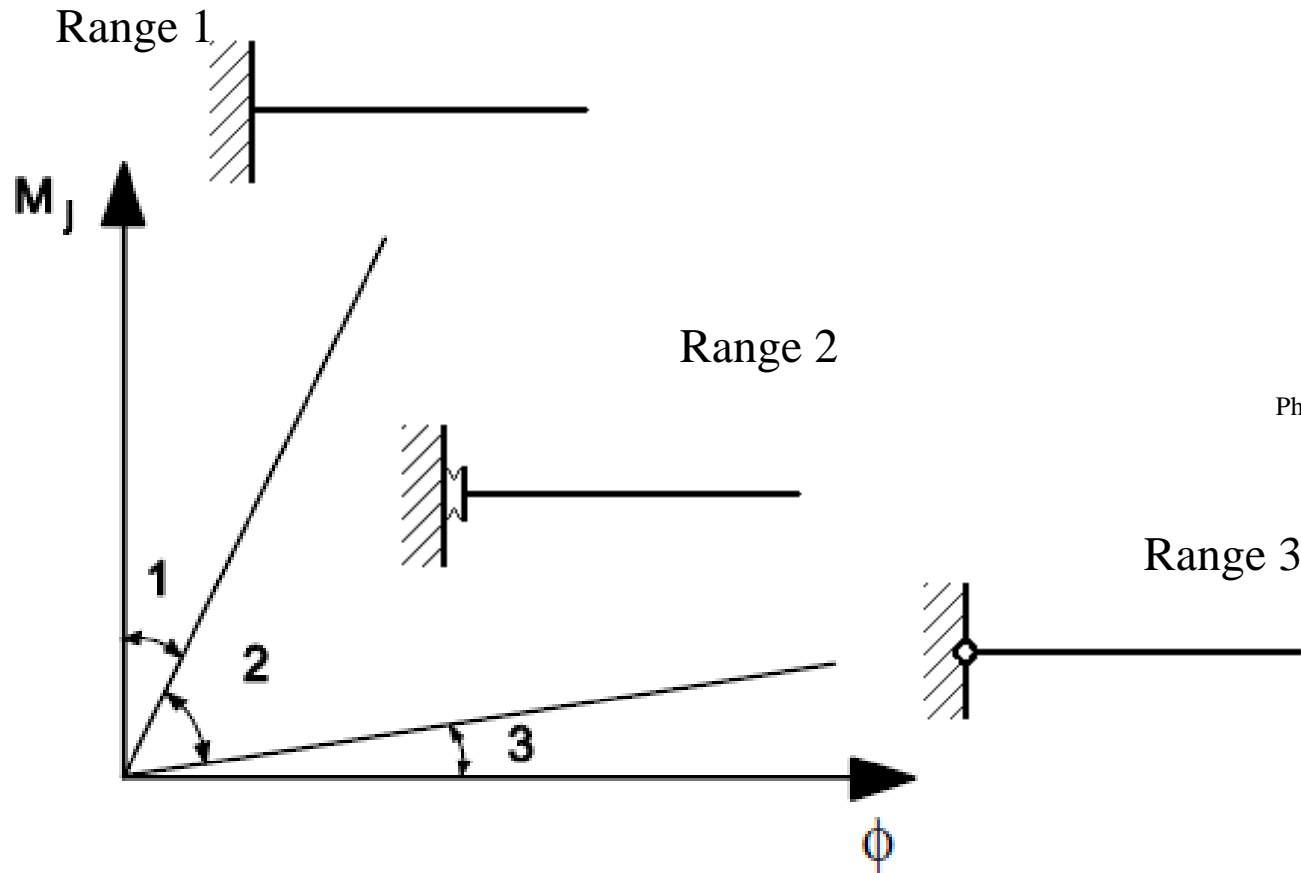


Photo: Author

Photo: EN 1993-1-8 fig 5.4

Siffness of the node was mentioned on Lec #13 (European Method, American Method...). But this stiffness was „global” stiffness. An now will be analysed „local” stiffness.

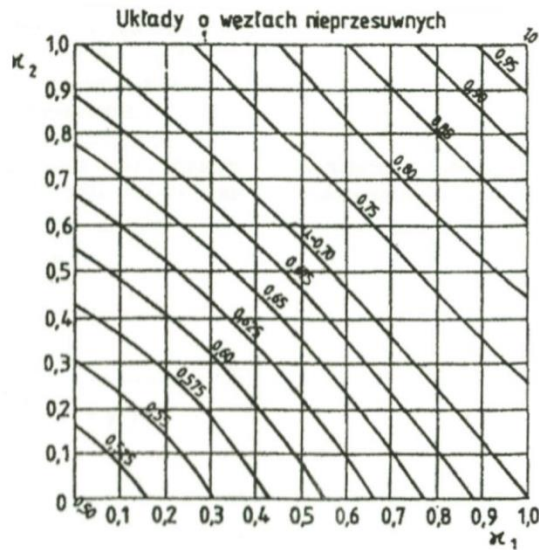


Photo: PN-B 3200 fig. Z1-3

Stiffness of the node

$$\kappa = \max [0,3 \ ; \ K_C / (K_C + K_0)]$$

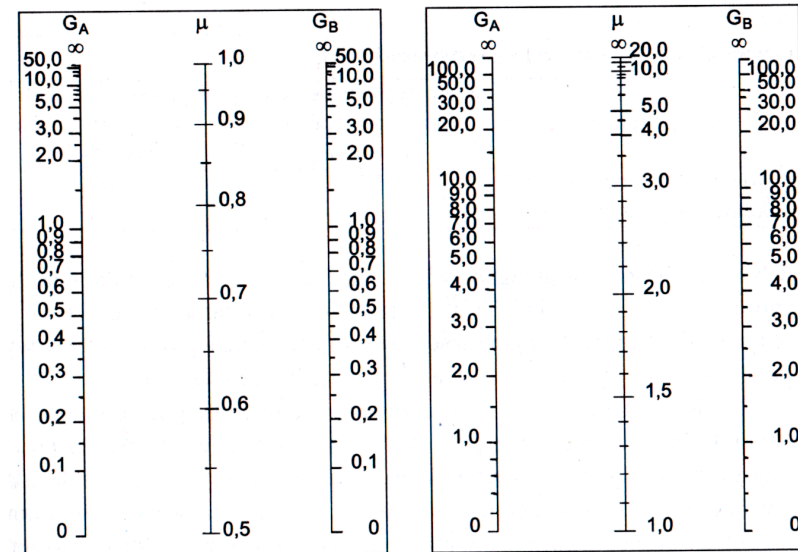
$$K_C = J_C / h$$

$$K_0 = \Sigma (\eta J_B / L)$$

Photo: M. Łubiński, W. Żółtowski
"Konstrukcje metalowe", Arkady,
Warszawa 2000

Siffness of the node

$$G = [\Sigma (J_C / h)] / [\Sigma (\eta J_B / L)]$$



„Global” stiffness: influence of adjacent parts on behavior of element. Only global rotation of nodes is taken into consideration. There is no change of initial angles between elements.

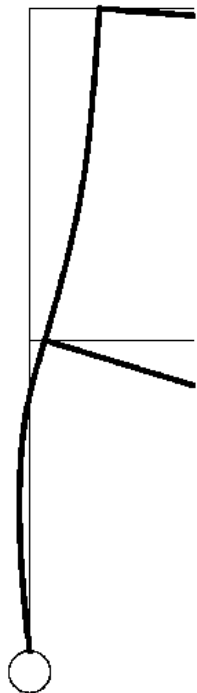
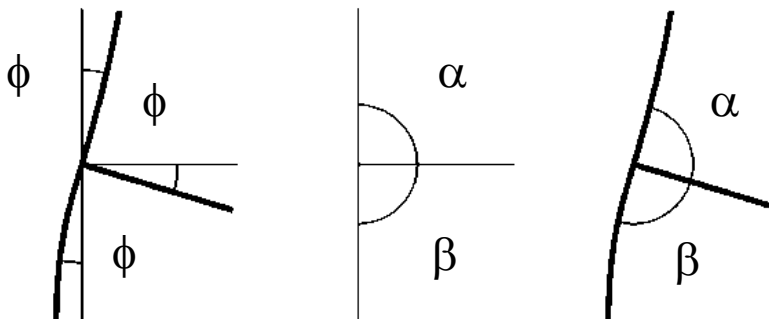


Photo: Author



„Local” stiffness: only change of initial angles between elements – as the effect of local deformations – is taken into consideration. Global rotation is not important for this way of analysis. „Local” stiffness defined type of joint.

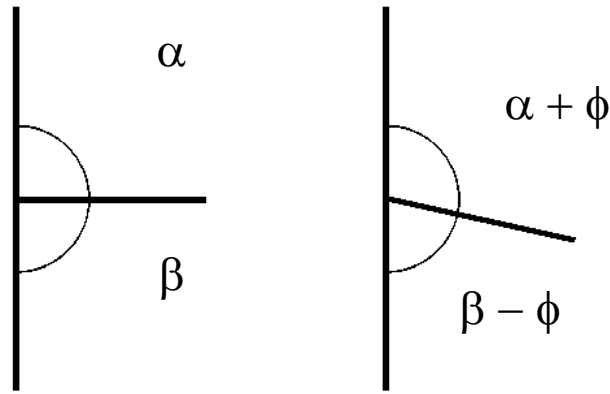
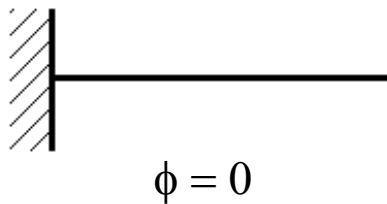
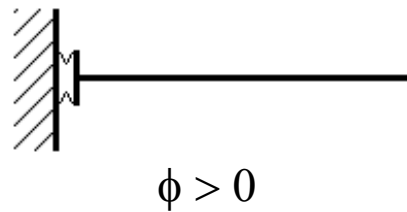


Photo: Author

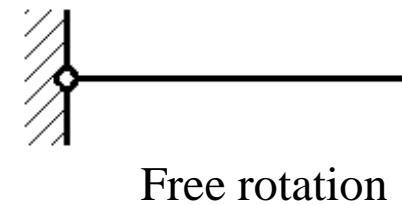
Rigid



Semi-rigid



Pinned



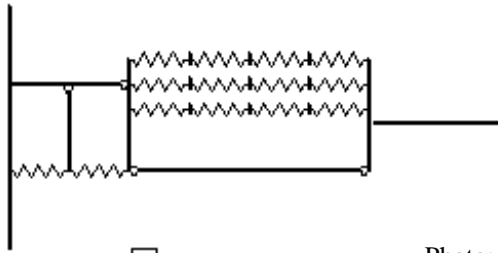
Conclusion

Global stiffness is important for stability (critical length) only.

Local stiffness can change distribution of cross-sectional forces. It is important for resistance of cross-sections and for stability of members.

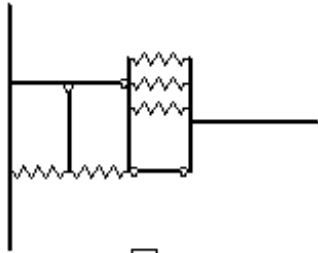
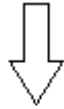
Local stiffness depends on local parameters: number of bolts, diameter of bolts, thickness of plates...

Each sub-part of joint important for its behavior is presented as spring.

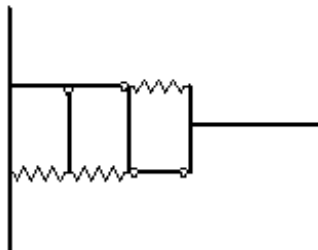
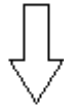


Stiffness of joint: full spring model.

Photo: Author



Recalculation from 3x(many springs) to 3x(effective spring).



Recalculation from 3x(effective spring) to one equivalent spring.

Based on this one, we calculate stiffness of joint.

Calculations according to Eurocode

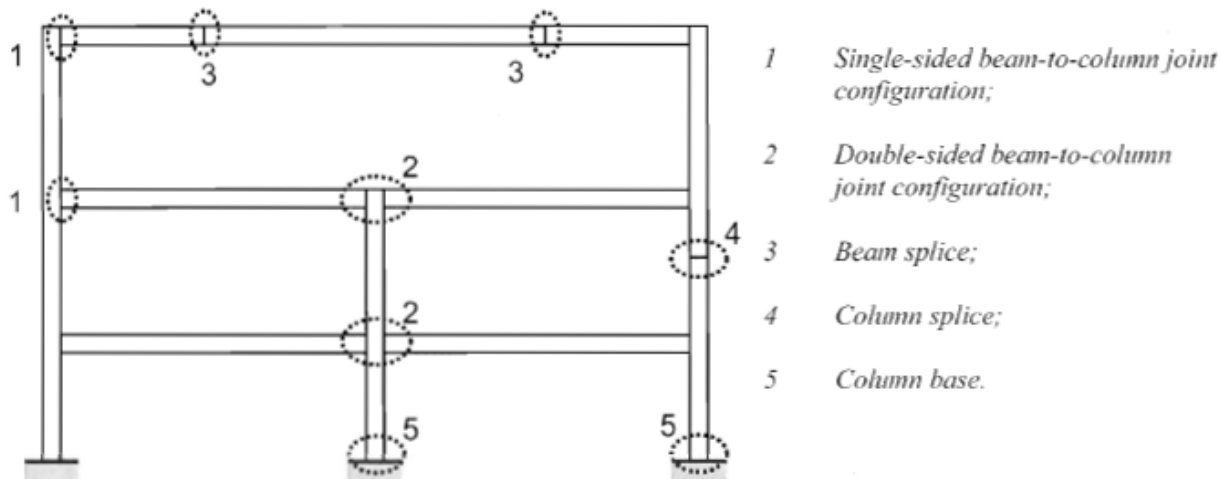
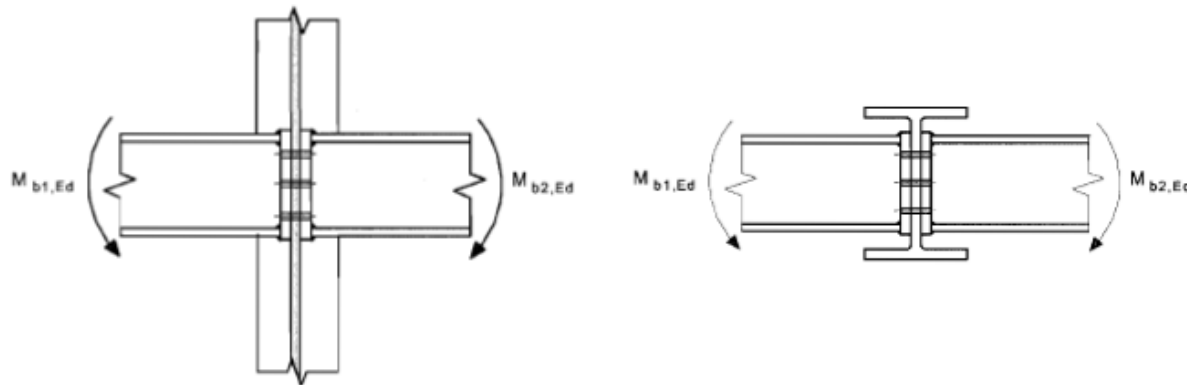


Photo: EN 1993-1-8 fig 1.2

a) Major-axis joint configurations



Double-sided beam-to-column joint configuration

Double-sided beam-to-beam joint configuration

b) Minor-axis joint configurations (to be used only for balanced moments $M_{b1,Ed} = M_{b2,Ed}$)

There are many inconsequences in Eurocode:

- According to fig 1.2 (#t / 73), we could execute minor axis joint - but in EN 1993-1-8 is no information about way of calculation for this one;
- According to fig 1.2 (#t / 73), we should calculate each joint - but in EN 1993-1-8 is information about stiffness of tension joint first of all;
- According to fig 1.2 (#t / 73), we should calculate beam-beam and column-column joints too - but in EN 1993-1-8 is information about stiffness of beam-column joint first of all;
- Formulas presented in EN 1993-1-8 are dedicated for joints with axial force smaller than 5% of connected member resistance; there is no information on which way calculate joints with bigger axial force;

- Values of k_i are presented only for joints of I-sections; joints of HS are discussed; there is no information about joints between other types of cross-sections;
- There is information in EN 1993-1-8 about k_i for shear joint, but global formula is dedicated to tension joint; there is no information about Ψ and $S_{j, \text{end}}$ (\rightarrow #t / 79);
 - There is no clear, if pinned joints should be checked (pinned \leftrightarrow semi rigid);
 - There is no information about way of springs models making in EN 1993-1-8;
- Generally, on literature are presented first of all springs model for tension bolted beam-column joint.

According to EN 1993-1-8, relationship $M-\phi$ for rigid support is approximated by
linear - nonlinear - linear
functions. This is similar to bi-linear relationship $\sigma-\epsilon$.

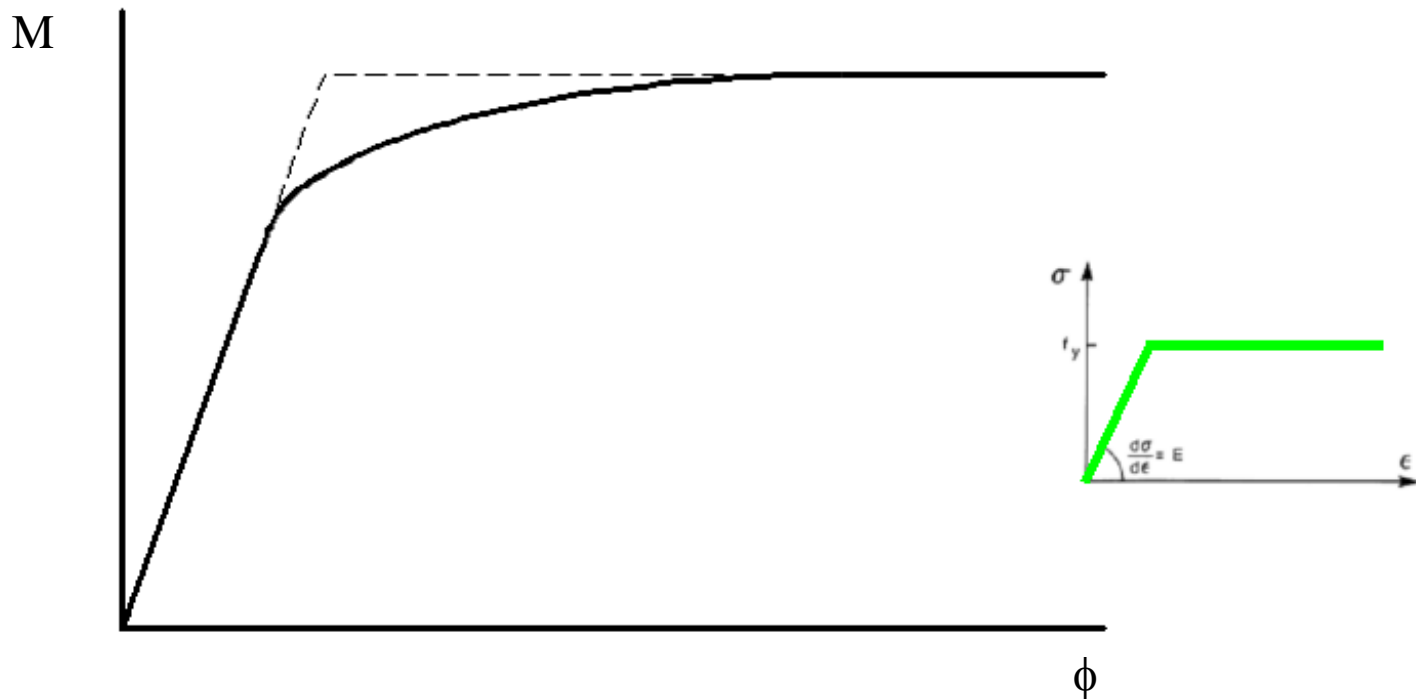
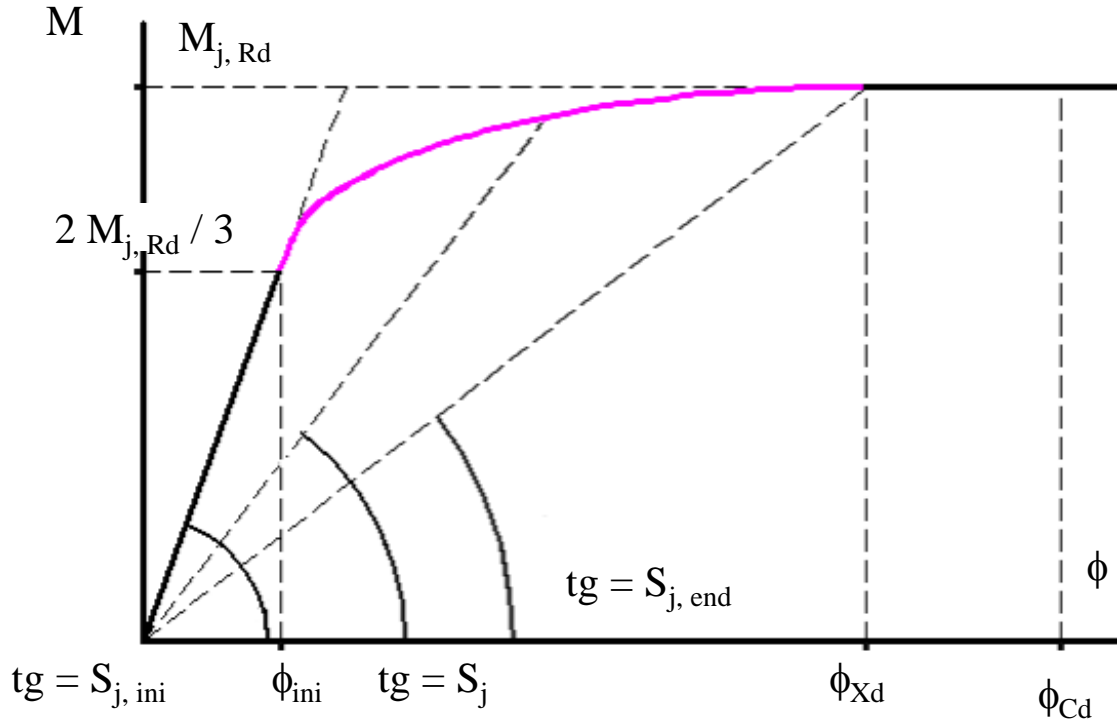


Photo: Author

EN 1993-1-8 5.1.2.(3), (4)

EN 1993-1-8 (6.27), (6.28a), (6.28b)

EN 1993-1-8 fig 6.1



$$\phi_{ini} = 0,667 M_{j,Rd} / S_{j,ini}$$

$$\phi_{Xd} = 1,5^\Psi M_{j,Rd} / S_{j,ini}$$

$$\phi_{Cd} = \phi_{max}$$

Photo: Author

$$M_{j,Ed} = S_j \phi$$

$S_{j,ini} \rightarrow$ geometry (lectures #15)

For $M_{j,Ed} > 2 M_{j,Rd} / 3 :$

$$S_j = S_{j,ini} / \mu$$

$$\mu = (1,5 M_{j,Ed} / M_{j,Rd})^\Psi$$

$$\mu_{end} = 1,5^\Psi$$

$M_{j,Rd} \rightarrow$ geometry (lecture #19)

$$M_{j,Ed} = S_j \phi = S_{j,ini} \phi / \mu$$

$$M_{j,Ed} \mu = S_{j,ini} \phi$$

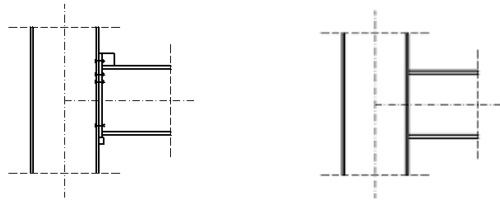
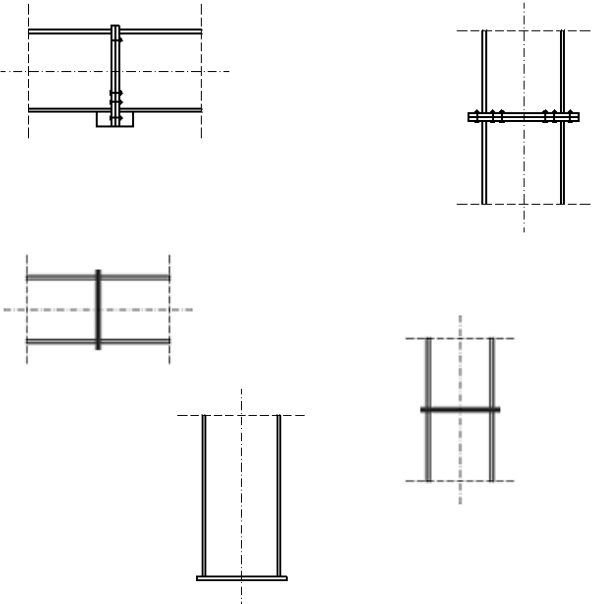
$$M_{j,Ed} (1,5 M_{j,Ed} / M_{j,Rd})^\Psi = S_{j,ini} \phi$$

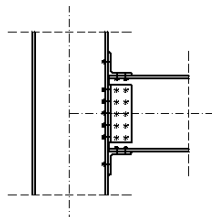
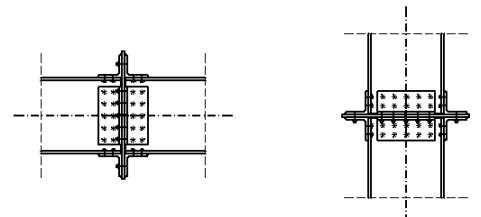
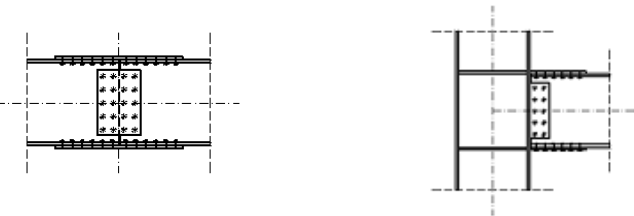
$$M_{j,Ed}^{\Psi+1} = S_{j,ini} M_{j,Rd}^\Psi \phi / 1,5^\Psi$$

$$M_{j,Ed}^{\Psi+1} = \phi (S_{j,ini} M_{j,Rd}^\Psi / 1,5^\Psi)$$

$$M_{j,Ed} = (\Psi+1 \sqrt[\Psi]{\phi}) A$$

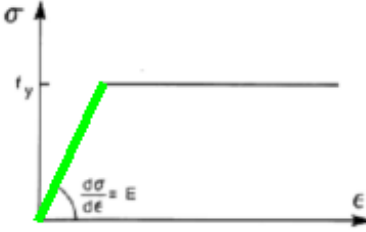
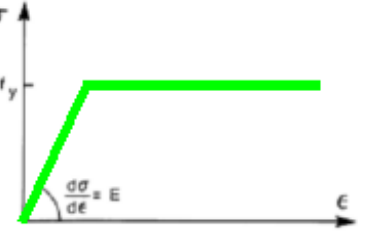
$A \rightarrow$ geometry

Type of joint	Ψ	$S_{j, end}$ (rigid)	$S_{j, end}$ (semi-rigid)
	2,7	$S_{j, ini} / 3,0$	$S_{j, ini} / 2,0$
		$S_{j, ini} / 3,0$	$S_{j, ini} / 3,0$

Type of joint	Ψ	$S_{j, end}$ (rigid)	$S_{j, end}$ (semi-rigid)
	3,1	$S_{j, ini} / 3,5$	$S_{j, ini} / 2,0$
		$S_{j, ini} / 3,5$	$S_{j, ini} / 3,5$
	?	$S_{j, ini} / ?$	$S_{j, ini} / ?$

Elastic and plastic analysis

Photo: Author

Analysis	Class of corss-section → #3 / 85	Stress-strain relationship EN 1993-1-1 fig. 5.3
Elastic	I, II, III, IV	 <p>A stress-strain diagram with stress σ on the vertical axis and strain ϵ on the horizontal axis. A green line starts at the origin and rises linearly to a yield stress f_y. The slope of this line is labeled $\frac{d\sigma}{d\epsilon} = E$. After reaching f_y, the line becomes horizontal, indicating a constant stress level.</p>
Plastic	I	 <p>A stress-strain diagram with stress σ on the vertical axis and strain ϵ on the horizontal axis. A green line starts at the origin and rises linearly to a yield stress f_y. The slope of this line is labeled $\frac{d\sigma}{d\epsilon} = E$. After reaching f_y, the line continues horizontally, representing a constant stress level during plastic deformation.</p>

Different formulas for resistance for elastic and plastic analysis

Elastic and plastic analysis for stiffness of joint:

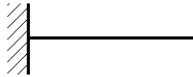
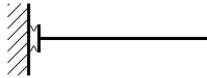
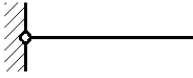
Analysis	Elastic	Model of joint	Plastic
Joints classified according to	$S_{j, ini}$		$M_{j, Rd}$ and ϕ_j
Types of joint	Rigid		Full-strength
	Semi-rigid		Partial-strength
	Pinned (hinged)		Nominally-pinned

Photo: Author

Elastic analysis of joint (Lec # 14, 15): initial, linear part of relationship
Plastic analysis of joint (advance analysis): second part of relationship

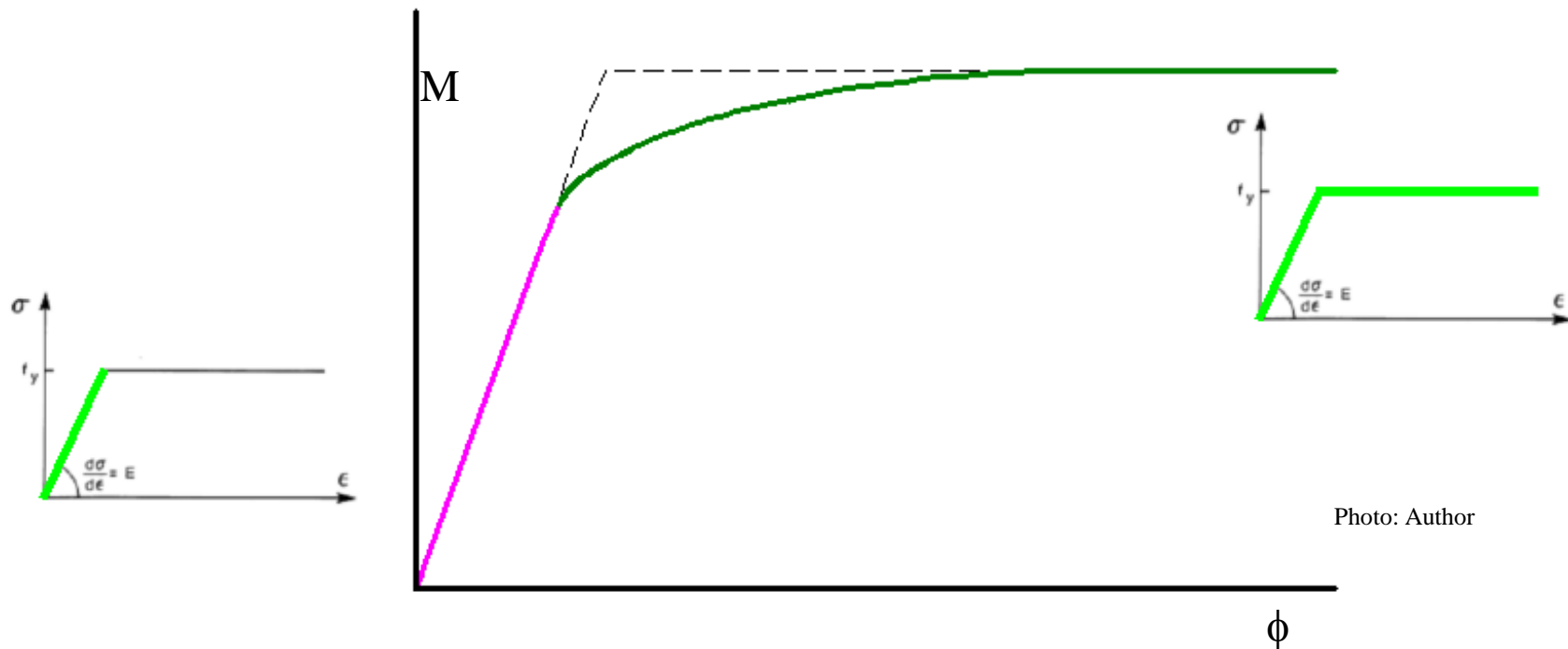


Photo: Author

Both types of analysis (elastic, plastic) are possible, but, based on experience, only one is recommended.

Calculations	Elastic (simpler)	Plastic (more economical)
Design of new structure	Recommended	Possible
Recalculation of existing structure (expertise or change of load)	Possible	Recommended

Examples of calculations according to elastic and plastic analysis in design projects during Ist step of study:

Design project	Elastic analysis	Plastic analysis of	
		elements	joints
I st (truss)	Total	No	No
II nd (floor girder)	Main part	Primary beam: redistribution of bending moments	No
III rd (frame)	Total	No	No

Redistribution of bending moments: Lec # 11 / 78 – 94, Des #2

Calculations according to elastic analysis

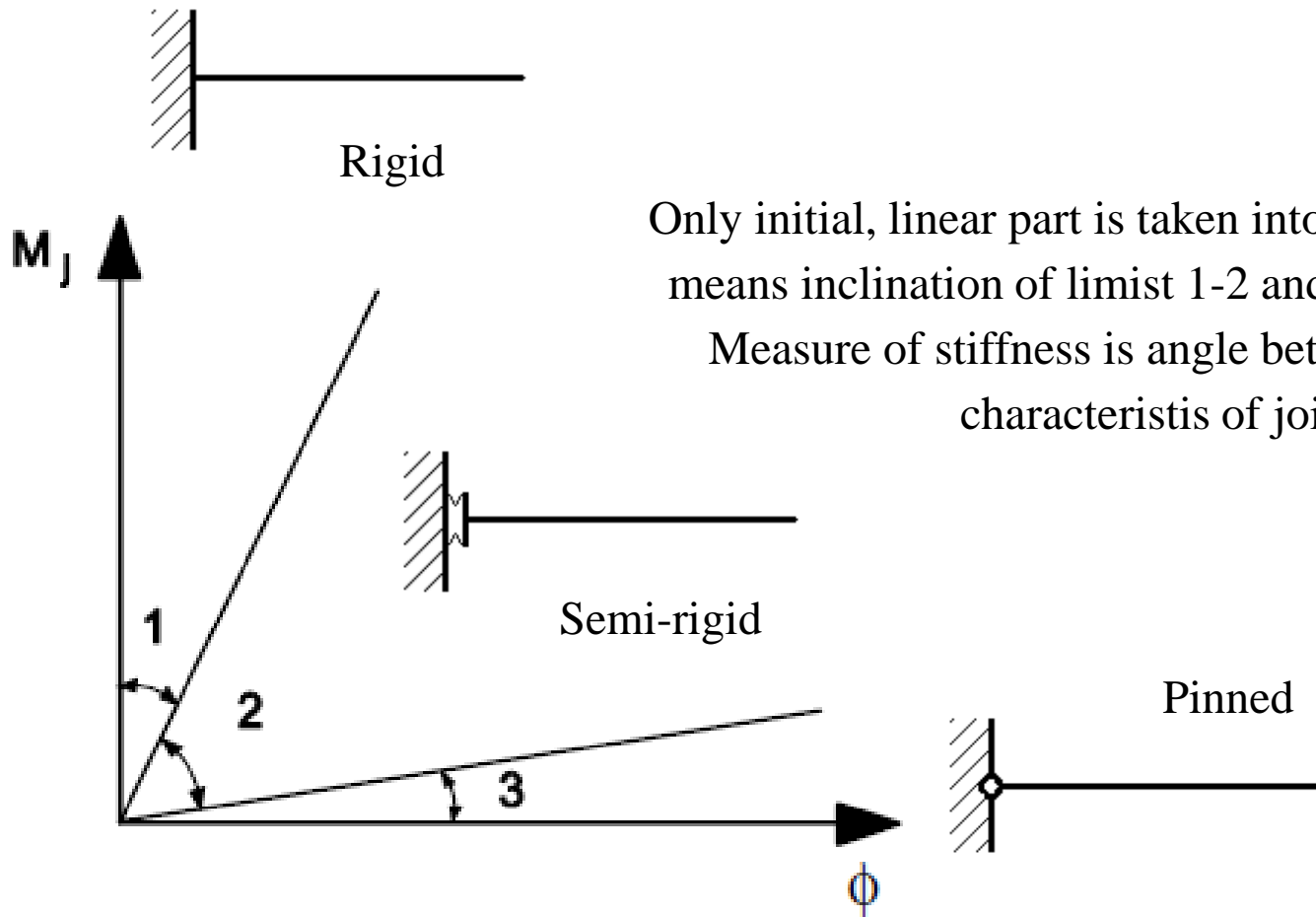


Photo: Author

We must analyze two questions:

A. What are limits S_{1-2} and S_{2-3} between three types of joints? → #t / 87- 95

B. What is characteristic $S_{j, ini}$ for analysed joint? → #t / 96 & lecture #15

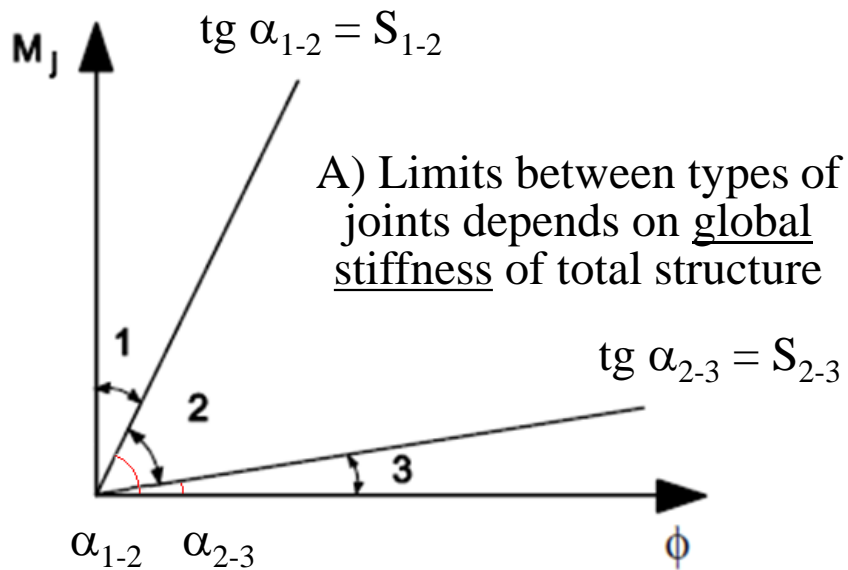
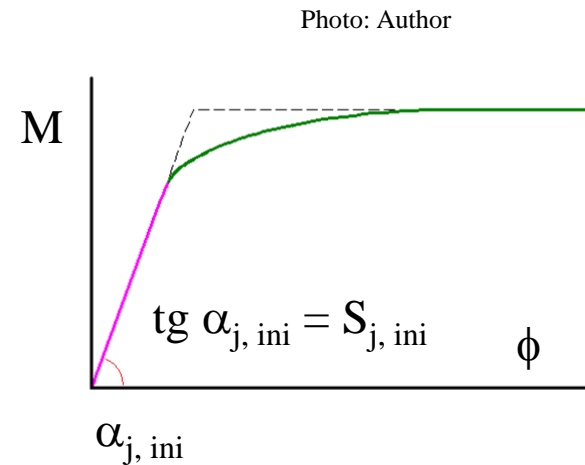


Photo: EN 1993-1-8 fig 5.4



B) **Characteristic** of joint depends on local stiffness of sub-parts of joint

According to Eurocode, we can divide all joints into two groups:

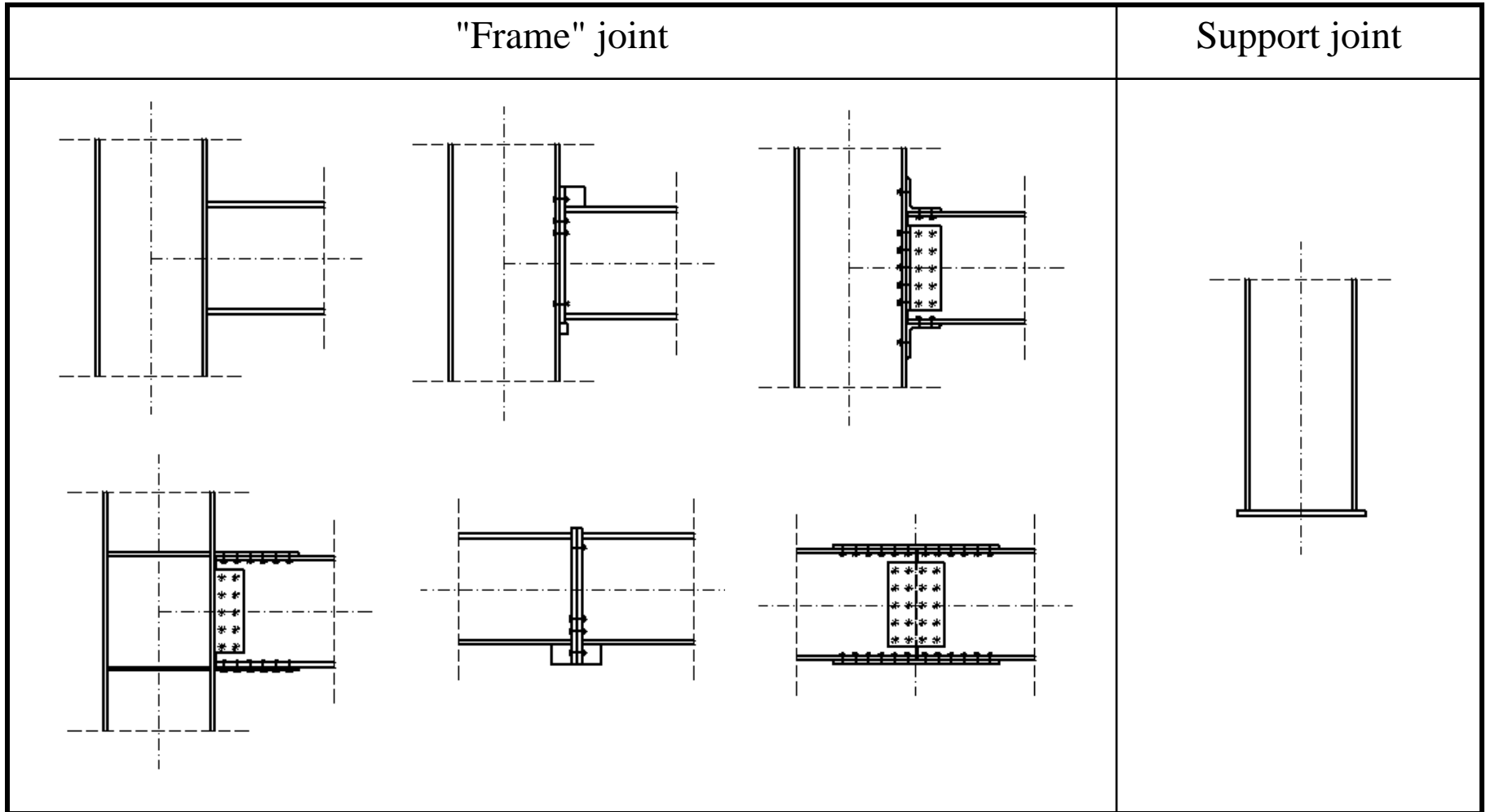
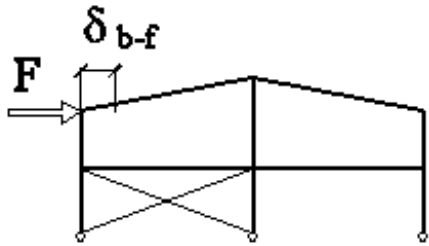
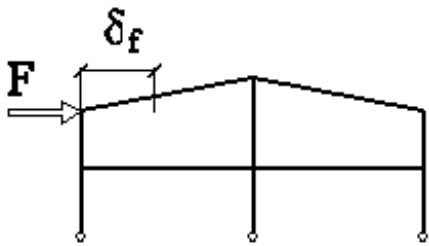
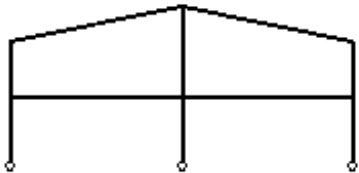
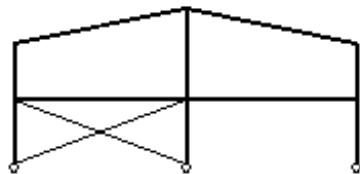


Photo: Author

Wall in-plane bracing



$$\delta_f / \delta_{b-f} \leq 5$$



Non-braced frame

$$\delta_f / \delta_{b-f} > 5$$



→ #13 / 47

Braced frame - no need second-order analysis

Second-order analysis

Photo: Author

When we must make second-order analysis (PN B 03200)

Limits 1-2, 2-3

Joints beam-column, column-column or beam-beam
("frame" joints):

$$M_j = S \phi$$

$$S_{2-3} = 0,5 E J_b / l_b$$

$$S_{1-2} = k_b E J_b / l_b$$

EN 1993-1-8 5.2.2.5 (1)

$$k_b = ? \rightarrow \#t / 90 - 91$$

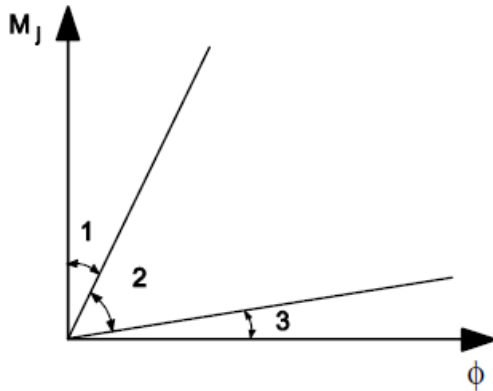


Photo: EN 1993-1-8 fig 5.4

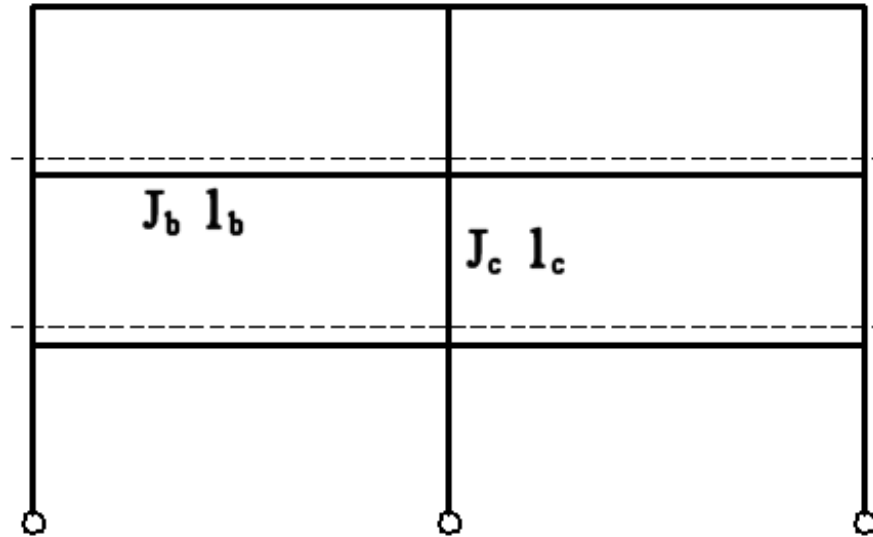
Difference between braced and non-braced frame is important for stiffness of joints:

braced frame $\rightarrow k_b = 8$

non-braced frame $\rightarrow \#t / 91$

EN 1993-1-8 5.2.2.5 (1)

Photo: Author



$$K_b = \text{average } (J_b / l_b)$$

$$K_c = \text{average } (J_c / l_c)$$

$K_b / K_c < 0,1$	$K_b / K_c \geq 0,1$
Each joints are semi-rigid	$k_b = 25$

EN 1993-1-8 5.2.2.5 (1)

Joints column-base (support joints)

$$M_j = S \phi$$

$$S_{2-3} = 0,5 E J_c / l_c$$

EN 1993-1-8 5.2.2.5 (1), (2)

$$S_{1-2} \rightarrow \#t / 93 - 94$$

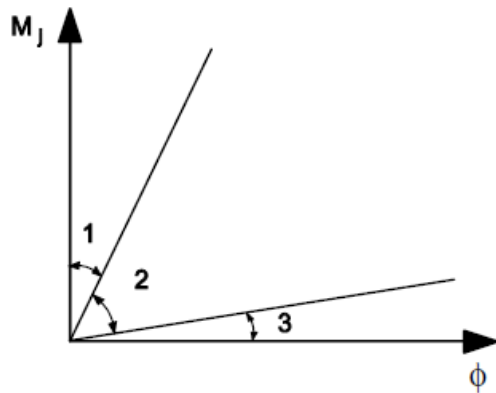


Photo: EN 1993-1-8 fig 5.4

Difference between braced and non-braced frame is important for stiffness of joints:

braced frame $\rightarrow \#t / 94$

non-braced frame $\rightarrow S_{1-2} = 30 E J_c / l_c$

EN 1993-1-8 5.2.2.5 (2)

$$\lambda_c = L_c / (93,9 \varepsilon i)$$

$\lambda_c \leq 0,5$	$0,5 < \lambda_c < 3,93$	$3,93 \leq \lambda_c$
$S_{1-2} = 0$ (Each joints are rigid)	$S_{1-2} = 7 (2 \lambda_c - 1) E J_c / l_c$	$S_{1-2} = 48 E J_c / l_c$

EN 1993-1-8 5.2.2.5 (2)

Summation: characteristic $M-\phi : S_{j, ini}$

$$S_{j, ini} \leq S_{1-2} \rightarrow \text{Pinned}$$

$$S_{1-2} < S_{j, ini} < S_{2-3} \rightarrow \text{Semi-rigid}$$

$$S_{2-3} \leq S_{j, ini} \rightarrow \text{Rigid}$$

"frame" joints:

$$S_{j, ini} = E z^2 / [\Sigma (1 / k_i)]$$

support joints:

$$S_{j, ini} = \{ E z^2 / [\Sigma (1 / k_i)] \} e / (e + e_k)$$

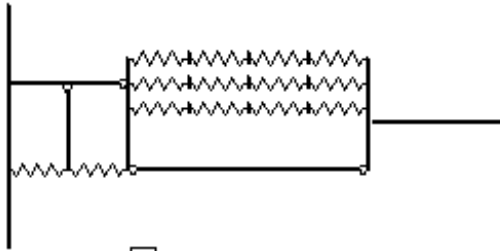
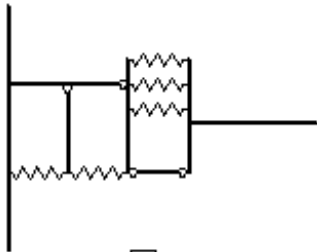
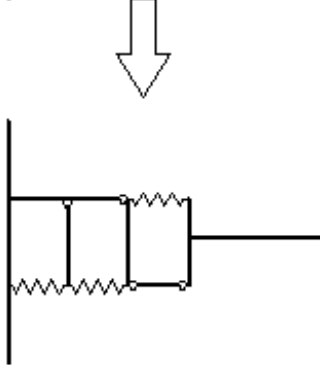


Photo: Author

Each sub-part of joint important for its behavior is presented as spring.



Each spring is described by stiffness factor k_i



Distances between sub-parts are named z , e , e_k

Values of k_i , z , e , e_k , and way of recalculation for effective and equivalent factors will be presented on Lec # 15.

Examination issues

Importance of stiffness in calculation of joints

Technical solutions for various types of joints

General difference between hinged and rigid joint

„Global” and „local” stiffenes of joint

Limits between rigid, semirigid and pinned joints - algorithm

Connection – połączenie, styk
Joint - węzeł
Bearing - docisk
Component method – metoda składowa
Pinned joint - węzeł przegubowy
Semi-rigid joint - węzeł podatny

Thank you for attention

© 2025 Tomasz Michałowski, PhD

tmichal@pk.edu.pl