Metal Structures

Lecture XIII

Columns

Contents

General information

Beam and column - similar type of cross-section, similar calculations.

Differences:

Beam - bending moments are the most important

Comumn - axial force is the most important

→ #11 / 3

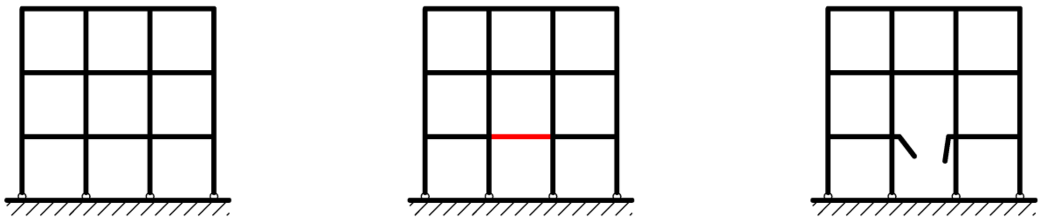
Cross-sectional forces

act on different types of members

	N_{Ed}	M_{Ed}	V_{Ed}
Truss bar	+	(+)	(+)
Bracing bar	+	(+)	(+)
Beam	(+)	+	+
Column	+	+	+

Destruction of one beam

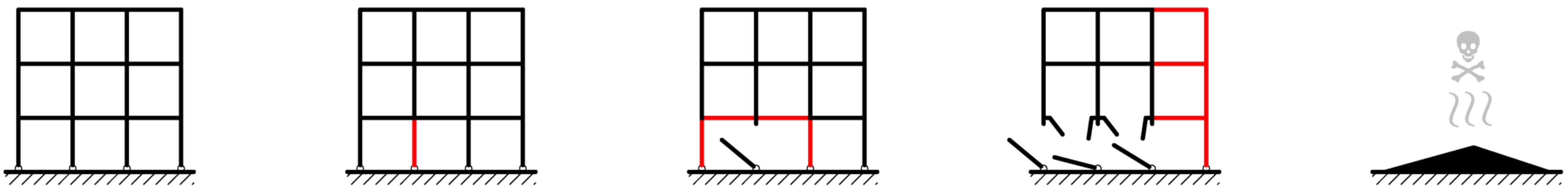
Photo: Autor



→ #7 / 58

Local damage

Destruction of one column



Global collapse

Conclusiom: column - more important for structure, calculation must be more accurate than for beams, few additional phenomenons must be analyzed.

Analysed phenomenons to check

	Beam	Column
Compressive axial force	Sometimes	Always
Flexural buckling	Sometimes	Always
Bending moment	Always	Very often
Shear force	Always	Very often
Interaction shear force – bending moment	Always	Very often
Lateral buckling	Always	Very often
Interaction axial force – bending moment	Sometimes	Very often
Interaction flexural – lateral buckling	Sometimes	Very often
Imperfections*	No	Yes
IInd order effect*	No**	Yes

* Auxiliary role for instability.

** Phenomenon acts on columns first of all; secondary effect on beams.

<table>
<tr><th colspan="2">ULS:</th><th rowspan="2">SLS:</th></tr>
<tr><th>Resistance of cross-section:</th><th>Stability of member:</th></tr>
<tr><td>Axial force
→ #t / 9 – 11, 15 – 16</td><td>Flexural buckling
→ #t / 24 – 71</td><td rowspan="6">Sway
→ #t / 93 – 94</td></tr>
<tr><td>Shear force
→ #t / 9 – 11, 15 – 16</td><td rowspan="4">Lateral buckling
→ #t / 72 – 76</td></tr>
<tr><td>Bending moment
→ #t / 9 – 11, 15 – 16</td></tr>
<tr><td>Bi-axial bending
→ #t / 11, 13 – 22</td></tr>
<tr><td>Interaction shear force - bending moment
→ #t / 11, 13 – 17, 21 – 22</td></tr>
<tr><td>Interaction axial force - bending moment
→ #t / 11, 13 – 22</td><td>Interaction flexural - lateral buckling
→ #t / 77 – 92</td></tr>
</table>

Resistance

There are used the same rules as for beams, in case of interactions between different types of cross-sectional force:

Ist, IInd, IIIrd class of cross-section → Lec #11

IVth class of cross-section → Lec #12

Formulas of resistance

Steel - different formulas for different class of cross-section

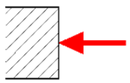
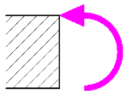
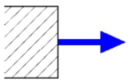
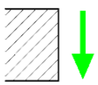
LOAD	I^{st} class	II^{nd} class	III^{rd} class	IV^{th} class
	$N_{Ed} / N_{c,Rd\,(1\text{-}3)} \leq 1{,}0$			$N_{Ed} / N_{c,Rd\,(4)} \leq 1{,}0$
	$M_{Ed\,(1)} / M_{Rd\,(1\text{-}2)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(1\text{-}2)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(3)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(4)} \leq 1{,}0$
	$N_{Ed} / N_{t,Rd} \leq 1{,}0$			
	$V_{Ed} / V_{Rd\,(1\text{-}3)} \leq 1{,}0$			$V_{Ed} / V_{Rd\,(4)} \leq 1{,}0$

Photo: Author

→ #4 / 83

$$N_{c,Rd\,(1\text{-}3)} = A\ f_y / \gamma_{M0}$$

$$N_{c,Rd\,(4)} = A_{eff}\ f_y / \gamma_{M0}$$

$$M_{Rd\,(1\text{-}2)} = W_{pl}\ f_y / \gamma_{M0}$$

$$M_{Rd\,(3)} = W_{el}\ f_y / \gamma_{M0}$$

$$M_{Rd\,(4)} = W_{eff}\ f_y / \gamma_{M0}$$

$$V_{Rd\,(1\text{-}3)} = A_v\ f_y / (\gamma_{M0} \sqrt{3})$$

$V_{Rd\,(4)}$ = impact of local instability + nonlinear relations with $M_{Rd\,(4)}$ and $N_{c,Rd\,(4)}$

$$N_{t,Rd} = A\ f_y / \gamma_{M0}$$

In addition to bending resistance (and lateral buckling), there must be checked shear resistance and resistance for axial force - compressive or tensile. In many cases, the interactions of cross-sectional forces cause additional complications.

C-S force	Interactions → #11 / 51							
$M_{Ed, y}$								
$V_{Ed, z}$								
$M_{Ed, z}$								
$V_{Ed, y}$								
$N_{Ed, t}$								
$N_{Ed, c}$								
T_{Ed}								

Phenomenons, important for IVth class of cross-section:

- Four forms of **local** instability:
  - Under axial compresssive stresses (effective cross-section);
  - Under axial compresssive stresses (flange induces buckling);
  - Under shear force;
  - Under transverse force

- Nonlinear interactions between M_{Ed}, V_{Ed}, N_{Ed}, F_s;

Interaction between bending moment and axial force, bi-axial bending, bi-axial bending and axial force is calculated according to very similar formulas. Additional effect of simultaneous action of compressive axial force and bending moment is interaction between two form of instability: flexural buckling and lateral buckling. Axial force is important, first of all, for column. Because of this, information about such phenomena will be presented on lecture # 13.

Calculation for

- interaction between bending moment and axial force;
- bi-axial bending;
- bi-axial bending and axial force

is made according to the same general rules and is very similar once another.

Basis is resistance for bending moment or resistance for bi-axial bending.

Influence of shear force: resistance for shera force and reduction resistance for bending moment.

Influence of axial force: resistance for axial force and reduction resistance for bending moment.

Level of cross-sections:

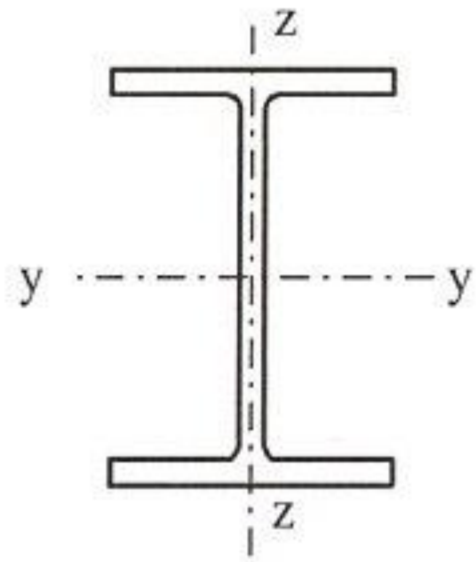


Photo: Author

F – geometrical characteristic of cross-section

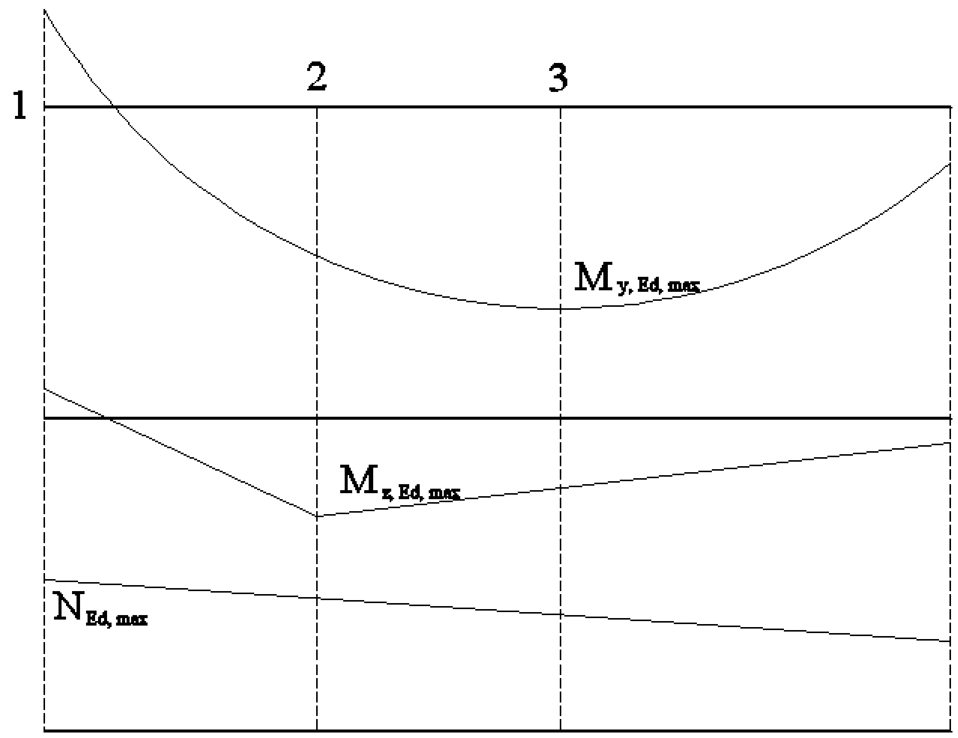
$R = F\ f_y$

$E / R \leq\ 1{,}0$

Elements, nodes - when instability is not important; bolts, rivets, pins

(~ 40% of calculation's conditions)

Photo: Author



Resistance of cross-sections = level of cross-section; resistance in point 1, in point 2, in point 3 for max value of $M_{Ed, y}$, $M_{Ed, z}$ and $N_{Ed,}$. These calculations are done three times; one time for each cross-sections.

I[st] or II[nd] class of cross-section, general formula:

$$(M_{y,\,Ed} / M_{N,\,y,\,Rd})^{\alpha} + (M_{z,\,Ed} / M_{N,\,z,\,Rd})^{\beta} \leq 1{,}0$$

EN 1993-1-1 (6.41)

$V_{Ed} > 0{,}5\ V_{Rd}$ → interaction between M_{Ed} and V_{Ed} → I[st] recalculation M_{Rd} → interaction between M_{Ed} and N_{Ed} → II[nd] recalculation M_{Rd}

Rectangular solid cross-section:

$$\alpha = \beta = 1{,}0$$

$$M_{N,\,y,\,Rd} = M_{y,\,pl,\,Rd}\,[\,1 - (N_{Ed} / N_{pl,\,Rd})^2\,]$$

$$M_{N,\,z,\,Rd} = M_{z,\,pl,\,Rd}\,[\,1 - (N_{Ed} / N_{pl,\,Rd})^2\,]$$

EN 1993-1-1 (6.41)

Doubly symmetrical I- H- cross-section:

$$\alpha = 2 \qquad \beta = \max\,(5n \;;\; 1{,}0)$$

$$n = N_{Ed} / N_{pl,\,Rd}$$

$$a = \min\,[\,0{,}5 \;;\; (A - 2\,b\,t_f) / A\,]$$

	$N_{Ed} \leq \min\,(\,0{,}25\,N_{pl,\,Rd} \;;\; 0{,}5\,h_w\,t_w\,f_y / \gamma_{M0})$	$N_{Ed} > \min\,(0{,}25\,N_{pl,\,Rd} \;;\; 0{,}5\,h_w\,t_w\,f_y / \gamma_{M0})$
$M_{N,\,y,\,Rd}$	$M_{pl,\,y,\,Rd}$	$\min\,[\,M_{pl,\,y,\,Rd} \;;\; M_{pl,\,y,\,Rd}\,(1 - n) / (1 - 0{,}5\,a)\,]$

	$N_{Ed} \leq h_w\,t_w\,f_y / \gamma_{M0}$	$N_{Ed} > h_w\,t_w\,f_y / \gamma_{M0}$	
		$n \leq a$	$n > a$
$M_{N,\,z,\,Rd}$	$M_{pl,\,z,\,Rd}$	$M_{pl,\,z,\,Rd}$	$M_{pl,\,z,\,Rd}\,\{1 - [(n - a) / (1 - a)]^2\}$

EN 1993-1-1 (6.33) - (6.41)

C - R- hollow and welded box cross-section:

$$M_{N, y, Rd} = \min [\ M_{pl, y, Rd} \quad ; \quad M_{pl, y, Rd} (1 - n) / (1 - 0,5\ a_w)\ ]$$

$$M_{N, z, Rd} = \min [\ M_{pl, z, Rd} \quad ; \quad M_{pl, z, Rd} (1 - n) / (1 - 0,5\ a_f)\ ]$$

	Welded box	RHS	CHS
a_w	min [0,5 ; (A - 2 b t_f) / A]	min [0,5 ; (A - 2 b t) / A]	0,5
a_f	min [0,5 ; (A - 2 h t_w) / A]	min [0,5 ; (A - 2 h t) / A]	
$\alpha = \beta$	min [6 ; 1,66 / (1 - 1,13 n^2)]		2

EN 1993-1-1 (6.33) - (6.41)

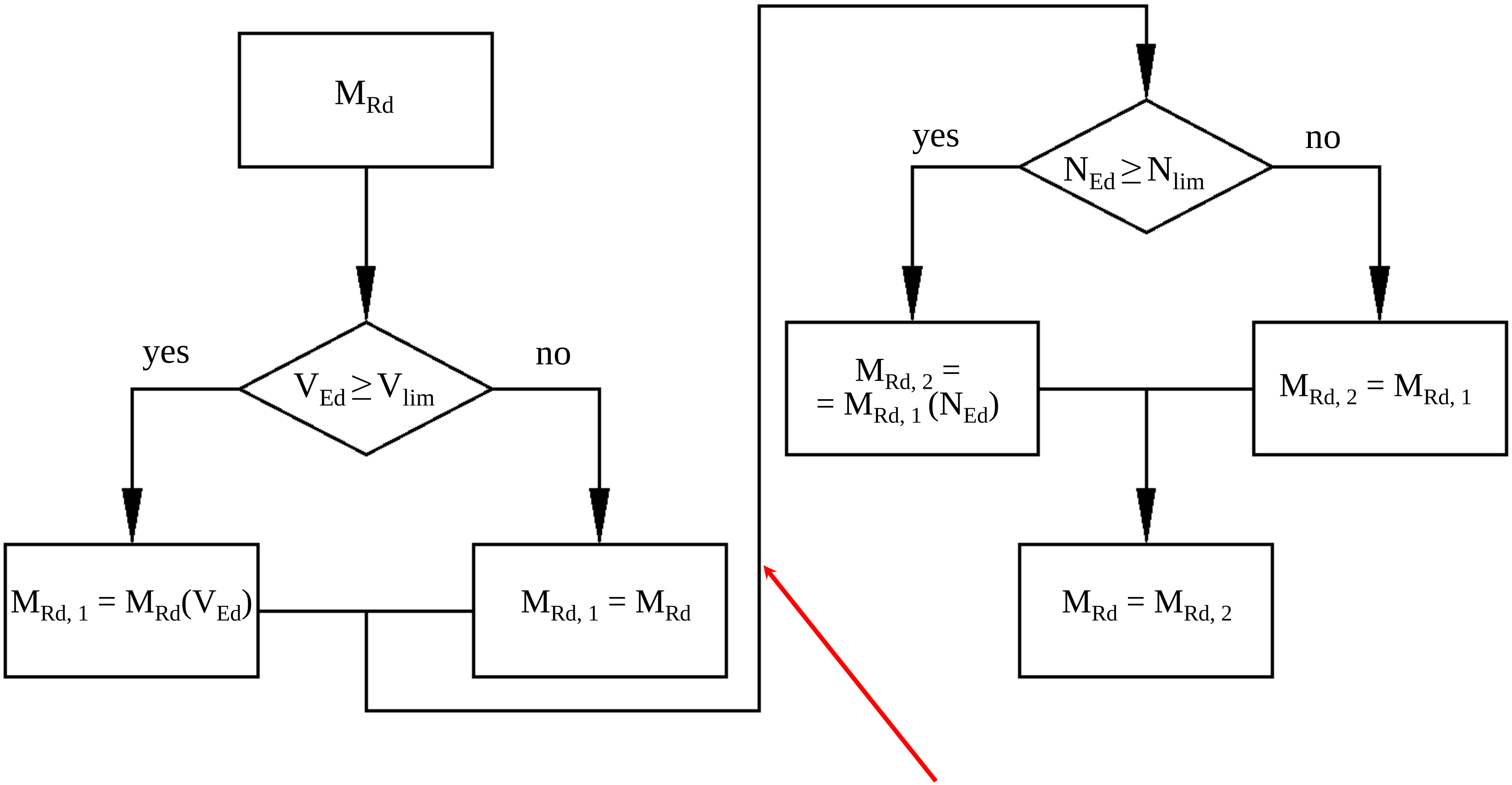


Photo: Author

This transition is not clearly outlined in Eurood, but such recalculation is safe.

IIIrd class of cross-section:

$$N_{Ed} / (A\ f_y / \gamma_{M0}) + M_{y,\ Ed} / (W_{y,\ min}\ f_y / \gamma_{M0}) + M_{z,\ Ed} / (W_{z,\ min}\ f_y / \gamma_{M0}) \leq 1{,}0$$

EN 1993-1-1 (6.42)

IVth class of cross-section:

$$N_{Ed} / (A_{eff}\ f_y / \gamma_{M0}) + (M_{y,\ Ed} + N_{Ed}\ e_{Ny}) / (W_{eff,\ y,\ min}\ f_y / \gamma_{M0}) +$$

$$+ (M_{z,\ Ed} + N_{Ed}\ e_{Nz}) / (W_{eff,\ z,\ min}\ f_y / \gamma_{M0}) \leq 1{,}0$$

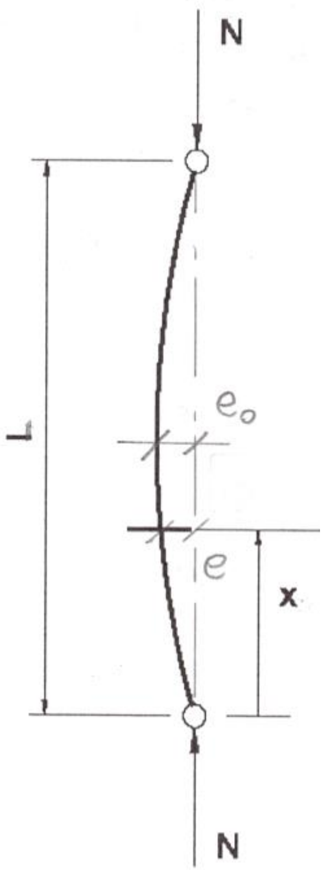
EN 1993-1-1 (6.44)

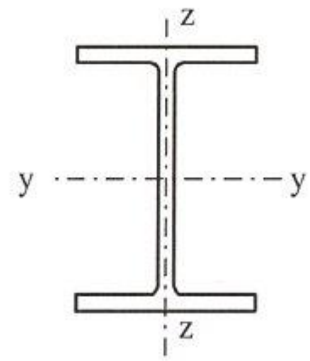
Instability

Simple cases (→ #5)		Interaction
$N_{Ed,c}$:	M_{Ed}:	$N_{Ed,c}$ + M_{Ed}:
flexural χ_y	lateral χ_{LT}	flexural χ_y + lateral χ_{LT}
flexural χ_z		flexural χ_z + lateral χ_{LT}
torsional χ_T		torsional χ_T + lateral χ_{LT}
flexural-torsional χ_{zT}		flexural-torsional χ_{zT} + lateral χ_{LT}

This calculation is done for level of element.

Level of elements:





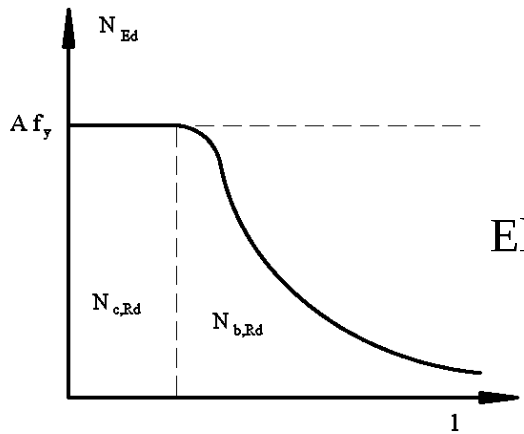


Photo: Author

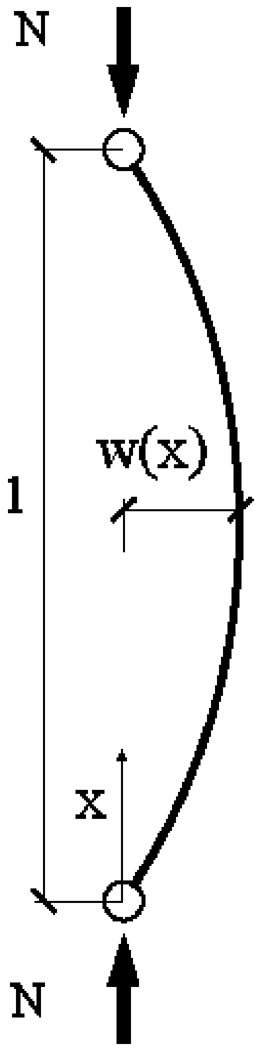
F – geometrical characteristic of cross-section
χ - instability coefficient (depends on element geometry)

$$R = \chi \, F \, f_y$$

$$E / R \leq 1{,}0$$

Elements, nodes - when instability is important

(~ 60% of calculation's conditions)



Flexural buckling

According to Mechanics of Materials:

$$M(x) = N\, w(x)$$

$$d[w(x)]^2 / dx^2 = -M(x) / EJ \rightarrow M(x) = -w''(x)\, E\, J$$

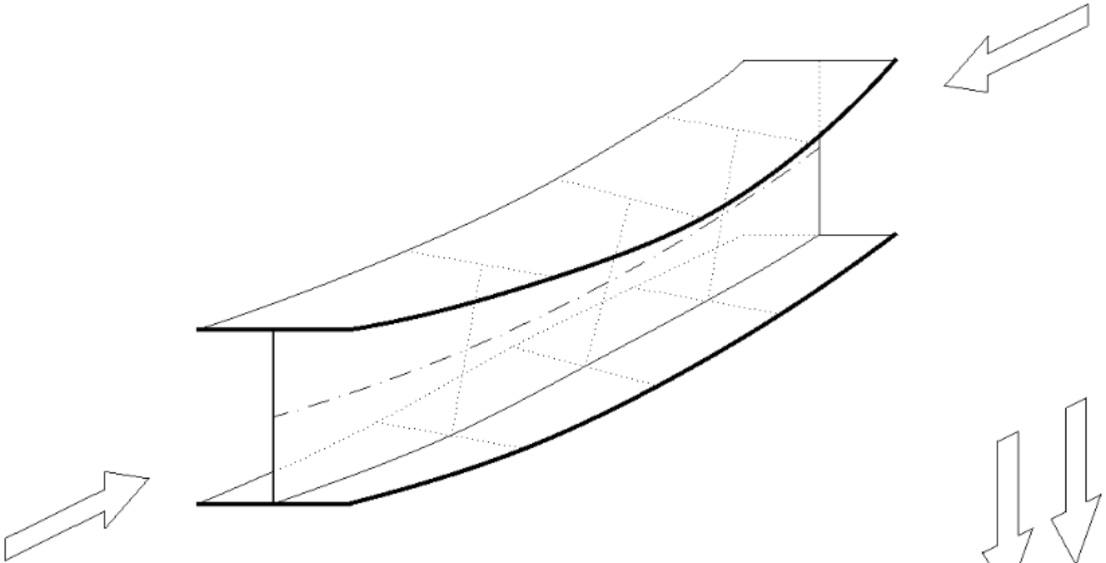
$$-w''(x)\, E\, J = N\, w(x)$$

$$w''(x) = -\, k^2\, w(x)$$

$$k = \sqrt{(N / EJ)}$$

Photo: Author

Flexural-torsional buckling versus lateral buckling



Flexural-torsional buckling

The same shape of
deformations, but other reasons

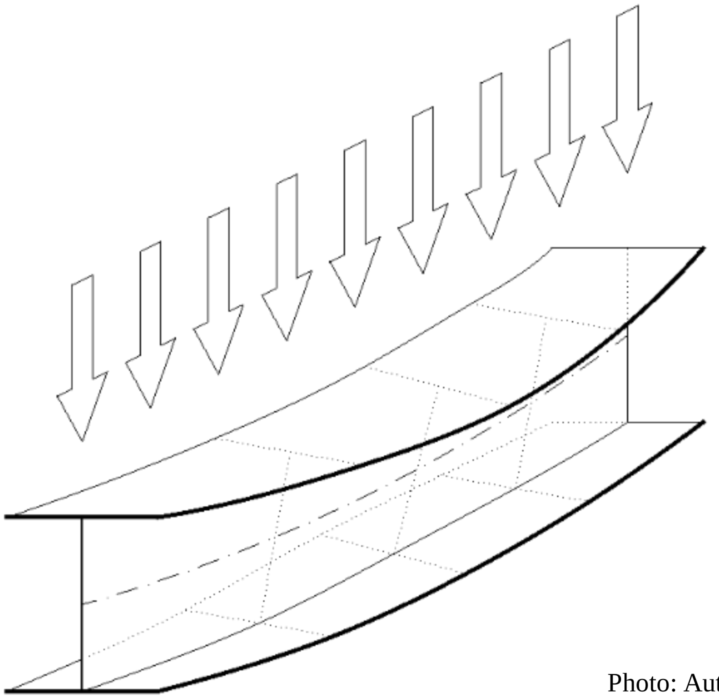


Photo: Author

Lateral buckling

There is very big difference between simple cases of instability and interaction flexural-lateral instability.

For simple cases, instability always reducts resistance:

$$R_{critical} = \mathrm{R}\ \chi$$

$$\chi \leq 1{,}0 \ \longrightarrow R_{critical} \leq \mathrm{R}$$

Because of this, there is no sense to perform two separate checks (for resistance and for instability) for simple cases:

$$\mathrm{E} / \mathrm{R} \leq 1{,}0$$

$$\mathrm{E} / (\mathrm{R}\ \chi) \leq 1{,}0$$

The second is always more important, closer to 1,0.

For simple cases, we calculate only E / (R χ) ≤ 1,0.

For interaction, there is possible than various modes of instability could mutually strengthen or weaken. Because of this, we don't know, which situation is more dangerous for structure:

complex form of loss of stability $E / R_{critical} \leq 1{,}0$

or

exceeding resistance $E / R \leq 1{,}0$

For situation, when interaction flexural-lateral buckling can occurs, both check – for resistance and for stability - must be done.

Ist, IInd, or IIIrd class of cross-section – the same geometry of cross-section for resistance and instability.

IVth class of cross-section: effective geometry for resistance, global geometry for instability.

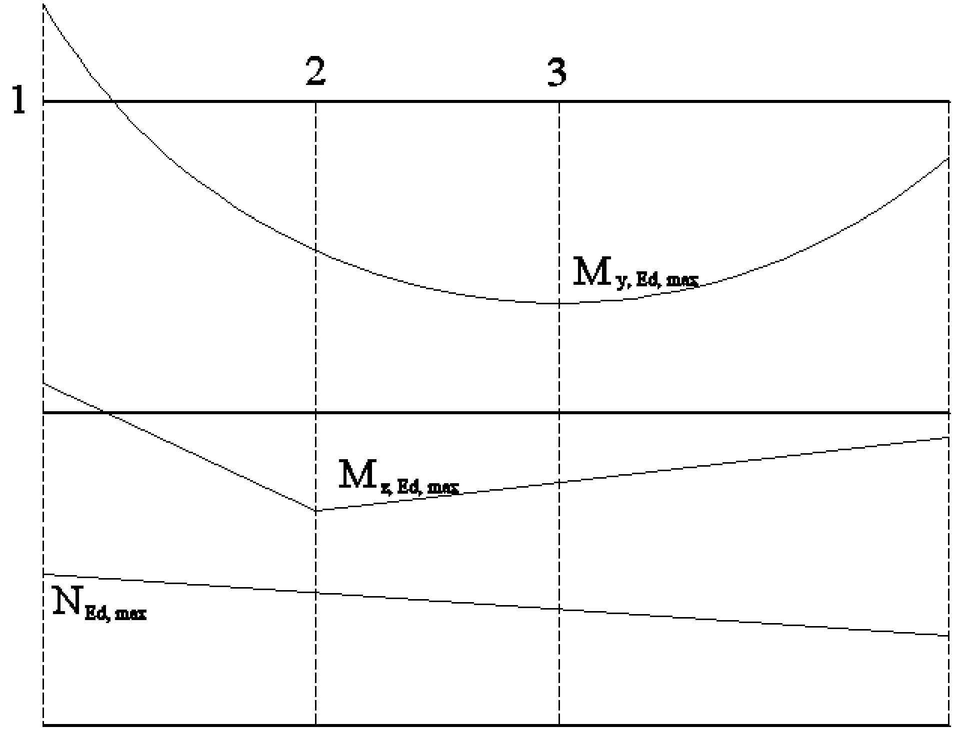


Photo: Author

Resistance of cross-sections = level of cross-section; resistance in point 1, in point 2, in point 3 for max value of $M_{Ed, y}$, $M_{Ed, z}$ and $N_{Ed,}$. These calculations are done three times; one time for each cross-sections.

Stability of element = level of element; one global calculation for maximum values of N_{Ed}, $M_{y, Ed}$, $M_{z, Ed}$ even if they are in three different cross-sections.

Critical length is very important for various modes of instability. For example, this matter was analysed in II^nd^ design project: critical length for truss cords in two various planes.

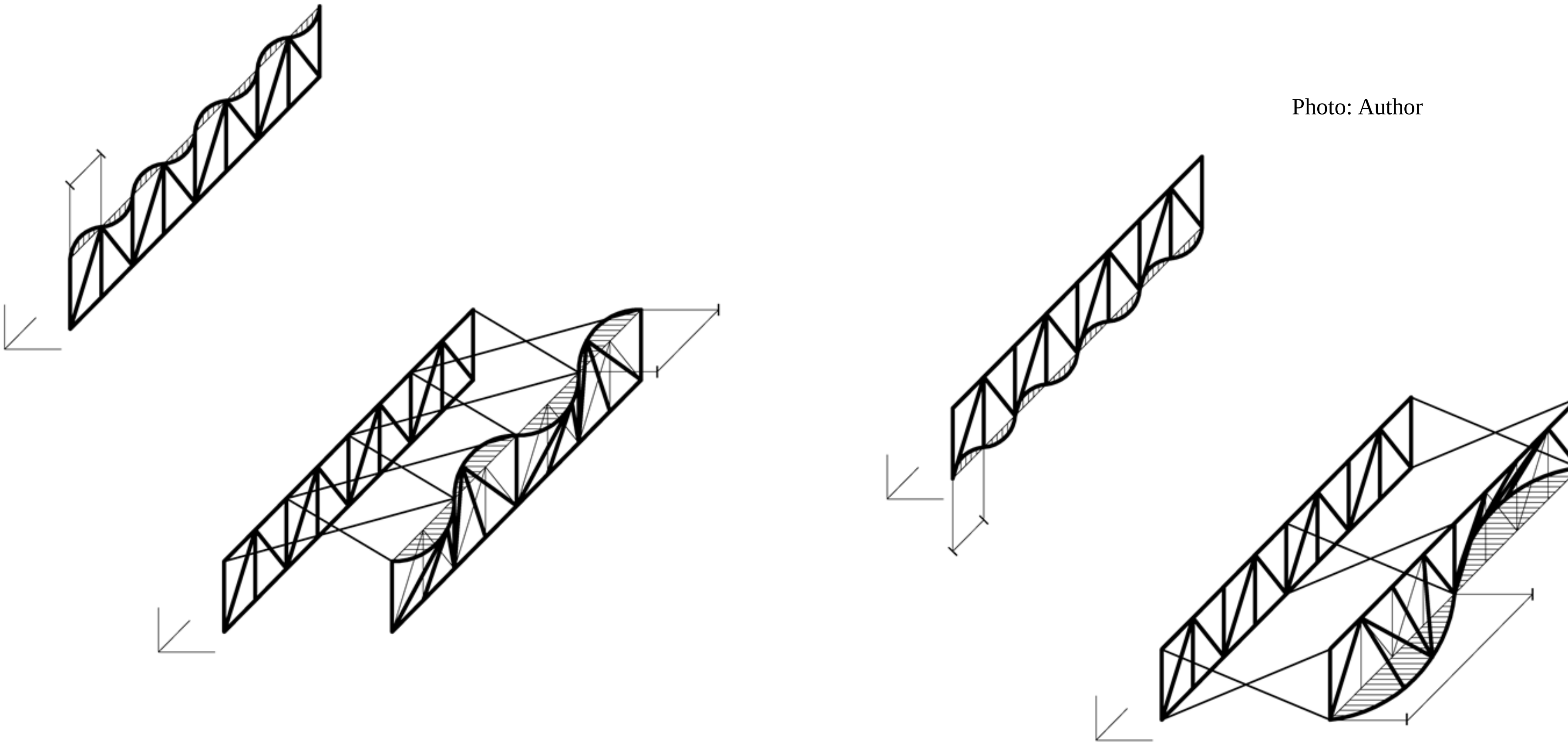
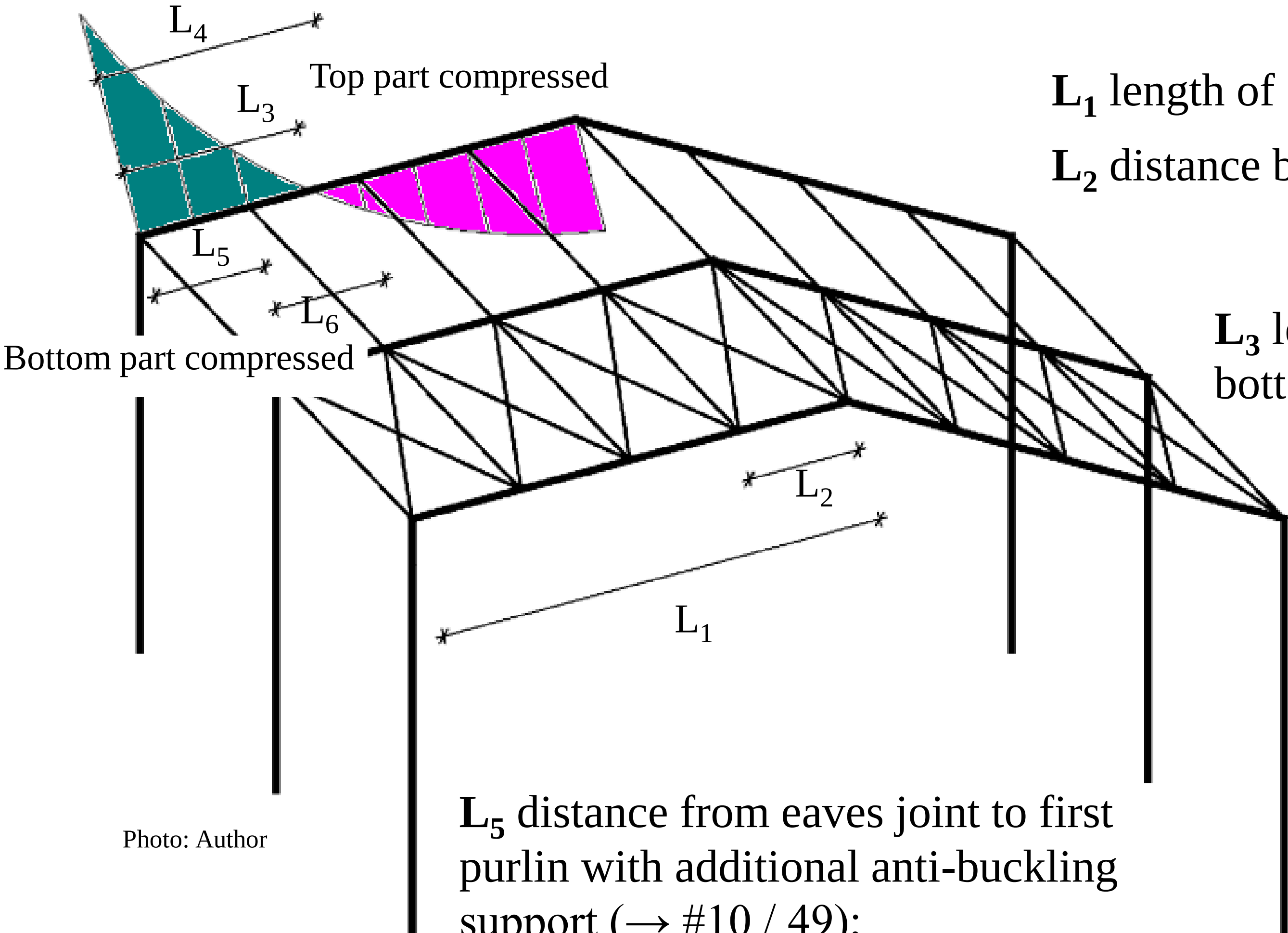


Photo: Author

Analysis of critical length for columns is much more complicated.

→ #10 / 13 I-beam roof girder



L_1 length of girder;

L_2 distance between bracings

L_3 length of girder where bottom part is compressed;

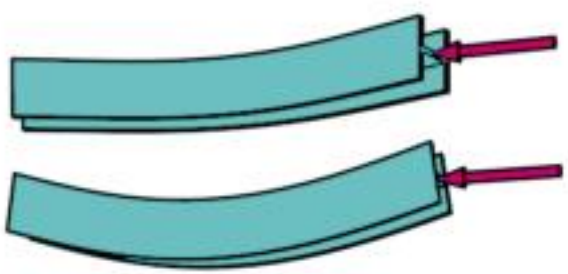
L_4 distance from eaves joint to first purlin in top part compressed;

L_5 distance from eaves joint to first purlin with additional anti-buckling support (→ #10 / 49);

L_6 distance between the closest purlins with additional anti-buckling supports (→ #10 / 49);

Case of instability		Distance between supports
Flexural buckling	in plane of frame	L_1
	out of plane of frame	L_2
Lateral buckling	if additional anti-buckling supports are applied (→ #10 / 49)	max (L_5 ; L_6)
	if additional anti-buckling supports are **not** applied (→ #10 / 49)	max (L_3 ; L_4)

Flexural buckling



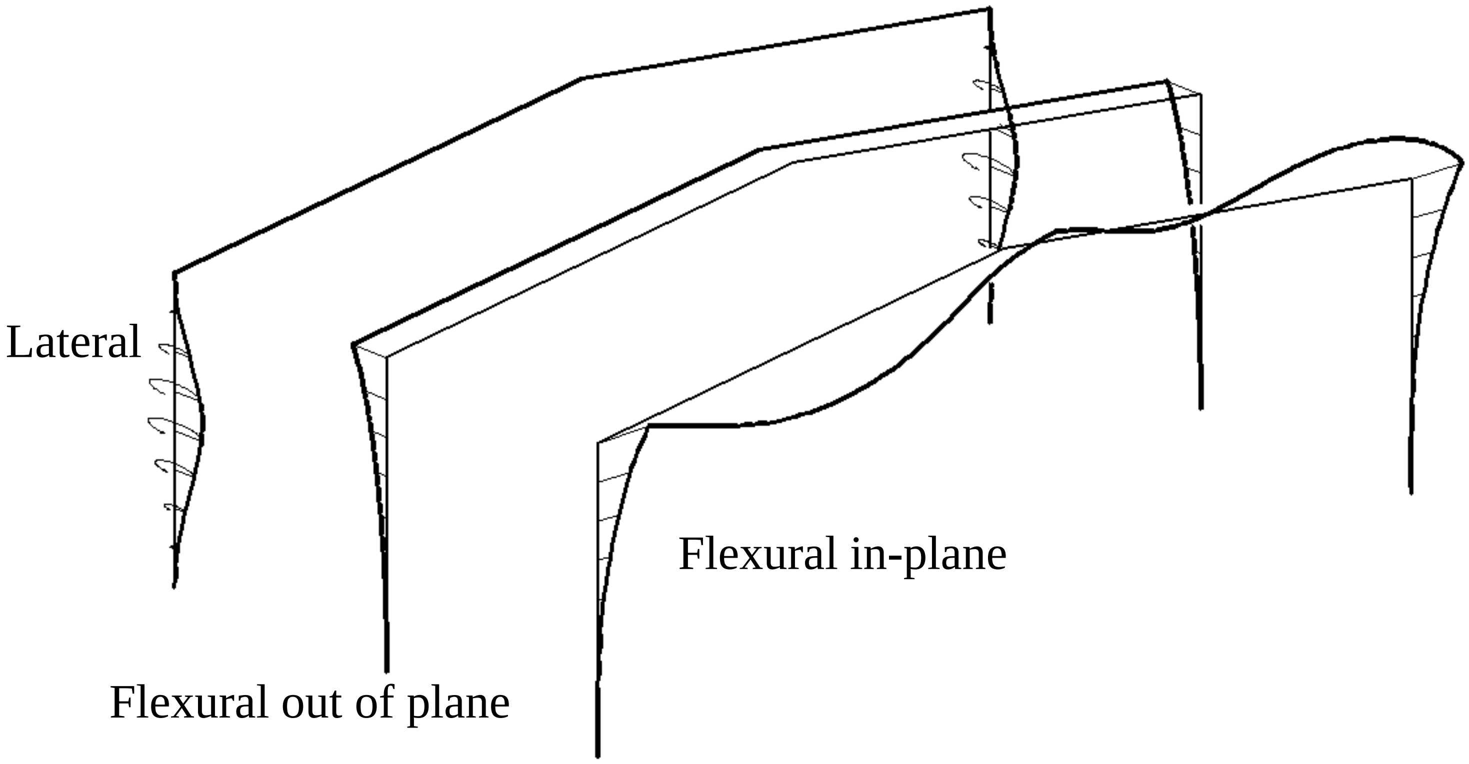
Lateral buckling

Photo: calcresource.com

→ #10 / 14

There are three most important modes of instability for columns: flexural buckling in-plane of frame, flexural buckling out of plane and lateral buckling. Additionally, if column is not hot-rolled I-beam section, there are torsional buckling and flexural-torsional buckling.

Photo: Author



There are different critical length for each of these buckling modes.

According to #t / 33, few various types of instability should be taken into consideration:

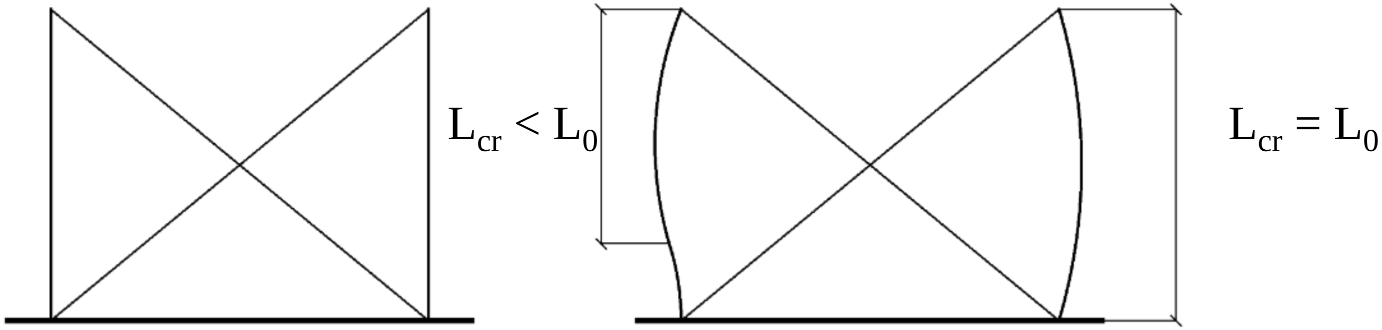
Buckling	Hot-rolled I-beam	Welded I-beam
Flexural out of plane	#t / 35 – 36	
Flexural in-plane	#t / 37 – 71	
Torsional	No necessary	#t / 72 – 76
Flexural-torsional		
Lateral		

Flexural buckling out of plane

Critical length depends on vertical bracings in side walls.

Photo: ed808.com

Photo: dreamstime.com

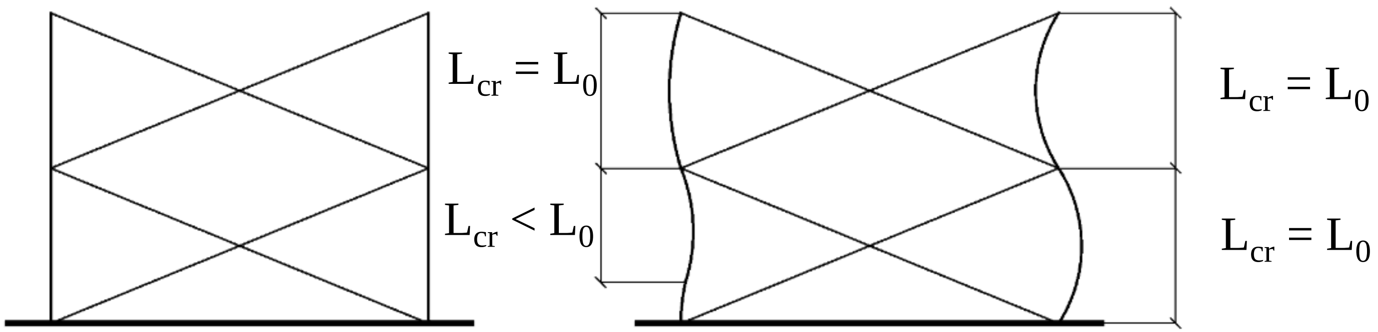


Rigid support

Hinged support

Photo: Author

Critical length out of plane is always not longer than length of member.



Rigid support

Hinged support

Flexural buckling in-plane

Flexural buckling in plane of frame can be analised using one of three methods:

• according to EN 1993-1-1 5.2.2. (3.a), (7.a) (Procedure „A”);
• according to EN 1993-1-1 5.2.2. (3.b), (7.b) (Procedure „B”);
• according to EN 1993-1-1 5.2.2. (3.c), (7.c) (Procedure „C”).

„A”, „B”, „C” – „copyright” names, used in this lectures only, for distiction three ways of calculation.

In theory (no imperfections, no second-order effects) for stability important is critical force only:

$$\pi^2 EJ / L_{cr}^2$$

In fact, we must take into consideration that columns are not perfectly straight and not perfectly vertical from beginning.

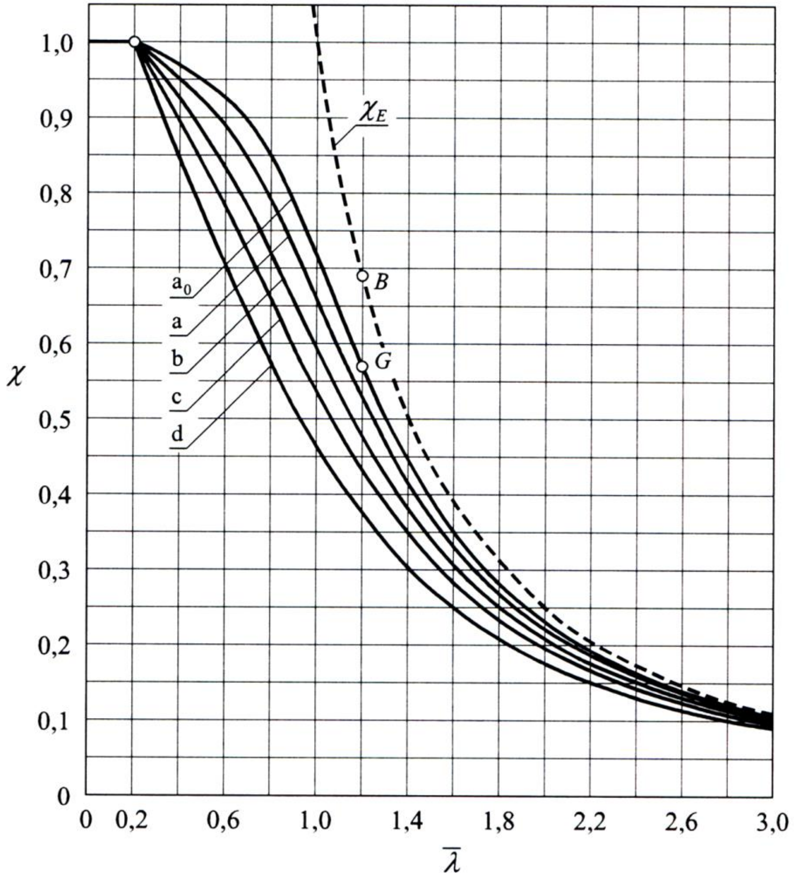


Photo: Zagadnienia stateczności konstrukcji metalowych, K. Rykaluk, DWE Wrocław 2012

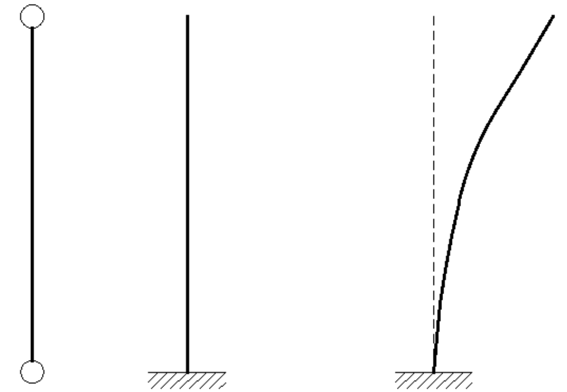
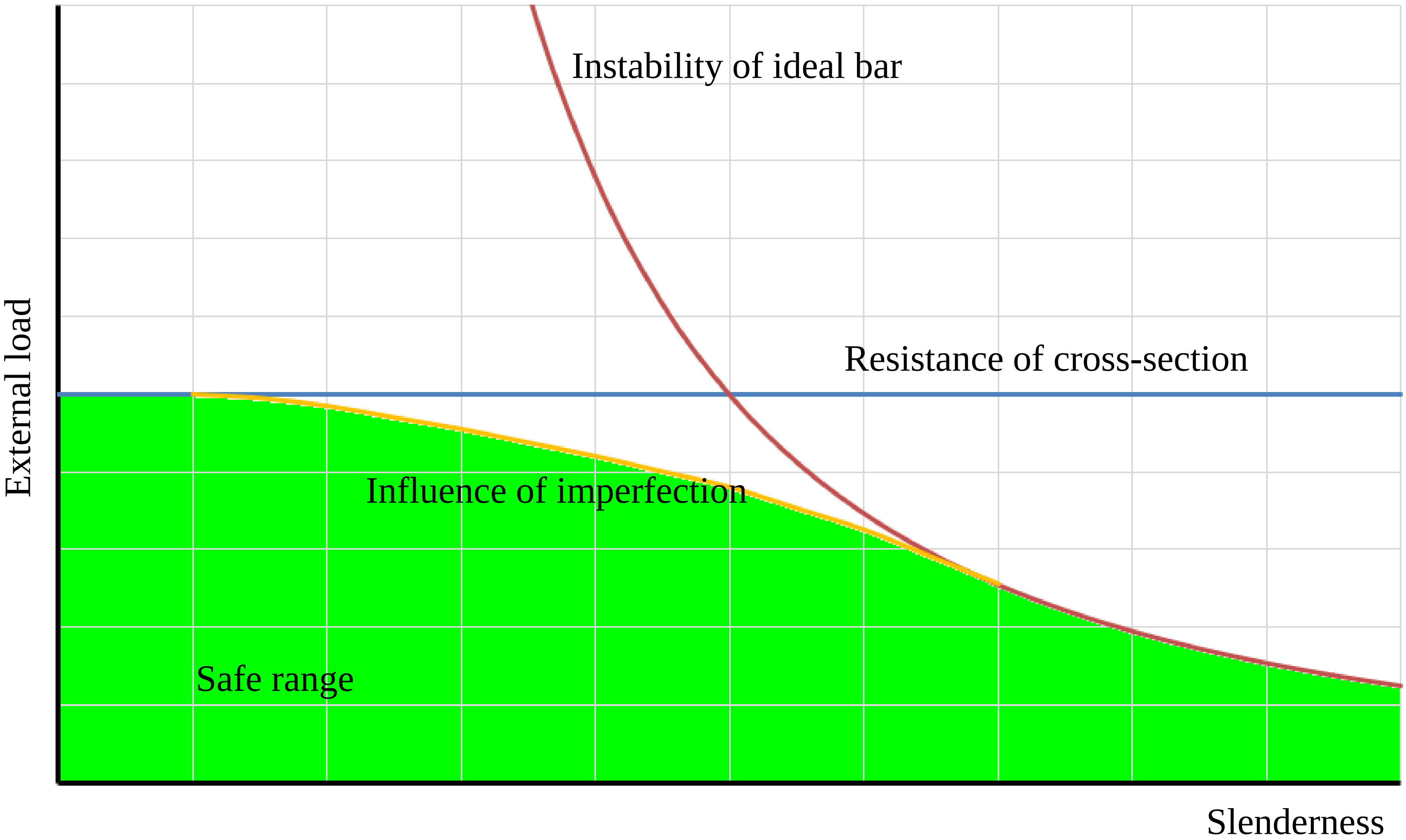


Photo: Author

In theory, instability is important for slenderness $\bar{\lambda} > 1{,}0$.

In fact, instability is important even for slenderness $\bar{\lambda} > 0{,}2$.

Fot: Author



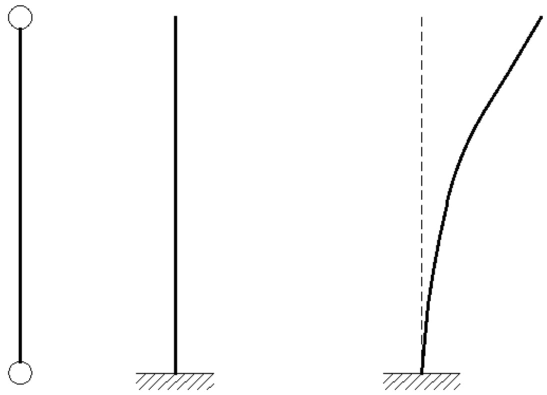
General rule for buckling (flexural, torsional, lateral)

According to EN 1993-1-1, for analysis of non-vertican non-straigh column three methods of calculation are used:

• taking into account additional bending moments appearing in compression of a non-vertical column with curve axis (Procedure „A”)

• taking into account exact value of buckling length factor for non-straigth and non-vertical column (Procedure „C”: formulas, tables, or results of numerical calculations);

• taking into account partly both procedures (Procedure „B”).



Rys: Autor

There are two groups of reasons for curvature and non-verticality of column :

• bow imperfections (initial curvature);
• sway imperfections (initial non-verticality);

• IInd order effects (non-verticality from exploitation).

When analyzing the loss of column stability (interaction of flexural and lateral buckling), influence of curvature and non-verticality on value of bending moments and on buckling lengths should be considered. Thus, imperfections can be considered simultaneously in two ways:

- in determining buckling factor - via the parameter α;
- in analysis of influence on value of bending moment - by equivalent loads.

	Procedure "A"	Procedure "B"	Procedure "C"
Loads	• „normal” • from IInd order effects • from sway imperfections • from bow imperfections	• „normal” • from sway imperfections	• „normal”
Calculations	• Resistance	• Resistance • $\mu_y = 1{,}0$ • Stability	• Resistance • Value of μ_y • Stability
Notes	Big amount of calculations for loads.	Medium amount of calculations for loads, medium amount of calculations for stability.	Small amount of calculations for loads, big amount of calculations for stability.

EN 1993-1-1 5.2.2. (3 a, b, c), (7a, b)

<u>Choice of method - individual designer's decision</u>. For frames susceptible to IInd order effects, procedure "A" is recommended.

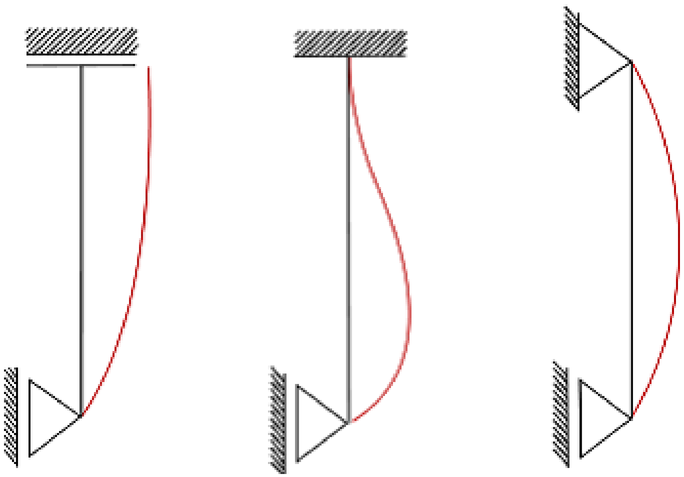
Calculation of μ_y for method "C" is the most complicated. First of all, mode of instability of structure must be analised. Generally, there are two main groups of structure:

- sway frame
- non-sway frame

Sway frame can be identified with non-braced frame; non-sway frame with braced frame.

Common mistake: mode of column global in-plane instability is analysed as basic modes:

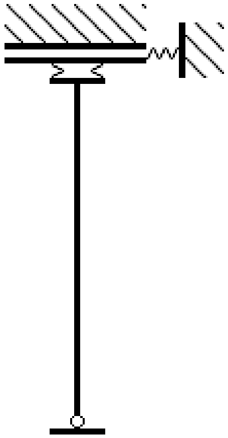
Photo: wikipedia



$\mu = 2,0$ $\mu = 0,7$ $\mu = 1,0$

In frames, there are spring supports for rotation and horizontal translation. Rigid top support is possible only for infinite stiffness of roof girders.

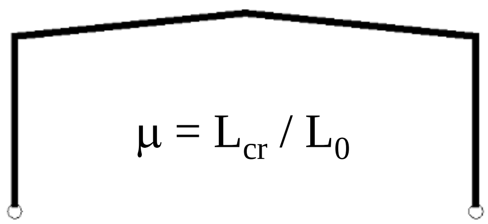
Photo: Author



Because of spring supports, there is often $\mu_y >> 2,0$

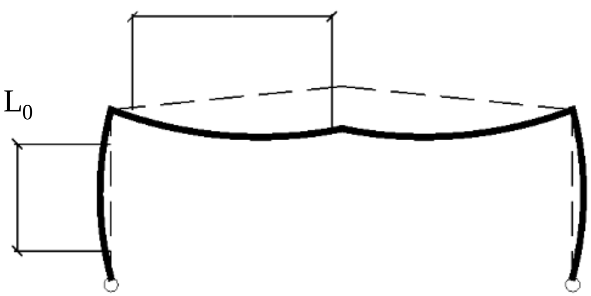
Braced frame ↔ non-braced frame,
first mode of instability

Braced frame: horizontal displacements are non 0 of course, but very small and can be neglected.



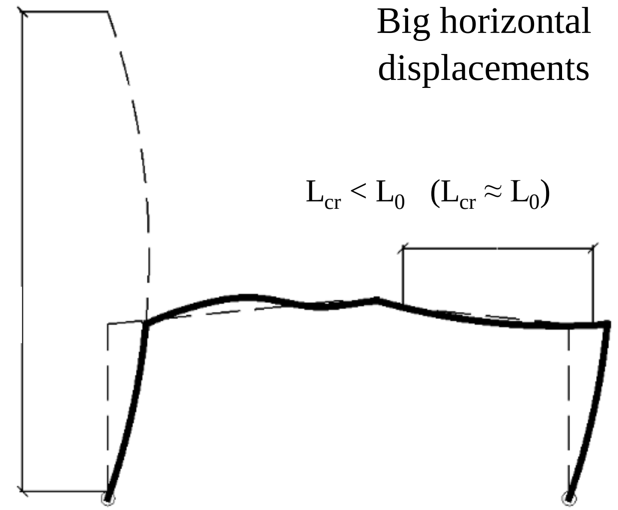
$\mu = L_{cr} / L_0$

Small horizontal displacements



braced frame = non-sway frame

Big horizontal displacements



non-braced frame = sway frame

Photo: Author

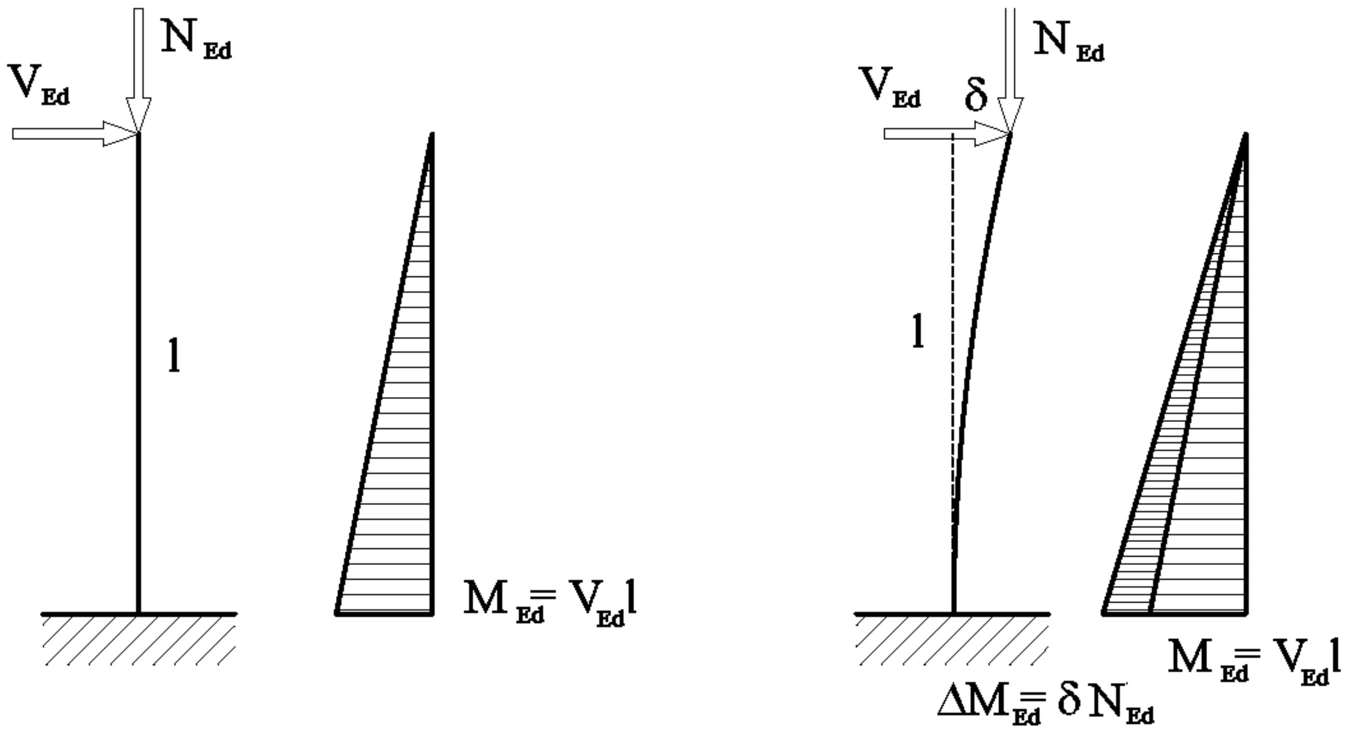
$\mu_{y,\ column} \leq 1{,}0$

$\mu_{y,\ column} \geq 1{,}0$

→ #3 / 86

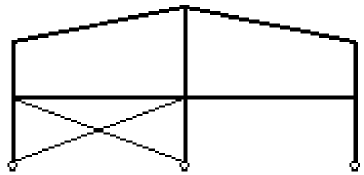
There is additional bending moment from axial force for very flexible structures

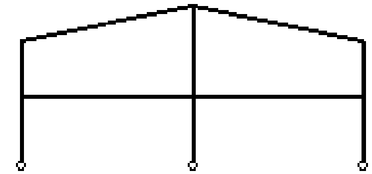
Photo: Author



For calculations, new value of horizontal force is applied: $V_{Ed}^{*} = V_{Ed}\ \alpha^{*}$

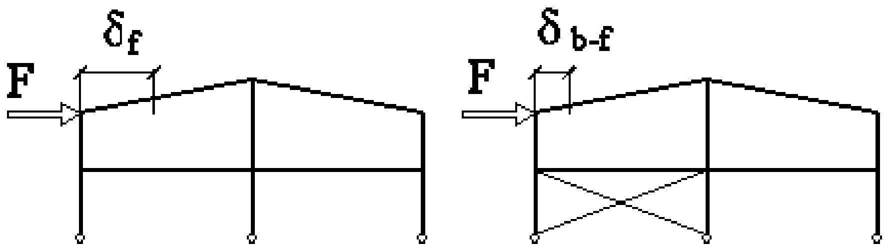
Wall with in-plane bracing





↓

↓



$\delta_f / \delta_{b\text{-}f} \leq 5$

→

Non-braced frame

$\delta_f / \delta_{b\text{-}f} > 5$

↓

↓

Braced frame - no need second-order analysis

Second-order analysis

Photo: Author

When we must make second-order analysis (PN B 03200)

For non-braced frame:

$\alpha_{cr} \approx (H_{Ed}\ h) / (V_{Ed}\ \delta_{H,Ed})$

$\alpha_{cr} = F_{cr} / F_{ed}$ α_{cr} – „distance" between cross-sectional force from external loads and critical force for instability		
$\alpha_{cr} > 10$	$10 \geq \alpha_{cr} \geq 3$	$\alpha_{cr} < 3$
No need analysis	Simple analysis	Advanced analysis

Simple analysis:

$Q_{Ed}^{*} = Q_{Ed}\ \alpha^{*}$

$\alpha^{*} = 1 / (1 - 1 / \alpha_{cr})$

Q_{Ed}, H_{Ed} , V_{Ed} – from one analysed storey only

EN 1993-1-1 5.2.1.(3),
EN 1993-1-1 5.2.2.(5)B

EN 1993-1-1 (5.1), (5.2), (5.4)

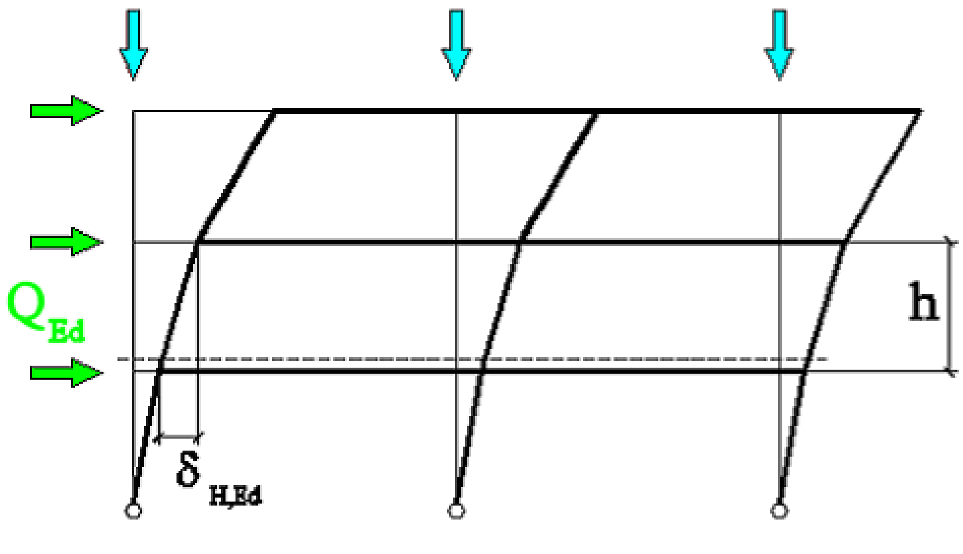
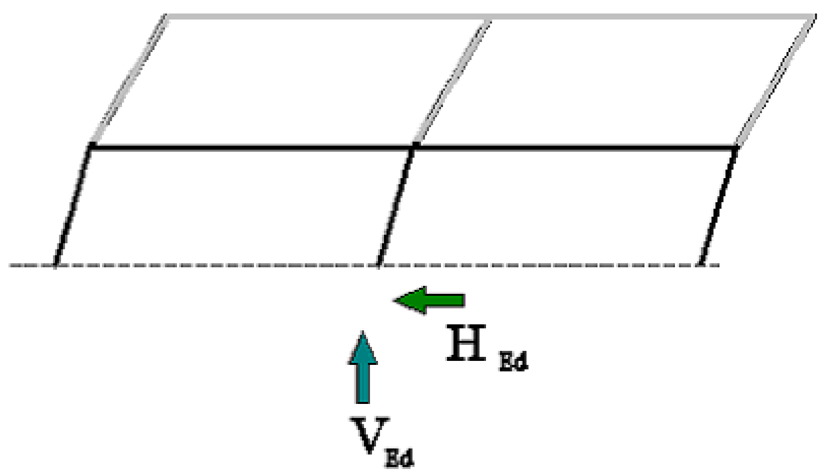


Photo: Author



α_{cr}	α^*
20	1,053
15	1,071
10	1,111
7	1,167
5	1,250
4	1,333
3	1,500
2	2,000
1	$\rightarrow \infty$

Of course, α_{cr} can be also calculated by computer programme

$$\alpha_{cr} = F_{cr} / F_{Ed} \rightarrow F_{cr} = \alpha_{cr} F_{Ed}$$

Based on F_{cr}, computer programme can analyses stabilities of column.

α_{cr} increases when:

there is big difference between critical buckling force and load
or
there are low horizontal forces and displacements.

Simple analysis - linear statical calculations with enlarged value of horizontal loads α^*.

Advanced analysis - nonlinear statical calculations (computer programmes with geometrical nonlinearity = lagre displacement range) without enlarged value of horizontal loads.

Analysis of imperfections (sway and bow) is important for metods "A" and "B".

Common mistake is identification of sway imperfection and second order analysis. For both phenomenon word "sway" is used, but there are two completely different phenomenon.

Sway imperfection: equivalent deformation (makes equivalent loads) for mamy various types of geometrical imperfections (→ Lec #6). Imperfections always exist in real structures.

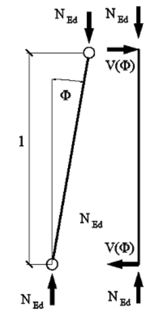


Photo: Author

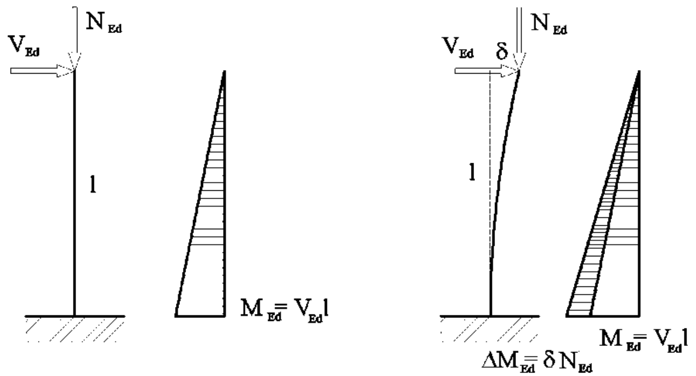


Photo: Author

Second order analysis: analysis of susceptibility to big values of deformations under horizontal loads. Sway makes additional cross-sectional forces (vertical loads on eccentricities). Second order effects exist only for flexible structures (for example masts, towers, flexible frames).

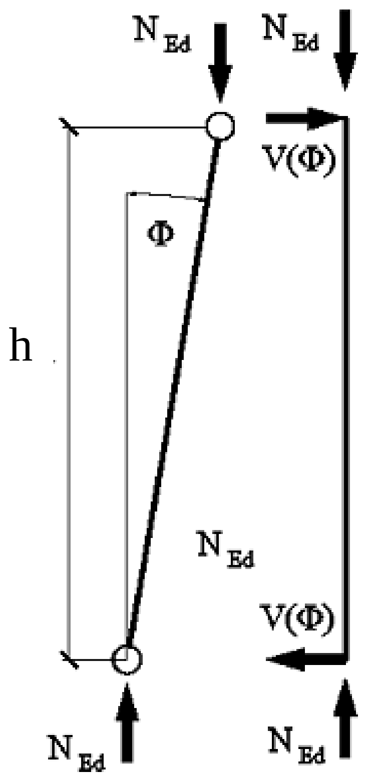
IInd order analysis is very important for distinction between braced and non-braced frame. Generally, IInd order analysis is very important for calculation of $\mu_{y,\ column}$ on two procedurs.

Procedure	Importance of IInd order analysis	
„A”		Calculation of multiplication factor α^{*} for horizontal actions (→ #t / 48)
„B”	No	
„C”	Distinction between sway and non-sway frame (→ #t / 45)	

Sway imperfection

→ #6 / 83

$\Phi = \Phi_0 \, \alpha_h \, \alpha_m$



EN 1993-1-1 fig. 5.4

Photo: Author

$\Phi_0 = 1 / 200$

$\alpha_h = \max\{ 2 / 3 \;\; ; \;\; \min[(2 / \sqrt{h}) \;\; ; \;\; 1,0]\}$

h – heigh of structure [m]

$\alpha_m = \sqrt{[0,5 (1 + 1 / m)]}$

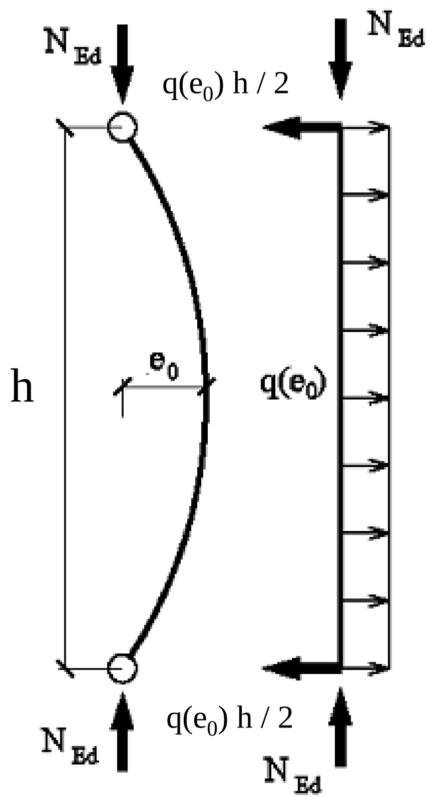
m - number of columns (elements) in a row, including only those columns, which carry a vertical load N_{Ed} not less than 50% of the average value of the column in the vertical plane considered

N_{Ed} – axial force in column or in braced member (chord of truss, etc).

$V(\Phi) \, h = M = N_{Ed} \, h \, \Phi$

$V(\Phi) = N_{Ed} \, \Phi$

e_0 depends on buckling curve (→ Lec #5)



EN 1993-1-1 fig. 5.4

Photo: Author

Buckling curve	Elastic analysis	Plastic annalysis
a_0	l / 350	l / 300
a	l / 300	l / 250
b	l / 250	l / 200
c	l / 200	l / 150
d	l / 150	l / 100

N_{Ed} – axial force in column or in braced member (chord of truss, etc).

$N_{Ed}\, e_0 = M = q(e_0)\, h^2 / 8$

$q(e_0) = 8\, N_{Ed}\, e_0 / h^2$

Sometimes there is no need to take imperfections into consideration.

Imperfections:	Can be neglectet, when:	
Sway	$H_{Ed} \geq 0,15\ V_{Ed}$	EN 1993-1-1 (5.7)
Bow	$(L / i)\ (1 / \lambda_1) \leq 0,5 \sqrt{(A\ f_y / N_{Ed})}$ $\lambda_1 = 93,9\ \varepsilon$	EN 1993-1-1 (5.8)

When imperfections can be neglected, we rather should use procedure "C".

EN 1993-1-1 N.A. 9 – single storey frames could be treated as braced frame without sway imperfections.

But, of course, it is safer to calculate it as non-braced frame with imperfections.

Procedure "A"

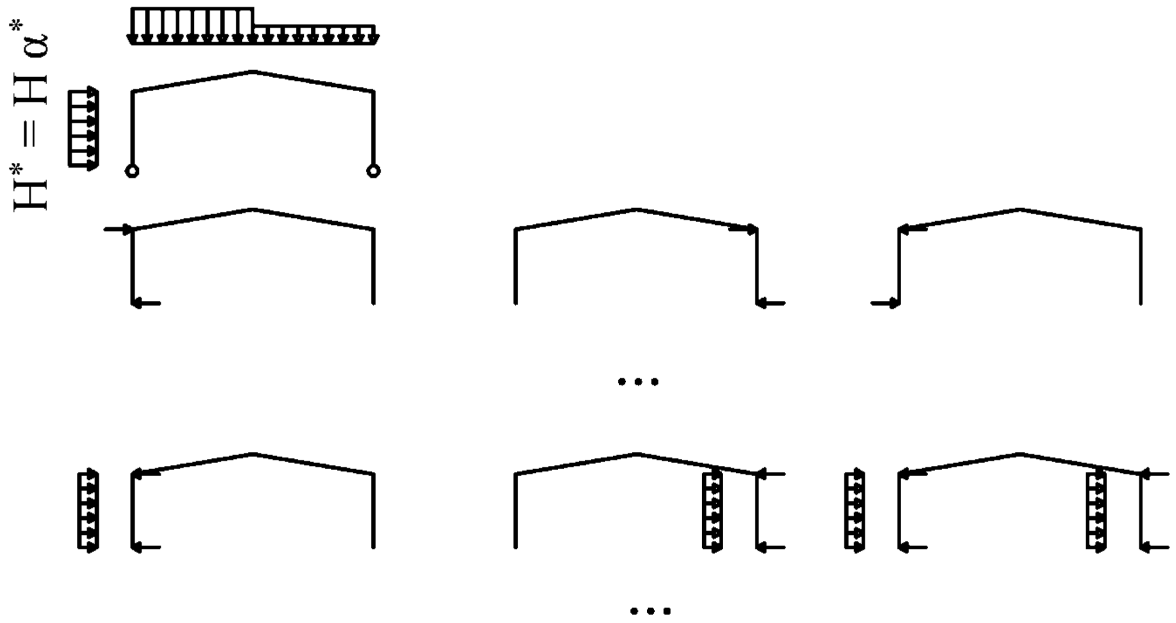


Photo: Author

„Normal” loads multiplied by factor α^*;

Different combinations of sway imperfection load (8 combinations for presented structure);

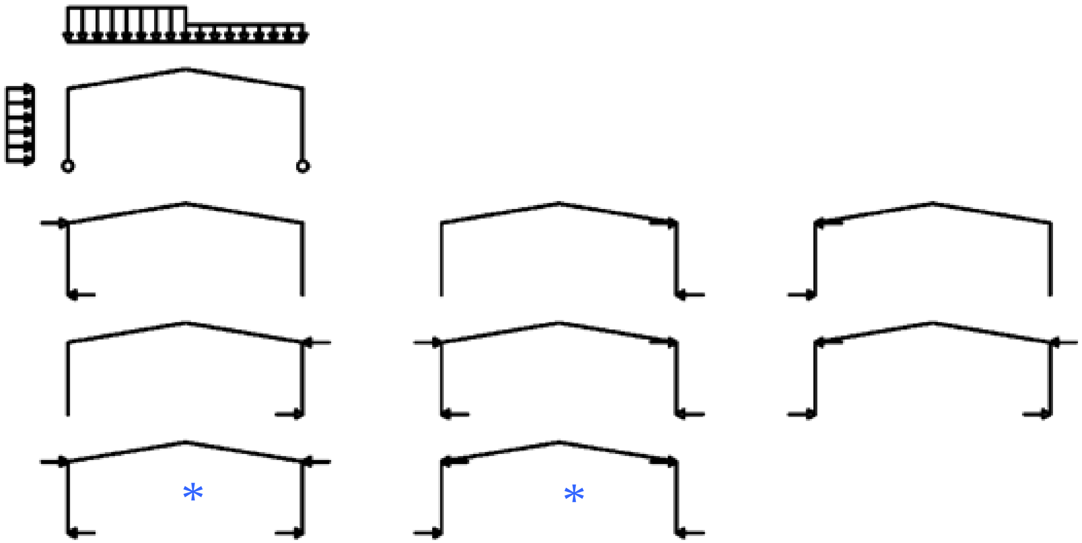
Different combinations of bow imperfection load (8 combinations for presented structure);

Simultaneously applying of opposite directions of loads from imperfections is not recommended;

Simultaneously applying of sway and bow imperfections is not recommended.

Procedure "B"

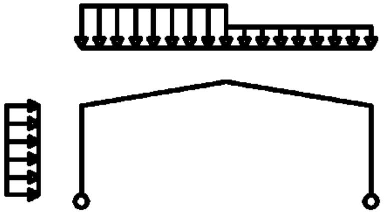
Photo: Author



Different combinations of sway imperfection load (8 combinations for presented structure);

Simultaneously applying of opposite directions of loads from sway imperfections is not recommended.

Procedure "C"



„Normal” loads are taken into consideration only.

Photo: Author

Value μ for different types of members:

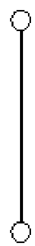
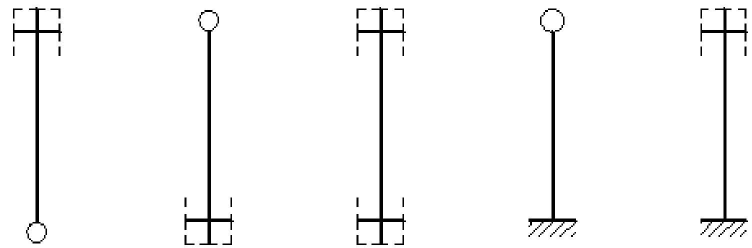
Type of element	Column		Beam
			
Calculations	$\mu = 1,0$	Procedure "A" - no analysis of stability	$\mu = 1,0$
		Procedure "B": $\mu = 1,0$	
		Procedure "C": $\mu = ?$	

Photo: Author

There are three methods of calculation value of α^* for procedure „A”:

- Geometrically nonlinear calculations (range of large displacements) → #t / 71
- Computer calculations of α_{cr} → #t / 48 - 49
- Simplified formula of α_{cr} → #t / 48 - 49

There are four methods of calculation value of μ for procedure „C”:

- Table for single-storey frames (treated as sway) → #t / 60 - 64
- European table for multistorey frames (distinction sway / non-sway) → #t / 65 - 67
- American table for multistorey frames (distinction sway / non-sway) → #t / 68 - 69
- Computer calculations (automatically distinction sway / non-sway) → #t / 70

Single-storey frame

"Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984

($\nu = \mu$)

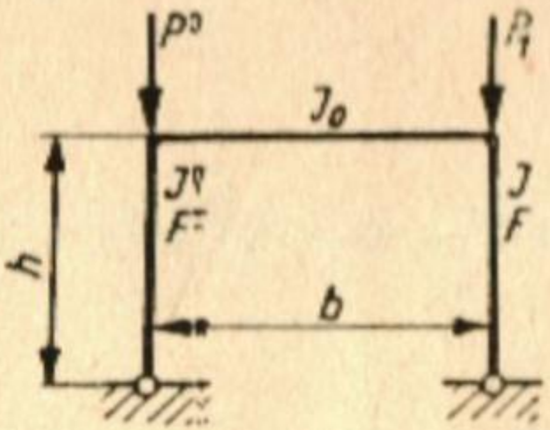
Rodzaj ramy	Oznaczenia	Współczynnik
	$n = \frac{P_1}{P}$ $c = \frac{Jb}{J_0 h}$ $s = \frac{4J}{b^2 F}$	$\sqrt{0,5(1+n)} \times$ $\times \sqrt{4+1,4(c+6s)+ +0,02(c+6s)^2}$

Photo: "Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984

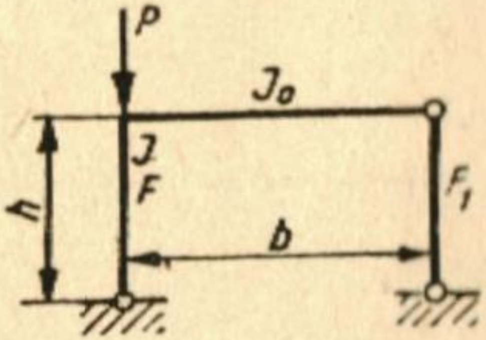
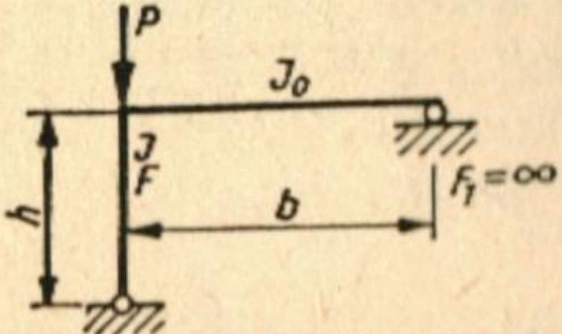
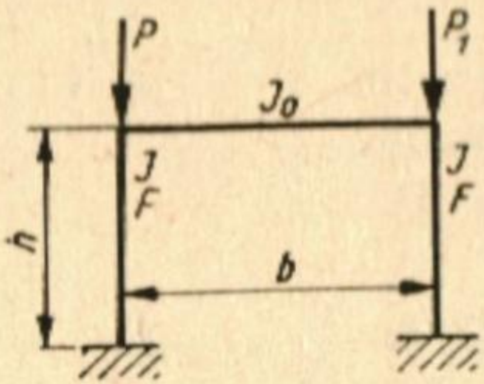
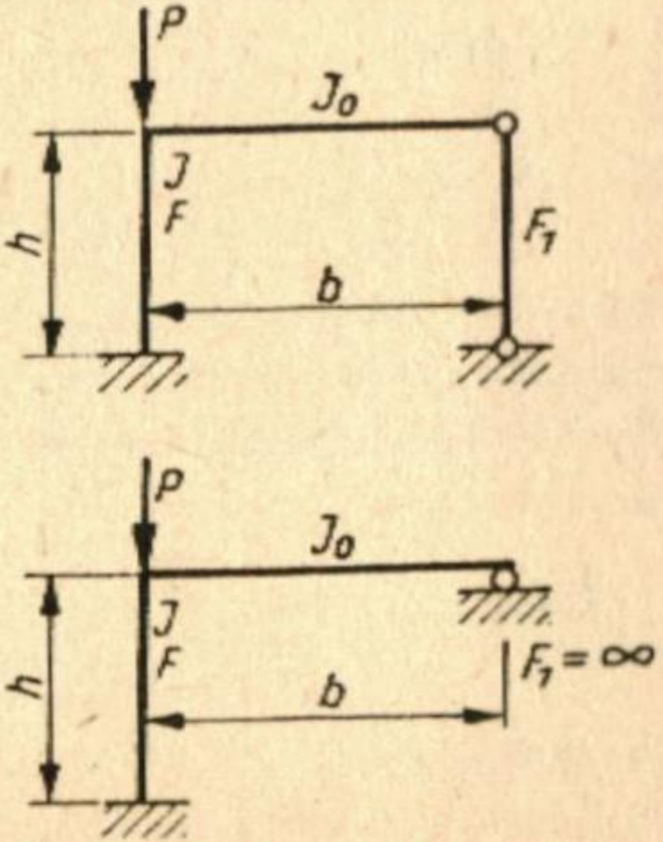
 	$c = \frac{2Jb}{J_0 h}$ $s = \frac{J}{b^2} \times$ $\times \left(\frac{1}{F} + \frac{1}{F_1} \right)$	$\sqrt{4+1{,}4(c+6s)+ +0{,}02(c+6s)^2}$
	$n = \frac{P_1}{P}$ $c = \frac{Jb}{J_0 h}$ $s = \frac{4J}{b^2 F}$	$\sqrt{0{,}5(1+n)} \times$ $\times \sqrt{1+0{,}35(c+6s)- -0{,}017(c+6s)^2}$

Photo: "Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984



$$c = \frac{2Jb}{J_0 h}$$

$$s = \frac{J}{b^2} \times$$

$$\times \left(\frac{1}{F} + \frac{1}{F_1} \right)$$

$$\sqrt{\begin{array}{l} 1 + 0{,}35(c+6s) - \\ \quad - 0{,}017(c+6s)^2 \end{array}}$$

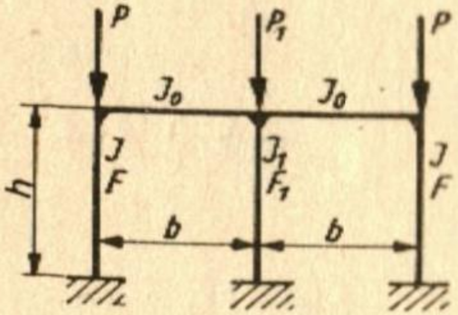
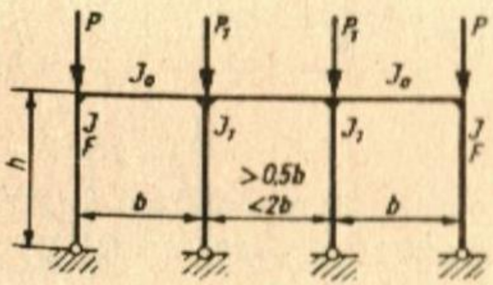
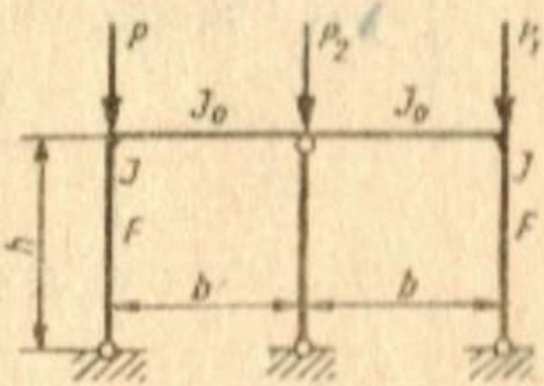
	$n = \frac{P_1}{P}$ $t = \frac{J_1}{J}$ $c = \frac{Jb}{J_0 h}$ $s = \frac{4J}{b^2 F}$ $c_n = c + \frac{9}{4} s$	dla słupów skrajnych $\frac{1+0{,}4c_n}{1+0{,}2c_n}\sqrt{\frac{2+n}{2+t}}$, dla słupów środkowych $\frac{1+0{,}4c_n}{1+0{,}2c_n}\sqrt{\frac{2+n}{2+t}}\times$ $\times\sqrt{\frac{t}{n}}$, ważne dla $\nu \leqslant 3$
		dla słupów skrajnych $\frac{1+0{,}4c_n}{1+0{,}2c_n}\sqrt{\frac{1+n}{1+t}}$, dla słupów środkowych $\frac{1+0{,}4c_n}{1+0{,}2c_n}\sqrt{\frac{1+n}{1+t}}\times$ $\times\sqrt{\frac{t}{n}}$, ważne dla $\nu \leqslant 3$

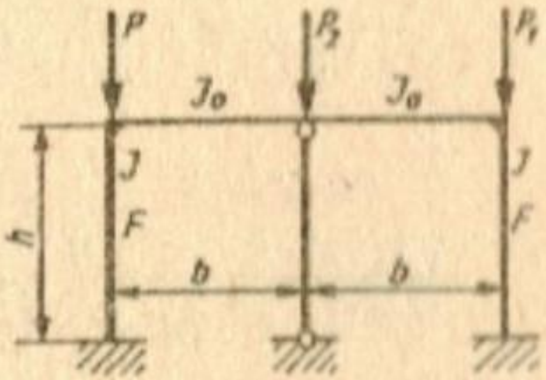
Photo: "Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984

Photo: "Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984



$$\sqrt{0,5(1+m)}\times\sqrt{4+1,40(c+6s)-0,02(c+6s)^2}\times\sqrt{1+0,48n}$$

$$n=\frac{P_2}{P}\qquad m=\frac{P_1}{P}\qquad c=\frac{Jb}{J_0h}\qquad s=\frac{4J}{b^2F}$$



$$\sqrt{0,5(1+m)}\times\sqrt{1+0,35(c+6s)-0,017(c+6s)^2}\times\sqrt{1+0,43n}$$

European method, multistorey frame

Old Polish standard PN-B 3200 and many National Appendixes to EN 1993-1-1

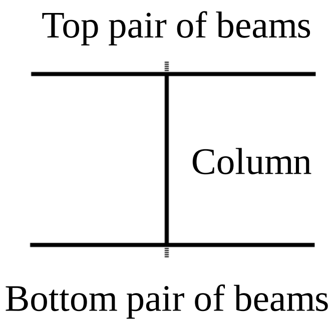
Stiffness of the node

$$\kappa = \max \left[0{,}3 \quad ; \quad K_C / (K_C + K_0) \right]$$

$$K_C = J_C / h$$

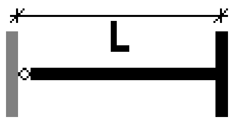
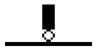
$$K_0 = \Sigma \left(\eta \, J_B / L \right)$$

Photo: Author

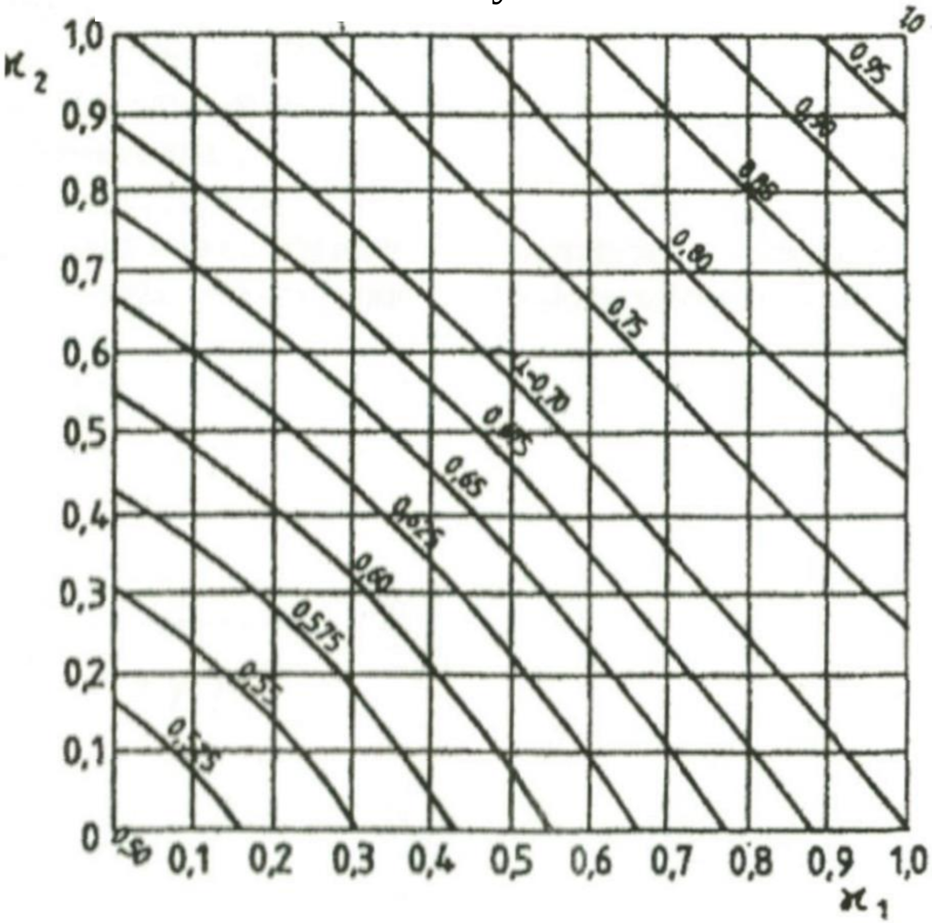


- beams are taken into consideration for both nodes;
- parts of column over top pair of beams (higher part) and under bottom pair of beams (lower part), are not taken into consideration;

Photo: Author

Influence of beam (analysed column, opposite end of beam):	η	
	Braced (non-sway) fram	Non-braced (sway) frame
 	1,5	0,5
	2,0	1,0
	0	
	$K_0 = 0,1\ K_C$	
	$K_0 = K_C$	

Non-sway frame



Sway frame

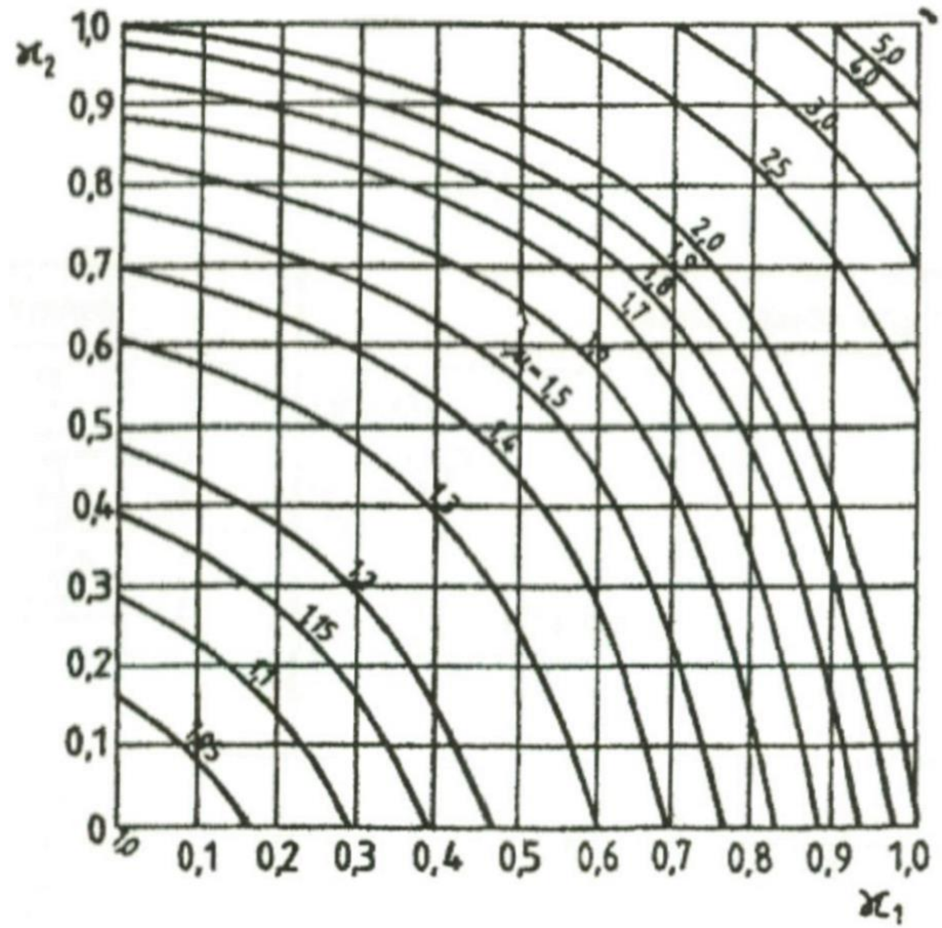
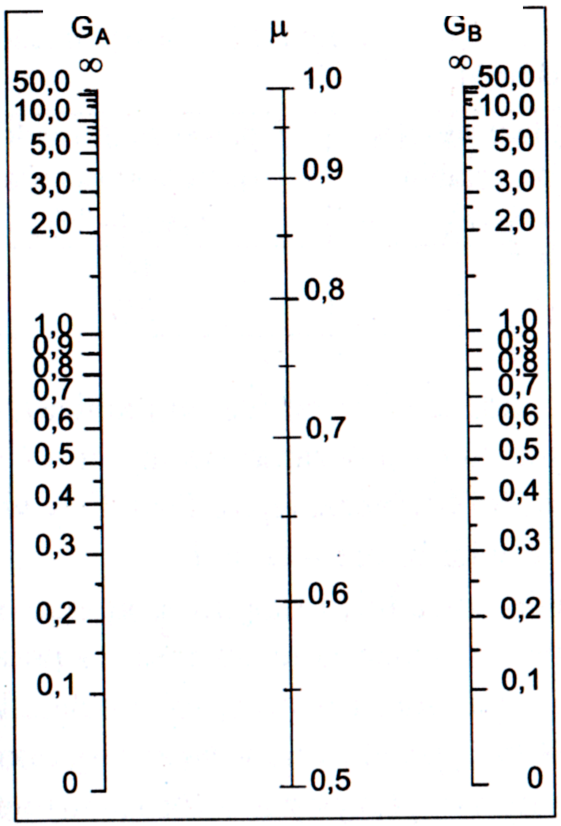
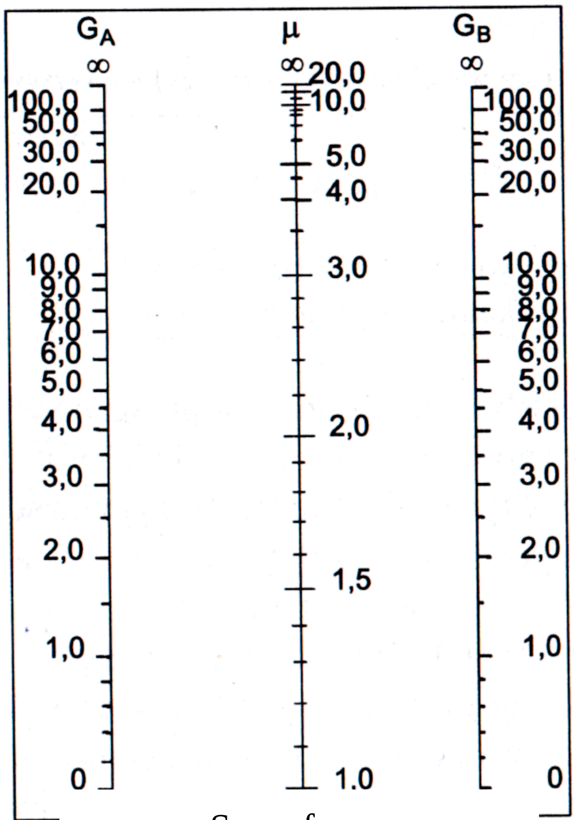


Photo: PN-B 3200 fig. Z1-3

American method, multistorey frame

Non-sway frame



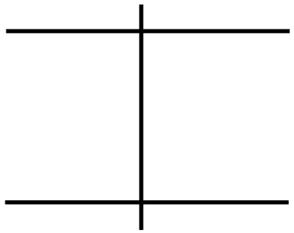


Sway frame

Photo: M. Łubiński, W. Żółtowski "Konstrukcje metalowe", Arkady, Warszawa 2000

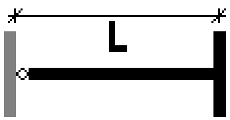
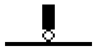
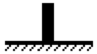
Siffeness of the node

$$G = [\Sigma\ (J_C / h)] / [\Sigma\ (\eta\ J_B / L)]$$



- beams are taken into consideration for both nodes;
- higher and lower part of column are taken into consideration;

Photo: Author

Influence of beam (analysed column, opposite end of beam):	η	
	Braced (non-sway) fram	Non-braced (sway) frame
	1,500	0,500
	2,000	0,667
	0	
	G = 10,000	
	G = 1,000	

Computer calculations

There are various results of computer calculations for various computer programme. Generally, there are four types of results:

- **N_{cr} (critical force)**;
- **μ (buckling length factor)**;
- **L_{cr} (critical length)**;
- **α_{cr} (critical instability factor)**

$$L_{cr} = H_{column}\,\mu$$

$$N_{cr} = N_{Ed}\,\alpha_{cr}$$

According to algorithm, presented in Lec # 5 / 41, slenderness of column is defined as

$$\lambda_k = (L_{cr,\,k} / i_k)\,(1 / \lambda_1) \quad ; \quad \lambda_1 = 93{,}9\,\varepsilon \quad ; \quad k = y, z$$

or as

$$\lambda_k = \sqrt{(A_{(eff)}\,f_y / N_{cr,\,k})} \quad ; \quad k = y, z$$

Each result from nummerical analysis could be used in formulas of Eurocode.

Second order effects in nummerical analysis are taken into consideration by geometrically nonlinear analysis.

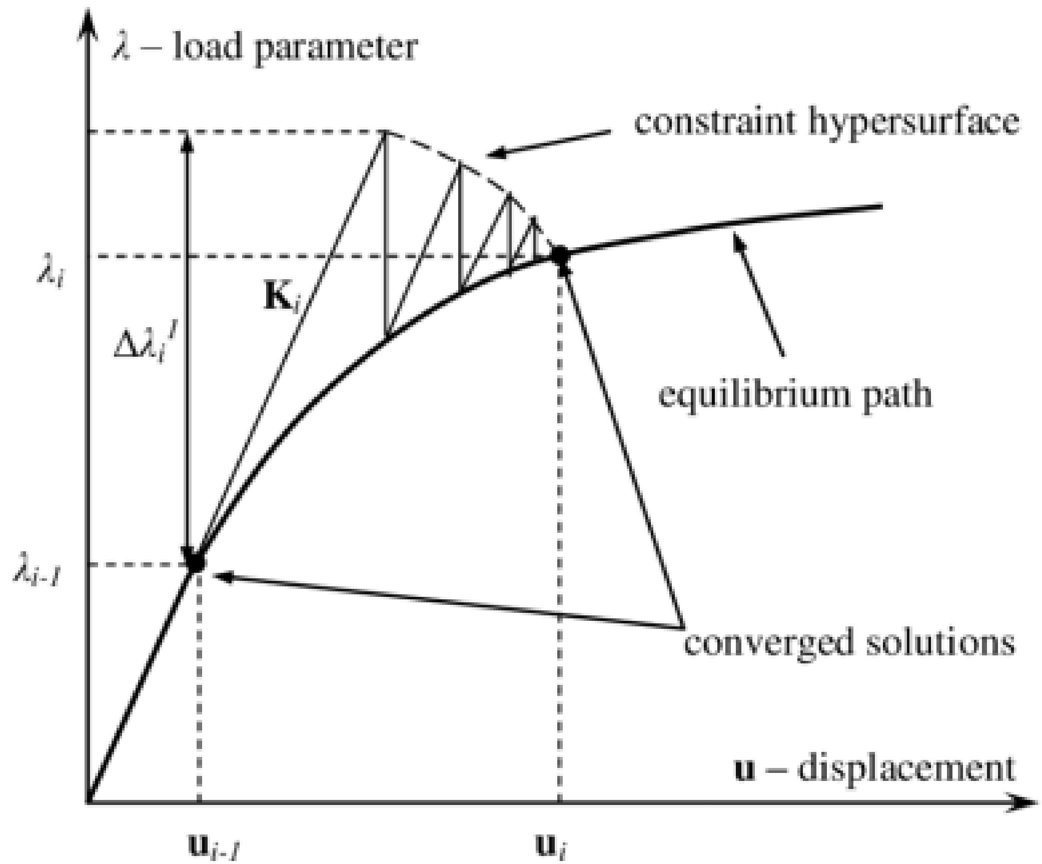


Photo: Simulation of beam-column stability with automatic strain incrementation, J. Szalai, F. Papp, Conference: Proceedings of the sixth conference on Computational structures technology

Lateral buckling

Loss of stability during bending; simultaneous torsional deformation and flexural deformation about the weak axis.

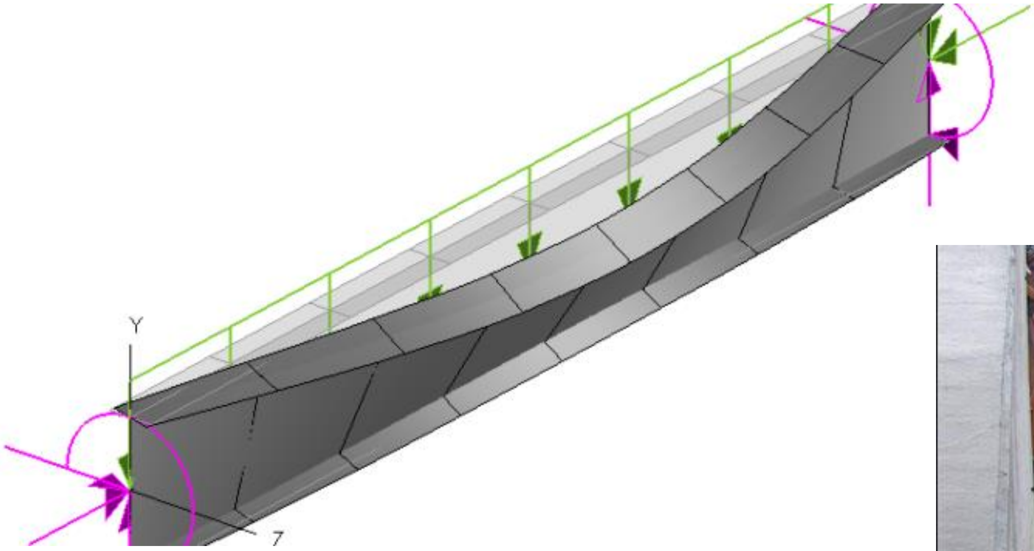


Photo: ssrcweb.org

Photo: civildigital.com

Torsional deformation is important the same for torsional buckling and torsional-flexural buckling.

Photo: resources.scia.net

Hinged or rigid support of base prevents both types of deformation at the bottom end of column.

The same – because of cooperation with beams and girders – ocurs in each poit of connection column-girder.

Photo: j-p.com.ua

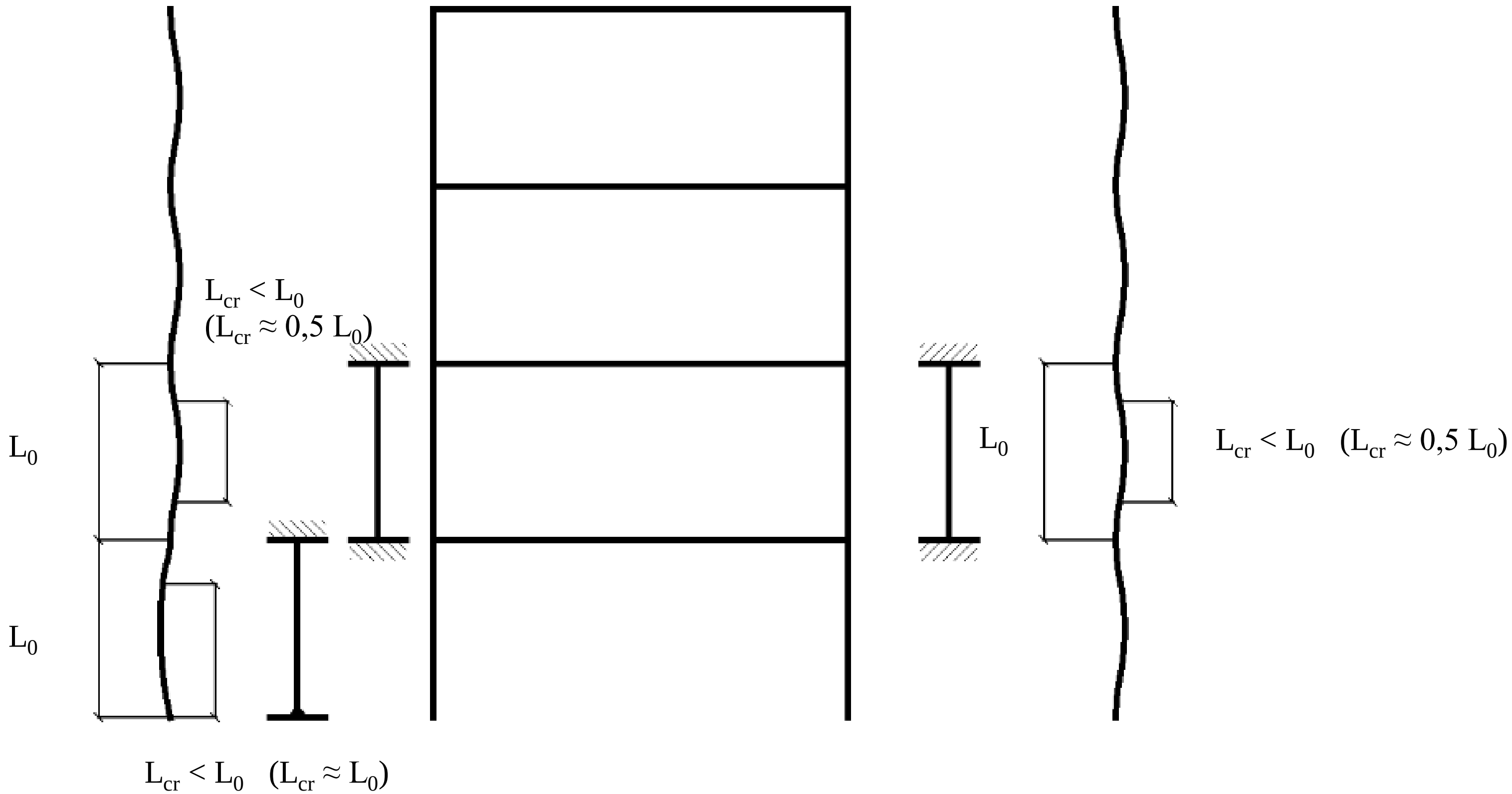
Photo: srt251fpaler.blogspot.com

Photo: moellerengineering.com

Flexural mode instability

Torsional mode instability

(angle of cross-section rotation)



In both cases, because of safety, could be taken $L_{cr} = L_0$

Photo: Author

The most often case for column: compression for internal flange. Compression of flange is very important for lateral buckling; bracings (if are necessary) should be applied to internal flanges.

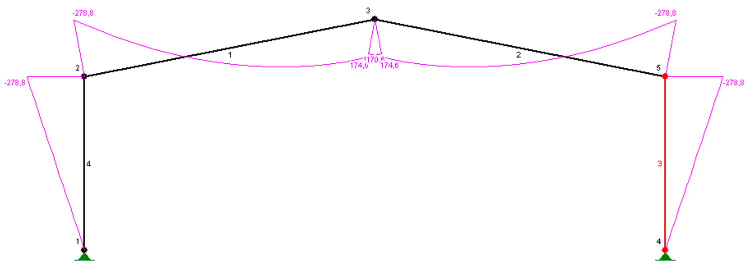


Photo: Author

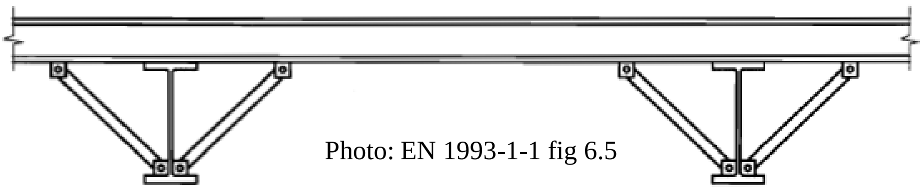
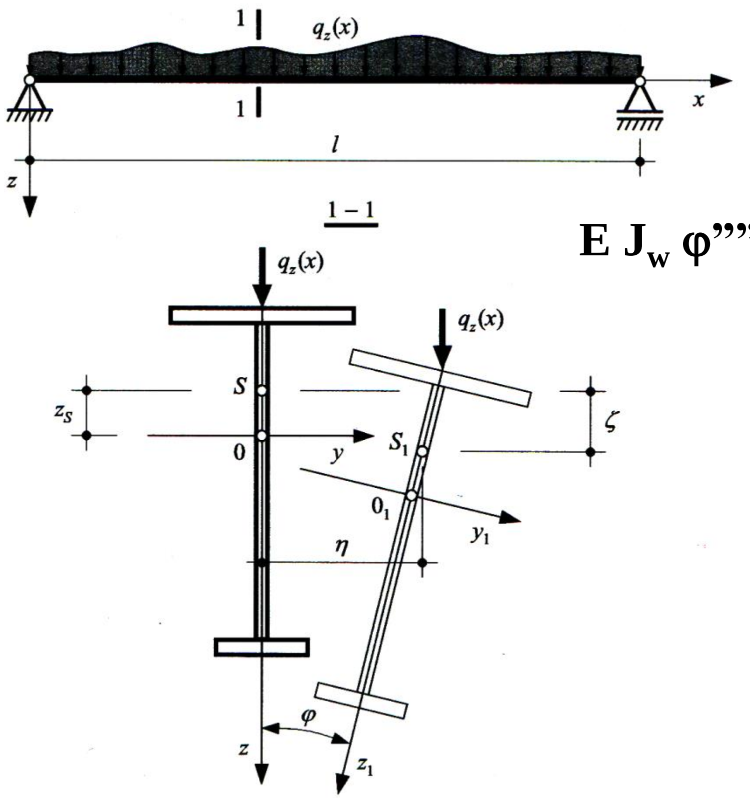


Photo: EN 1993-1-1 fig 6.5

Photo: nowbuildings.com.au

Anti lateral bracings for columns could be applied in the same way as for roof girders:

- purlins and bar bracings for roof girders;
- wall girts and bar bracings for columns.



Rys: K. Rykaluk, Zagadnienia stateczności konstrukcji metalowych, DWE 2012

Full formula for analisis of lateral buckling:

$$\mathbf{E\,J_z\,\eta'''' + (M_y\,\varphi)'' = 0}$$

$$\mathbf{E\,J_w\,\varphi'''' - [(2\,\beta_z\,M_y + G\,J_T)\,\varphi']' + q_z\,(e_z - z_s)\,\varphi + M_y\,\eta'' = 0}$$

$$\mathbf{\beta_z = \{\int_A [\,z\,(y^2 + z^2)\,dA] - z_s\} / (2\,J_y)}$$

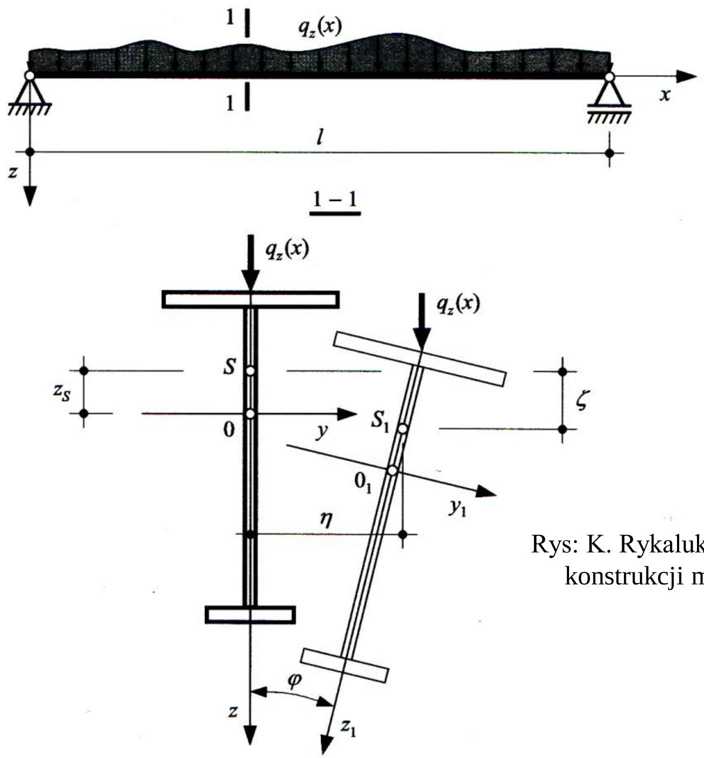
Simplified formula:

$$\mathbf{M_{cr} = i_s\,\sqrt{}\,(N_{cr,\,z}\,N_{cr,\,T})}$$

$$\mathbf{N_{cr,\,z} = \pi^2\,EJ_z / (\mu_z\,l_{0z})^2}$$

$$\mathbf{N_{cr,\,T} = [\pi^2\,EJ_w / (\mu_T\,l_{0T})^2 + GJ_t] / i_s^2}$$

Critical lengths of **flexural** and **torsional** instability are in both formulas analised separately: in functions **η(x)** and **φ(x)** in full formula, or by critical length factors μ_z and μ_T in simplified one. There is **NO** common critical length for lateral buckling; always must be separately analised **two critical lengths**.



Rys: K. Rykaluk, Zagadnienia stateczności konstrukcji metalowych, DWE 2012

Interactions of various modes of instability

Full derivative formulas for interaction of axial force and bending moments is very complicated (change in comparison to „pure" lateral buckling, without axial force):

$$E\,J_z\,\eta'''' + (M_y\,\varphi)'' - [N\,(\eta' + \varphi'\,z_s)]' = 0$$

$$E\,J_z\,\zeta'''' + (M_z\,\varphi)'' - [N\,(\zeta' - \varphi'\,y_s)]' = 0$$

$$E\,J_w\,\varphi'''' - [(2\,\beta_z\,M_y + G\,J_T)\,\varphi' + r_s^2\,N - 2\,\beta_z\,M_z]' + [q_y\,(e_y - y_s) + q_z\,(e_z - z_s)]\,\varphi +$$

$$+ M_y\,\eta'' - z_s(N\,\eta')' + y_s(N\,\zeta')' = 0$$

$$\beta_z = \{ \int_A [\,z\,(y^2 + z^2)\,dA] - z_s \} / (2\,J_y)$$

Solution os this problem is possible only in few specific cases. Solution presented in Eurocode bases not on accuracte derivative description, but on old-fashion method – extension of cross-section resistance problem:

$$N_{Ed} / A + M_{Ed,\, y} / W_y + M_{Ed,\, z} / W_z \leq f_y$$

Which could be presented as:

$$n + m_y + m_z \leq 1{,}0$$

n, m_x, m_y – efforts under axial force and bending moments.

Stability is presented, in analogy, as:

$$\mathbf{n / \chi_{x.\, y} + k_{ij}\, m_y / \chi_{LT} + k_{ij}\, m_z \leq 1{,}0}$$

$\chi_{x.\, y}$, χ_{LT} – flexural buckling fasctors, lateral buckling factor;
k_{ij} – interactions factors.

Buckling factos χ_i are calculated based on value buckling length factor (critical length factor) μ_i, (→ Lec #5).

Interaction of flexural – lateral buckling is analised for method "B" and "C" by three methods:

"German" method → #t / 80 - 85

or

"French" method → #t / 80 - 81, 86 - 90

or

"Polish" method → #t / 80,   91 - 92

Methods comes from old national Standards, have used before Eurocodes.

"German" (German and Austrian old national Standards) method (EN 1993-1-1, App. B) – recommended, simpler than "French".

"French" (French and Belgian old national Standards) method (EN 1993-1-1, App. A) – more complicated calculations in comparison to "German" method.

"Polish" (Polish old national Standard PN B 3200 for Metal Structures) method (EN 1993-1-1, NA 20) – quick but inaccurate; effort calculated according to "Polish" method is oftem bigger than according to "German" method – cross-section designed according to "Polish" method is oversized and uneconomical.

"French" and "German" methods:

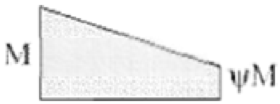
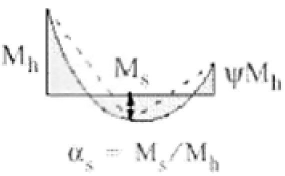
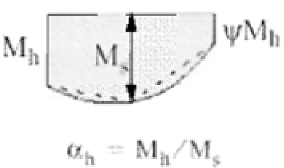
$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) + \\ + k_{yz} (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk} / \gamma_{M1}) \leq 1,0$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + k_{zy} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) + \\ + k_{zz} (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk} / \gamma_{M1}) \leq 1,0$$

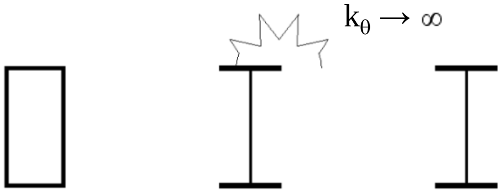
EN 1993-1-1 (6.61), (6.62)

	k_{yy} , k_{yz} , k_{zy} , k_{zz}
"French" method	EN 1993-1-1 App. A, table A1, A2
"German" method	EN 1993-1-1 App. B, table B1, B2, B3

"German" method EN 1993-1-1 tab. B.3

<table>
<tr><th rowspan="2">Moment diagram</th><th rowspan="2" colspan="2">Range</th><th colspan="2">C_{my} and C_{mz} and C_{mLT}</th></tr>
<tr><th>Uniform loading</th><th>Concentrated loading</th></tr>
<tr><td></td><td colspan="2">$-1 \leq \Psi \leq 1$</td><td colspan="2">max $(0{,}6 + 0{,}4\,\Psi \;;\; 0{,}4)$</td></tr>
<tr><td rowspan="3"></td><td>$0 \leq \alpha_s \leq 1$</td><td>$-1 \leq \Psi \leq 1$</td><td>max $(0{,}2 + 0{,}8\,\alpha_s \;;\; 0{,}4)$</td><td>max $(0{,}2 + 0{,}8\,\alpha_s \;;\; 0{,}4)$</td></tr>
<tr><td rowspan="2">$-1 \leq \alpha_s < 0$</td><td>$0 \leq \Psi \leq 1$</td><td>max $(0{,}1 - 0{,}8\,\alpha_s \;;\; 0{,}4)$</td><td>max $(-\,0{,}8\,\alpha_s \;;\; 0{,}4)$</td></tr>
<tr><td>$-1 \leq \Psi < 0$</td><td>max $[\,0{,}1\,(1 - \Psi) - 0{,}8\,\alpha_s \;;\; 0{,}4)$</td><td>max $[\,0{,}2\,(-\Psi) - 0{,}8\,\alpha_s \;;\; 0{,}4)$</td></tr>
<tr><td rowspan="3"></td><td>$0 \leq \alpha_h \leq 1$</td><td>$-1 \leq \Psi \leq 1$</td><td>$0{,}95 + 0{,}05\,\alpha_h$</td><td>$0{,}9 + 0{,}1\,\alpha_h$</td></tr>
<tr><td rowspan="2">$-1 \leq \alpha_h < 0$</td><td>$0 \leq \Psi \leq 1$</td><td>$0{,}95 + 0{,}05\,\alpha_h$</td><td>$0{,}9 + 0{,}1\,\alpha_h$</td></tr>
<tr><td>$-1 \leq \Psi < 0$</td><td>$0{,}95 + 0{,}05\,\alpha_h\,(1 + 2\,\Psi)$</td><td>$0{,}9 - 0{,}1\,\alpha_h\,(1 + 2\,\Psi)$</td></tr>
<tr><td colspan="5">For members with sway buckling mode, the equivalnet uniform moment factor should be taken $C_{my} = 0{,}9$ or $C_{mz} = 0{,}9$ respectively.</td></tr>
<tr><td colspan="5">C_{my}, C_{mz} and C_{mLT} should be obtained according to the bending moment between the relevant braced points as follows:
Moment factor: bending axis: points braced in direction:
C_{my} y-y z-z
C_{mz} z-z y-y
C_{mLT} y-y y-y</td></tr>
</table>

For **"German" method**, members are divided into susceptible to torsional deformations and not susceptible to torsional deformations.

Cross-section	Comments
	No lateral buckling, no need for buckling interaction analysis
 Short member: $1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] \geq 1{,}0$	Member not susceptible to torsional deformations, EN 1993-1-1 tab. B.1
 $1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] < 1{,}0$	Member susceptible to torsional deformations, EN 1993-1-1 tab. B.2

EN 1993-1-1 tab. B.1

"German" method, interaction factors

$$n_y = N_{Ed}\,\gamma_{M1} / (\chi_y\, N_{Rd})$$

$$n_z = N_{Ed}\,\gamma_{M1} / (\chi_z\, N_{Rd})$$

Interaction factors	Cross-section	I^{st}, II^{nd} class	III^{rd}, IV^{th} class
k_{yy}	I, H, RHS	$C_{my} \cdot \min\{1 + 0{,}6\,\lambda_y\, n_y \;;\; 1 + 0{,}6\, n_y\}$	$C_{my} \cdot \min\{1 + (\lambda_y - 0{,}2)\, n_y \;;\; 1 + 0{,}8\, n_y\}$
k_{yz}	I, H, RHS	$0{,}6\, k_{zz}$	k_{zz}
k_{zy}	I, H, RHS	$0{,}6\, k_{yy}$	$0{,}8\, k_{yy}$
k_{zz}	I, H	$C_{mz} \cdot \min\{1 + (2\,\lambda_z - 0{,}6)\, n_z \;;\; 1 + 1{,}4\, n_z\}$	$C_{mz} \cdot \min\{1 + 0{,}6\,\lambda_z\, n_z \;;\; 1 + 0{,}6\, n_z\}$
	RHS	$C_{mz} \cdot \min\{1 + (\lambda_z - 0{,}2)\, n_z \;;\; 1 + 0{,}8\, n_z\}$	

For I- and H-sections and RHS, under axial compression and unaxial bending $M_{y,\,ED}$, the coefficient k_{zy} may be = 0

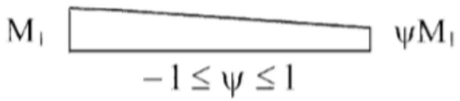
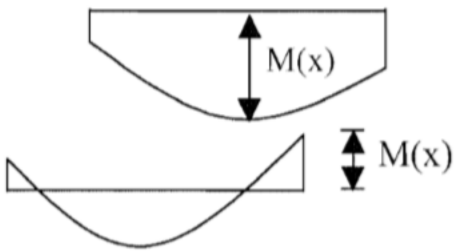
EN 1993-1-1 tab. B.2

"German" method,
interaction factors

$n_y = N_{Ed}\,\gamma_{M1} / (\chi_y\, N_{Rd})$ $n_z = N_{Ed}\,\gamma_{M1} / (\chi_z\, N_{Rd})$ $C = 0{,}05 / (C_{mLT} - 0{,}25)$

Interaction factors	I^{st}, II^{nd} class	III^{rd}, IV^{th} class
k_{yy}	The same as in Tab. B1	
k_{yz}		
k_{zy}	$\lambda_z \geq 0{,}4$: min $\{1 - 2\,C\,\lambda_z\,n_z$; $1 - 2\,C\,n_z\}$	min $\{1 - C\,\lambda_z\,n_z$; $1 - C\,n_z\}$
	$\lambda_z < 0{,}4$: min $\{0{,}6 - \lambda_z$; $1 - 2\,\lambda_z\,C\,n_z\}$	
k_{zz}	The same as in Tab. B1	

"French" method EN 1993-1-1 tab. A.2

Moment diagram	$C_{mi,0}$ (i = y, z)
	$0{,}79 + 0{,}21\,\Psi_i + 0{,}36(\Psi_i - 0{,}33)(N_{Ed} / N_{cr,i})$
	$1 + \{[\pi^2 E\, J_i\, \lvert\delta_x\rvert / L^2\, \lvert M_{i,Ed}(x)\rvert] - 1\}(N_{Ed} / N_{cr,i})$ $M_{i,Ed}(x)$ = max $M_{y,Ed}$ or max $M_{z,Ed}$ δ_x = max deflection
	$1 - 0{,}18\, N_{Ed} / N_{cr,i}$
	$1 + 0{,}33\, N_{Ed} / N_{cr,i}$

"French" method EN 1993-1-1 tab. A.1, part I

$N_{cr,\,y}$, $N_{cr,\,z}$, $N_{cr,\,T}$, $N_{cr,\,z\text{-}T}$ → #5 / 35

$\varepsilon_y = (M_{y,\,Ed}\ A) / (N_{Ed}\ W_{el,\,y})$ – I[st], II[nd], III[rd] class of cross-section

$\varepsilon_y = (M_{y,\,Ed}\ A_{eff}) / (N_{Ed}\ W_{eff,\,y})$ – IV[th] class of cross-section

$C_1 = 1 / k_c^2$

k_c → #5 / 71

$a_{LT} = \max\,(1 – J_t / J_y\ \ ;\ \ 0)$

λ_0 – slenderness for lateral buckling, M = const (→ #5 / 61)

λ_{LT} – slenderness for lateral buckling for real distribution of bending moment

$\lambda_{max} = \max\,(\lambda_y\ \ ; \lambda_z)$

$n_y = N_{Ed} / N_{cr,\,y}$

$n_z = N_{Ed} / N_{cr,\,z}$

$n_T = N_{Ed} / N_{cr,\,T}$

$n_{z\text{-}T} = N_{Ed} / N_{cr,\,z\text{-}T}$

"French" method EN 1993-1-1 tab. A.1, part II

$$\lambda_{comp} = 0{,}2\ (\sqrt{C_1})\ \{\sqrt[4]{[(1 - n_z)\ (1 - n_{z\text{-}T})]}\}$$

$\lambda_0 \leq \lambda_{comp}$	$\lambda_0 > \lambda_{comp}$
$C_{my} = C_{my,\,0}$ $C_{mz} = C_{mz,\,0}$ $C_{mLT} = 1{,}0$	$C_{my} = C_{my,\,0} + (1 - C_{my,\,0})\ (a_{LT}\ \sqrt{\varepsilon_y})\ /\ (1 + a_{LT}\ \sqrt{\varepsilon_y})$ $C_{mz} = C_{mz,\,0}$ $C_{mLT} = \max\ (1{,}0\ \ ;\ \ C_{my}^2\ a_{LT}\ /\ \{\sqrt{[(1 - n_z)\ (1 - n_{z\text{-}T})]}\}\ )$

$$m_y = M_{y,\,Ed}\ /\ M_{pl,\,y,\,Rd}$$

$$m_z = M_{z,\,Ed}\ /\ M_{pl,\,z,\,Rd}$$

$$w_y = \min\ (W_{pl,\,y}\ /\ W_{el,\,y}\ \ ;\ \ 1{,}5)$$

$$w_z = \min\ (W_{pl,\,z}\ /\ W_{el,\,z}\ \ ;\ \ 1{,}5)$$

$$n_{pl} = N_{Ed}\ /\ N_{Rd}$$

$$b_{LT} = 0{,}5\ a_{LT}\ \lambda_0^2\ m_y\ m_z\ /\ \chi_{LT}$$

$$c_{LT} = 10\ a_{LT}\ \lambda_0^2\ m_y\ /\ [\chi_{LT}\ C_{my}\ (5 + \lambda_z^4)]$$

"French" method EN 1993-1-1 tab. A.1, part III

$$d_{LT} = 2\, a_{LT}\, \lambda_0\, m_y\, m_z / [\chi_{LT}\, C_{my}\, C_{mz}\, (0{,}1 + \lambda_z^4)]$$

$$e_{LT} = 1{,}7\, a_{LT}\, \lambda_0\, m_y / [\chi_{LT}\, C_{my}\, (0{,}1 + \lambda_z^4)]$$

$$C_{yy} = \max\, (1 + (w_y + 1)\, \{[2 - 1{,}6\, C_{my}^2\, (\lambda_{max} + \lambda_{max}^2) / w_y]\, n_{pl} - b_{LT}\} \quad ; \quad 1 / w_y)$$

$$C_{yz} = \max\, (1 + (w_y - 1)\, \{[2 - (14\, C_{my}^2\, \lambda_{max}^2) / w_z^5]\, n_{pl} - c_{LT}\} \quad ; \quad 0{,}6\, \sqrt{[1 / (w_y\, w_z)]}\,)$$

$$C_{zy} = \max\, (1 + (w_y - 1)\, \{[2 - (14\, C_{my}^2\, \lambda_{max}^2) / w_z^5]\, n_{pl} - d_{LT}\} \quad ; \quad 0{,}6\, \sqrt{[1 / (w_y\, w_z)]}\,)$$

$$C_{zz} = \max\, (1 + (w_z - 1)\, \{[2 - 1{,}6\, C_{mz}^2\, (\lambda_{max} + \lambda_{max}^2) / w_z - e_{LT}]\, n_{pl}\} \quad ; \quad 1 / w_z)$$

$$\mu_y^* = (1 - n_y) / (1 - \chi_y\, n_y)$$

$$\mu_z^* = (1 - n_z) / (1 - \chi_z\, n_z)$$

"French" method EN 1993-1-1 tab. A.1, part IV

Interaction factors	I^{st}, II^{nd}, III^{rd} class of cross-section	IV^{th} class of cross-section
k_{yy}	$C_{my}\, C_{mLT}\, \mu_y^{*} / (1 - n_y)$	$C_{my}\, C_{mLT}\, \mu_y^{*} / [C_{yy}\, (1 - n_y)]$
k_{yz}	$C_{mz}\, \mu_y^{*} / (1 - n_z)$	$0{,}6\, [\sqrt{(w_z / w_y)}]\, C_{mz}\, \mu_y^{*} / [C_{zy}\, (1 - n_z)]$
k_{zy}	$C_{my}\, C_{mLT}\, \mu_z^{*} / (1 - n_y)$	$0{,}6\, [\sqrt{(w_y / w_z)}] C_{my}\, C_{mLT}\, \mu_z^{*} / [C_{zy}\, (1 - n_y)]$
k_{zz}	$C_{mz}\, \mu_z^{*} / (1 - n_z)$	$C_{mz}\, \mu_z^{*} / [C_{zz}\, (1 - n_z)]$

"Polish" method

I- H- beam, RHS: EN 1993-1-1 NA.20

$$N_{Ed} / (\chi_y N_{Rk}) + C_{my} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk}) + $$
$$+ (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk}) \leq 1{,}0 - \Delta_{0, y}$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + C_{mz} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk}) + $$
$$+ (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk}) \leq 1{,}0 - \Delta_{0, z}$$

	$\Delta_{0, y}$	$\Delta_{0, z}$
Ist, IInd class of cross-section	$0{,}1 + 0{,}2 [(W_{pl, y} / W_{el, y}) - 1)]$	$0{,}1 + 0{,}2 [(W_{pl, z} / W_{el, z}) - 1)]$
IIIrd, IVth class of cross-section	0,1	

C_{my}, C_{mz} - EN 1993-1-1 App. B, table B3

"Polish" method

EN 1993-1-1 NA.20

CHS:

$$N_{Ed} / (\chi_y N_{Rk}) + \sqrt{\{ k_{yy} [(M_{y, Ed} + \Delta M_{y, Ed}) / (M_{y, Rk})]^2 + } \\ \overline{+ C_{mz} [(M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk})]^2\}} \leq 1,0$$

$$N_{Ed} / (\chi_z N_{Rk}) + \sqrt{\{ k_{zz} [(M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk})]^2 + } \\ \overline{+ C_{my} [(M_{y, Ed} + \Delta M_{y, Ed}) / (M_{y, Rk})]^2\}} \leq 1,0$$

C_{my}, C_{mz} - EN 1993-1-1 App. B, table B3

Deformations of column

It is recommended that horizontal displacements do not exceed the following limits:

• H / 150 for single-storey structures (without cranes);

• H / 500 for multistorey structures;

H - level of considered girder to the top of the foundation

EN 1993-1-1 NA.23

In case of multistorey structure, horizontal displacements should be checqued for each level, not only for top point of structure.

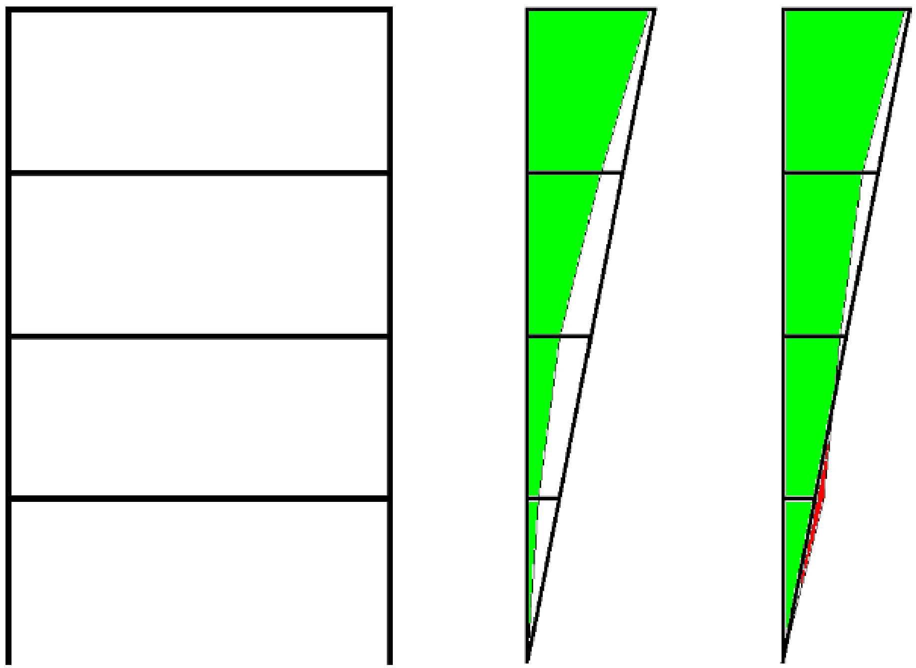


Photo: Author

Summary

There are few general part for full algorithm of calculations of column:

• Decision about method of calculation: A, B, C, C_1 (C_1 means C for single storey frame, treated as braced frame, without imperfections, without IInd order effects);

• Decision about way of calculation μ_y: T (Table), EU (European), Am (American), Co (Computer);

• Calculation of additional forces from IInd order effects and sway or bow imperfections (s.b.i, s.i) according to method A or B;

• Checking R (resistance);

• Checking interaction of various modes of instability according to method G (German), F (French) or P (Polish);

• Checking D (deformations).

There are two different algorithms: for single-storey frame and for multistorey frame.

Single-storey frame

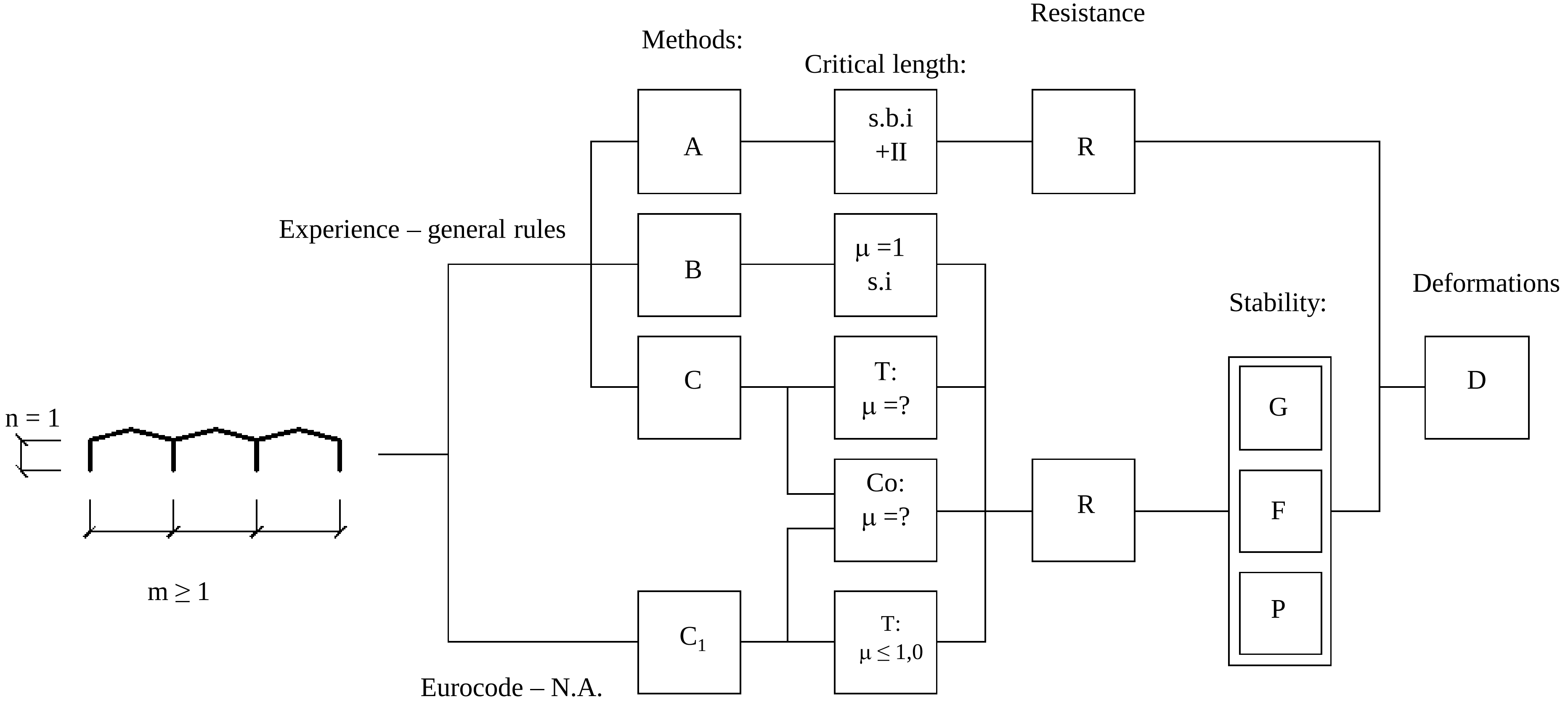


Photo: Author

Multistorey frame

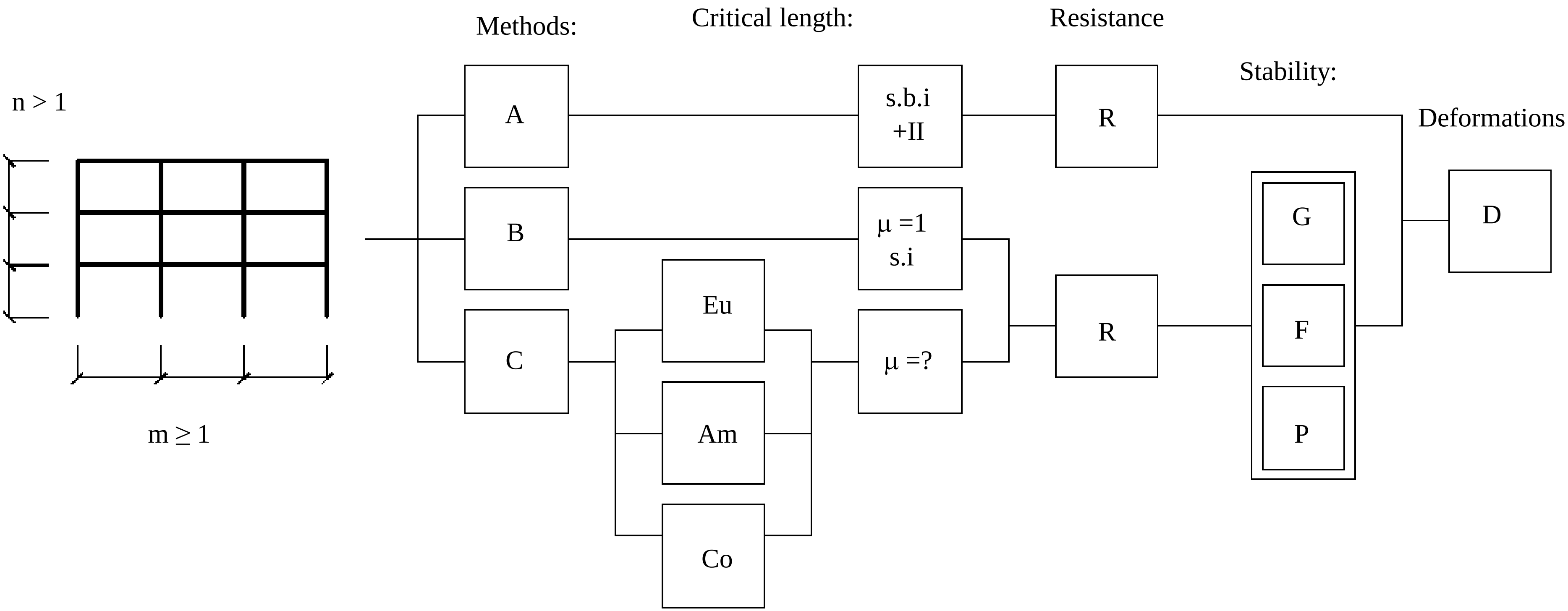


Photo: Author

Examination issues

Differences between second order effects and imperfections

Role of second order effects for stability of structure and way of calculation

Hot-rolled I-column under bi-axial bending and shear forces - algorithm of calculation

Three ways of analysis for instability of column

Thank you for attention

tmichal@pk.edu.pl