Metal Structures

Lecture XI

Hot rolled beams

Contents

<u>Cross-sectional forces</u>

act on different types of members

	N_{Ed}	M_{Ed}	V_{Ed}
Truss bar	+	(+)	(+)
Bracing bar	+	(+)	(+)
Beam	(+)	+	+
Column	+	+	+

Dependences between loads, cross-sectional forces and types of stresses

q

↓ ↓

$q(x) = dV(x)/dx$ $q(x) = d^2M(x)/dx^2$ **N**

↓ ↓ ↓

V_{Ed} ↔ $dM(x)/dx = V(x)$ ↔ **M_{Ed}** ← Second order effects ← **N_{Ed}**

↓ ↓ ↓

τ **σ**

Eurocode does not separate beams and columns. Both types of elements are treated the same: they work under bending moments, shear forces and axial force. In addition, the interactions between these forces should be considered.

In the course of the lecture:

- hot-rolled beams (mainly bended members, #11)
- welded beams (mainly bended members, #12)
- columns (mainly compressed members, #13)

will be presented separately.

Geometry of cross-section

For beams, the most important are bending moments. Because of this, steel beams have specific shape of cross-ection, analysed in Ist Laboratory.

Photo: discountsteel.com

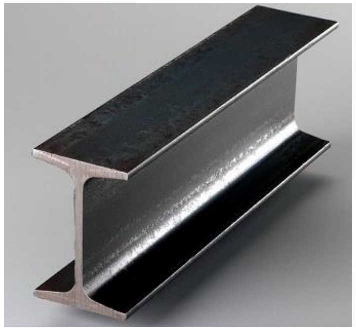


Photo: weldingweb.com

Geometrical characteristics - rectangular cross-section

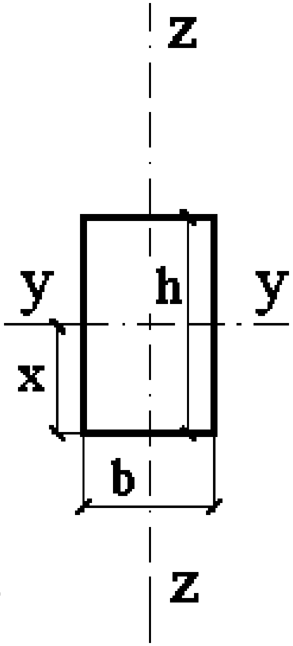


Photo: Autor

$$A = b\,h$$

$$J_y = b\,h^3 / 12$$

$$W_y = J_y / z_{max} = J_y / (0{,}5h) = b\,h^2 / 6$$

$$i_y = \sqrt{(J_y / A)} = h / (2\sqrt{3})$$

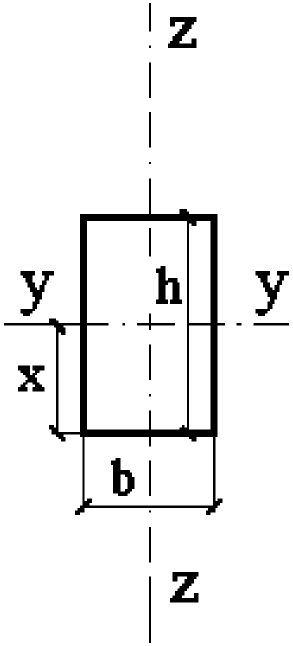


Photo: Autor

What is the best type of cross-section, if we have limitations as follow:

A → small (small dead weight)

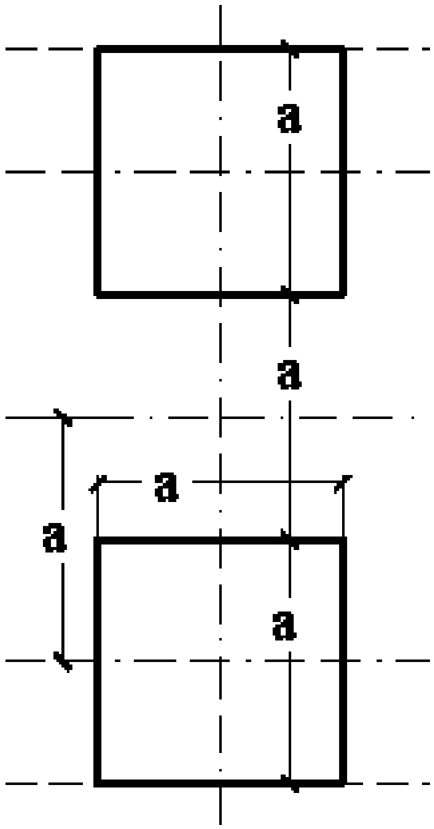
W_y → large (large resistance of cross-section)

h → limited (design limitation)

For example:

$A = 2\ a^2$

$h \leq 3a$



J_y = 2 [b h^3 / 12] + 2 [A_1 d^2]

Own stiffness Steiner's theorem

b = a h = a d = a x = 1,5 a $A_1 = a^2$

J_y = 2 [b h^3 / 12] + 2 [A_1 d^2] =

= 0,167 a^4 + 2,000 a^4 = 2,167 a^4

$W_y = J_y / z = 1{,}444\ a^3$

$i_y = \sqrt{(J_y / A)} = 1{,}041\ a$

$M_{y\ max} = 1{,}444\ a^3 f_y$

Photo: Autor

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Steiner's theorem can be used only when exist rigid connection between separated part of cross-section.

Upper and lower branch of cross-section do not cooperate each other, when there is no connecting element. In this case, load acts only on one branch of cross-section.

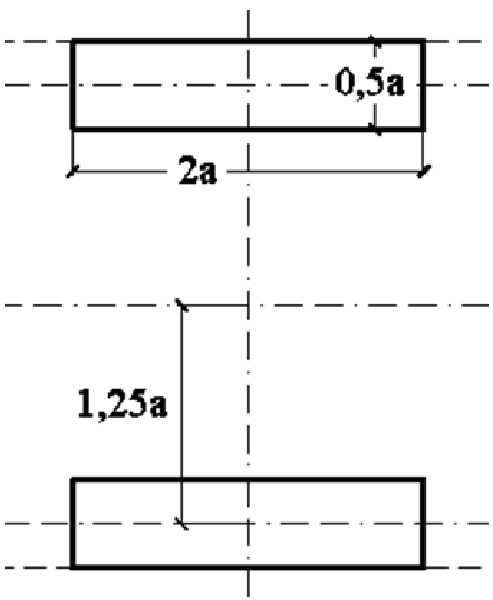


Photo: Autor

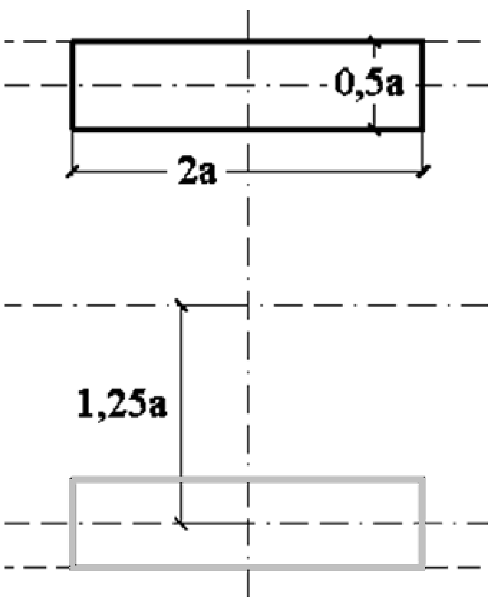
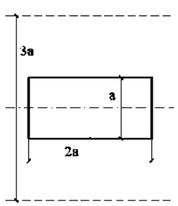
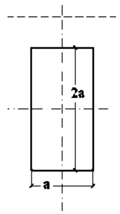
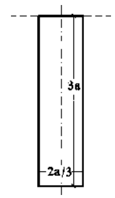
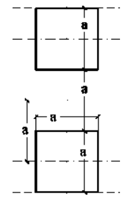
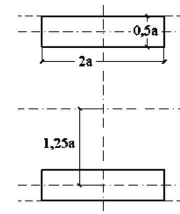
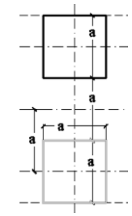
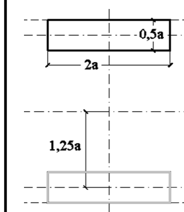


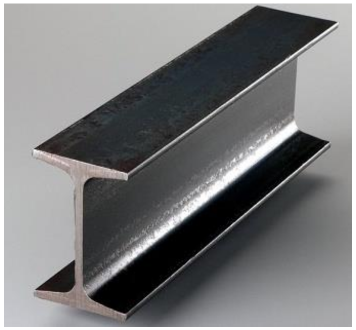
Photo: Autor

				Steiner's theorem		No steiner's theorem	
							
A	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$
J_y	0,167 a^4	0,667 a^4	1,500 a^4	2,167 a^4	3,167 a^4	0,167 a^4	0,042 a^4
W_y	0,333 a^3	0,667 a^3	1,000 a^3	1,444 a^3	2,111 a^3	0,111 a^3	0,028 a^3
i_y	0,289 a	0,577 a	0,866 a	1,041 a	1,258 a	0,289 a	0,145 a
$M_{y\ max}$	0,333 $a^3 f_y$	0,667 $a^3 f_y$	1,000 $a^3 f_y$	1,444 $a^3 f_y$	2,111 $a^3 f_y$	0,111 $a^3 f_y$	0,028 $a^3 f_y$
J_y	1,000	4,000	9,000	13,000	19,000	0,083	0,021
W_y	1,000	2,000	3,000	4,333	6,333	0,056	0,014
i_y	1,000	2,000	3,000	3,602	4,221	0,204	0,102
$M_{y\ max}$	1,000	2,000	3,000	4,333	6,333	0,056	0,014

The best shape of cross-section for bending member:

I

Photo: discountsteel.com



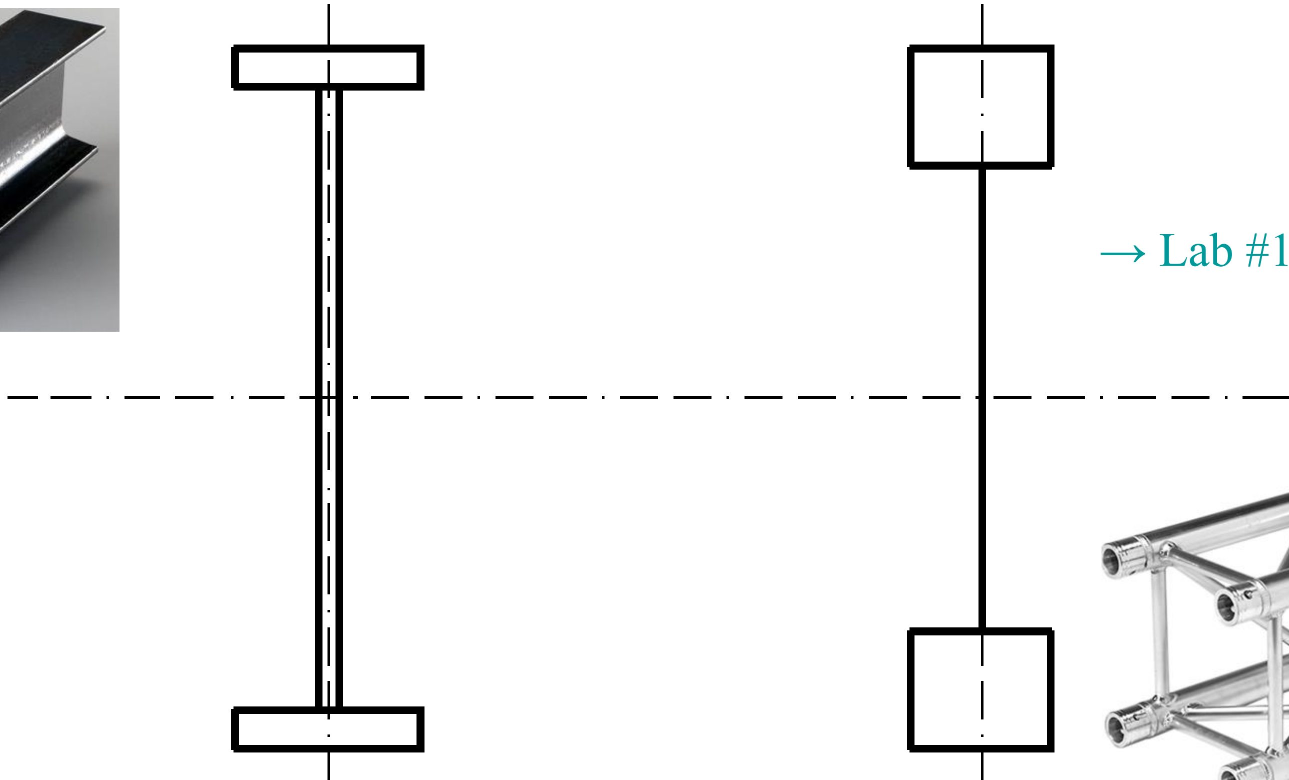


Photo: Autor

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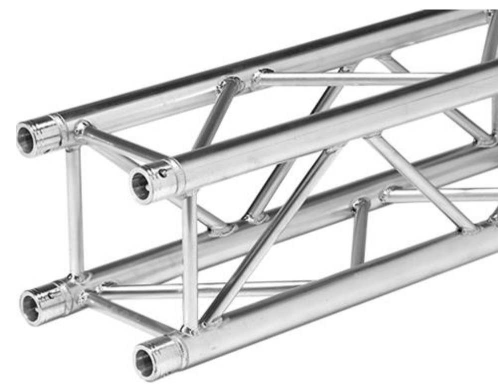

Photo: conference-truss-hire.co.uk

Cross-section of I-beam

Cross-section of truss

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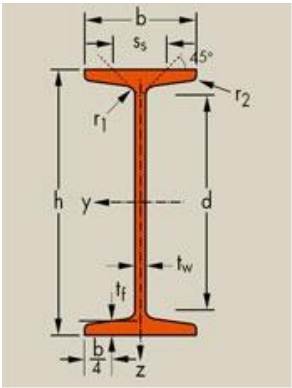
Hot rolled:

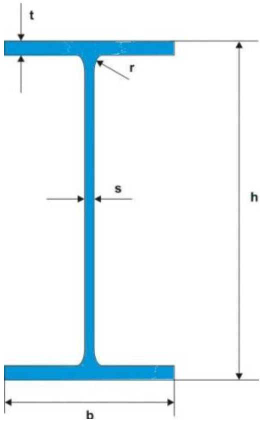
I H

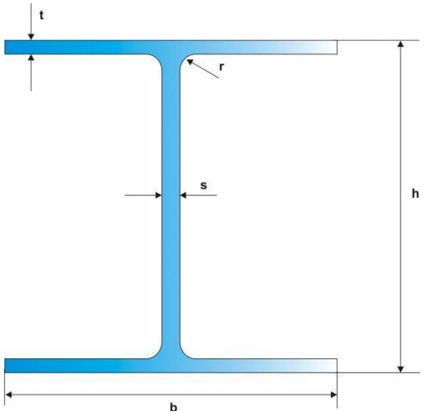
IP

HE

Photo: optimax.pl







IPN

IPE, IPE-A, IPE-AA IPE-O

HEB, HEA, HEAA, HEM

Photo: hmsteel.pl

Photo: hmsteel.pl

Welded:

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- Plane web
    - IKS
    - HKS

Photo: weldingweb.com

- Corrugated web

Photo: hxssvic.en.ec21.com

For which type of loads are recommended different types of cross-sections:

	IP	HE
N_{Ed}		👍
$M_{y,\,Ed}$	👍	
$M_{z,\,Ed}$	👎	
$M_{y,\,Ed} + M_{z,\,Ed}$	👍	👍
$N_{Ed} + M_{y,\,Ed}$	👍	👍
$N_{Ed} + M_{z,\,Ed}$	👎	👍
$N_{Ed} + M_{y,\,Ed} + M_{z,\,Ed}$		👍

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Proportion of cross-sections

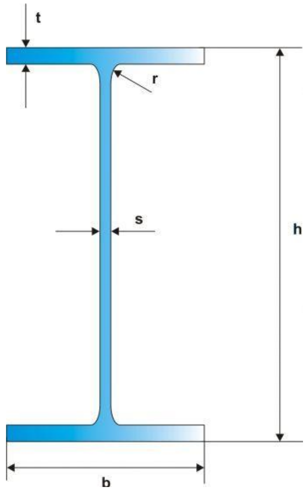
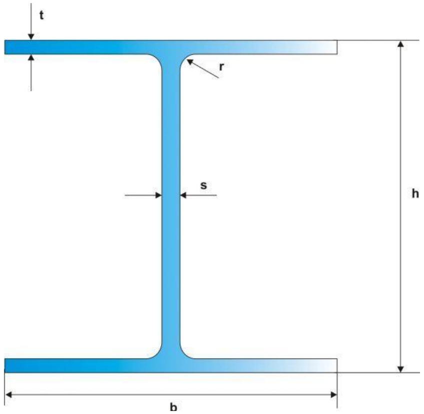


Photo: hmsteel.pl



Cross-sections	J_z / J_y	
	100 - 300	> 300
IP	~ 1 / 13	~ 1 / 13 - 1 / 30
HE	~ 1 / 3	~ 1 / 3 - 1 / 40

Photo: Setro Metal Group

Photo: Trasko-Stal Sp. z o. o.

L < 25 - 30 m → hot rolled I-beam IP

L > 25 - 30 m → welded I-beam IK

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Steel skeletons (3d frames)

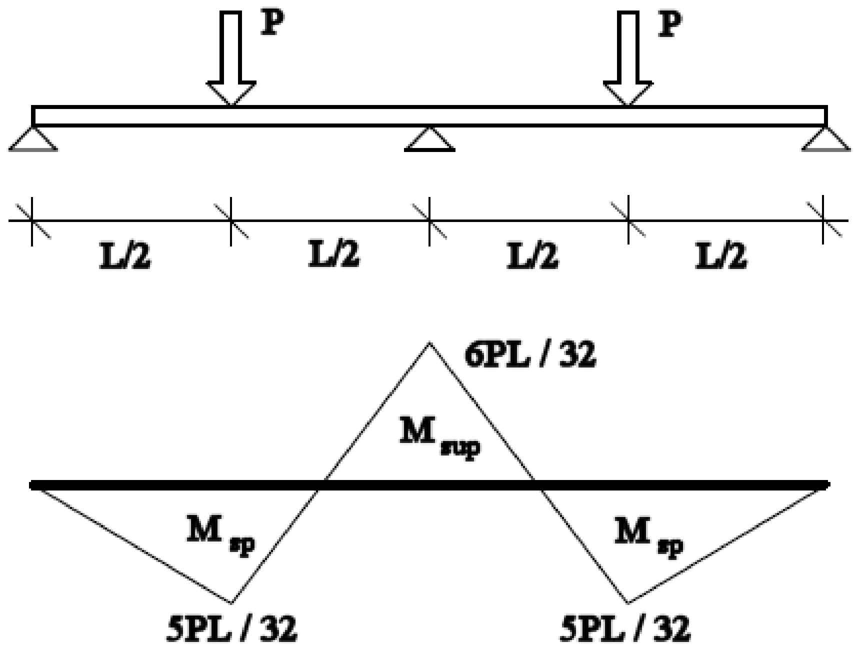
Photo: metroland.com.au

Beams, girders → hot rolled IP, welded IK

Colums → hot rolled IP, HE, welded IK, HK

Experiment

Let's make an experiment: two-span continous I-beam; forces P can change its value.



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$$M_{sup} = 6\ PL / 32$$

$$M_{sp} = 5\ PL / 32$$

$$M_{max} = M_{sup}$$

$$\sigma_{max} = \sigma(M_{sup})$$

Photo: Author

What will happened with beams of different cclasses?

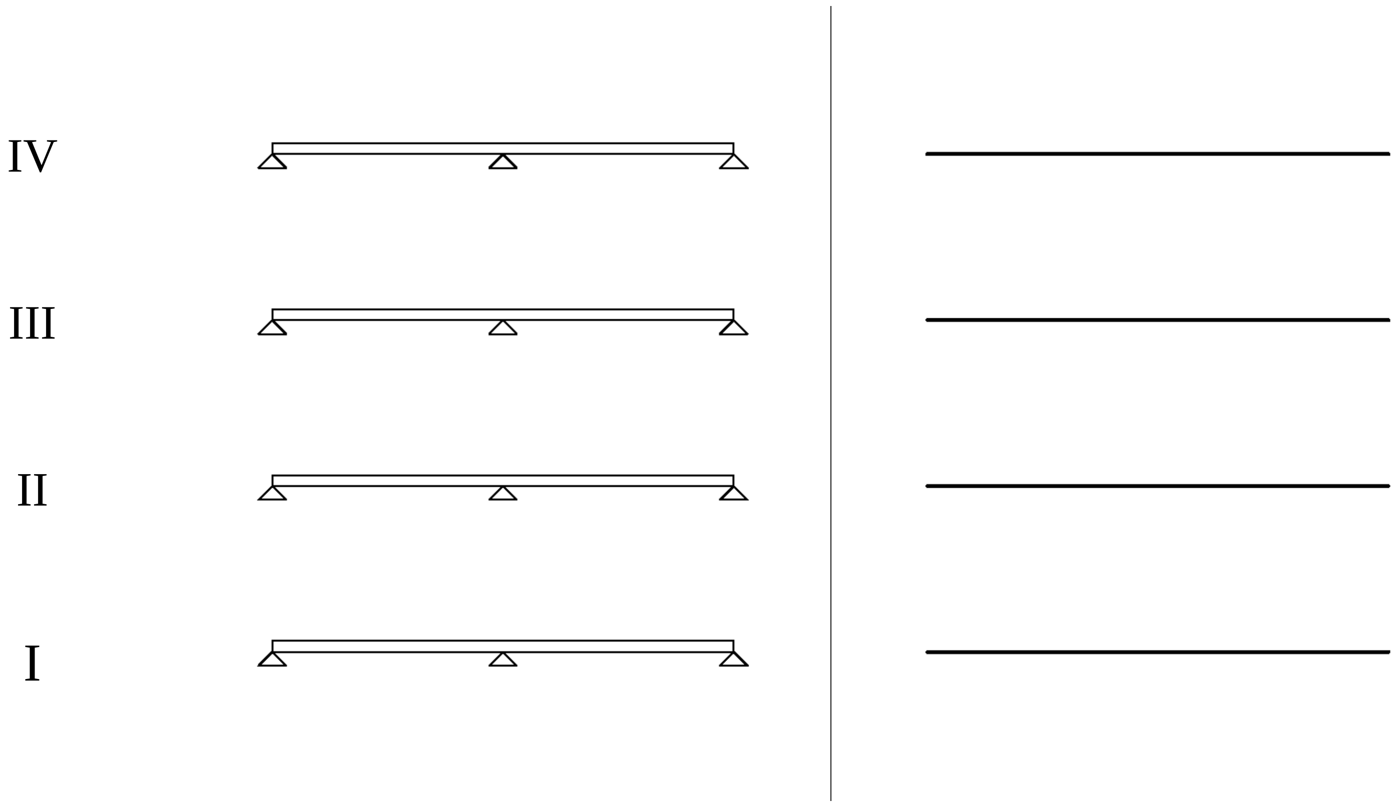

Photo: Author

$P_0 = 0 \quad M = 0$

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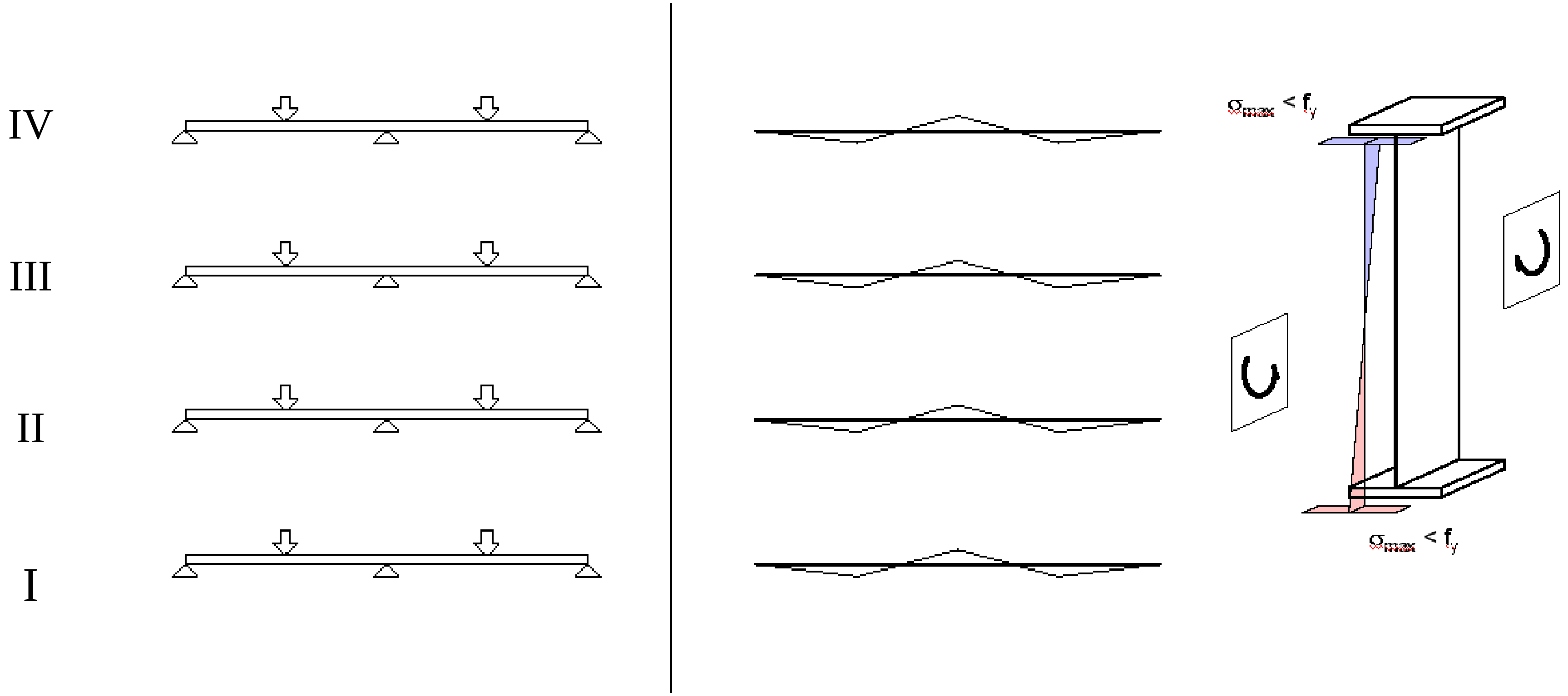

$P_1 \neq 0$

$$M_{sup} = 6\ P_1\ L\ /\ 32$$

Photo: Author

$$M_{sp} = 5\ P_1\ L\ /\ 32$$

M_{sup} / M_{sp} = 1,2 = constant for each beam

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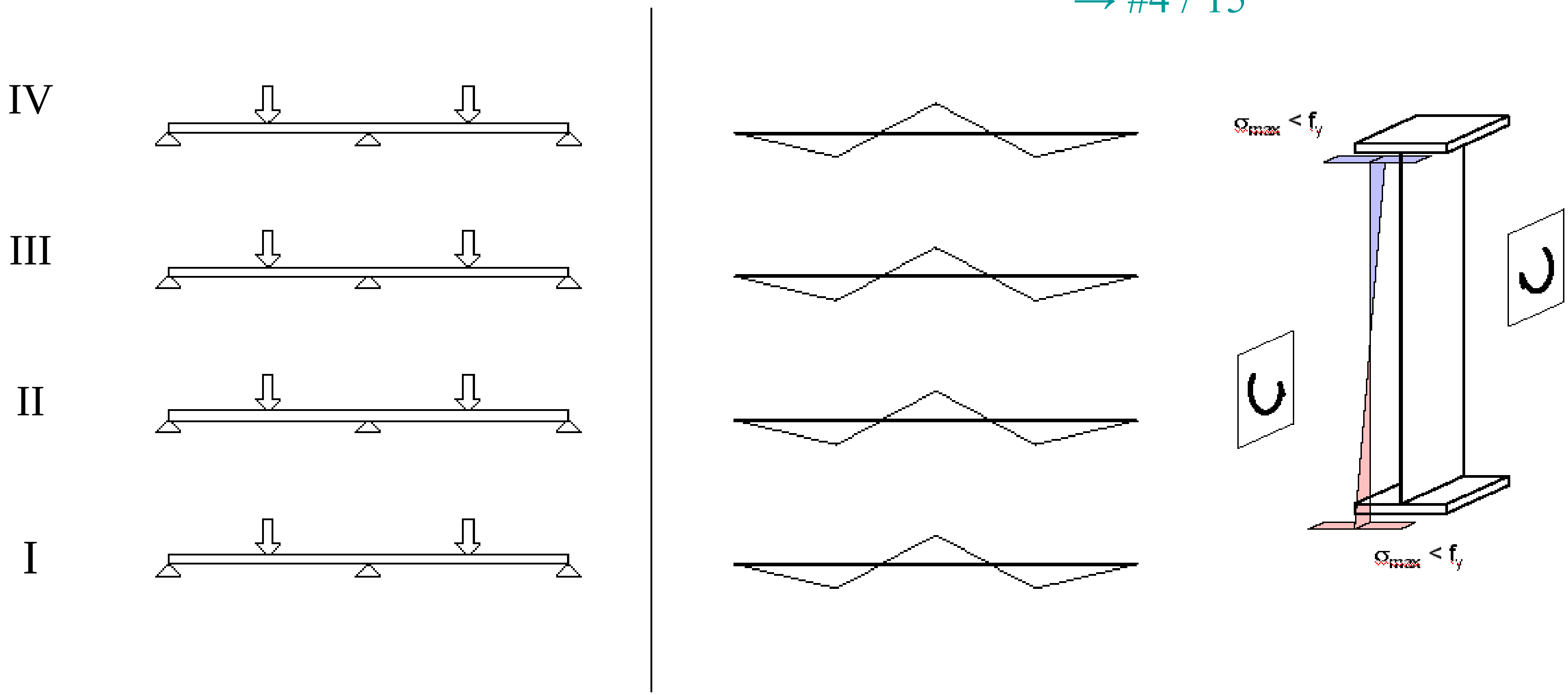

Photo: Author

$$P_2 = P_1 + \Delta P$$

$$M_{sup} = 6\ P_2\ L\ /\ 32$$

$$M_{sp} = 5\ P_2\ L\ /\ 32$$

M_{sup} / M_{sp} = 1,2 = constant for each beam

Photo: Author

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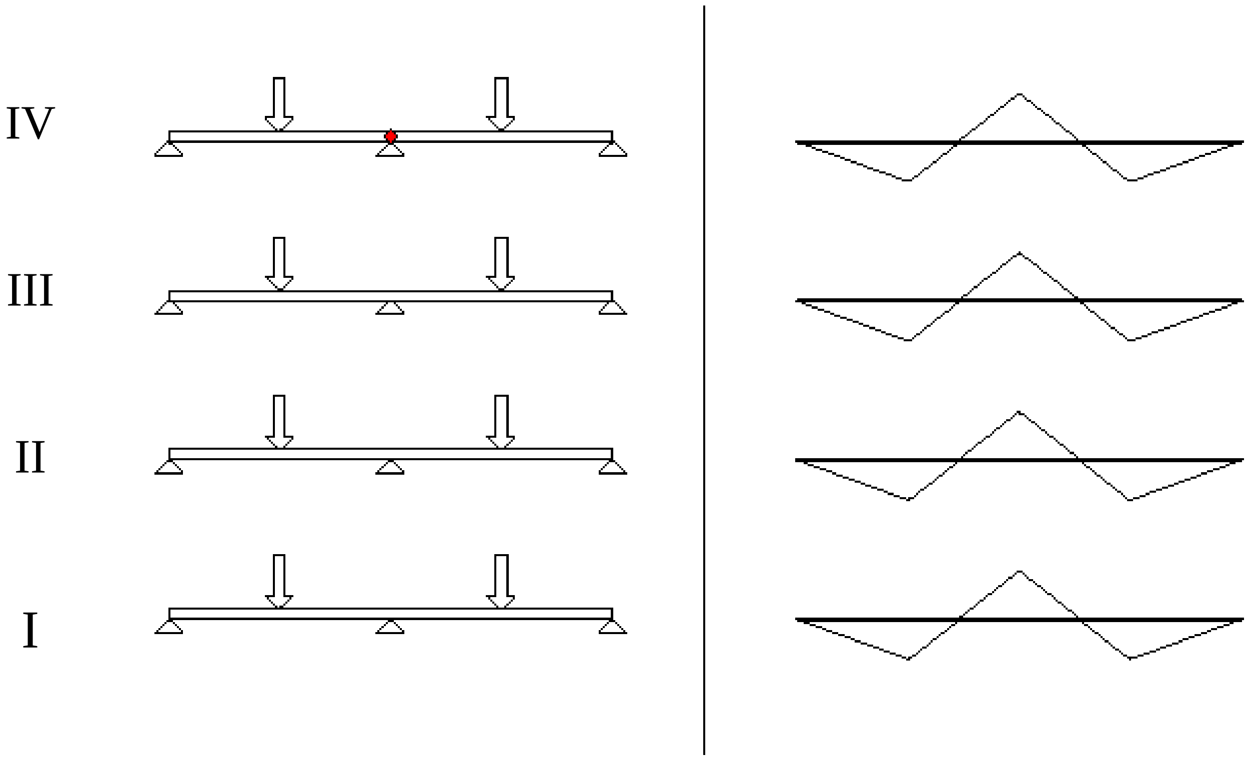
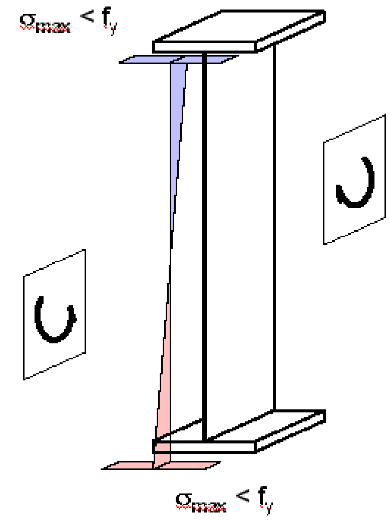
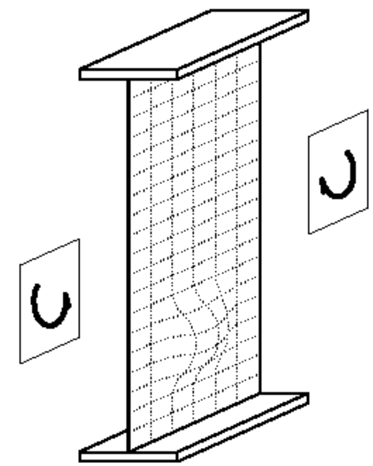


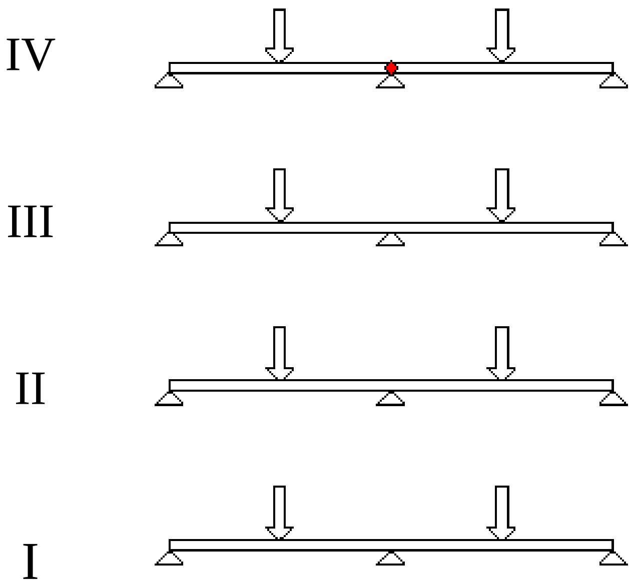


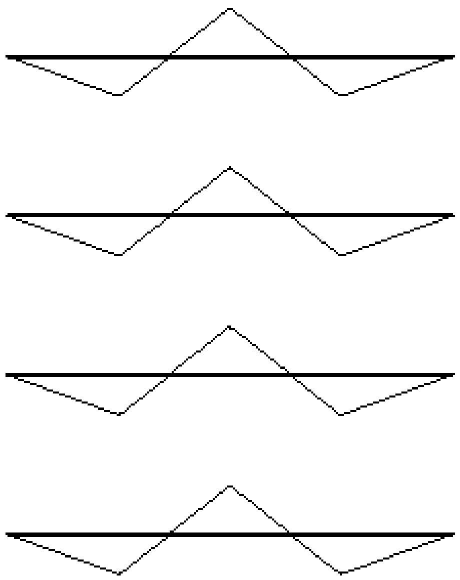


$$P_3 = P_2 + \Delta P$$

There is local instability for IV class I-beam in compressed part of cross-section. Local instability occurs for cross-section of the max value of bending moment.

End of resistance for IV class I-beam.





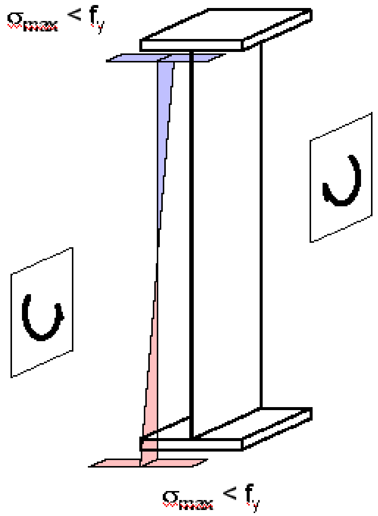


Photo: Author

$$P_4 = P_3 + \Delta P$$

$$M_{sup} = 6\ P_4\ L\ /\ 32$$

$$M_{sp} = 5\ P_4\ L\ /\ 32$$

M_{sup} / M_{sp} = 1,2 = constant for I, II, III beams

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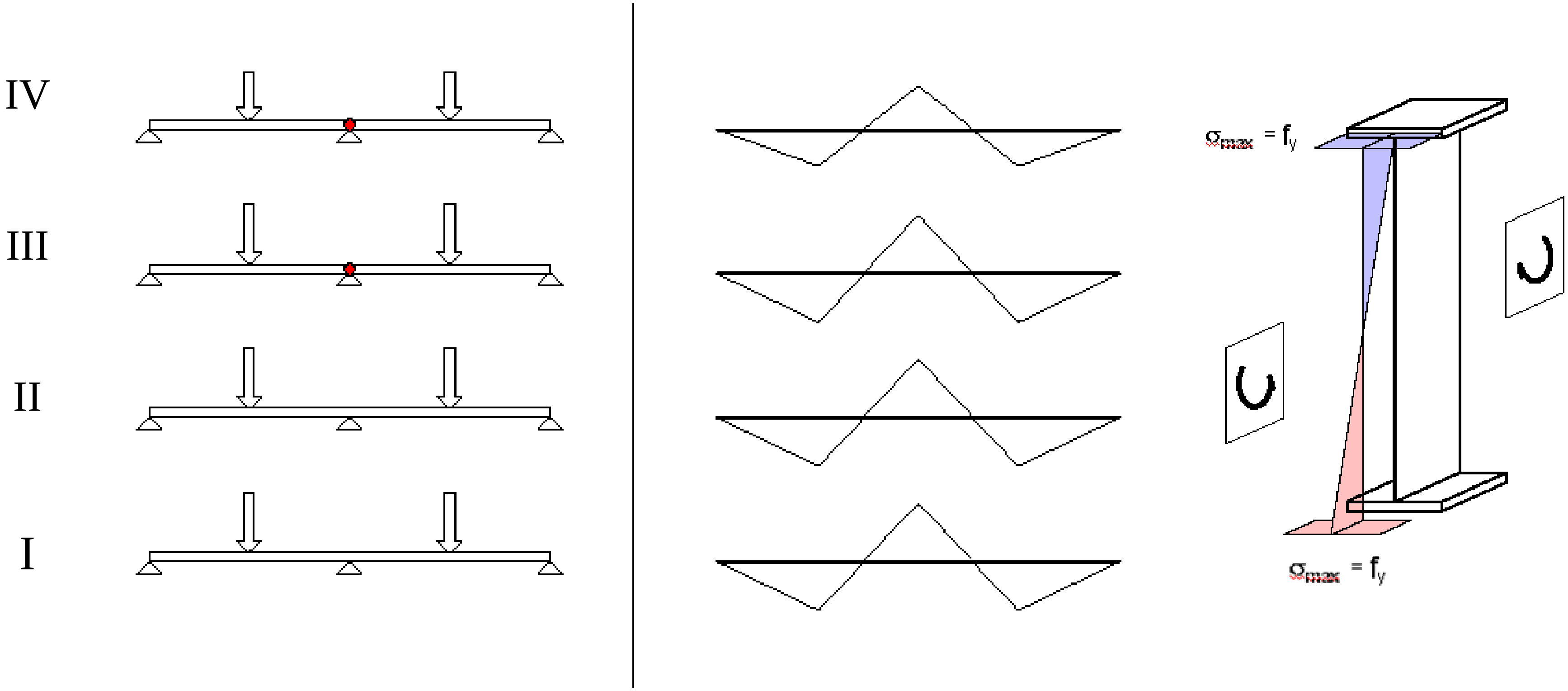

$$P_5 = P_4 + \Delta P$$

Photo: Author

$$\sigma_{max,\ comp} = \sigma(M_{sup}) = f_y$$

End of resistance for III class I-beam.

End of elastic work of cross-section for I and II beams.

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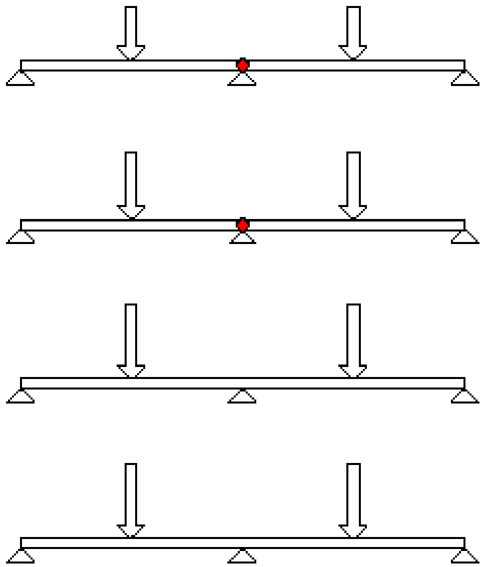
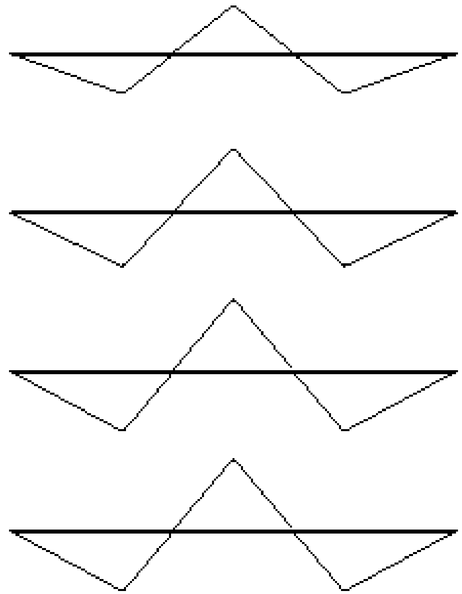
IV

III

II

I





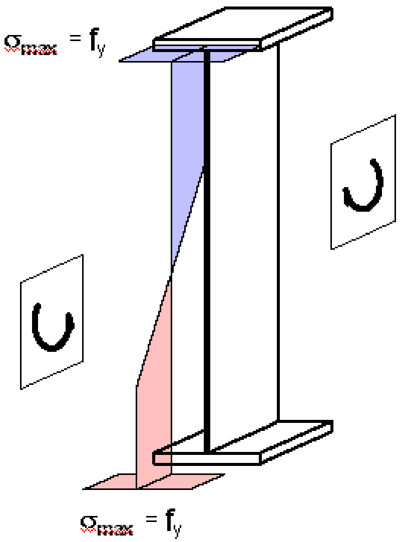


Photo: Author

$$P_6 = P_5 + \Delta P$$

$$M_{sup} = 6\ P_6\ L / 32$$

$$M_{sp} = 5\ P_6\ L / 32$$

M_{sup} / M_{sp} = 1,2 = constant for I, II beams

Elasto-plastic work of cross-section for I and II beams

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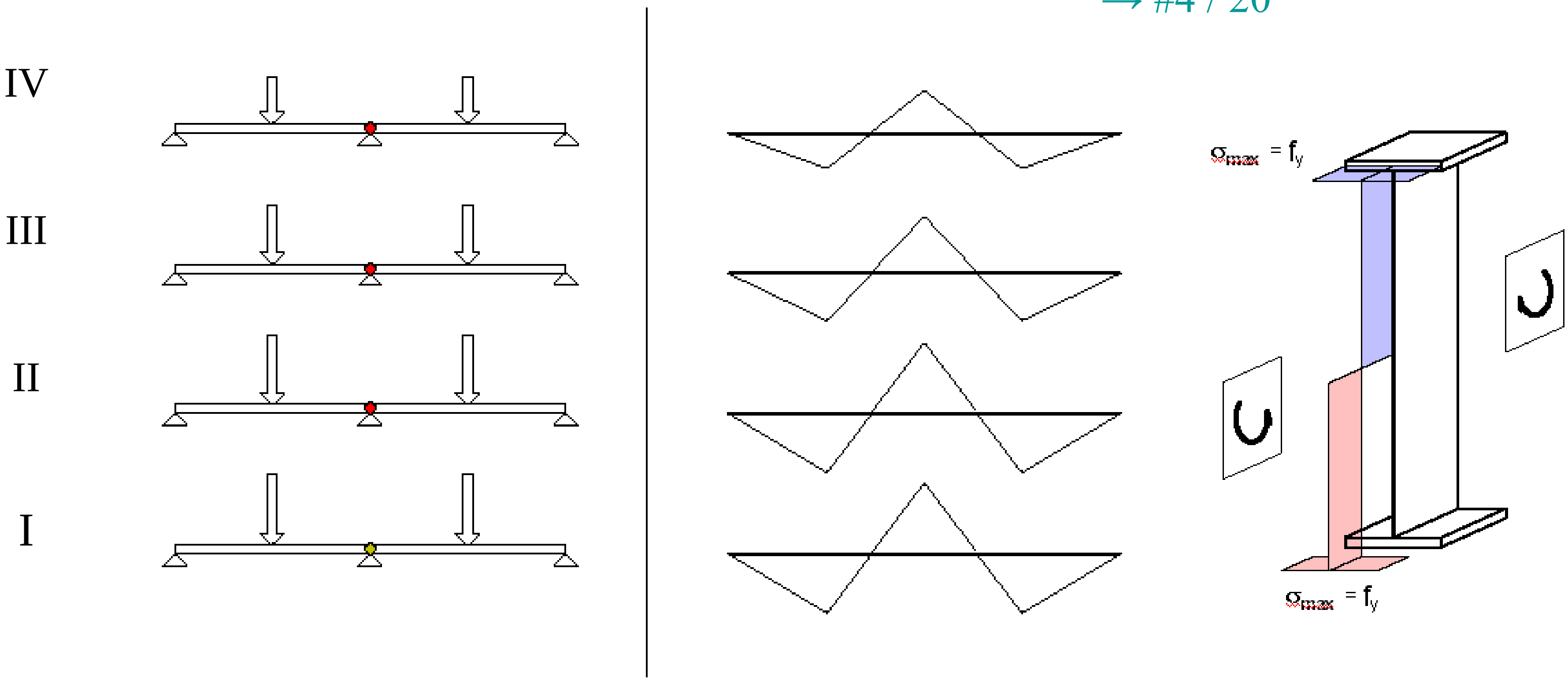
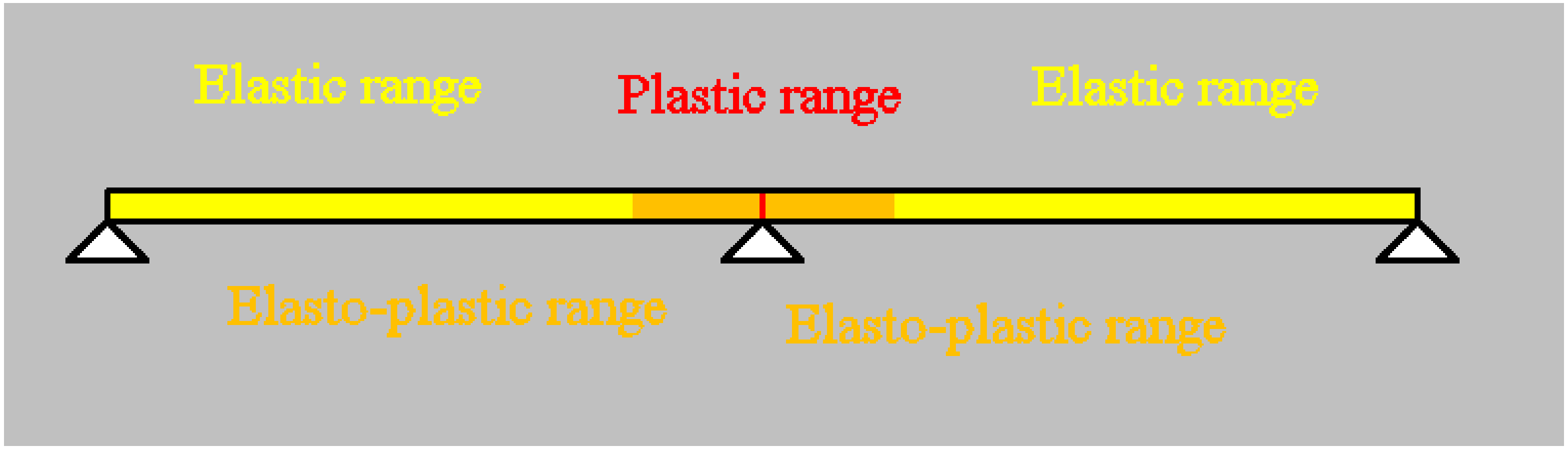


Photo: Author

$$P_7 = P_6 + \Delta P$$

Whole cross-section with maximum bending moment in plastic range.

End of resistance for II class I-beam.



Cross-section in plastic range behaves the same way, as hinge. Plastic shelf = no limit for deformations (rotation).

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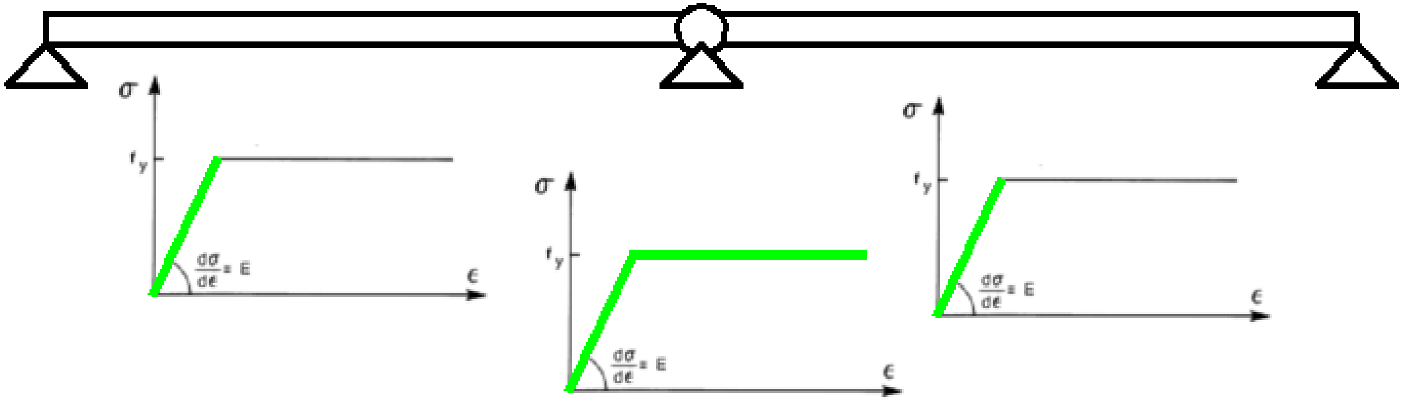
Photo: Author



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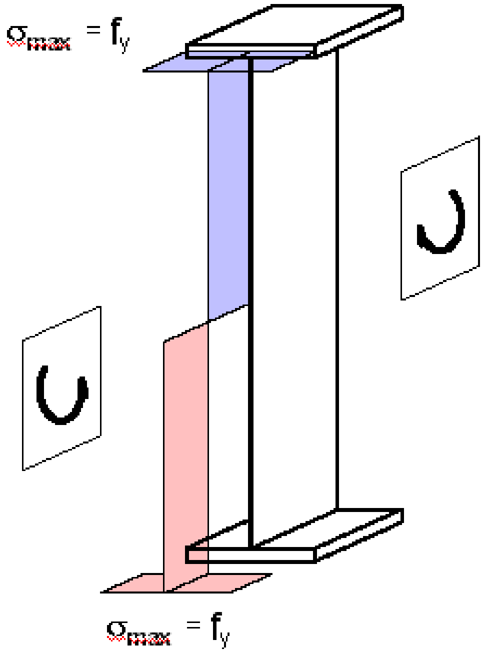
Cross-section in plastic range = plastic hinge

"Normal" hinge → $M = 0$

Plastic hinge → $M = M_{pl} \neq 0$

Stresses for cross-section with M_{pl} are as follow:



M_{pl} is maximum bending moment, which can be carried by cross-section

Photo: Author

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For $P < P_7$:

Bending moments are calculated as for statically indeterminate structure

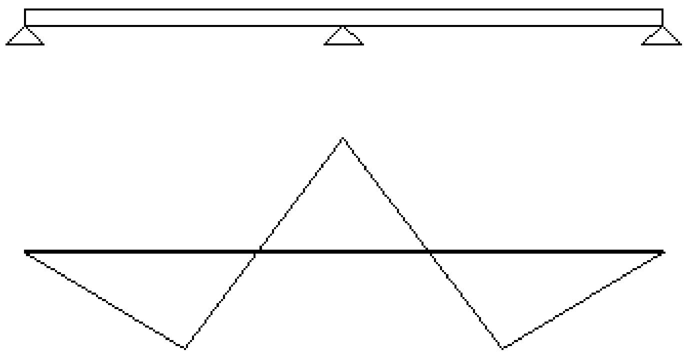

Photo: Author

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For P = P_7 :

Static scheme changes:

Statically indeterminate two span continous beam → two single-span beams supported by common support

Loads change:

Pair of identical forces P → pair of identical forces P and bending moment M_{pl} in plastic hinge

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For $P = P_7$:

Bending moments are sum from P and M_{pl}

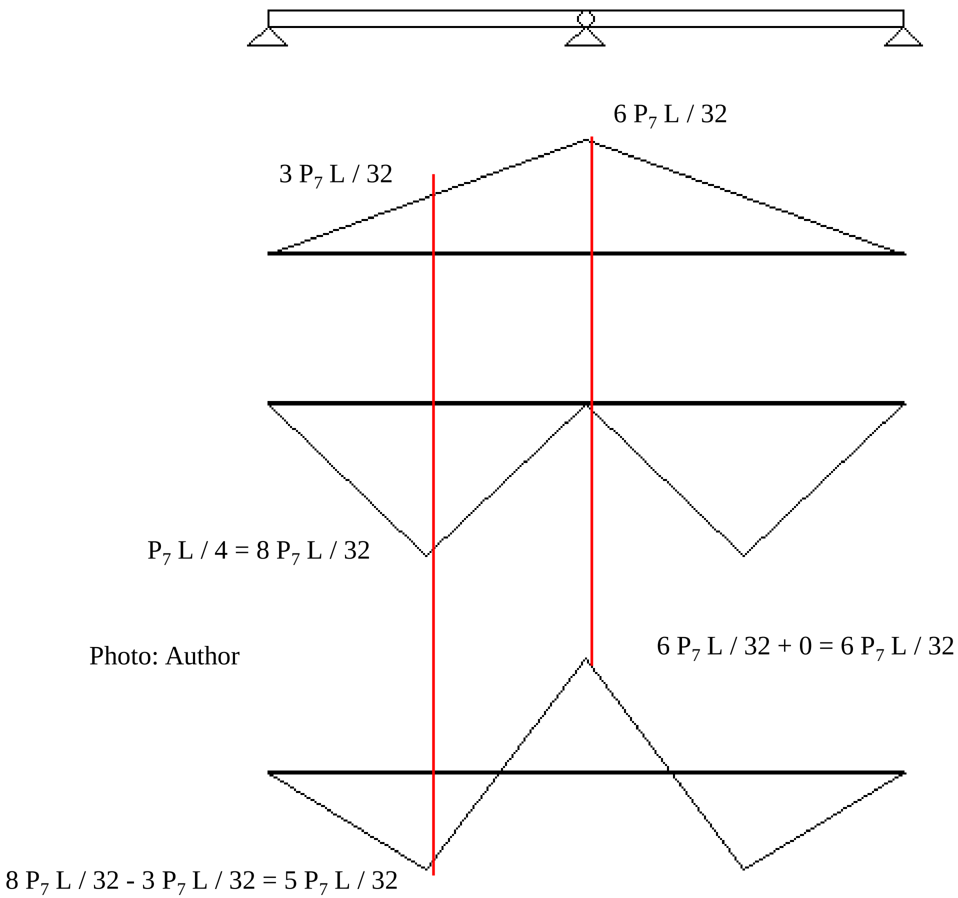

Photo: Author

Bending moments from M_{pl} calculated for two single-span beams

Bending moments from P calculated for two single-span beams

Sum: the same as for $P < P_7$

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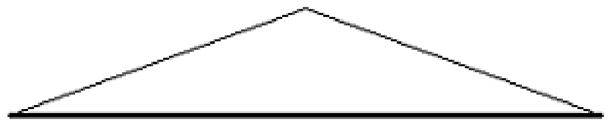
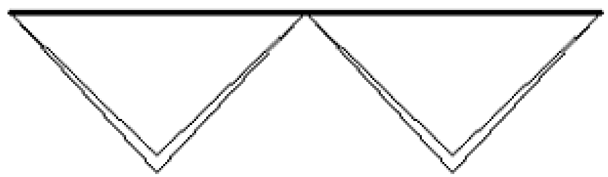
For $P > P_7$:

There is still possible to increase value of P. But part of bending moment from M_{pl} will be still the same. Part of bending moment from P will increases only.

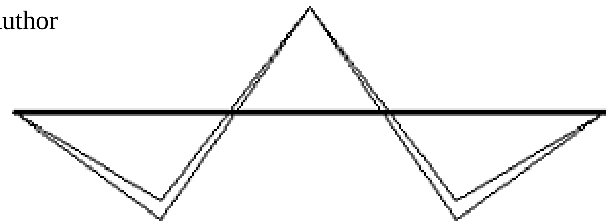


Bending moments from M_{pl} calculated for two single-span beams



Bending moments from P calculated for two single-span beams

Photo: Author



Sum

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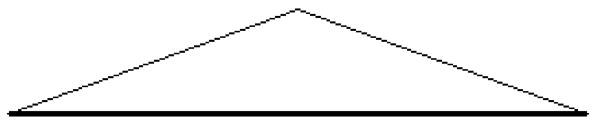
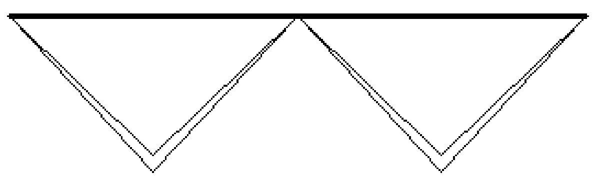
For $P > P_7$:

End of increasing possibility will be, when summarized bending moment reaches the maximum value of bending moment, which can be carried by the cross section. If cross-section is the same for all beam, $M_{max} = M_{pl} = const$

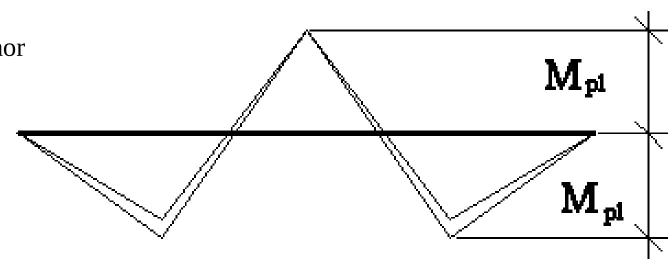


Bending moments from M_{pl} calculated for two single-span beams



Bending moments from P calculated for two single-span beams

Photo: Author



Sum

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For $P > P_7$:

For each cross-section, where $M_{max} = M_{pl}$, forms plastic hinge. End of resistance for I beam comes, when our structure transform into mechanism.

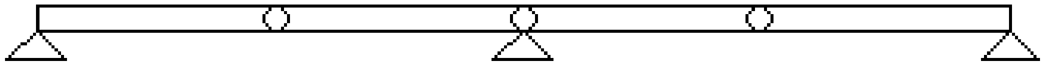

Photo: Author

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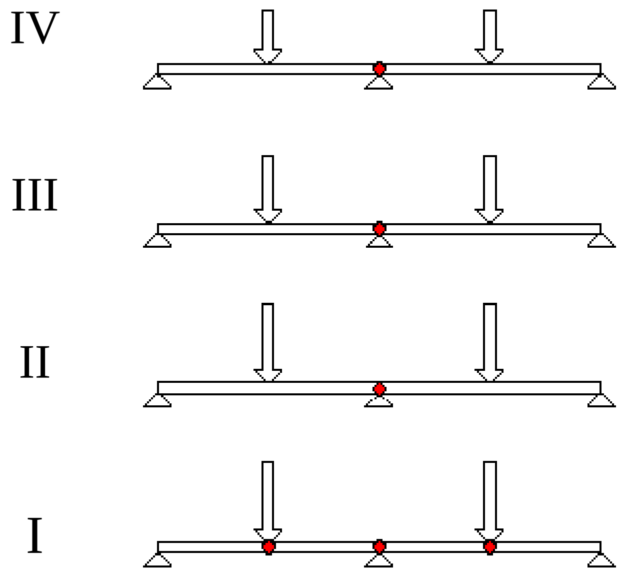

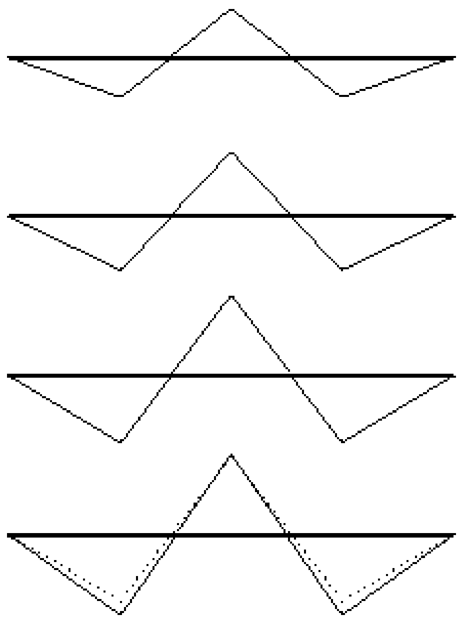

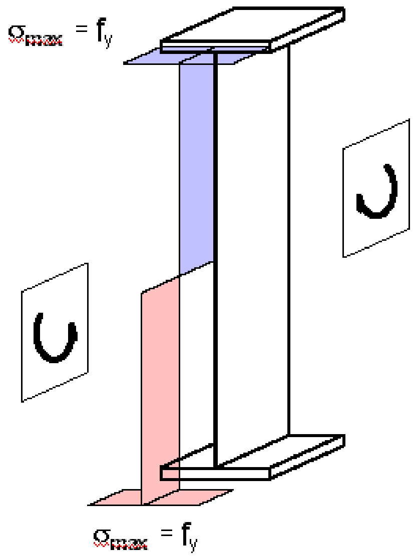

$$P_8 = P_7 + \Delta P$$

$$M_{sup} = 6\ P_8\ L\ /\ 32$$

$$M_{sp} = 6\ P_8\ L\ /\ 32$$

Photo: Author

$M_{sup}\ /\ M_{sp}$ = 1,0 I beams

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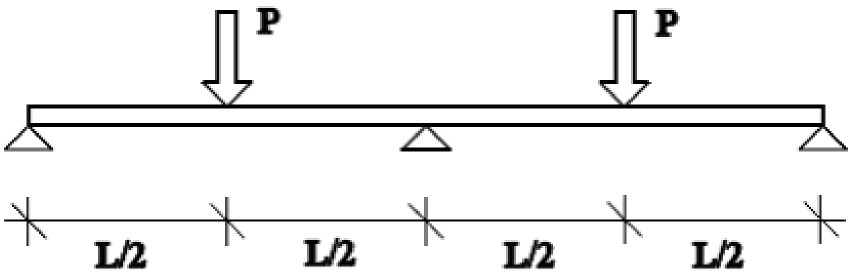
Summary

Class of cross-section	Destruction by / end of resistance	Statical calculations
IV	Local instability for compressed part of cross-section	"Normal" statical calculations (The Force Method, The Displacement Method, computer calculations...)
III	$\sigma_{max,\ comp} = f_y$	
II	First plastic hinge	
I	Transform structure into mechanism	Recalculation according to change of static scheme → plastic redistribution of bending moment

Photo: Author

→ #4 / 32



For analysed structure (if geometrical characteristisc are the same for each beams):

$P_3 < P_5$ - destruction of IV beam

P_5 - destruction of III beam

$P_7 \approx (1,1 \div 1,2)\ P_5$ - destruction of II beam

$P_8 \approx (1,25 \div 1,35)\ P_5$ - destruction of I beam

Proportions for I-beam in bending about stron axis

Experiment was made according to initial assumptions: geometrical characteristics (G) for each beams are the same, only classes of cross-section are various. We calculated, what value of load could be applied to known cross-section. Experiment shows, that we have bigger and bigger resistance (R): from the smallest for IVth class to the biggest for Ist class.

Situation in design process is completely opposite: we know, what is value of load (P) and looking for cross-section. The most often situation: if we want the same resistance (R) for each class, we must applied bigger and bigger depth of cross-section (h): from the smallest, the most massive for Ist class to the biggest, the most slenderness for IVth class.

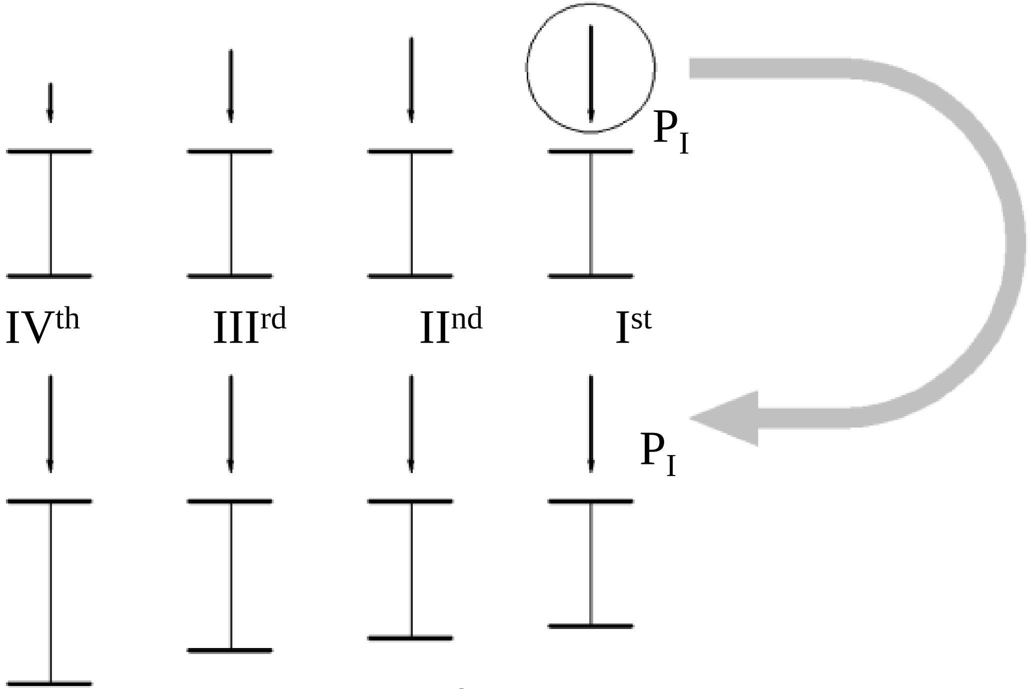


Photo: Autor

Experiment:

$G = const, h \approx const$

$R_{IV} < R_{III} < R_{II} < R_I \rightarrow$

$\rightarrow P_{IV} < P_{III} < P_{II} < P_I$

If we want applicated P_I for each class of cross-section, we must change depth of cross sections

$h_{IV} > h_{III} > h_{II} > h_I$

Formulas of resistance

Steel - different formulas for different class of cross-section

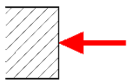
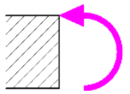
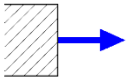
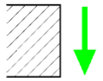
LOAD	I^{st} class	II^{nd} class	III^{rd} class	IV^{th} class
	$N_{Ed} / N_{c,Rd\,(1\text{-}3)} \leq 1,0$			$N_{Ed} / N_{c,Rd\,(4)} \leq 1,0$
	$M_{Ed\,(1)} / M_{Rd\,(1\text{-}2)} \leq 1,0$	$M_{Ed} / M_{Rd\,(1\text{-}2)} \leq 1,0$	$M_{Ed} / M_{Rd\,(3)} \leq 1,0$	$M_{Ed} / M_{Rd\,(4)} \leq 1,0$
	$N_{Ed} / N_{t,Rd} \leq 1,0$			
	$V_{Ed} / V_{Rd\,(1\text{-}3)} \leq 1,0$			$V_{Ed} / V_{Rd\,(4)} \leq 1,0$

Photo: Author

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Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

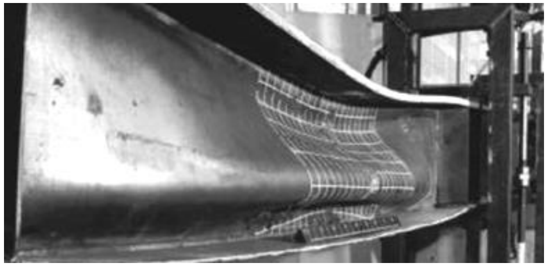
IVth class of cross-section:

Effective cross-section, local instability

Example of calculations: Laboratory #2

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013



Rys: Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

IIIrd class of cross-section:

Resistance for bending moment depends on elastic sectional modulus

$W_{el,\,y}$

Notations pages 104-108 / Bezeichnungen Seiten 104-108

Désignation Designation Bezeichnung		Valeurs statiques / Section properties / Statis							
		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse		
	G kg/m	I_y cm⁴	$W_{el.y}$ cm³	$W_{pl.y}$♦ cm³	i_y cm	A_{vz} cm²	I_z cm⁴	$W_{el.z}$ cm³	$W_{pl.z}$ cm³
IPE A 100	6.9	141.2	28.81	32.98	4.01	4.44	13.12	4.77	7.
IPE 100	8.1	171.0	34.20	39.41	4.07	5.08	15.92	5.79	9.
IPE A 120	8.7	257.4	43.77	49.87	4.83	5.41	22.39	7.00	10.
IPE 120	10.4	317.8	52.96	60.73	4.90	6.31	27.67	8.65	13.

Photo: europrofil.lu

I[st] and II[nd] class of cross-section:

Resistance for bending moment depends on plastic sectional modulus

$W_{pl,\,y}$ (Laboratory #1)

Notations pages 104-108 / Bezeichnungen Seiten 104-108

Désignation Designation Bezeichnung		Valeurs statiques / Section properties / Statis							
		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse		
	G kg/m	I_y cm⁴	$W_{el.y}$ cm³	$W_{pl.y}$♦ cm³	i_y cm	A_{vz} cm²	I_z cm⁴	$W_{el.z}$ cm³	$W_{pl.z}$ cm³
IPE A 100	6.9	141.2	28.81	32.98	4.01	4.44	13.12	4.77	7.
IPE 100	8.1	171.0	34.20	39.41	4.07	5.08	15.92	5.79	9.
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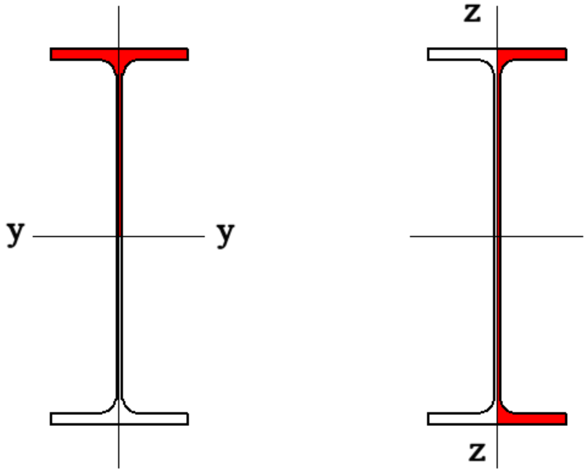
IPE 120	10.4	317.8	52.96	60.73	4.90	6.31	27.67	8.65	13.

Photo: europrofil.lu

Photo: Author



$$W_{y,\,pl} = 2\,S_y\,(1/2\ I)$$

$N_{c,Rd\ (1\text{-}3)} = A\ f_y / \gamma_{M0}$

$N_{c,Rd\ (4)} = A_{eff}\ f_y / \gamma_{M0}$

$M_{Rd\ (1\text{-}2)} = W_{pl}\ f_y / \gamma_{M0}$

$M_{Rd\ (3)} = W_{el}\ f_y / \gamma_{M0}$

$M_{Rd\ (4)} = W_{eff}\ f_y / \gamma_{M0}$

$V_{Rd\ (1\text{-}3)} = A_v\ f_y / (\gamma_{M0} \sqrt{3})$

$V_{Rd\ (4)}$ = impact of local instability + nonlinear relations with $M_{Rd\ (4)}$ and $N_{c,Rd\ (4)}$

$N_{t,Rd} = A\ f_y / \gamma_{M0}$

Bending moment only:

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$$M_{Ed} / M_{Rd} \leq 1{,}0$$

Class of cross-section	IVth	IIIrd	IInd	Ist
Distribution of σ across cross-section	Elastic		Plastic	
Effects	Local instability	Resistance of cross-section		
M_{Ed} =	From „normal" static calculations of structure			From special recalculation to new static scheme and new loads (**redistribution**)
M_{Rd} =	$W_{eff}\ f_y / \gamma_{M0}$	$W_{el}\ f_y / \gamma_{M0}$	$W_{pl}\ f_y / \gamma_{M0}$	

W_{eff} – IInd laboratory

W_{el} – tables for design

W_{pl} – Ist laboratory

Resistances for different classes of cross-section can be presented as envelope of resistance:

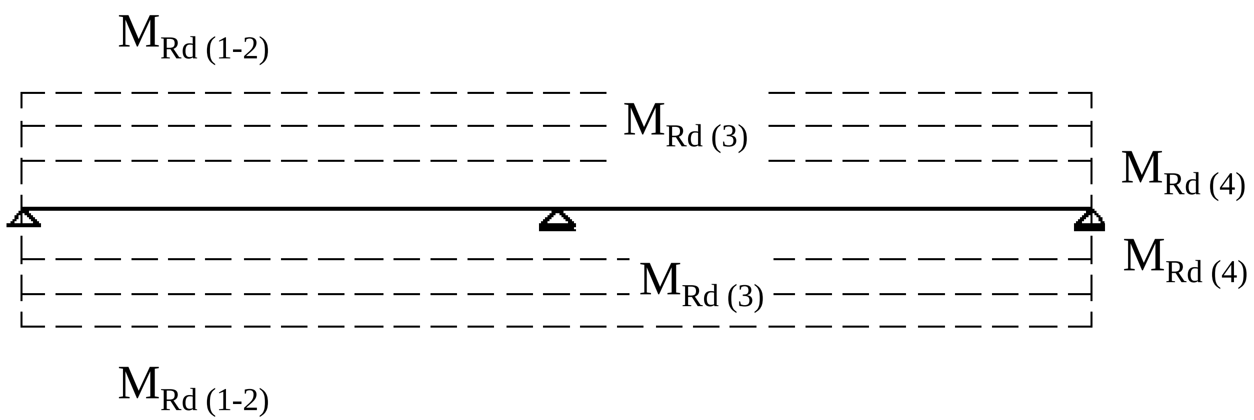


Photo: Author

$$M_{Rd\,(1-2)} = W_{pl}\, f_y / \gamma_{M0}$$

$$M_{Rd\,(3)} = W_{el}\, f_y / \gamma_{M0}$$

$$M_{Rd\,(4)} = W_{eff}\, f_y / \gamma_{M0}$$

IV[th], III[rd], II[nd] class of cross-section: M_{Ed} from „normal" static calculation, end of resistance, when $M_{Ed} / M_{Rd\,(4)} = 1{,}0$ in **one** point.

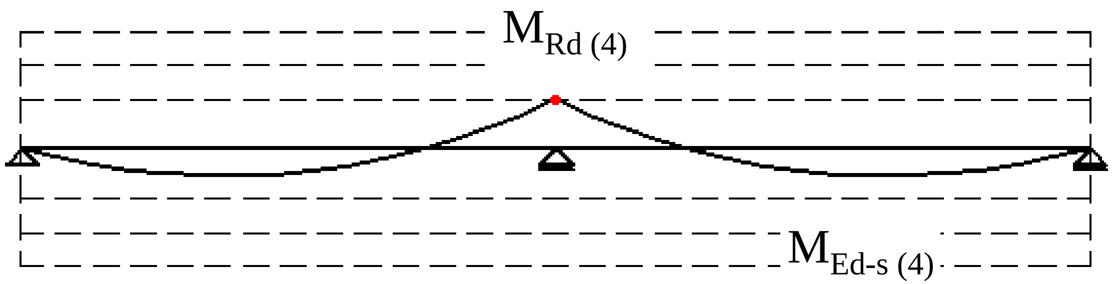
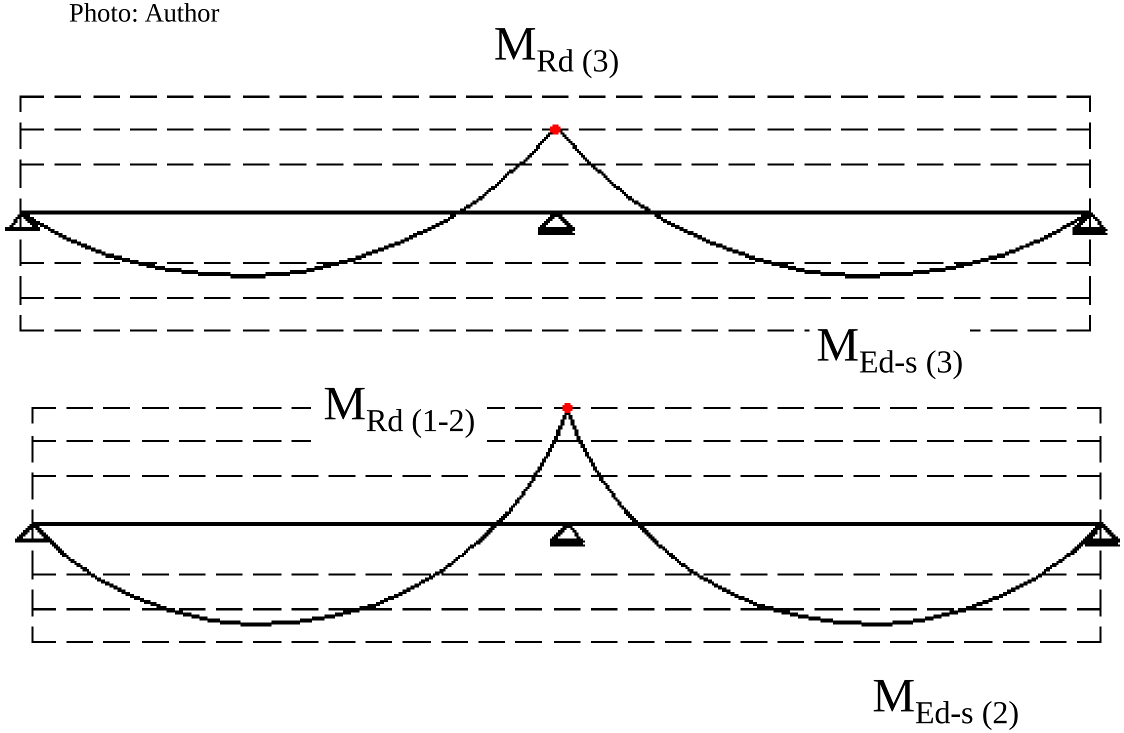


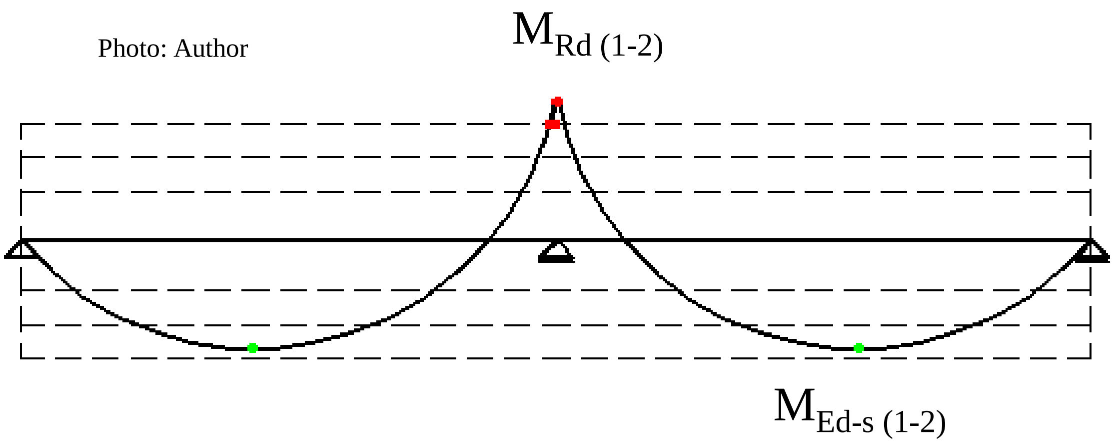
Photo: Author



Proportion between max bending moment over central support and max in span is constant.

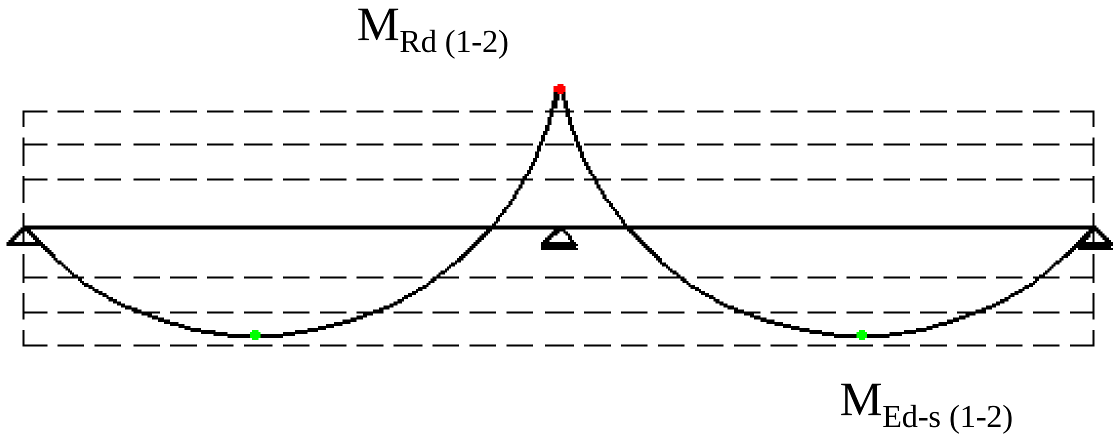
I[st] class of cross-section: bigger values of external action as for I[nd] can be applicated. But from „normal" static calculations, increasing values obf bending moments in span and over suport is proportional. X% bigger value of external actions → X% bigger values of bending moments.

Photo: Author



From „normal" static calculations, values of bending moments in spans are still smaller than resistance, but around central support is bigger than resistance not in one point, but in short distance.

<u>Redistribution of bending moments makes change shape of bending moments diagram.</u>

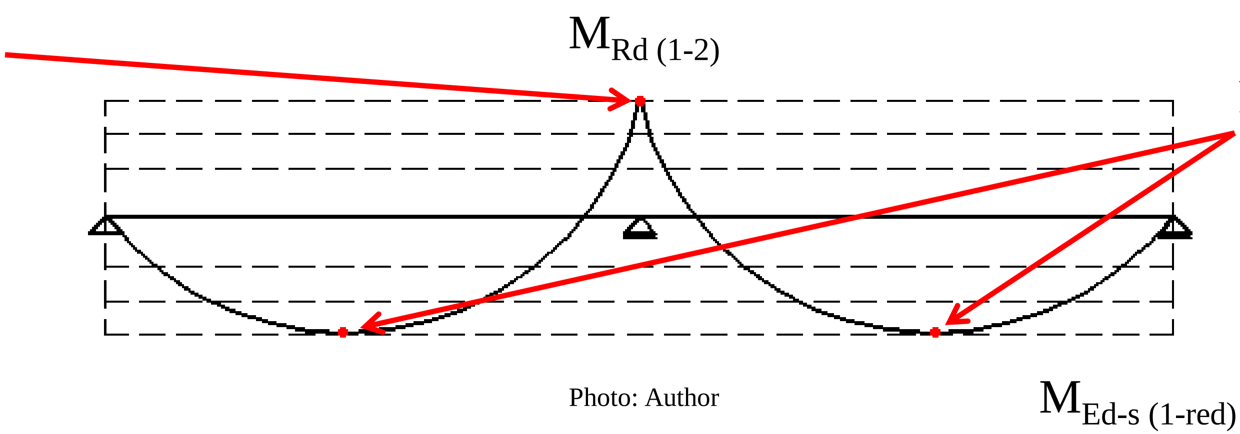


„Normal” calculation

Photo: Author

For I[st] class of cross-section is applied <u>redistribution of bending moment</u>. Because of redistribution, <u>diagram of bending moments has completely different shape and proportion as from "normal" static</u>. There should be used special methods of recalculations of bending moments for this situation. End of resistance, when $M_{Ed} / M_{Rd\ (1\text{-}2)} = 1{,}0$ in **one point over central support and one poin in few spans** (structure transforms into mechanism).

Redistribution: smaller than in „normal” calculation



Redistribution: biggerer than in „normal” calculation

Photo: Author

„Normal” static calculations, over support > redistribution, over support = redistribution, in span > „normal” static calculations, in span.

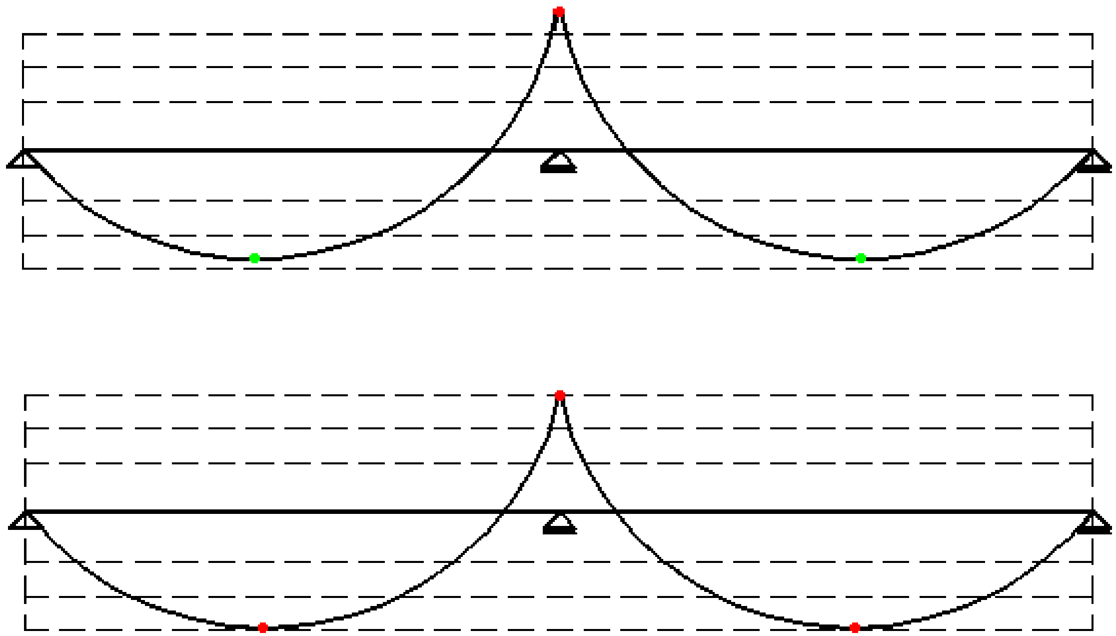


Photo: Author

After redistribution, values of bending moments over support is the same than in span.

In addition to bending resistance (and lateral buckling), there must be checked shear resistance and resistance for axial force - compressive or tensile. In many cases, the interactions of cross-sectional forces cause additional complications.

C-S force	Interactions							
$M_{Ed, y}$								
$V_{Ed, z}$								
$M_{Ed, z}$								
$V_{Ed, y}$								
$N_{Ed, t}$								
$N_{Ed, c}$								
T_{Ed}								

Axial force

$$N_{Ed} / N_{Rd} \leq 1{,}0$$

$$N_{c,Rd\ (1\text{-}3)} = A\, f_y / \gamma_{M0}$$

$$N_{c,Rd\ (4)} = A_{eff}\, f_y / \gamma_{M0}$$

$$N_{t,Rd} = A\, f_y / \gamma_{M0}$$

Additionally, for compressive force, flexural, torsional and flexural-torsional buckling is important.

Shear force

$$V_{Ed} / V_{c,Rd} \leq 1,0$$

$$V_{c,Rd} = ?$$

There are three ways of calculations according to Eurocode.

I[st] way of calculation: elastic analysis, no local buckling;

$$V_{c,Rd} = V_{Rd}(z) = J_y\, t(z)\, f_y / [\gamma_{M0}\, S_y(z) \sqrt{3}]$$

EN 1993-1-1 (6.19), (6.20)

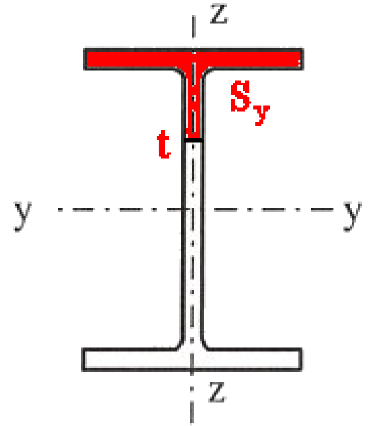


Photo: Author

II[nd] way of calculation: elastic analysis, I-beam, $A_f / A_w \geq 0,6$;

$$V_{c,Rd} = A_w\, f_y / (\gamma_{M0} \sqrt{3})$$

$$A_w = h_w\, t_w$$

EN 1993-1-1 (6.21)

III[rd] way of calculation: plastic analysis, I class of cross-section, I-beam, no torsional moment:

$$V_{c,Rd} = V_{pl,Rd} = A_v f_y / (\gamma_{M0} \sqrt{3})$$

$$A_v \rightarrow \#t / 56$$

EN 1993-1-1 (6.17), (6.18)

(3) The shear area A_v may be taken as follows:

a) rolled I and H sections, load parallel to web $A - 2bt_f + (t_w + 2r)t_f$ but not less than $\eta h_w t_w$

b) rolled channel sections, load parallel to web $A - 2bt_f + (t_w + r)t_f$

c) rolled T-section, load parallel to web

- for rolled T-sections: $A_v = A - bt_f + (t_w + 2r)\frac{t_f}{2}$
- for welded T-sections: $A_v = t_w(h - \frac{t_f}{2})$

There is difference between EN and PN-EN for red part - no information about T-sections in PN-EN

d) welded I, H and box sections, load parallel to web $\eta \sum (h_w t_w)$

e) welded I, H, channel and box sections, load parallel to flanges $A - \sum (h_w t_w)$

f) rolled rectangular hollow sections of uniform thickness:

load parallel to depth $Ah/(b+h)$

load parallel to width $Ab/(b+h)$

g) circular hollow sections and tubes of uniform thickness $2A/\pi$

where A is the crosssectional area;

b is the overall breadth;

h is the overall depth;

h_w is the depth of the web;

r is the root radius;

t_f is the flange thickness;

t_w is the web thickness (If the web thickness in not constant, t_w should be taken as the minimum thickness.).

η see EN 1993-1-5.

NOTE η may be conservatively taken equal 1,0.

Additionally:

$$h_w / t_w \leq 72\,\varepsilon / \eta$$

EN 1993-1-1 (6.21)

Local stability of web under shear force

η according to EN 1993-1-5 (generally = 1,0)

Bending and shear force - interaction:

$$V_{Ed} / V_{c,Rd} \leq 0,5$$

No reduction of bending moment resistance

$$0,5 < V_{Ed} / V_{c,Rd} \leq 1,0$$

Reduction of bending moment resistance

$$\rho = [\,2\ (V_{Ed} / V_{c,Rd}) - 1]^2$$

$$M_{V,Rd} = \min \{M_{Rd}\ ;\ [W_{pl} - (\rho\, h_w^2\, t_w / 4)]\, f_y / \gamma_{M0}\}$$

EN 1993-1-1 (6.29), (6.30)

Interaction between bending moment and axial force, bi-axial bending, bi-axial bending and axial force is calculated according to very similar formulas. Additional effect of simultaneous action of compressive axial force and bending moment is interaction between two form of instability: flexural buckling and lateral buckling. Axial force is important, first of all, for column. Because of this, information about such phenomena will be presented on lecture # 13.

→ Lab #1 / 52

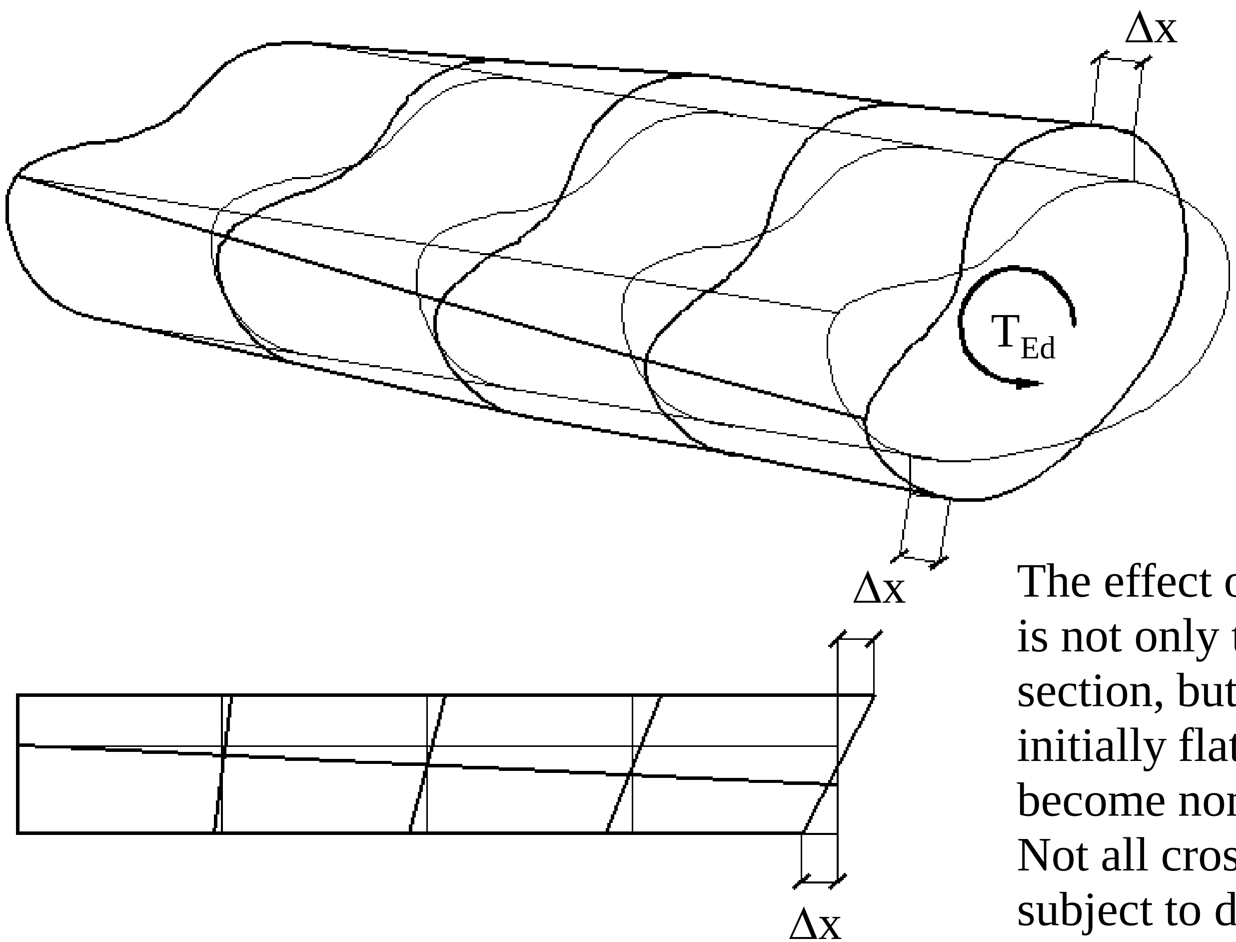
Torsion is the most complex type of load. They should be considered in several different ways:

- round bars;
- circular hollow sections;
- square or rectangular hollow sections;
- simple cross-sections (L T ┼);
- rest open cross-sections.

Differentiation consists in various deformations of twisted bars, various stress distributions or various formulas for geometric characteristics.

General case:

→ Lab #1 / 53



Rys: Autor

The effect of the torsional moment is not only the rotation of the section, but also its deplanation - initially flat and parallel sections become non-flat and non-parallel. Not all cross-sectional shapes are subject to deplanation, and for those parts which are subject to deplanation have negligible values.

Cross-section	Deplanation		Torsional moment T_{Ed}	Remarks
Round (bar, hollow section)	Does not exists		$T_{Ed} = T_{t,\,Ed}$	-
L T +	Very small	Free	$T_{Ed} = T_{t,\,Ed}$	-
		Restricted by supports	$T_{Ed} \approx T_{t,\,Ed}$	-
Rest	Important	Free	$T_{Ed} = T_{t,\,Ed}$	-
		Restricted by supports	$T_{Ed} = T_{t,\,Ed} + T_{w,\,Ed}$	In addition, B_{Ed} it should be taken into account

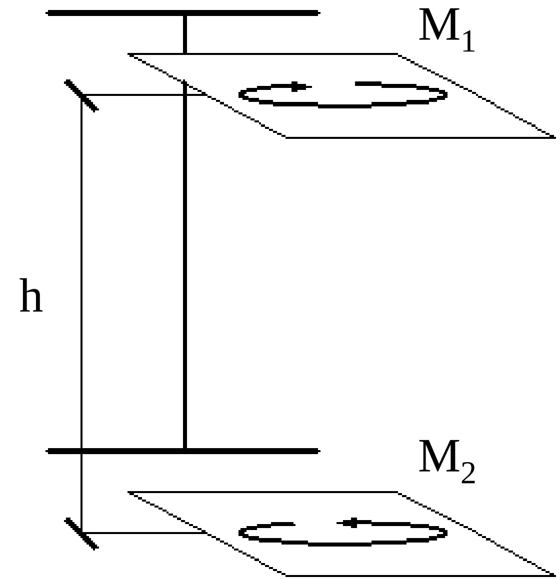
$T_{t,\,Ed}$ – St Venant torsion moment (free deplanation of cross-section);

$T_{w,\,Ed}$ – warping torsional moment (restricted deplanation of cross-section);

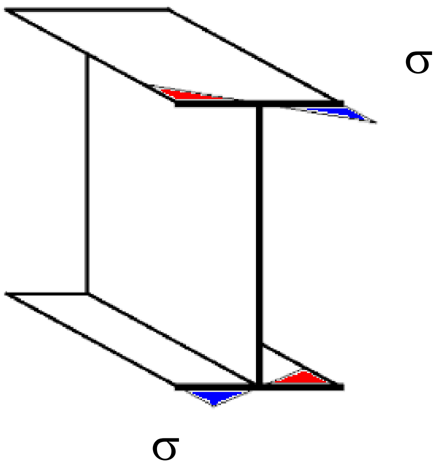
B_{Ed} - bimoment

→ Lab #1 / 54

I-beam deplanation: both flanges are deformed in opposite directions.



In case of restricted torsion, specific distribution of stresses (σ not ony τ) in flanges is inducted.



Rys: Autor

Axial stresses can be presented as the effect of bi-moment [Nm^2]:

$$B = h\,M$$

$$M = |M_1| = |M_2|$$

Bi-moment is taken into consideration during analisis of thin-walled structures also.

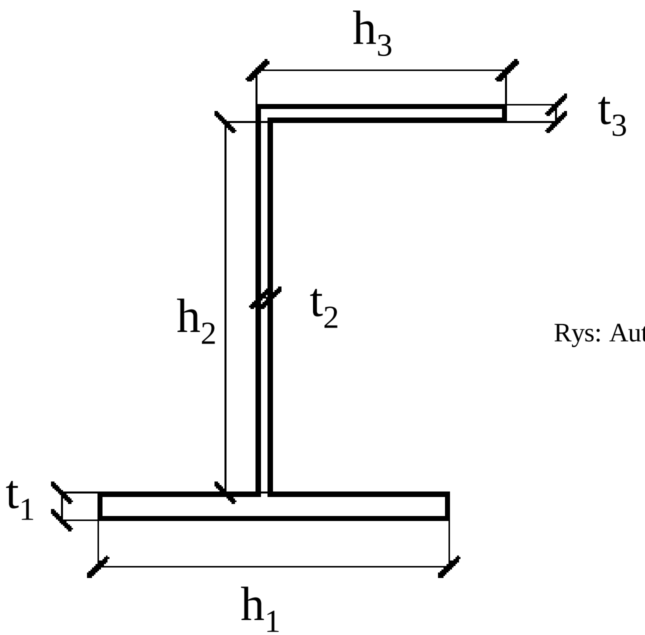
→ Lab #1 / 55

Different ways of bar deformation make it necessary to consider two separate groups of geometrical characteristics:

The first group (J_T $W_T{}^*$) concerns the torsion of the cross section and resistance due to the torsional moment.

The second one (J_ω W_ω) is related to the section deplanation and resistance due to the bi-moment.

→ Lab #1 / 56



Rys: Autor

In analogy to bending, the moment of inertia at torsion and the sectional modulus at torsion can be presented:

$$W_T{}^* = J_T \,/\, t_{max}$$

$$t_{max} = \max\,(\,t_1 \;;\; t_2 \;;\; t_3 \ldots)$$

Values for I-beam are presented in special tables.

→ Lab #1 / 57

Notations pages 104-108 / Bezeichnungen Seiten 104-108

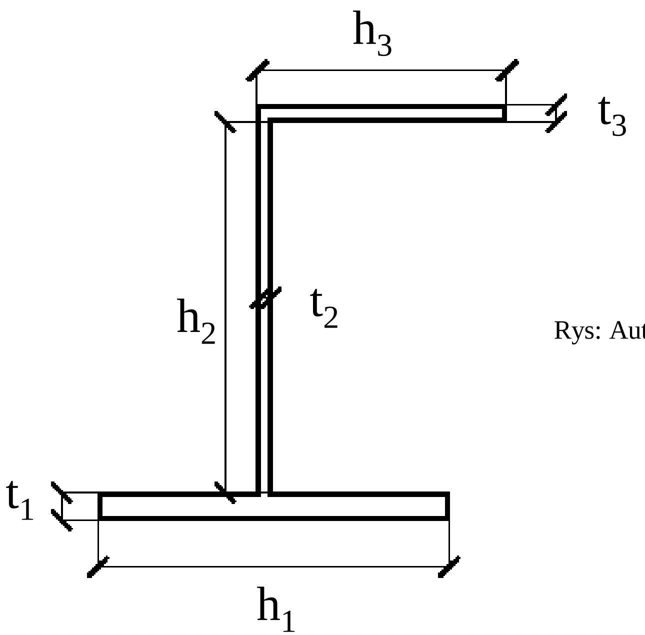
Désignation / Designation / Bezeichnung		Valeurs statiques / Section properties / Statische Kennwerte													Classification ENV 1993-1-1						HISTAR
		axe fort y-y / strong axis y-y / starke Achse y-y					axe faible z-z / weak axis z-z / schwache Achse z-z								pure bending y-y			pure compression			
	G kg/m	I_y cm^4	$W_{el.y}$ cm^3	$W_{pl.y}$♦ cm^3	i_y cm	A_{vz} cm^2	I_z cm^4	$W_{el.z}$ cm^3	$W_{pl.z}$♦ cm^3	i_z cm	s_s mm	I_t cm^4	$I_w \times 10^{-3}$ cm^6	S 235	S 355	S 460	S 235	S 355	S 460	
IPE A 360	50.2	14520	811.8	906.8	15.06	29.76	944.3	111.1	171.9	3.84	50.69	26.51	282	1	1	-	4	4	-	
IPE 360	57.1	16270	903.6	1019	14.95	35.14	1043	122.8	191.1	3.79	54.49	37.32	313.6	1	1	-	2	4	-	
IPE O 360	66.0	19050	1047	1186	15.05	40.21	1251	145.5	226.9	3.86	59.69	55.76	380.3	1	1	-	1	3	-	
IPE A 400	57.4	20290	1022	1144	16.66	35.78	1171	130.1	202.1	4.00	55.60	34.79	432.2	1	1	-	4	4	-	
IPE 400	66.3	23130	1156	1307	16.55	42.69	1318	146.4	229.0	3.95	60.20	51.08	490	1	1	-	3	4	-	
IPE O 400	75.7	26750	1324	1502	16.66	47.98	1564	171.9	269.1	4.03	65.30	73.10	587.6	1	1	-	2	3	-	

Rys: europrofil.lu

→ Lab #1 / 58

Approximated formulas could be used also:

$$J_T \approx \alpha \,(h_1 t_1^3 + h_2 t_2^3 + h_3 t_3^3 + \dots) / 3$$



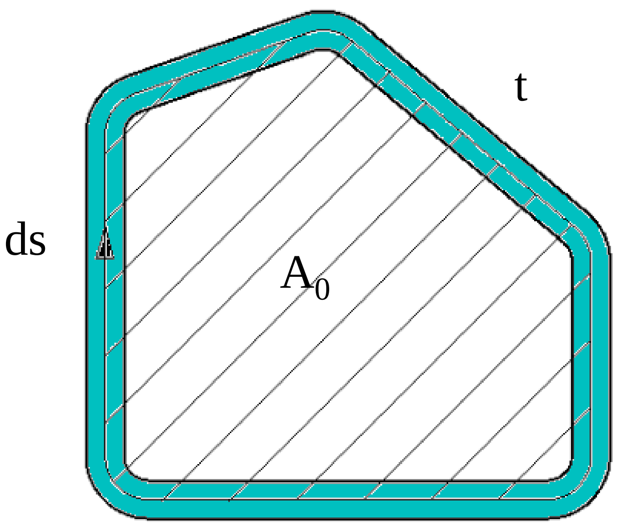
Rys: Autor

Cross-section	α
Hot-rolled I-beam	1,20
Welded I-beams with vertical stiffeners	1,50
L	1,00
C	1,12
T	1,40

For hollow cross-sections (circular, rectangular, square), the moment of inertia at torsion and the torsional sectional modulus are calculated differently:

$$J_T = 4\,A_0^2 / \int (ds / t)$$

$$W_T^* = 2\,t\,A_0$$



Rys: Autor

For round bars:

$$J_T = \pi\, r^4 / 2$$

$$W_T^* = \pi\, r^3 / 2$$

→ Lab #1 / 59

The most complicated topic in the field of calculating cross-sectional characteristics is to count those associated with deplanation. This issue is beyond the scope of the course of metal structures. It should be presented during lectures from Strength of Materials.

$$W_w = J_w / \omega$$

J_w – warping moment of inertia / warping constant [m^6]

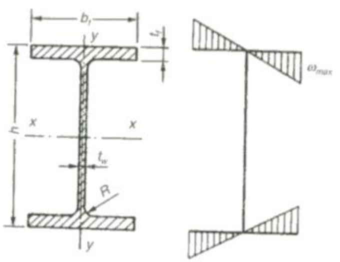
W_w – warping sectional modulus [m^4]

ω – sectional coordinate [m^2]

Warping sectional modulus is not such important for metal structures as sectional coordinate. Warping constant is the most important.

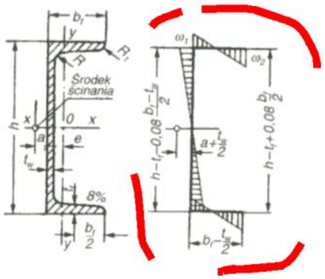
→ Lab #1 / 60

Geometrical characteristics associated with torsion and deplanation are important not only in the case of torsion, but also in the case of thin-walled sections (sometimes aluminum or cold-formed steel sections are calculated using this method), and for matters related to the loss of stability of the element under compression or bending (→ lecture # 5). For "basic" cross-sections (I-sections, C-sections), the warping of inertia and the sectional coordinate are given in the tables.



Rys.: Tablice do projektowania konstrukcji metalowych, W. Bogucki, M. Żyburtowicz, Arkady Wa-wa 1996

→ Lab #1 / 61



Wskaźnik wytrzymałości		Promień bezwładności		Pole wycinkowe	Wycinkowy moment bezwładności	Wycinkowy wskaźnik wytrzymałości	Moment bezwładności przy skręcaniu	Giętnokrętna charakterystyka 1/cm
W_x	W_y	i_x	i_y	ω_{max}	I_ω	W_ω	I_τ	$k=\sqrt{\frac{GI_\tau}{EI_\omega}}$
cm^3		cm		cm^2	cm^6	cm^4	cm^4	
20,0	3,69	3,24	1,06	8,6	118	13,7	0,70	0,0477
34,2	5,79	4,07	1,24	13,0	351	27,1	1,20	0,0363
53,0	8,65	4,90	1,45	18,1	889	49,1	1,74	0,0275
77,3	12,3	5,74	1,65	24,3	1980	81,5	2,45	0,0218
109	16,7	6,58	1,84	31,3	3958	126	3,61	0,0188

Wskaźnik wytrzymałości		Promień bezwładności		Pole wycinkowe		Wycinkowy moment bezwładności	Wycinkowy wskaźnik wytrzymałości	Moment bezwładności przy skręcaniu	Giętnokrętna charakterystyka 1/cm
W_x	W_y	i_x	i_y	ω_2	ω_2	I_ω	W_ω	I_τ	$k=\sqrt{\frac{GI_\tau}{EI_\omega}}$
cm^3		cm		cm^2		cm^6	cm^4	cm^4	
5,33	2,63	1,32	1,04	1,77	2,75	8,46	3,08	0,66	0,172
3,63	0,78	1,47	0,55	2,04	3,18	13,2	4,15	1,02	0,173
8,31	3,18	1,74	1,13	2,56	4,13	19,9	4,81	0,76	0,121
10,6	3,75	1,92	1,13	2,86	4,60	30,6	6,65	1,14	0,120
17,7	5,07	2,52	1,25	4,10	6,97	80,9	11,6	1,88	0,0946

Generally, difference between free and restricted deplanation concerns, first of all, stresses in cross-section:

Cross-section	Deplanation		Stresses
Round (bar, hollow section)	Does not exists		$\tau = T_{Ed} / W_T$
L T +	Very small	Free	
		Restricted by supports	
Rest	Important	Free	
		Restricted by supports	$\tau = T_{t, Ed} / W_T + T_{w, Ed} S_w / J_w t$ $\sigma_x = B_{Ed} \omega / J_w$

S_w - warping static moment [m^4]
$\Theta(x)$ – rotation angle
sh, ch – hyperbolic sin and cos

$$\Theta(x) = A \text{ sh } (k x) + B \text{ ch } (k x) + C x + D + \Theta_s(x)$$

$$k = \sqrt{[(G J_T) / (E J_w)]} \approx 0{,}62 \sqrt{(J_T / J_w)}$$

$$B_{Ed} = -E J_w \Theta''$$

$$T_{w, Ed} = -E J_w \Theta''' + G J_T \Theta'$$

Restricted deplanation is very-very complicated case. It can be avoided thanks to special technical solutions.

In case of steel halls: rigid frames, connected by hinges each other.

Photo: traskostal.pl

The same for floor girders: secondary beams are connected with primary beams by hinges.

Photo: image.made-in-china.com

There is big difference, if main frames are connected each other by rigid joints...

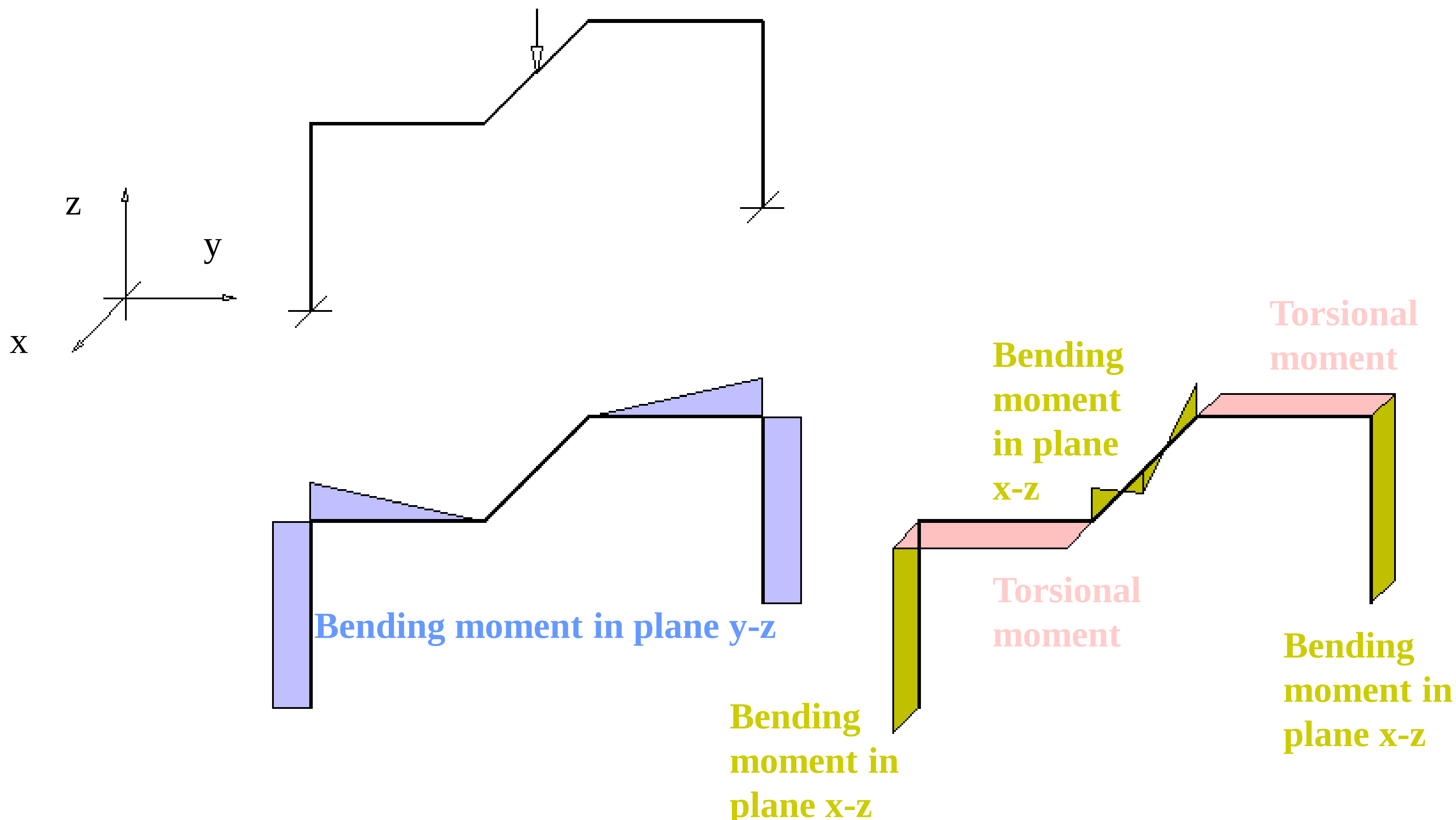


Photo: Author

...or if main frames are connected each other by hinged joints.

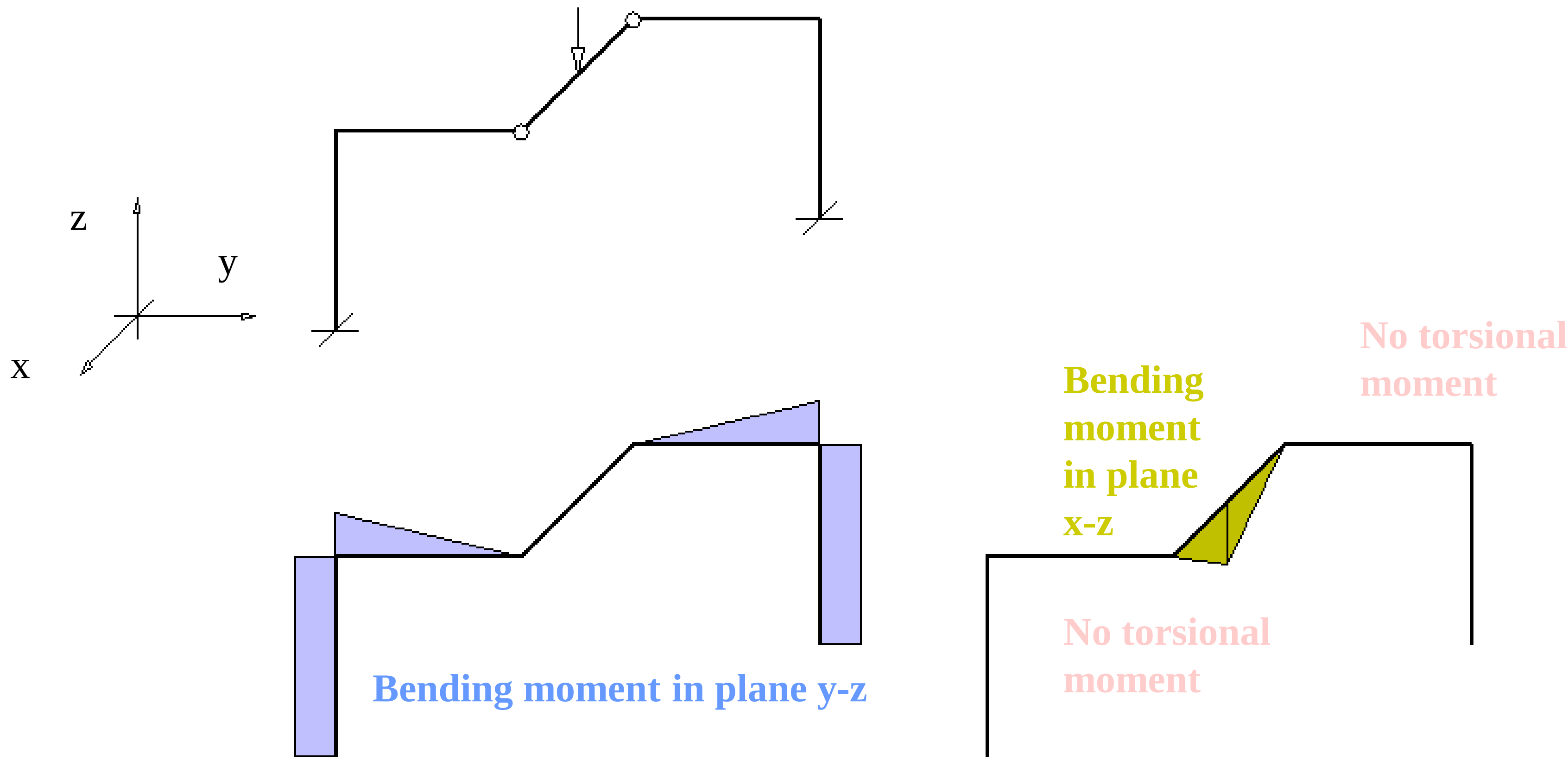


Photo: Author

Recommended solution for steel structures: hinged joints between main frames → no torsional moments in structures, no problem with deplanation.

Torsional moments are analysed, almost exclusively, for run-beam of crane supportes structures → Metal Structures 2.

Photo: hak.com.pl

Shear lag effect

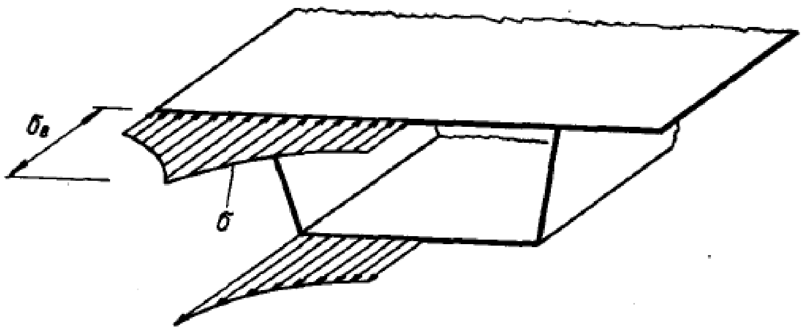


Photo: docplayer.no

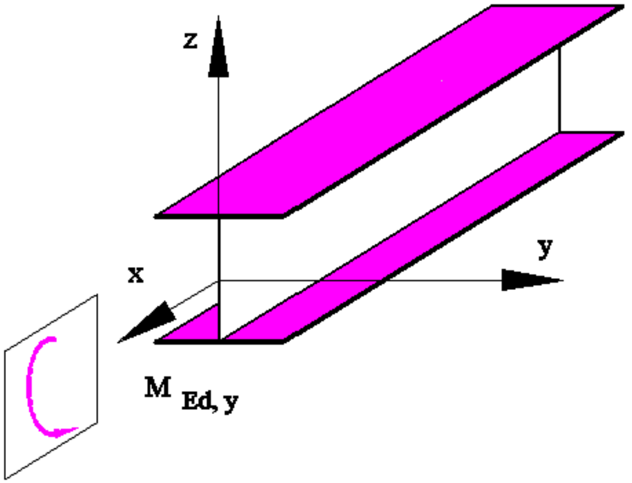


Photo: Author

There is nonlinear shape of stress diagram for very wide part of cross-section perpendicular to plane of bending moment (flanges). This nonlinear shape can be recalculated to linear → effective width of flanges.

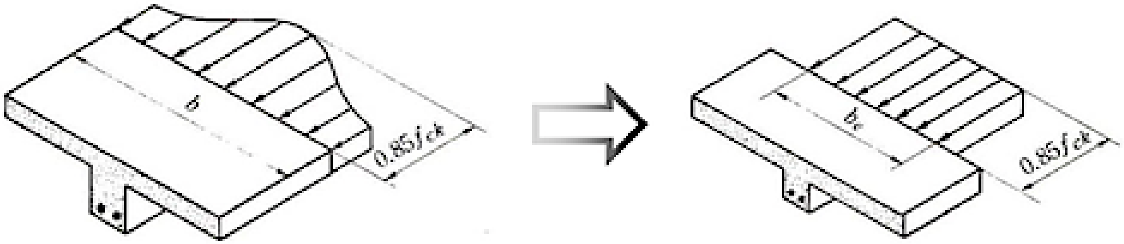


Photo:cfile3.uf.tistory.com

Additionally, effective width is different for different points along beam.

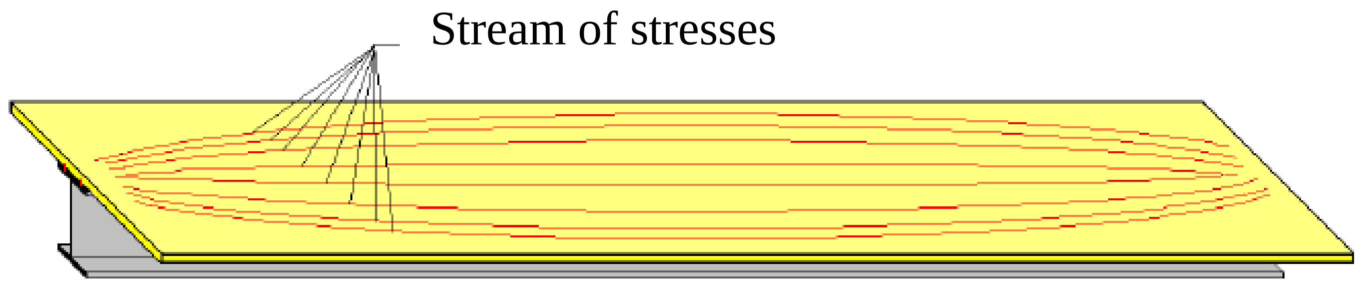
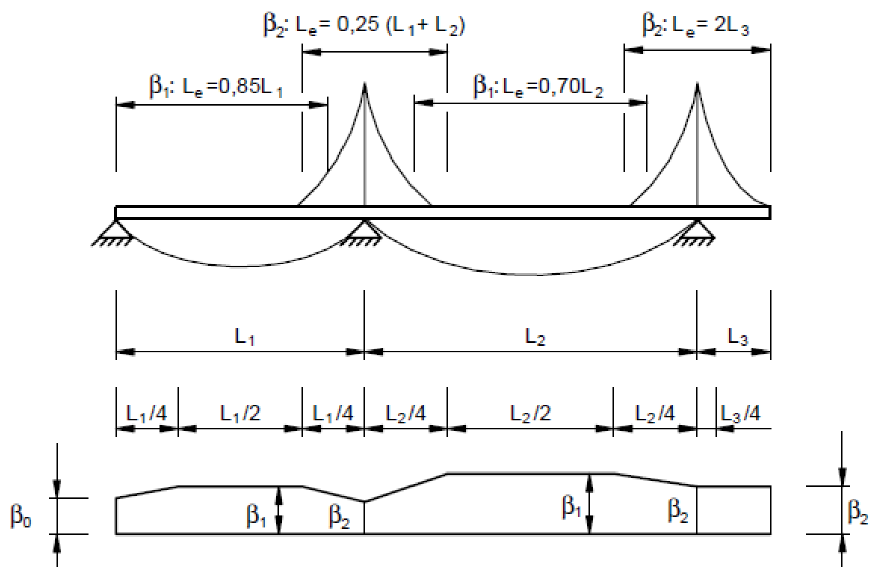


Photo:A Biegus, Projektowanie zespolonych konstrukcji stalowo-betonowych według Eurokodu 4, Politechnika Wrocławska



Value of reduction factor depends on static scheme of I-beam and point along beam.

→ Lab #2 / 25

Photo: EN 1993-1-5, fig 3.1

Calculation of shear lag is presented in Laboratory #2 and Lecture #12. Generally, this phenomenon is dangerous for beams, when member is under bending and

$$b_0 > L_e / 50$$

Phenomenon is very important for welded I-beams. For long hot-rolled I-beams, if $L > L_{s\text{-}l}$, it can be neglected.

I-beam	$L_{s\text{-}l}$
IPE 120	1,60
IPEAA 220	2,75
IPE 300	3,75
IPE 600	5,50
IPEO 600	5,70

I-beam	$L_{s\text{-}l}$
HEA 160	4,00
HEA 320	7,50
HEA 1000	7,50
HEB 1000	7,50
HEM 1000	7,55

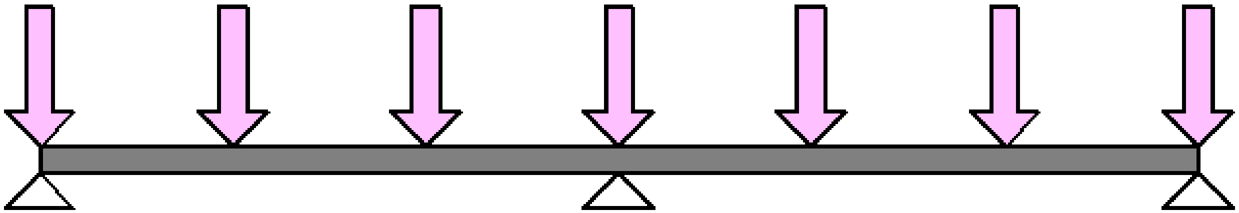
The most often, length of members is greater than that given above. Shear lag can be omitted in most cases.

Redistribution of bending moments

There are two methods of recalculations of bending moments as an effect of redistribution.

Method	Time	M_{Ed}	V_{Ed}	Accuracy
"Table"	👍	👍	👎	👎
"Graphical"	👎	👍	👍	👍

Example 1 - "table" method



L = 2 x 14,0 m

G = 15 kN

Q = 50 kN

Photo: Author

Table: PN B 3200

Value of bending moments after redistribution

6. Belki ciągłe o bisymetrycznym przekroju klasy 1, zabezpieczone przed zwichrzeniem, można projektować z uwzględnieniem plastycznej redystrybucji (wyrównania) momentów, obliczając ich ekstremalne wartości wg wzorów:
- przy obciążeniach równomiernie rozłożonych: g-stałym, q-zmiennym

G, g – dead-weight

$$M = C_g\, gl^2 + C_q ql^2 \qquad \text{(Z4-9)}$$

From forces G, Q in points

- przy obciążeniach skupionych: G - stałym, Q - zmiennym,

$$M = C_G\, Gl + C_Q\, Ql \qquad \text{(Z4-10)}$$

Q, q – live load

From continue loads g, q

gdzie C_g, C_q, C_G, C_Q - wg tabl. Z4-2.
Współczynniki C można również przyjmować, gdy rozpiętość i ekstremalne obciążenia przęseł różnią się nie więcej niż o 10%, przy czym do obliczenia momentu podporowego należy przyjmować wartości średnie rozpiętości i obciążeń przyległych przęseł.
Belki o liczbie przęseł większej niż 5 oblicza się analogicznie jak belki pięcioprzęsłowe, traktując wszystkie przęsła poza dwoma skrajnymi z obu stron jak przęsło środkowe (nr 3).

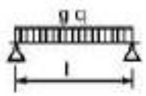
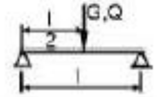
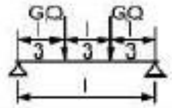
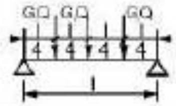
Moments (→ #t / 82):

1, 2, 3 – in spans ; A, B, C – over supports

Stiffness distribution along beam (→ #t / 82)

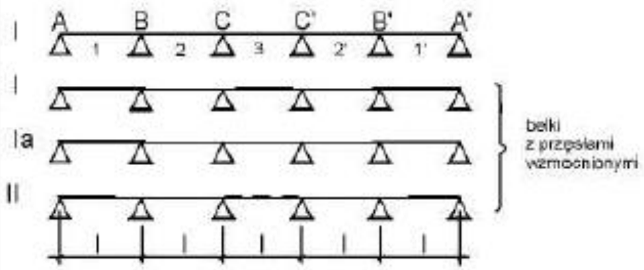
Type of action

Number of spans

Liczba przęseł	Rodzaj belki[1)]	Oznaczenie momentów								
			C_g	C_q	C_G	C_Q	C_G	C_Q	C_G	C_Q
2	I	M_1	0,086	0,105	0,167	0,198	0,250	0,292	0,334	0,412
		M_B	-0,086	-0,105	-0,167	-0,198	-0,250	-0,292	-0,334	-0,412
3	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,039	0,086	0,083	0,150	0,084	0,217	0,166	0,334
	II	M_1	0,096	0,111	0,188	0,213	0,278	0,308	0,375	0,437
		M_B	-0,063	-0,096	-0,125	-0,175	-0,167	-0,256	-0,250	-0,375
		M_2	0,063	0,096	0,125	0,175	0,167	0,256	0,250	0,375
4	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,222	0,367
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,222	-0,367
	II	M_1	0,096	0,110	0,188	0,212	0,278	0,306	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380

		M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
	I	M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,223	0,368
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,223	-0,368
		M_3	0,070	0,102	0,139	0,189	0,184	0,272	0,277	0,401
		M_1	0,096	0,110	0,188	0,212	0,278	0,307	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
5	II	M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_3	0,063	0,100	0,125	0,181	0,167	0,265	0,250	0,389
		M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
	Ia	M_2	0,051	0,092	0,146	0,164	0,139	0,246	0,209	0,360
		M_C	-0,063	-0,098	-0,125	-0,179	-0,167	-0,263	-0,250	-0,385
		M_3	0,063	0,098	0,125	0,179	0,167	0,263	0,250	0,385

1)



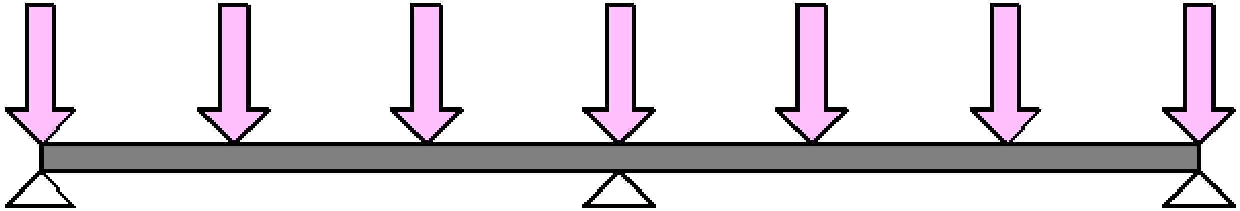


Photo: Author

L = 2 x 14,0 m

Number of span = 2

G = 15 kN

Q = 50 kN

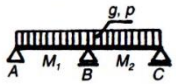
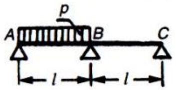
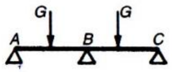
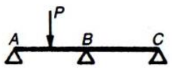
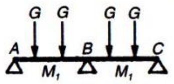
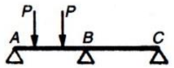
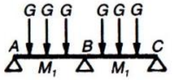
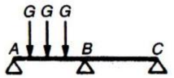
Forces in points, evenly every 1/3 of span length

Liczba przęseł	Rodzaj belki[1]	Oznaczenie momentów	g q		G,Q		G,Q G,Q		G,Q G,Q G,Q	
			C_g	C_q	C_G	C_Q	C_G	C_Q	C_G	C_Q
2	I	M_1	0,086	0,105	0,167	0,198	0,250	0,292	0,334	0,412
		M_B	-0,086	-0,105	-0,167	-0,198	-0,250	-0,292	-0,334	-0,412
3	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,039	0,086	0,083	0,150	0,084	0,217	0,166	0,334
	II	M_1	0,096	0,111	0,188	0,213	0,278	0,308	0,375	0,437
		M_B	-0,063	-0,096	-0,125	-0,175	-0,167	-0,256	-0,250	-0,375
		M_2	0,063	0,096	0,125	0,175	0,167	0,256	0,250	0,375
4	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,222	0,367
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,222	-0,367
	II	M_1	0,096	0,110	0,188	0,212	0,278	0,306	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380

$$M_1 = |M_B| = 0{,}250 \cdot 15\ [\mathrm{kN}] \cdot 14\ [\mathrm{m}] + 0{,}292 \cdot 50\ [\mathrm{kN}] \cdot 14\ [\mathrm{m}] = 256{,}9\ \mathrm{kNm}$$

Shear forces - Winkler's table (approximation: elsatic range, no redistribution)

Dwa przęsła

Lp.	Schematy obciążeń	Momenty przęsłowe		Momenty podporowe	Siły poprzeczne				
		M_1	M_2	M_B	Q_A	Q_{B_l}	Q_{B_p}	Q_B	Q_C
1		0,070	0,070	**–0,125**	0,375	**–0,625**	**0,625**	**1,250**	0,375
2		**0,096**	–0,025	–0,063	**0,437**	–0,563	0,063	0,625	–0,063
3		0,156	0,156	**–0,188**	0,312	**–0,688**	**0,688**	**1,376**	0,312
4		**0,203**	–0,047	–0,094	**0,406**	–0,594	0,094	0,688	–0,094
5		0,222	0,222	**–0,333**	0,667	**–1,334**	**1,334**	**2,677**	**0,667**
6		**0,278**	–0,056	–0,167	**0,833**	–1,167	0,167	1,334	–0,167
7		0,266	0,266	**–0,469**	1,032	**–1,968**	**1,968**	**3,936**	1,032
8		**0,383**	–0,117	–0,234	**1,266**	–1,734	0,234	1,968	–0,234

552

553

Before redistribution (elastic or elasto-plastic range)

$$R_A = R_C = 0{,}667 \cdot (15\ [kN] + 50\ [kN]\ ) + (15\ [kN] + 50\ [kN]\ ) = 108{,}333\ kN$$

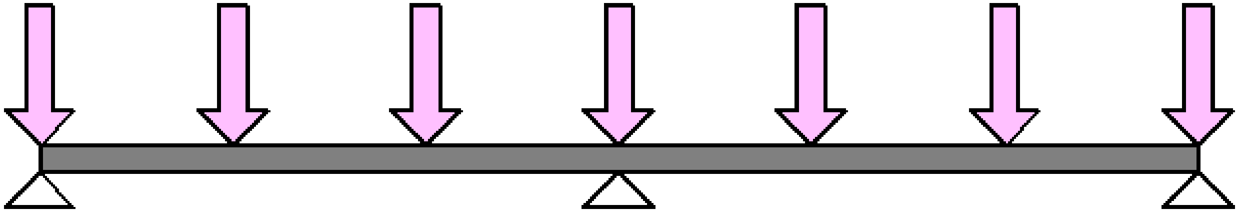
$$R_B = 2{,}667 \cdot (15\ [kN] + 50\ [kN]\ ) + (15\ [kN] + 50\ [kN]\ ) = 238{,}333\ kN$$

After redistribution (plastic range)

$$R^*_A = 1{,}1\ R_A = 119{,}167\ kN$$

$$R^*_B \approx R_B = 238{,}333\ kN$$

Example 2 - "graphical" method



L = 2 x 14,0 m

G = 15 kN

Q = 50 kN

Photo: Author

1. Dead-load only

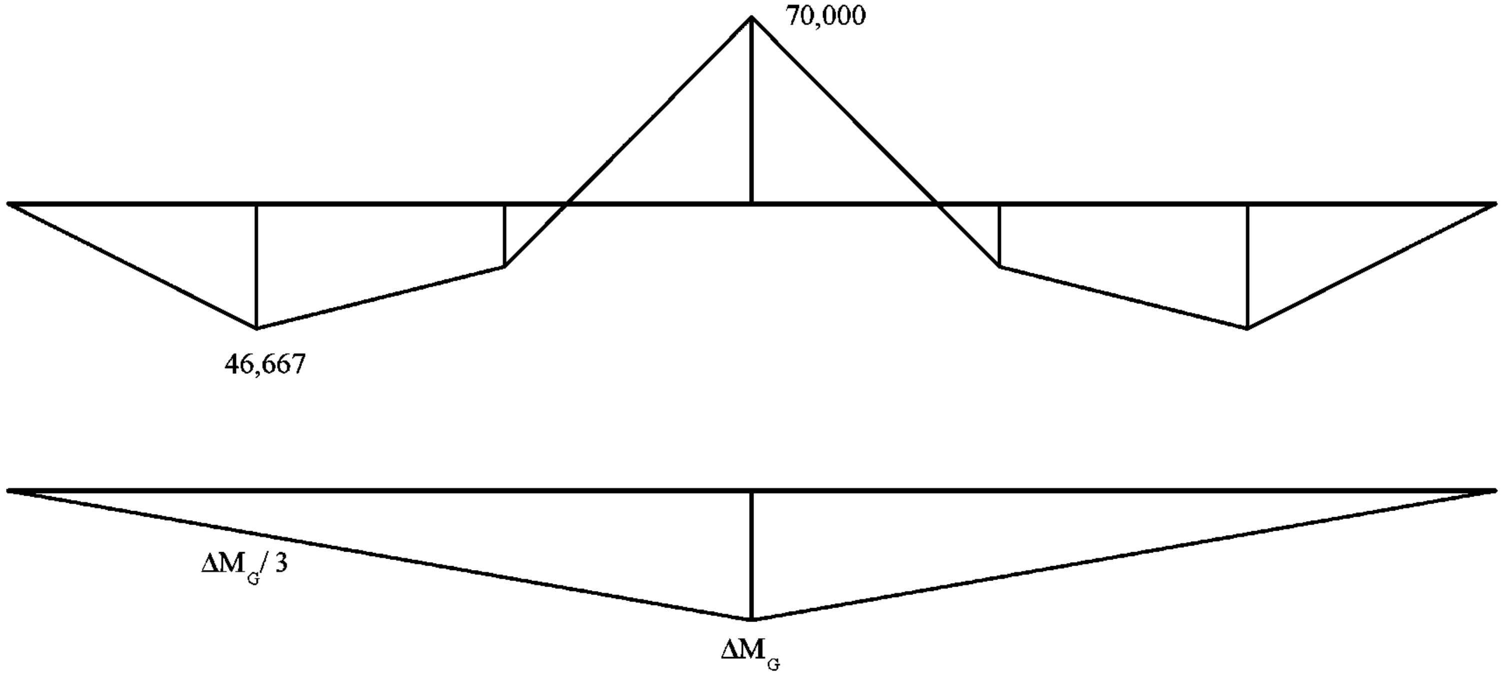


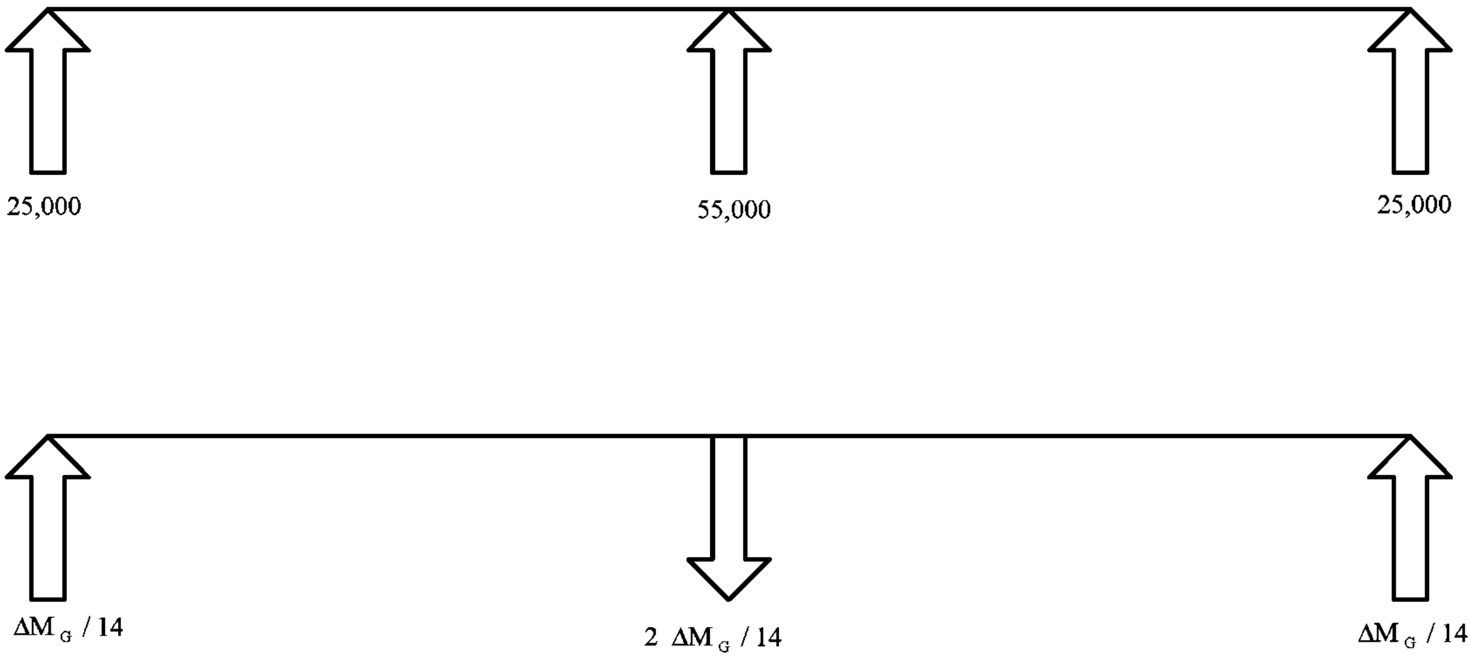
Photo: Author

$$46{,}667 + \Delta M_G / 3 = 70{,}000 - \Delta M_G$$

$$4\, \Delta M_G / 3 = 70{,}000 - 46{,}667$$

$$\Delta M_G = 17{,}500 \text{ kNm}$$

$$\mathbf{M_G = 46{,}667 + \Delta M_G / 3 = 70{,}000 - \Delta M_G = 52{,}500\ kNm}$$

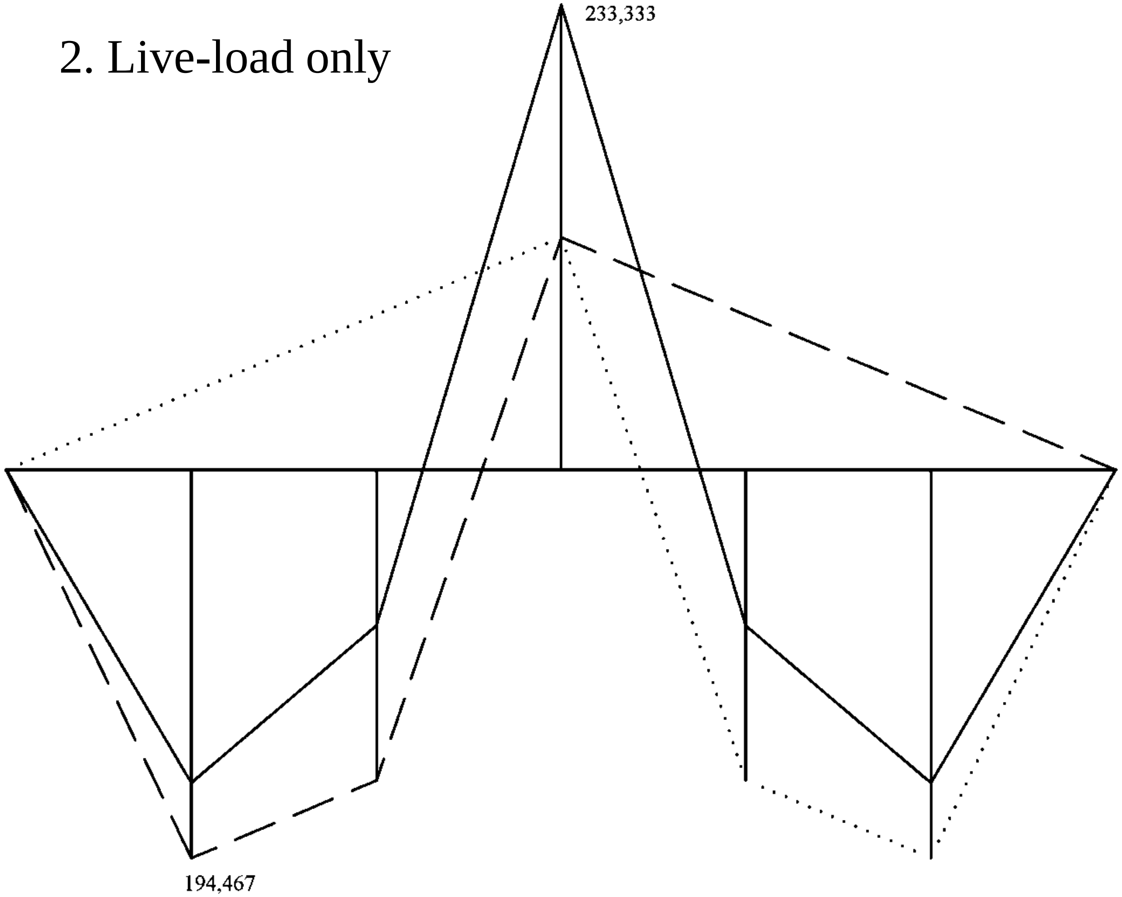


$$R_{AG} = 25{,}000 + \Delta M_G / 14 = 26{,}250 \text{ kN}$$

$$R_{BG} = 55{,}000 - 2\,\Delta M_G / 14 = 52{,}500 \text{ kN}$$

Photo: Author

2. Live-load only

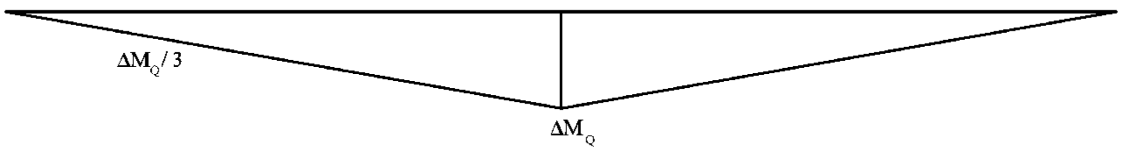


$$194{,}467 + \Delta M_Q / 3 =$$
$$= 233{,}333 - \Delta M_Q$$

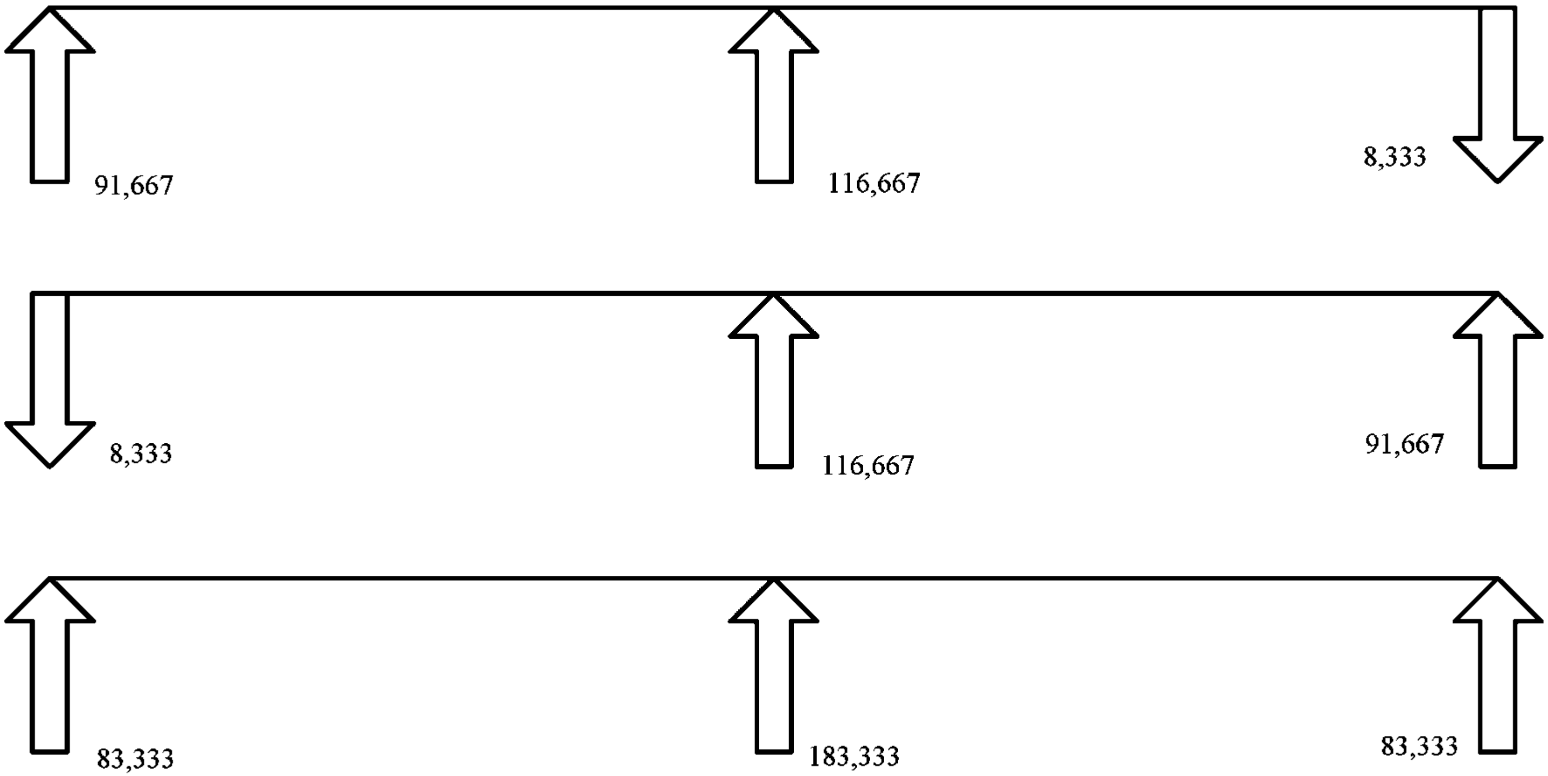
$$4\,\Delta M_Q / 3 =$$
$$= 233{,}333 - 194{,}467$$

$$\Delta M_Q = 29{,}150 \text{ kNm}$$

Photo: Author



$$\mathbf{M_Q = 194{,}467 + \Delta M_Q / 3 = 233{,}333 - \Delta M_Q = 204{,}183 \text{ kNm}}$$



$$R_{AQ\,max} = 83{,}333 + \Delta M_Q / 14 = 97{,}918\ kN$$

$$R_{BQ\,max} = 183{,}333 - 2\,\Delta M_Q / 14 = 181{,}250\ kN$$

Photo: Author

$$\mathbf{M_1 = |\,M_B| = M_G + M_Q = 52{,}500 + 204{,}183 = 256{,}683\ kNm}$$

$$R_{A\,max} = R_{AG} + R_{AQ\,max} = 120{,}000\ kN$$

$$R_{B\,max} = R_{BG} + R_{BQ\,max} = 233{,}750\ kN$$

Conclusions

Class of cross-section	M_1 [kNm] (span)	$\lvert M_B \rvert$ [kNm] (suport)	$R_{A\ max}$ [kN] (left & right supports)	$R_{B\ max}$ [kN] (central suport)
II, III, IV („normal” calculation)	241,133	303,333	108,333	238,333
I – table redistribution	256,900	256,900	119,167	238,333
I – graphical redistribution	256,683	256,683	120,000	233,750

Economy

	IPE 450 77,6 kg / m	IPE 450 A 67,2 kg / m	IPE 400 66,3 kg / m	IPE 400 A 57,4 kg / m	IPE 360 57,1 kg / m
as for III (303,333 kNm ; $\mathbf{W_{el}}$)	86,052 %	96,978 %	111,659 %	126,299 %	142,848 %
as for II (303,333 kNm ; $\mathbf{W_{pl}}$)	75,839 %	86,398 %	98,759 %	112,830 %	126,671 %
as for I (256,683 kNm ; W_{pl})	64,176 %	73,110 %	83,571 %	95,478 %	107,190 %
S235 ~5,0 zł / kg ≈ 1,0 € / kg	~390 zł / m	~335 zł / m as for III	~330 zł / m as for II	~290 zł / m as for I	~285 zł / m

(difference 8,9 kg / m → cheaper ~9 € / m ~45 zł / m) · (few thousands meters in total strucutre)

Deflections

EN 1993-1-1 N.A. 22

Member	w_{max} or w_3
Main roof girder (truss or beam)	L / 250
Purlin	L / 200
Corrugated sheel - roofing	L / 150
Floor girder: primary beam secondary beam	 L / 350 L / 250
Door head or window head	L / 500
w_{max} = netto (total - precamber) w_3 = from variable actions L -length of beam or 2x length of cantilever	

Summary

General algorithm of calculation:

- Resistance for shear force → #t / 53-56;
- Shear lag effect (reduction of cross-section, if is necessary) → #t / 75-77;
- Resistance for bending moment (reduction of resistance, if shear force is big) → #t / 44, 58;
- Stability under bending moment (lateral buckling) → #5, #13;
- Stability under shear force → #t / 57;
- Deflection → #t / 95;

Examination issues

Hot-rolled beam under bending moment and shear force - algorithm of calculation

Thank you for attention

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