

Metal Structures

Lecture IX

Steel trusses



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Definition

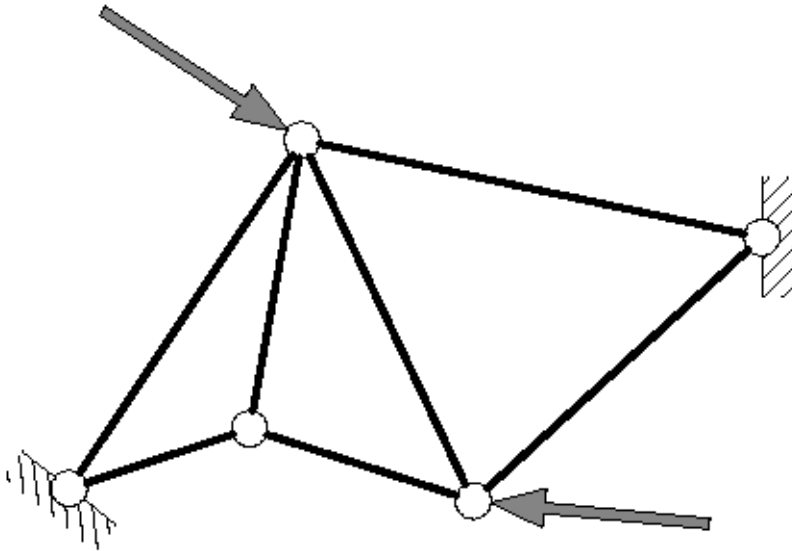


Photo: Author

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Truss – theory (idealization):

- Straight bars only;
- Forces in joints only;
- Hinged joints;

Schwedler-Żurawski formula for straight bar:

$$\begin{aligned}d M(x) / dx &= Q(x) \\d Q(x) / dx &= q(x)\end{aligned}$$

Forces in joints only → no loads along bar
($q(x) = 0$):

$$q(x) = 0 \rightarrow Q(x) = \text{const} = C \rightarrow M(x) = C x + A$$

Hinges:

$$M(0) = 0 \rightarrow A = 0 \quad ; \quad M(L) = 0 \rightarrow C = 0$$

$$M(x) = 0 \quad ; \quad Q(x) = 0$$

There are axial forces only

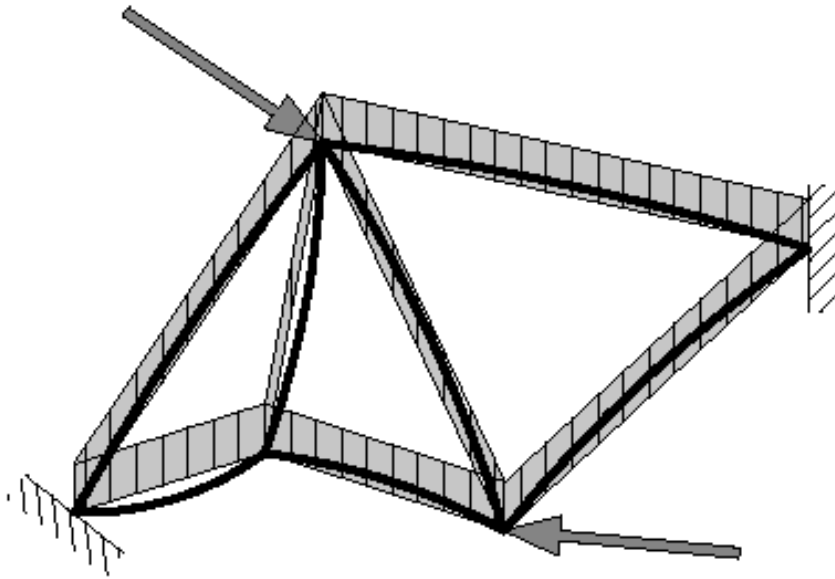


Photo: Author

Truss – real:

- Bars with imperfections;
- Gravity along bars;
- No ideal hinged joints;

It's rather frame

Effort (calculation as a truss) \approx Effort (calculation as a frame)

Time of calculations (truss) \ll Time of calculations (frame)

Because of these reasons, we calculate real truss as ideal truss.

But, because of this - sometimes very small difference between truss and frame - various ideal static models (truss, truss with continuous cords, frame) are applied to calculations of real truss (\rightarrow #t / 54).

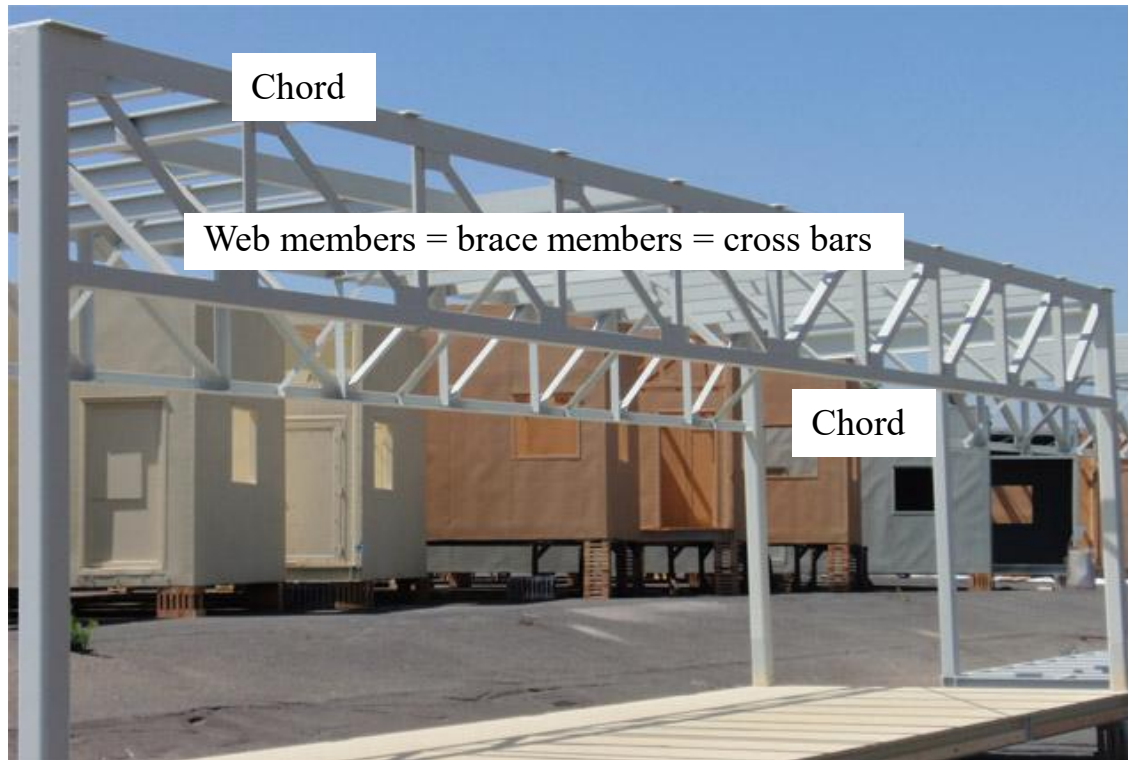


Photo: waldenstructures.com

There is common name for every not-chord bars (web member or brace member or cross bar). In Polish language exist two words: „słupki” (vertical bars) and „krzyżulce” (inclined bars). Common mistake is to design different type of cross-section only because of different name or direction - for web members important are only axial forces and lengths, not names.

Geometry and cross-section

There are many different shapes of trusses.

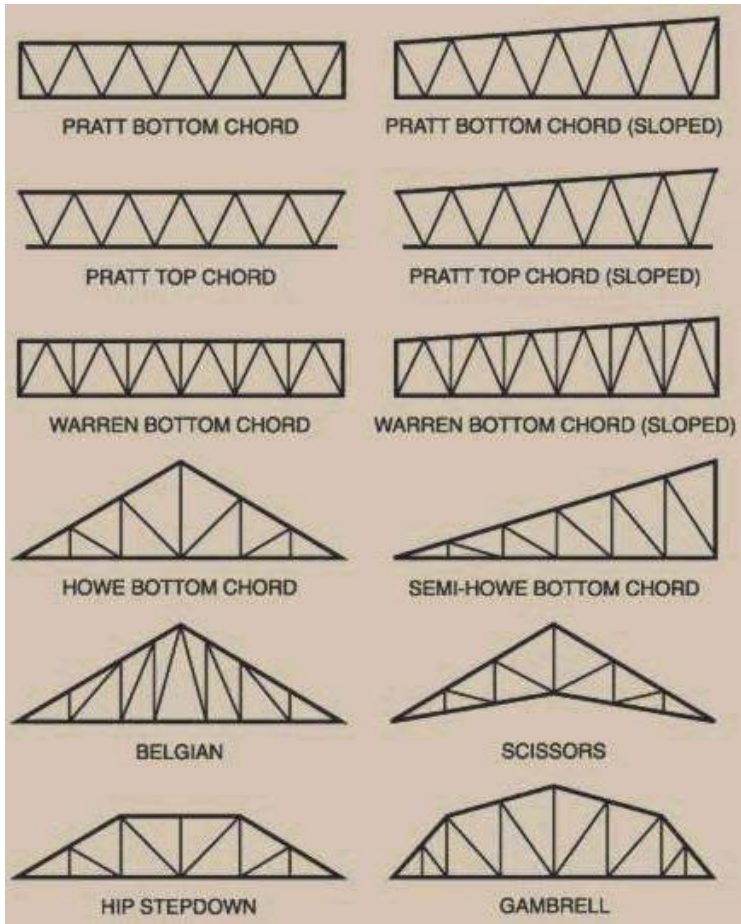


Photo: tridenttruss.com



Photo: steelconstruction.info



Photo: e-plytawarstwowa.pl



Photo: domgaz.com.pl



Photo: konar.eu



Photo: i435.photobucket.com

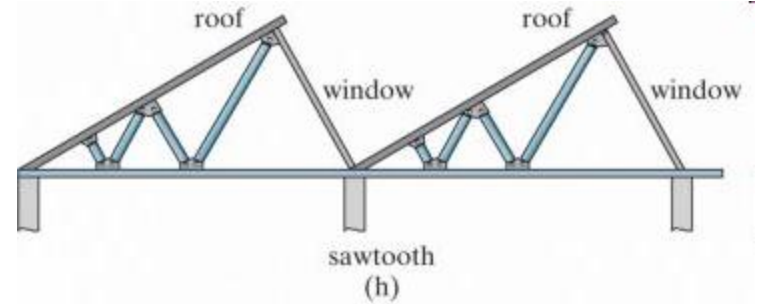


Figure: 03-03H

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Photo: community.fansshare.net

Specific subtype are trusses with paralell chords. For these trusses, forces in top and bottom chord are similar.

$$| N_{Ed, top} | / | N_{Ed, bottom} | \approx 0,90 \div 1,10$$



Photo: waldenstructures.com

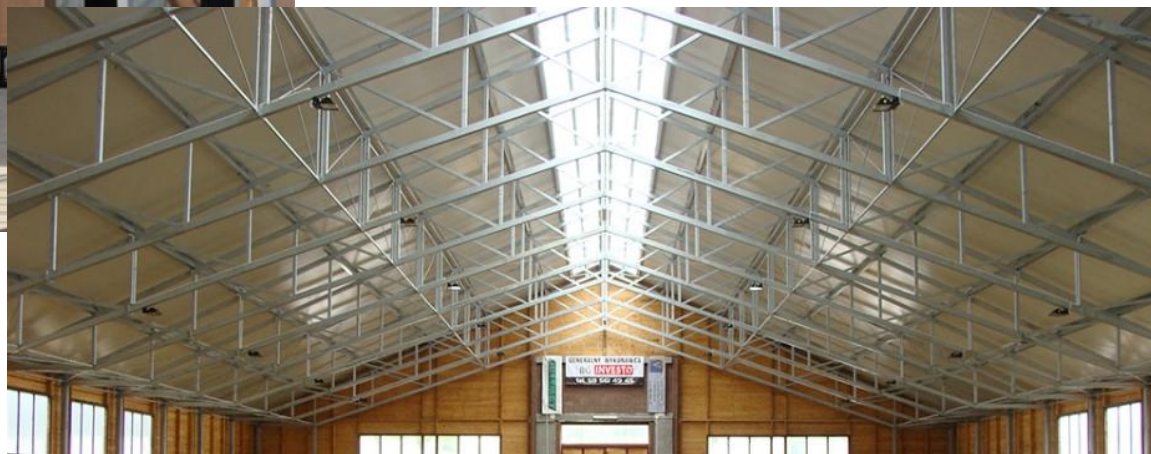


Photo: gillmet.com.pl

Initial assumptions about geometry:

→ Des #1 / 13

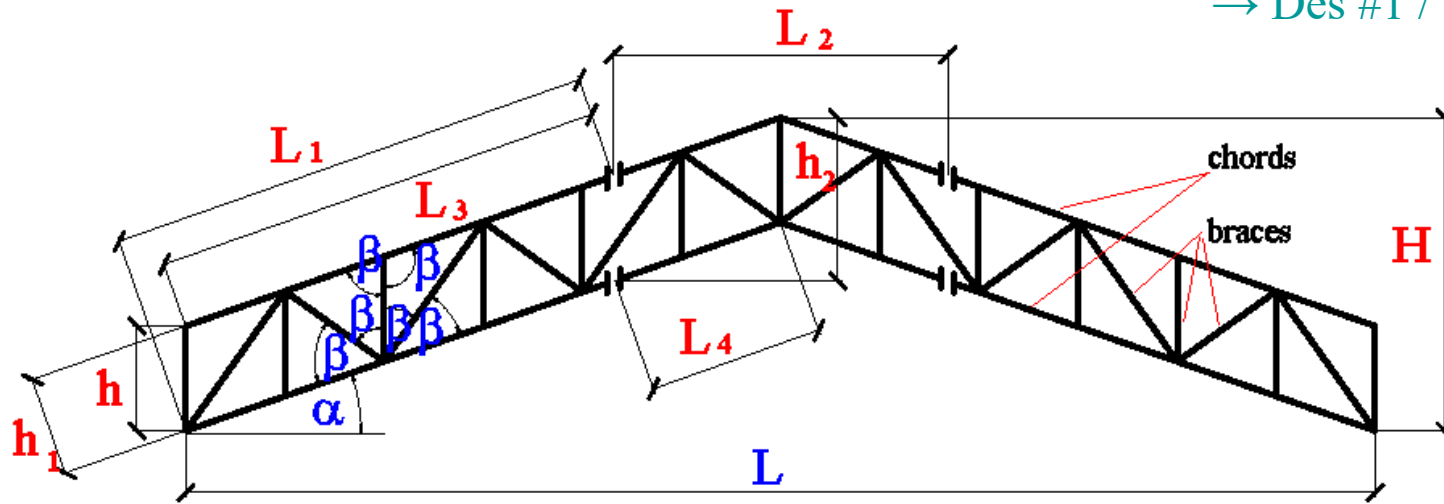


Photo: Author

$$h = L (1/10 \sim 1/15)$$

$$H = L (1/5 \sim 1/10)$$

$$\alpha \geq 5^\circ$$

$$30^\circ \geq \beta \geq 60^\circ \text{ or } \beta \approx 90^\circ$$

$$\max (h_1; h_2) \leq 3,20 \text{ m}$$

$$\max (L_1; L_2; L_3; L_4) \leq 12,00 \text{ m (road transport)}$$

Trusses and I-beams – two types of main girders in steel structures.

Stiffness of I-beam:

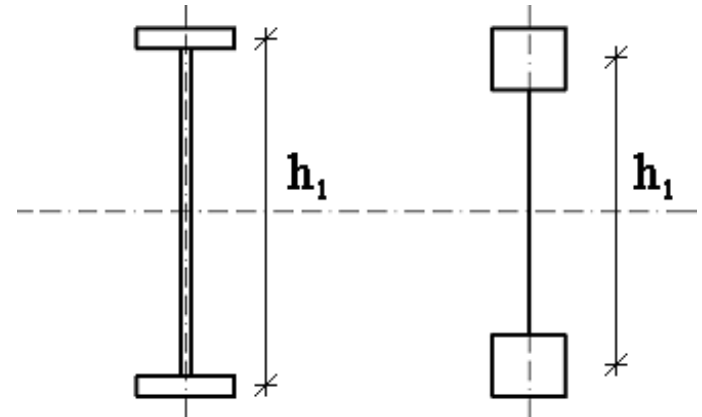
$$J_{\text{I-beam}} = J_{\text{top flange}} + J_{\text{web}} + J_{\text{bottom flange}}$$

For symmetrical cross-section:

$$J_{\text{top flange}} = J_{\text{bottom flange}} \approx 2 \left(h_{1, \text{I}} / 2 \right)^2 A_{\text{flange}}$$

$$J_{\text{I-beam}} \approx 0,5 h_{1, \text{I}}^2 A_{\text{flange}} + J_{\text{web}}$$

Photo: Author



Stiffness of truss (Konstrukcje metalowe, M. Łubiński, A Filipiak, W. Żółtowski, Arkady 2000):

$$J_{\text{truss}} \approx 0,7 [A_{\text{top chord}} \cdot A_{\text{bottom chord}} / (A_{\text{top chord}} + A_{\text{bottom chord}})] h_{1, \text{truss}}^2$$

For symmetrical cross-section:

$$A_{\text{top chord}} = A_{\text{bottom chord}} = A_{\text{chord}}$$

$$J_{\text{truss}} \approx 0,7 [A_{\text{chord}}^2 / (2 A_{\text{chord}})] h_{1, \text{truss}}^2 = 0,35 h_{1, \text{truss}}^2 A_{\text{chord}}$$

Stiffness is important, first of all, for SLS. SLS limits (accepted deflection) are the same for truss and beams. This means:

$$\text{stiffness of I-beam} = \text{stiffness of truss: } J_{\text{I-beam}} = J_{\text{truss}}$$

Stiffness of I-beam:

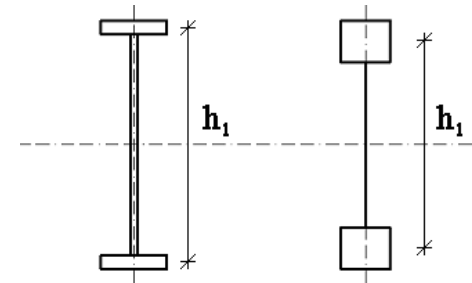
$$J_{\text{I-beam}} \approx 0,5 h_{1,I}^2 A_{\text{flange}} + J_{\text{web}}$$

For various types of I-beams, $J_{\text{web}} = 7\% - 25\%$ of J_{flange}

Stiffness of truss:

$$J_{\text{truss}} \approx 0,35 h_{1,\text{truss}}^2 A_{\text{chord}}$$

$$\text{If } A_{\text{flange}} = A_{\text{chord}} \text{ and } h_{1,I} = h_{1,\text{truss}} \quad J_{\text{I-beam}} \approx 1,53 \div 1,79 J_{\text{truss}}$$

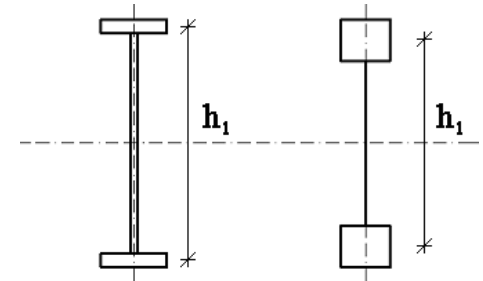


We need $J_{\text{I-beam}} = J_{\text{truss}}$; this means: $h_{1,\text{truss}} > h_{1,I}$ and maybe $A_{\text{chord}} > A_{\text{flange}}$

Photo: Author

$$h_{1, \text{truss}} > h_{1, \text{I}}$$

Initial assumptions about geometry of cross-sections:



$$h_{1, \text{Ibeam}} = L / 20 - L / 25 \quad ; \quad h_{1, \text{truss}} = L / 10 - L / 15 \rightarrow h_{1, \text{truss}} > h_{1, \text{Ibeam}}$$

Photo: Author

L [m]	$h_{1, \text{Ibeam}}$	$h_{1, \text{truss}}$
15,0	~ 650 mm	~ 1200 mm
20,0	~ 900 mm	~ 1600 mm
25,0	~ 1200 mm	~ 2000 mm

$$A_{\text{chord}} \leftrightarrow A_{\text{flange}}$$

L [m]	$h_{1, \text{Ibeam}}$	Cross-section
15,0	~ 650 mm	HEA 650 $A_{\text{flange}} = 78 \text{ cm}^2$
20,0	~ 900 mm	HEA 900 $A_{\text{flange}} = 90 \text{ cm}^2$
25,0	~ 1200 mm	HLA+ 1100 $A_{\text{flange}} = 124 \text{ cm}^2$

The same resistance:

L [m]	$h_{1, \text{truss}}$	Cross-section
15,0	~ 1200 mm	168,3/12,5 $A_{\text{chord}} = 61,2 \text{ cm}^2$
20,0	~ 1600 mm	193,7/12,5 $A_{\text{chord}} = 71,2 \text{ cm}^2$
25,0	~ 2000 mm	244,5/14,2 $A_{\text{chord}} = 103,0 \text{ cm}^2$

$$J_{\text{truss}} \approx 0,35 h_{1, \text{truss}}^2 A_{\text{chord}}$$

L [m]	J_{Ibeam}	J_{truss}
15,0	HEA 650 281 700 cm ⁴	308 500 cm ⁴
20,0	HEA 900 422 100 cm ⁴	637 950 cm ⁴
25,0	HLA 1100 867 400 cm ⁴	1 442 000 cm ⁴

Trusses are more stiff than I-beams, but space inside structure for trusses is bigger than for I-beams:

L [m]	$h_{1, \text{Ibeam}}$	$h_{1, \text{truss}}$
15,0	~ 650 mm	~ 1200 mm
20,0	~ 900 mm	~ 1600 mm
25,0	~ 1200 mm	~ 2000 mm

Dead-weight of truss

Ist proposal (PN B 02001):

$$g_T = [2 / a + 0,12 (g + q)] L / 100$$

g_T (dead-weight of all trusses per area of roof), g (roofing + purlins), q (snow + wind) → [kN/m²], characteristic values

a (distance between trusses), L (truss span) → [m]

IInd proposal:

$$g_{T1} = d_{\text{steel}} q_1 L^3 / (2 H f_y)$$

g_{T1} (dead-weight of one truss) → [kN], d_{steel} own weight of steel → [kN/m³], H – height of truss (vertical distance between top and bottom chord) → [m]

$$q_1 = a(g + q)$$

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$$g_{T1} \approx a L g_T$$

$a = 6,0 \text{ m}$, Ist proposal, $aLg_T \text{ [kN]} =$

q [kN / m]	L [m]		
	18,0	24,0	30,0
10,0	53,1	94,5	147,6
15,0	76,5	137,0	212,4

$a = 6,0 \text{ m}$, IInd proposal, $g_{T1} \text{ [kN]} =$

q [kN / m]	L [m]		
	18,0	24,0	30,0
10,0	48,7	86,6	135,3
15,0	73,1	129,9	202,9

Types of truss structures

Truss purlins

„Classical” trusses

Multi-chords trusses

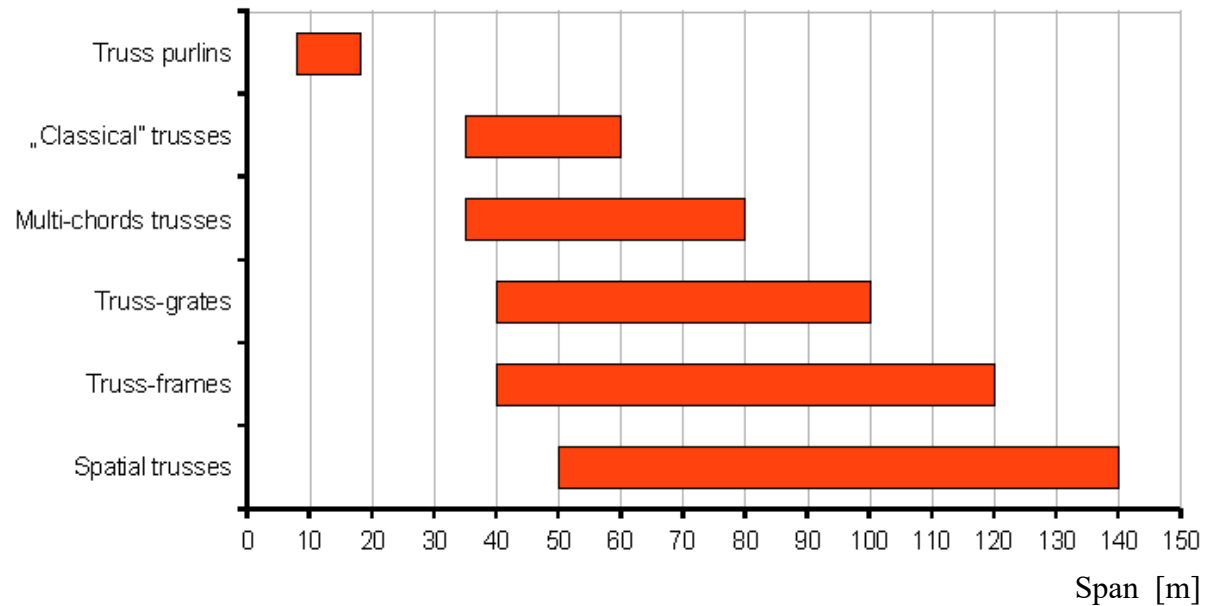
Truss-grates

Truss-frames

Spatial trusses

Laced columns

Photo: Author



Truss purlins

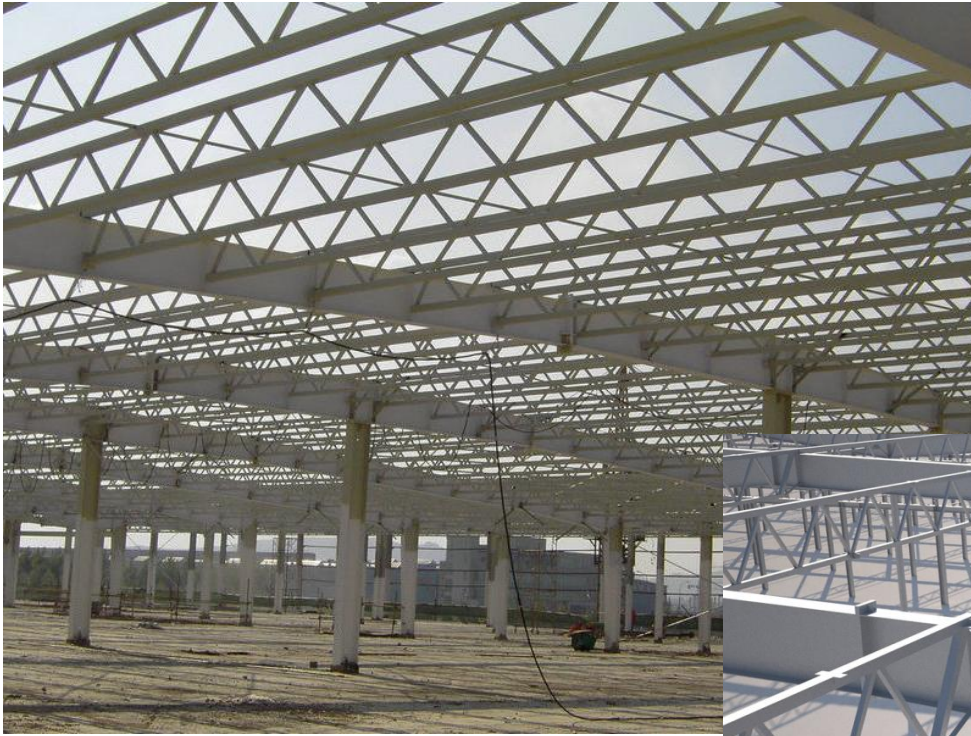


Photo: structural-steelbuilding.com



Photo: CoBouw Polska Sp. z o. o.

Truss purlin

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"Normal" truss - forces are applied in nodes; there are axial forces in chords and cross-bars only.

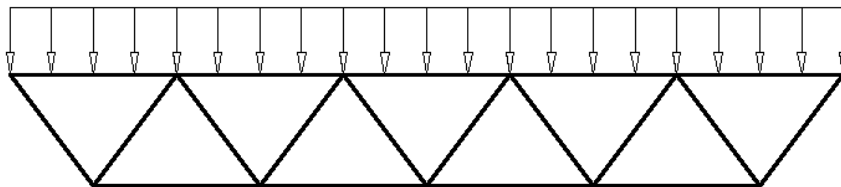
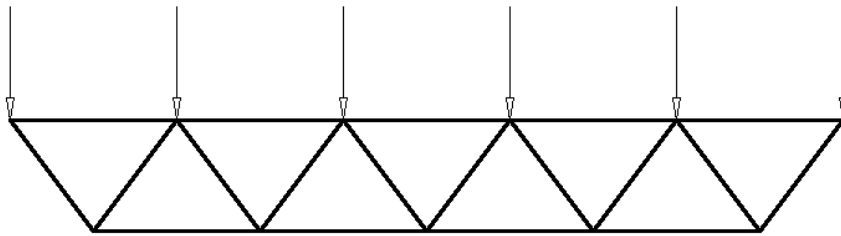
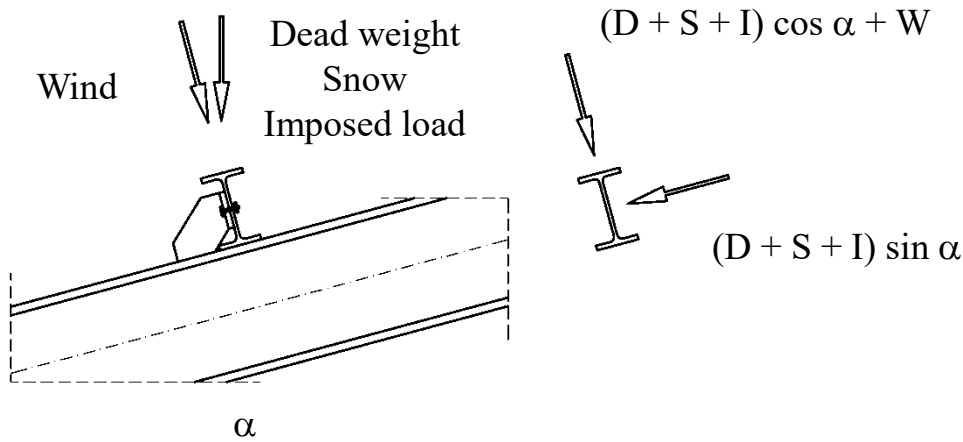


Photo: Author



Photo: construdare.com

Truss purlin - continuous load from roofing; there are axial forces in chords and cross-bars; in addition top chord is bending.



I-beam purlin: bi-axial bending.

→ #8 / 37

Truss purlin: horizontal force
 $W \sin \alpha$

has very small value and can be neglected
 (acts on roofing, not purlins). All loads act in
 plane of truss. There is need wedge to install
 truss purlin in vertical position.

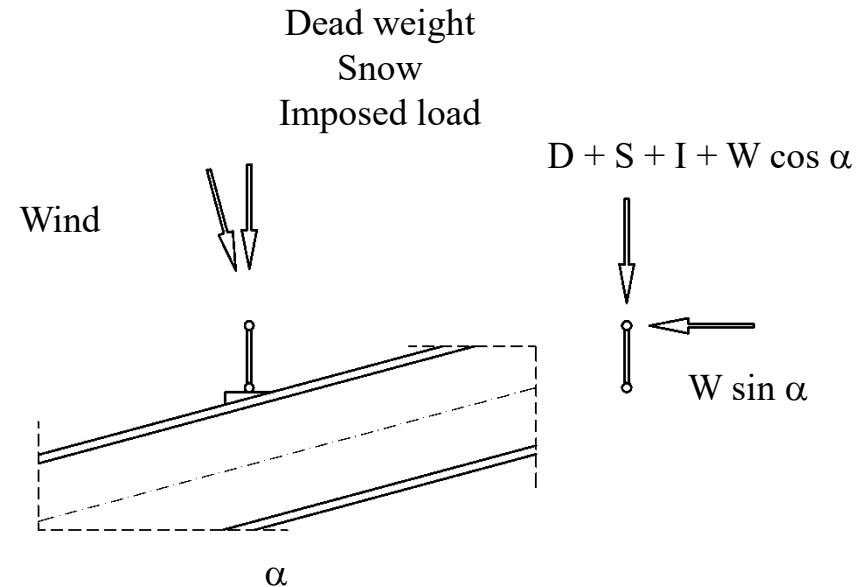


Photo: Author

„Classical” trusses



Photo: steelconstruction.info



Photo: domgaz.com.pl

2-chords flat trusses, used
as a single roof girders.



Photo: waldenstructures.com

Multi-chords trusses



Photo: steelconstruction.info

Photo: multimetalgb.ca



3 or 4 trusses connected each other in triangle or square cross-section. Often used for temporary structures (bandstand) or for masts and towers.



Photo: conference-truss-hire.co.uk



Photo: rktruss.com



Photo: stretchtents.com.au

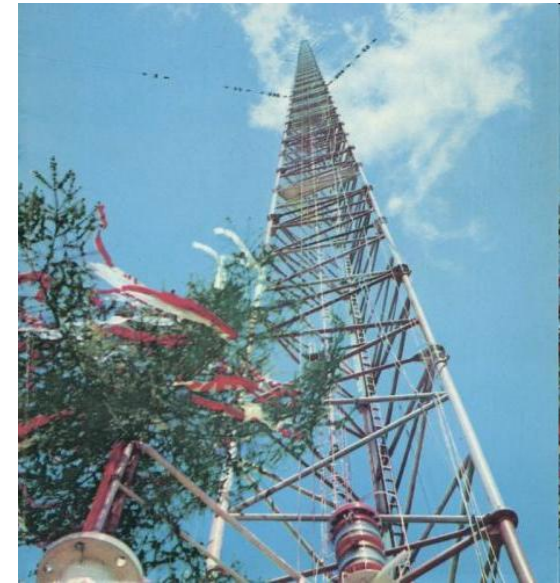


Photo: eioba.pl

Truss-grates



Photo: cdn8.muratorplus.smcloud.net



Photo: qdjinfei.en.made-in-china.com

Complex of trusses of the same height, perpendicular each other.

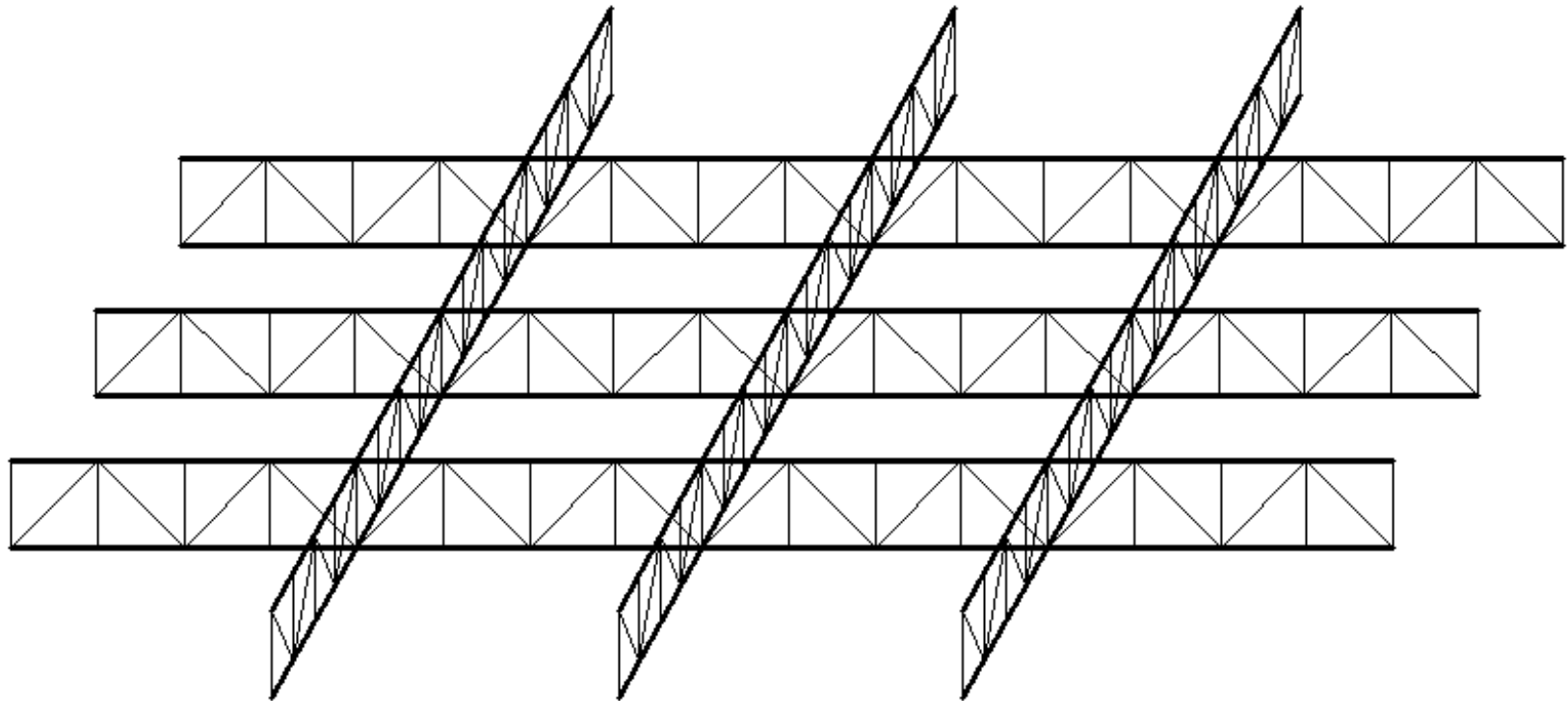


Photo: Author

Frame-truss

"Classical" portal frame (two columns and roof girder), but made of trusses, not I-beams.

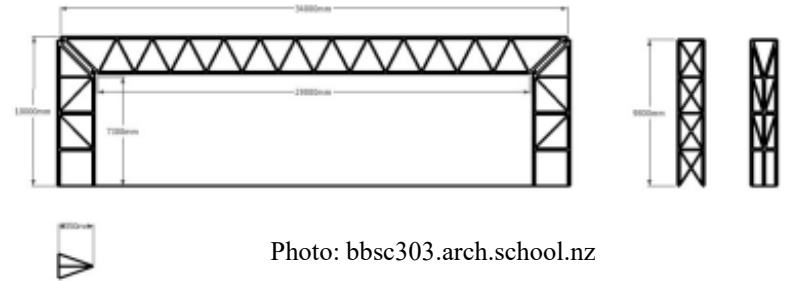


Photo: bbsc303.arch.school.nz



Photo: wikipedia

Spatial trusses



Photo: miripiri.co.in



Photo: urwishengineers.com

2 or 3 layers of bars + cross-bars
between layers.

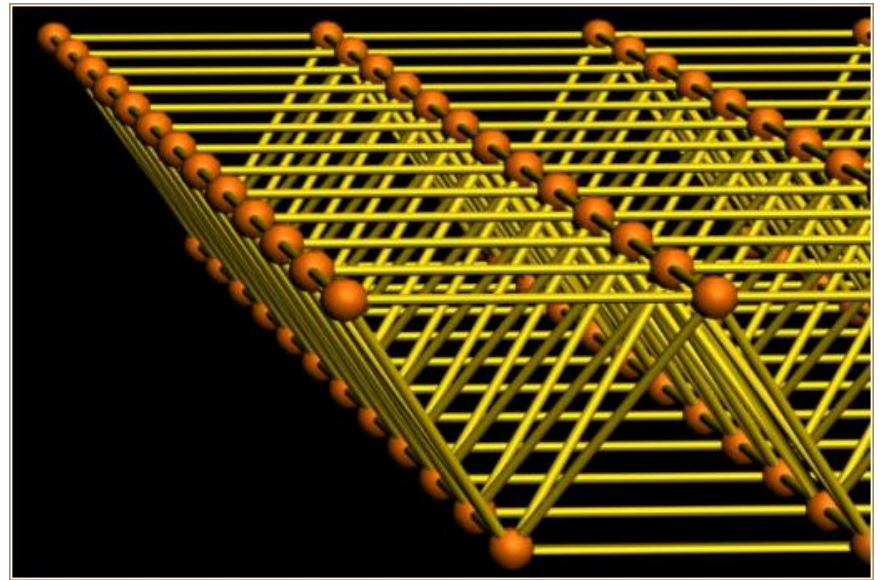
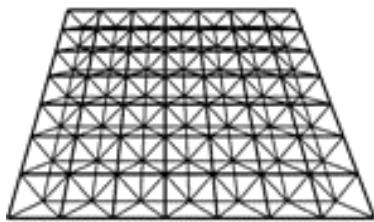
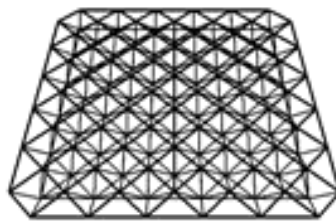


Photo: civiltech.ir

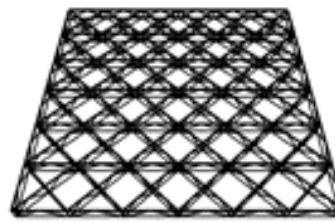
Photo: shreeengineering.in/



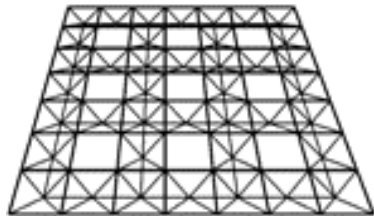
(a) Two-way on two-way grid



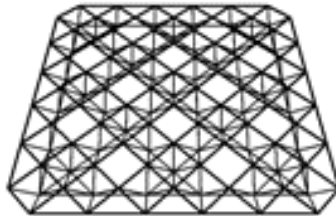
(b) Diagonal on diagonal grid



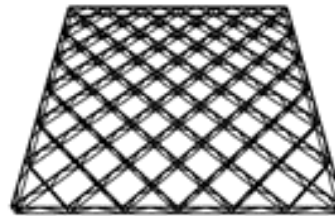
(c) Three-way truss grid



(d) Reduced two-way on
two-way grid



(e) Reduced diagonal on
diagonal grid



(f) Diagonal truss grid

Layers can be flat, cylindrical or spherical. More information about this type of structures will be presented on IInd step of studies.



Photo: wikipedia

Photo: cnxzf.com

Laced columns

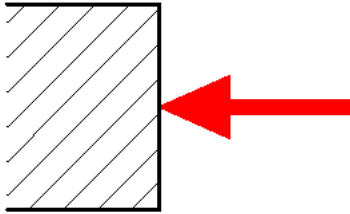


Photo: zksgrzelak.eu

Structure looks like truss, but its behaviour and calculation model is completely different than for truss.

For laced columns, battened columns and closely spaced build-up compression members we use specific algorithm of calculations. More information about this type of structures will be presented on IInd step of studies.

Calculations



$$N_{Ed} / N_{b,Rd} \leq 1,0$$

$$N_{b,Rd} = \chi A f_y / \gamma_{M0}$$

$$N_{Ed} / N_{t,Rd} \leq 1,0$$

$$N_{t,Rd} = A f_y / \gamma_{M1}$$

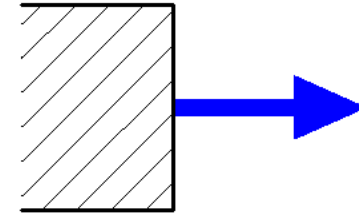


Photo: Author

χ - lecture #5:

- ◆ Flexural buckling
- ◆ Torsional buckling
- ◆ Flexural-torsional buckling

Critical length for compressed chord of truss is depend on plane of analysis. For vertical plane, critical length is equal distance between cross-bars. For horizontal plane, critical length is equal distance between bracings:

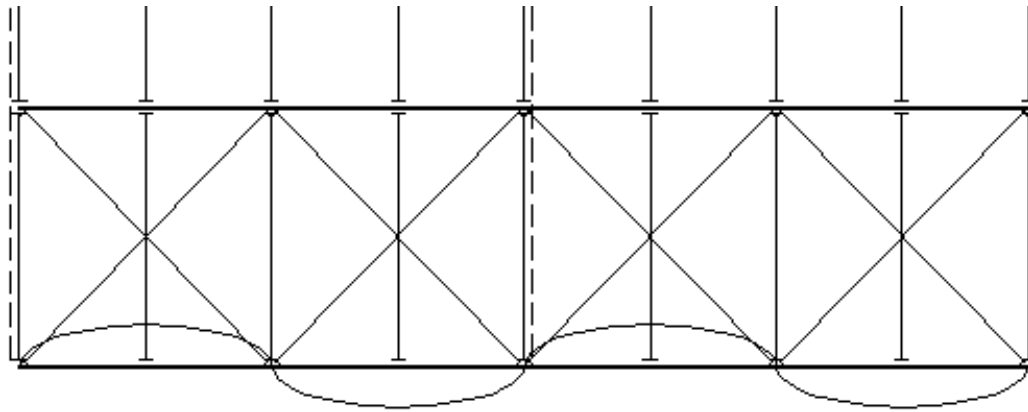
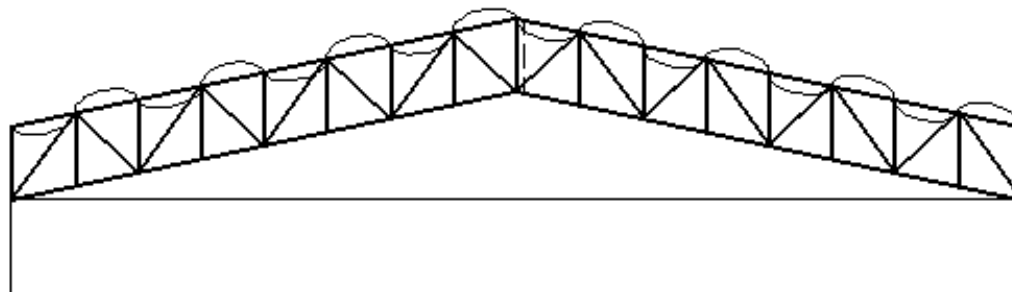


Photo: Author



Distance between supports for truss members is depend on plane of analysis and part of structure. For web members, in both planes distance between supports = distance between nodes.

Flexural buckling of chords:

→ Des #1 / 53

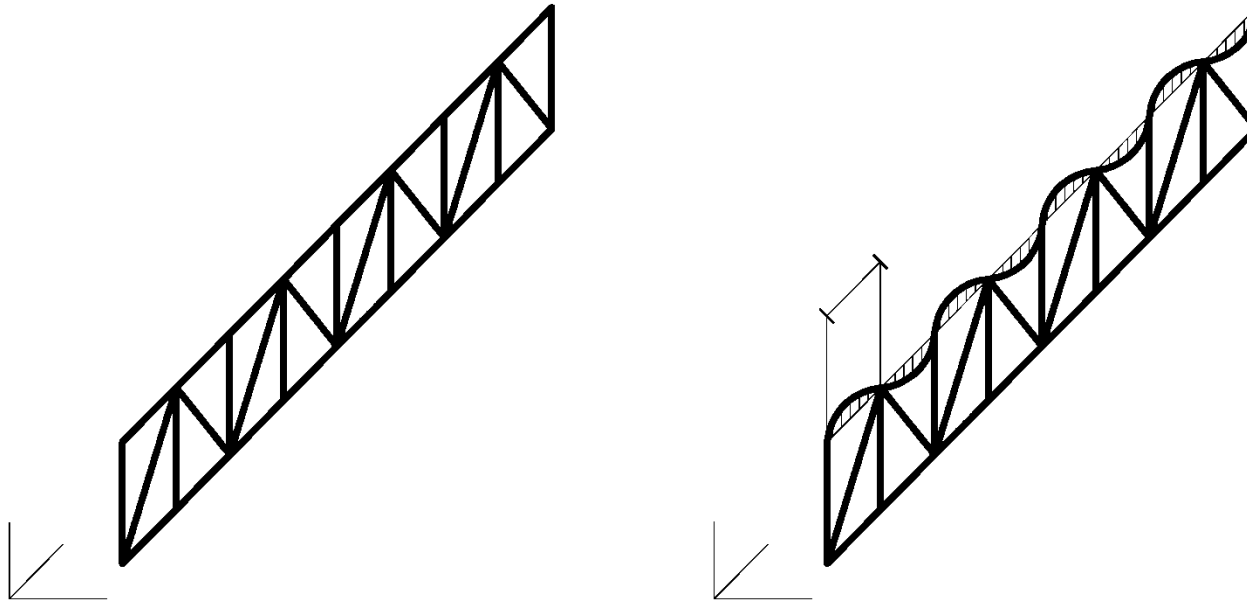


Photo: Author

Top chords in compression; buckling in plane: distance between supports = distance between nodes

Flexural buckling of chords:

→ Des #1 / 54

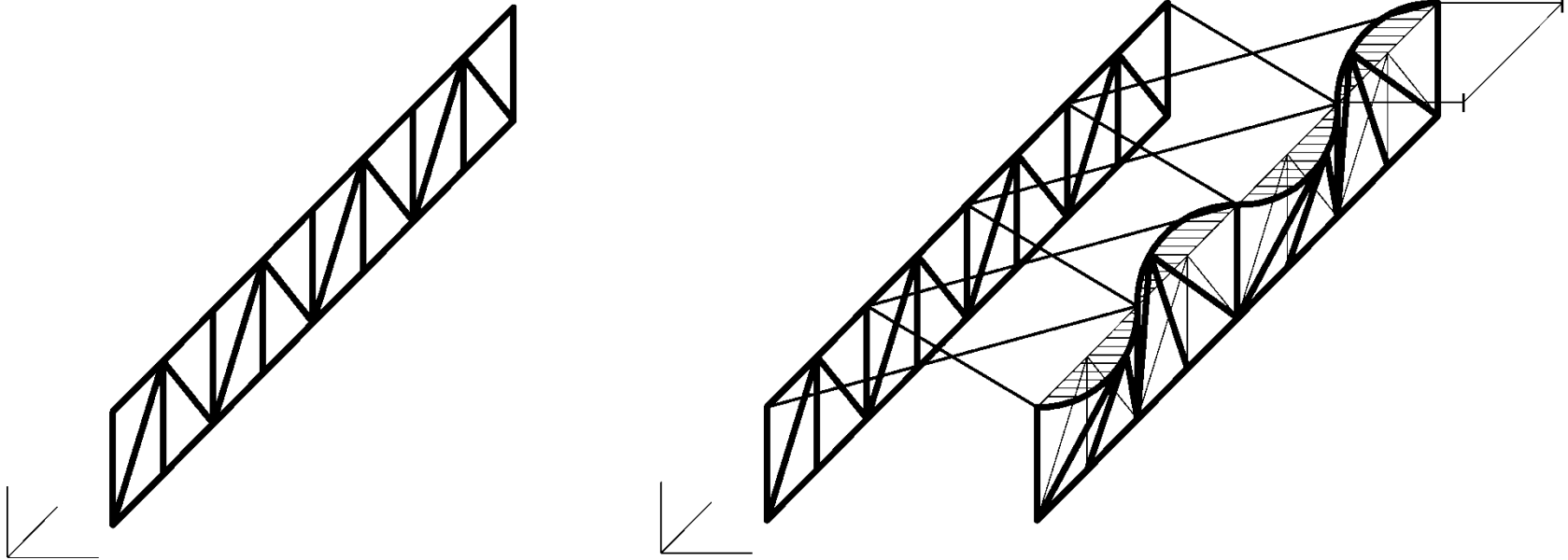


Photo: Author

Top chords in compression; buckling out of plane: distance between supports = distance between horizontal bracings

Flexural buckling of chords:

→ Des #1 / 55

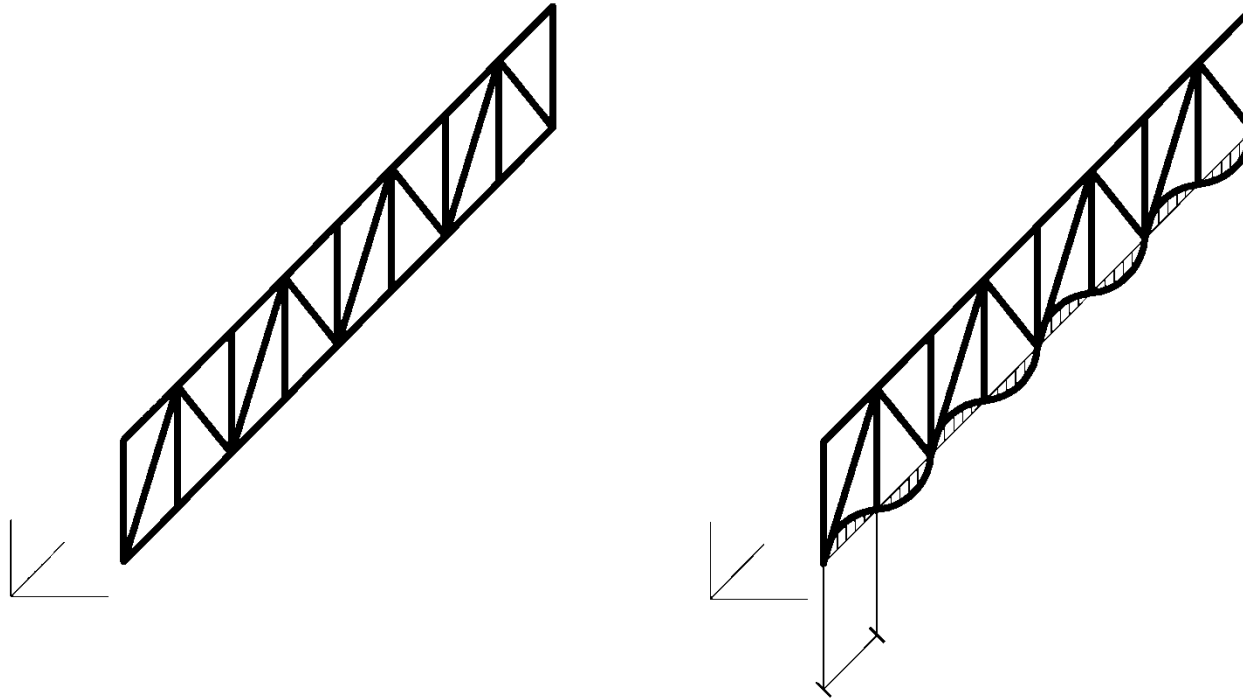


Photo: Author

Bottom chords in compression; buckling in plane: distance between supports = distance between nodes

Flexural buckling of chords:

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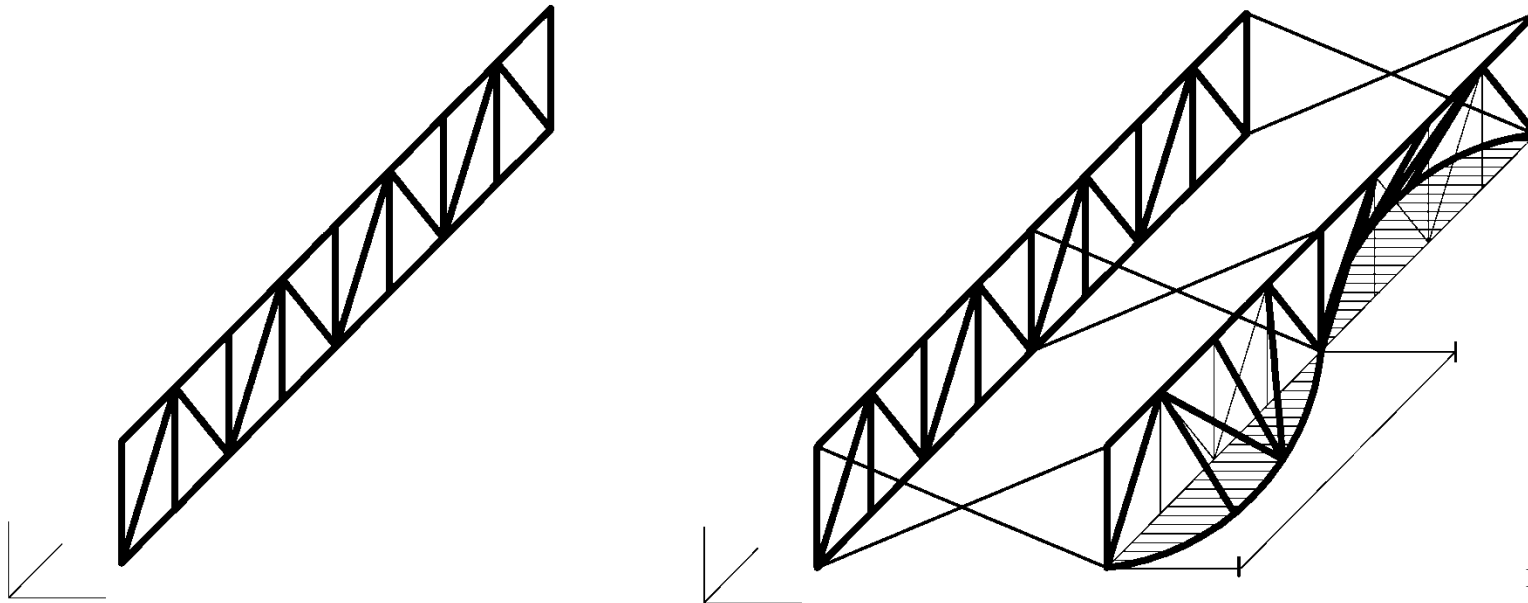


Photo: Author

Bottom chords in compression; buckling out of plane: distance between supports = distance between vertical bracings

Bracings are very important for behaviour of structures, including for trusses. For calculations, there are used special algorithms and models.

More information about bracings will be presented on lecture #10.

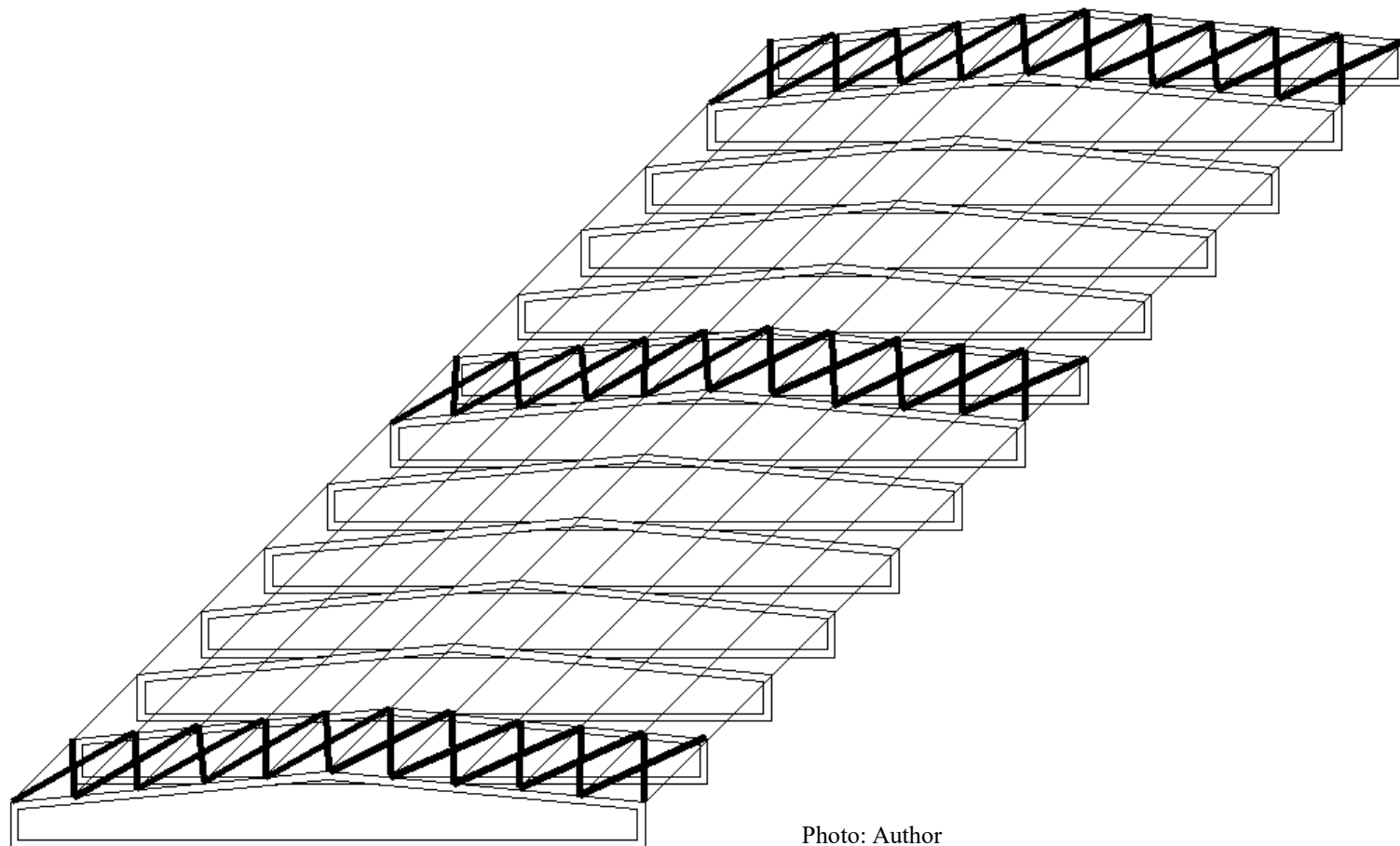

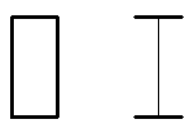



Photo: Author

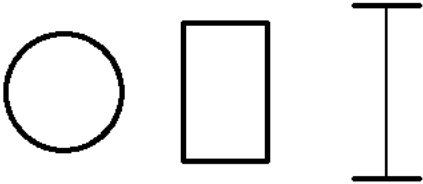
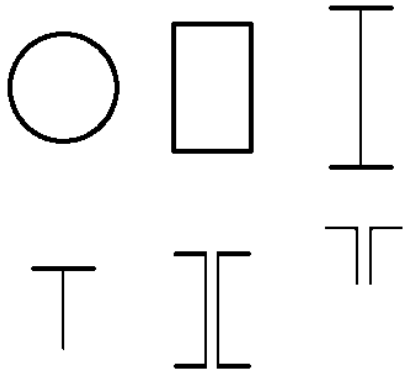

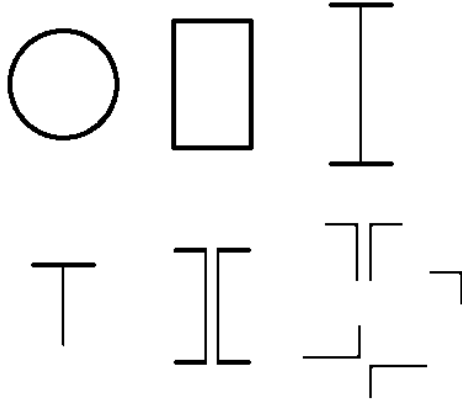
The result of calculations is buckling factor χ .

It is calculated in different way for different cross-sections.

Buckling		
Flexural		Flexural, torsional, flexural-torsional
	 <p>(hot rolled I)</p>	 <p>(welded I)</p>
$\chi = \chi_y = \chi_z$ (only if $l_{cr, y} = l_{cr, z}$)	$\chi = \min(\chi_y ; \chi_z)$	$\chi = \min(\chi_y ; \chi_z ; \chi_T ; \chi_{z, T})$

Cross-sections of truss members

	PN B 03200	EN 1993
Elements	Each types of cross-sections are accepted	
Joints	No additional requirements	Additional requirements → many types of cross-sections are not accepted

	Modern types of cross-sections (EN)	Old types of cross-sections (PN-B)
Chords		
Web members	 <p data-bbox="374 1168 529 1196">Photo: Author</p>	

Whenever we use hollow sections, we must hermetically close the ends by welding to prevent corrosion inside HS. Such type of corrosion can develop without apparent external symptoms and can cause damage of structure without warning signs.



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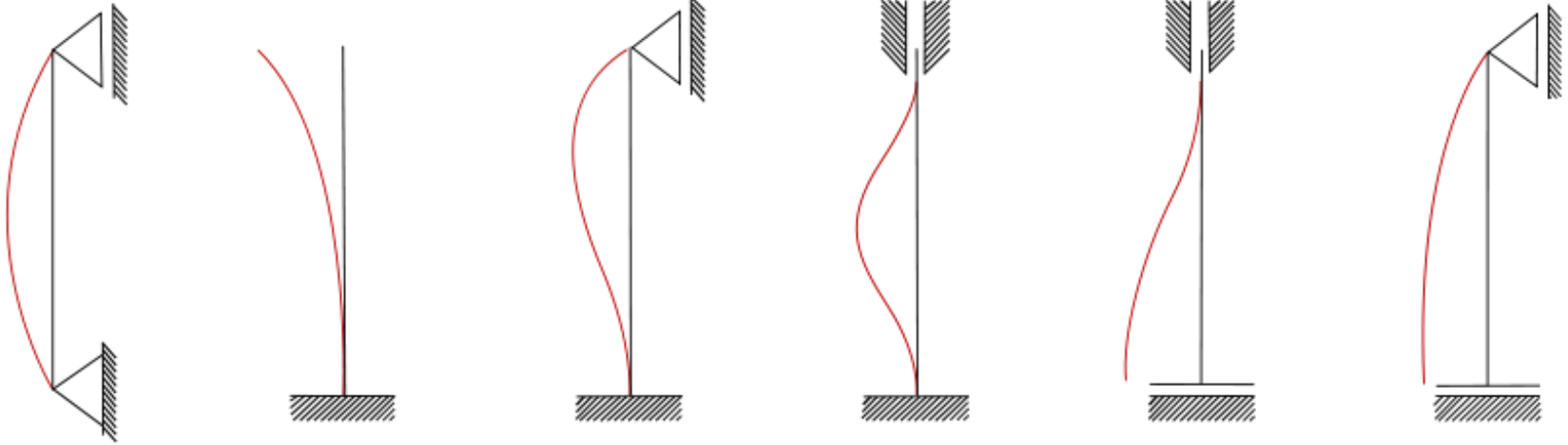
Photo: Błędy wykonawcze podczas realizacji konstrukcji stalowych, Litwin M, Górecki M, *Budownictwo i Architektura 4* (2009) 63-72

Number of various cross-sections, designed in truss: 2 - 5:

Chords	Brace members
<p>Top and bottom - the same I-beam</p> <p>or</p> <p>Two different I-beams, first for top, second for bottom</p>	1-3 different RHS
	1-3 different CHS
<p>Top and bottom - the same RHS</p> <p>or</p> <p>Two different RHS, first for top, second for bottom</p>	1-3 different RHS
	1-3 different CHS
<p>Top and bottom - the same CHS</p> <p>or</p> <p>Two different CHS, first for top, second for bottom</p>	1-3 different CHS

There are different factors for different types of support

Photo: wikipedia



μ	1,0	2,0	0,7	0,5	1,0	2,0
l_{cr}	$1,0 l_0$	$2,0 l_0$	$0,7 l_0$	$0,5 l_0$	$1,0 l_0$	$2,0 l_0$

Conclusions: different types of support is important factor affecting the calculation:

$$N_{cr} = \pi^2 EJ / (\mu l_0)^2$$

Buckling length ratio μ for truss members:

Element	Direction	Cross-section			
		I H	pipe	closely-spaced build-up members	other
Chord	In plane	0,9	0,9	1,0	1,0
	Out of plane	1,0	0,9	1,0	1,0
Brace	In plane	0,9	0,9; 1,0; 0,75	1,0; A	1,0; A
	Out of plane	1,0	1,0; 0,75	1,0; A	1,0; A

Bolted connections

Chords parallel and $d_{\text{truss bracing}} / d_{\text{chord}} < 0,6$

EN 1993-1-1 BB.1.1




L sections:

$$\bar{\lambda}_{\text{eff}, i} = 0,5 + 0,7 \bar{\lambda}_i \quad i = y, z$$

$$\bar{\lambda}_{\text{eff}, v} = 0,35 + 0,7 \bar{\lambda}_v \quad v = u, v$$

Not recommended type of cross-sections
because of requirements for joints

Additional requirements during analysis of truss / frame behavior can be divided into three groups (EN 1993-1-8 5.1.5):

- ◆ General and additional requirements, permissible shapes of joints (→ #t / 47 - 50); 
- ◆ Loads according to definition of ideal truss, in nodes only (→ #t / 50); 
- ◆ Acceptable values of eccentricities (→ #t / 47, 51 - 53). 

Consequences of satisfaction / dissatisfaction (partial satisfaction) are various, in dependence of group of requirements. Generally, 5 various static models of truss / frame is taken into consideration.

The more conditions are dissatisfied, the static model of the truss is getting closer to the frame with rigid joints.

Additional requirements for truss nodes

(EN 1993-1-8 7.1):

- ◆ Chords → □ □ ○ I; ■
- ◆ Web members → □ □ ○; ■
- ◆ Deformation ends of element are not accepted; ■
- ◆ f_y (□ □ ○) ≤ 460 MPa; ■
- ◆ f_y (□ □ ○) > 355 MPa → $f_{y, design} = 0,9 f_y$; ■
- ◆ t (□ □ ○) ≥ 2,5 mm; ■
- ◆ t_{chord} (□ □ ○) ≤ 25 mm; ■
- ◆ Compressed chords and web members → Ist or IInd class of cross-section; ■
- ◆ $\beta_i \geq 30^\circ$; ■
- ◆ Distances between web members (eccentricities) must be respected (→ #t / 51 - 53); ■
- ◆ Shape of joints must be respected (EN 1993-1-8 fig. 7.1), (→ #t / 48); ■
- ◆ (Length of member) / (depth of member) > 6 (EN 1993-1-8 5.1.5.(3)); ■



Photo: tatasteelconstruction.com

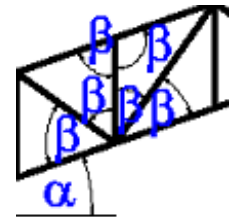
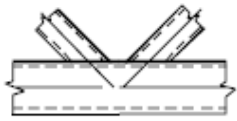
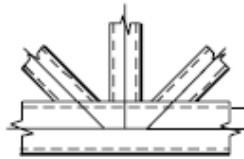


Photo: Author

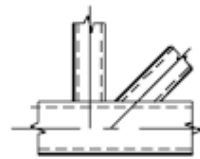




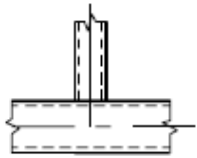
K joint



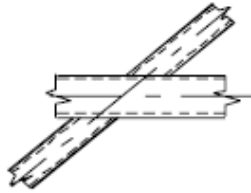
KT joint



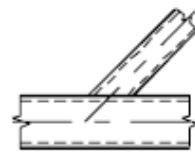
N joint



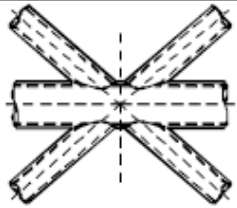
T joint



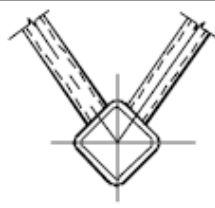
X joint



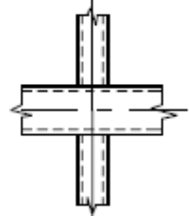
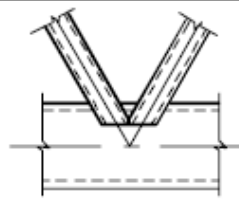
Y joint



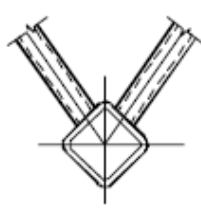
DK joint



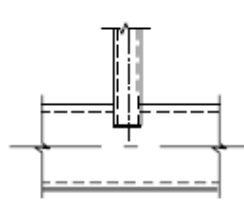
KK joint



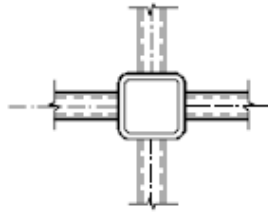
X joint



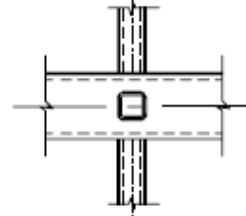
TT joint



DY joint



XX joint



Types of permissible joints

Photo: EN 1993-1-8 fig. 7.1

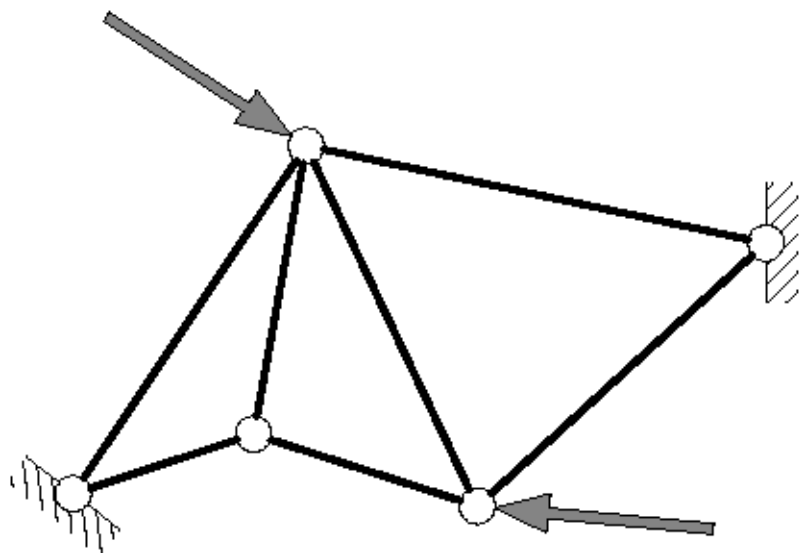
For each type of joint, many additional requirements must be satisfied. These requirements are presented in few tables in EN 1993-1-8; symbols are explained in EN 1993-1-8 1.5.(4), (5), (6).

Joint		Table	Comments
Chord	Web members		
CHS	CHS	7.1	-
RHS	CHS, RHS	7.8, 7.9	-
I-beam	CHS, RHS	7.20	-
C-section	CHS, RHS	7.21	C-section for chord is acceptet, but in this situation local bending moments must be taken into consideration (this means, this structure is not ideal truss).

Geneally, requirements presentes in tables, are as follow:

$$\min \leq (\text{depth of HS}) / (\text{thickness of its wall}) \leq \max$$

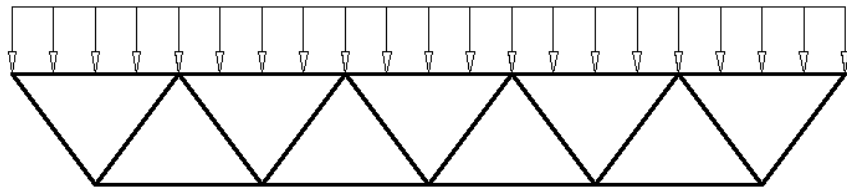




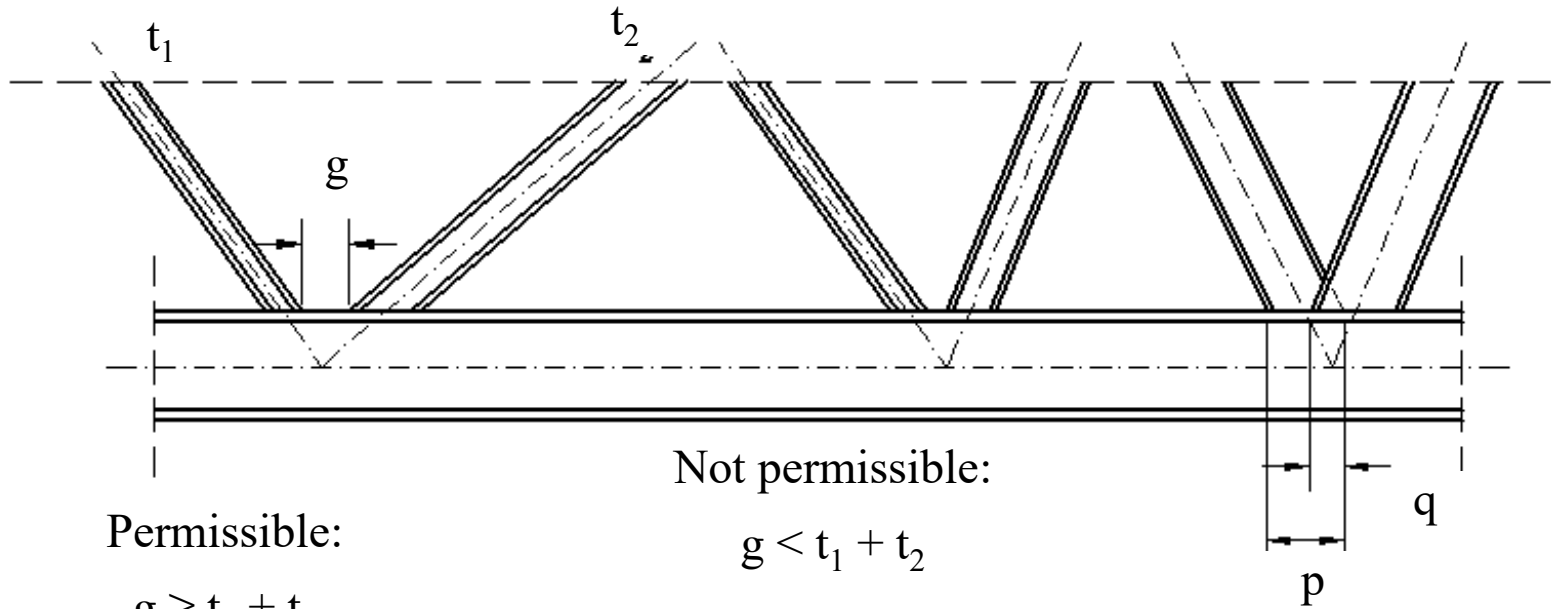
Truss – theory (idealization):

- Straight bars only; ■
- Forces in joints only; ■
- Hinged joints; ■

Photo: Author



Truss purlin not satisfy requirement „forces in joints only”. The same in case of force applied out of joint.



Permissible:

$$g \geq t_1 + t_2$$

Not permissible:

$$g < t_1 + t_2$$

or

$$q / p < 0,25$$

Permissible:

$$q / p \geq 0,25$$

EN 1993-1-8 7.1

Photo: Author



Results:

There is possible, that we must trace other axis of member to satisfy requiremen for

$$g \geq t_1 + t_2 \text{ or } q / p \geq 0,25$$

It makes eccentricities. Eccentricities make non-zero values of bending moment.

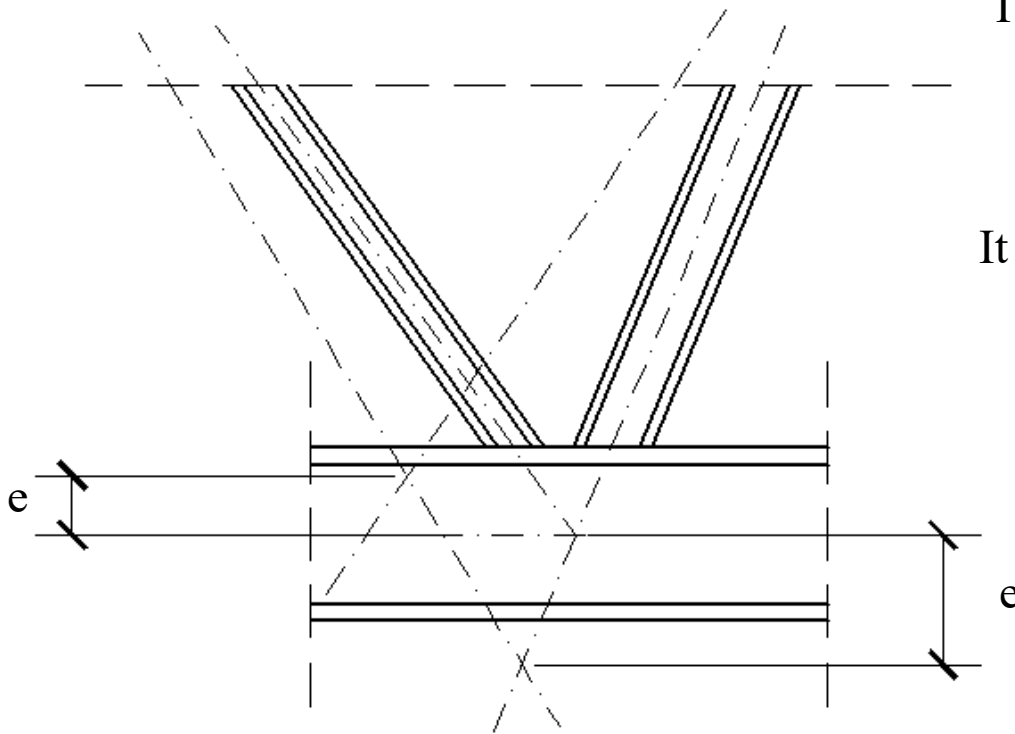
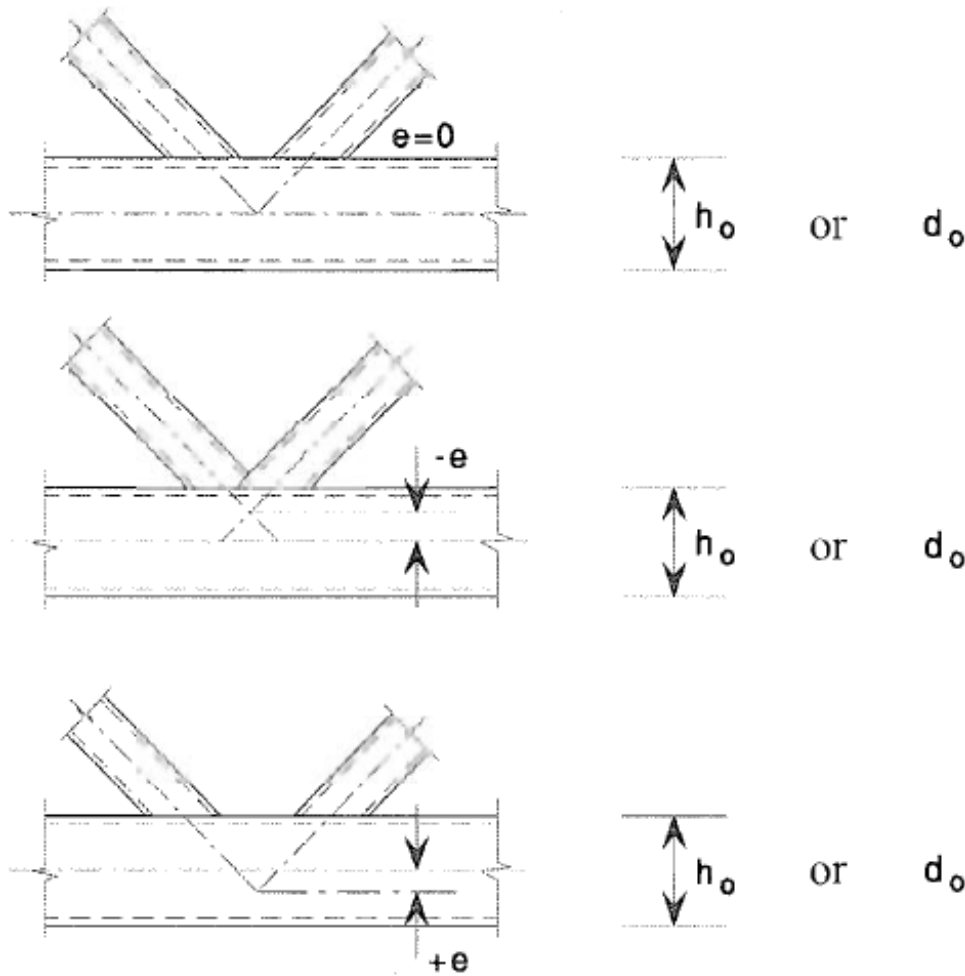


Photo: Author



Photo: EN 1993-1-8 fig. 5.3



Acceptable limits for eccentricities:

$$-0,55 a_0 \leq e \leq +0,25 a_0$$

$$a_0 = h_0 \text{ or } d_0$$

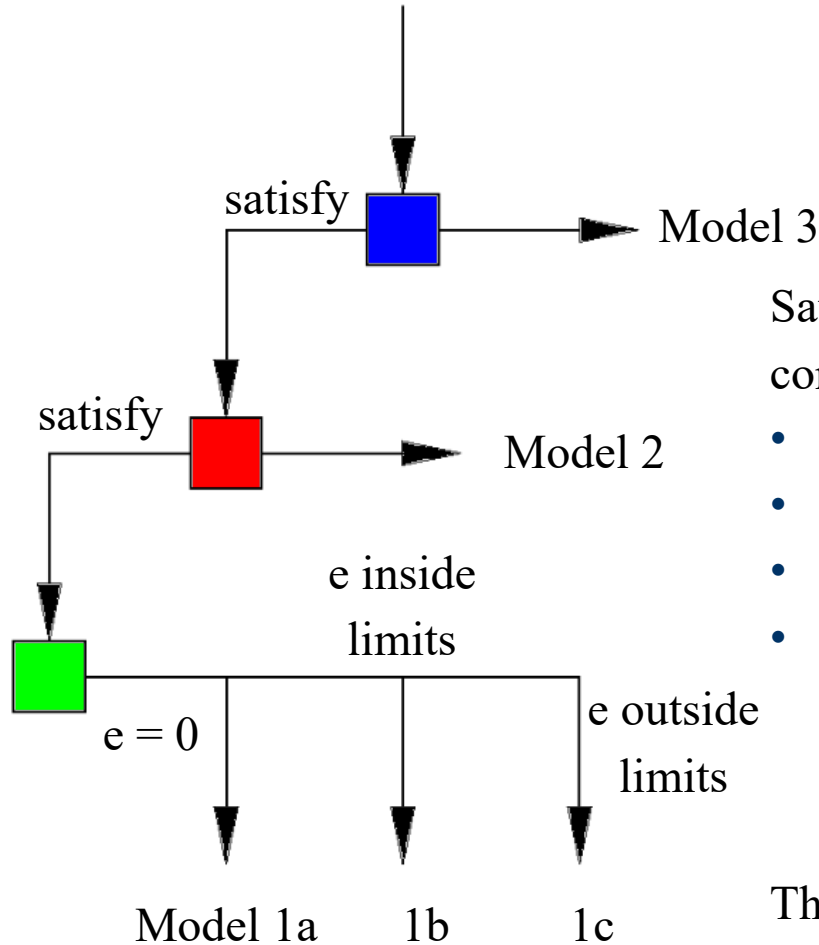
EN 1993-1-8 (5.1a), (5.1b)

Three possibilities must be taken into account:

- no eccentricities;
- eccentricities inside limits;
- eccentricities outside limits,

Truss

Stiffness



Satisfaction or dissatisfaction of three groups of conditions qualifies truss nodes as:

- all rigid;
- rigid and pinned;
- all pinned with secondary bending moments;
- all perfectly pinned.

→ Des #1/ 76

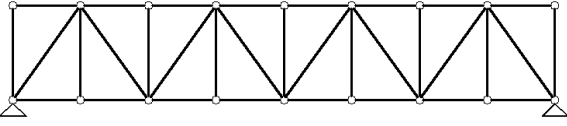
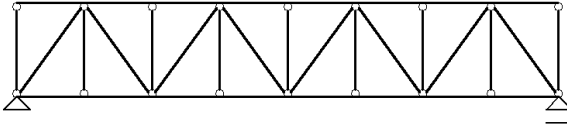
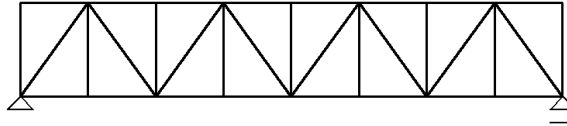
Third group of condition („green” condition) will be analysed in Your range of project only.

More information will be presented on lecture #9.

Photo: Author

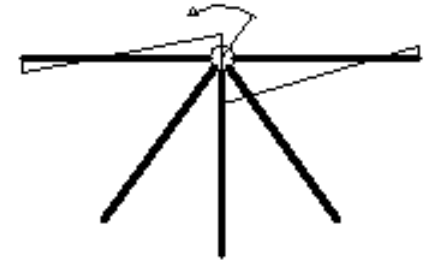
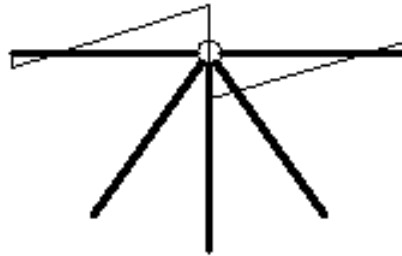
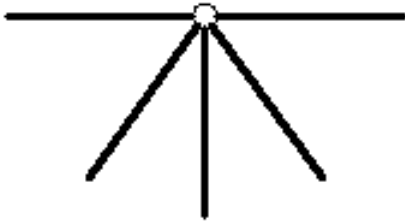
5 various models of truss behavior:

Photo: Author

		
<p>1. Ideal truss.</p>	<p>2. Continous top / bottom chord.</p>	<p>3. Frame.</p>
<p>Subtypes: 1a, 1b, 1c.</p>	<p>No subtypes.</p>	<p>No subtypes.</p>
<p>Hinged joints only.</p>	<p>Hinged and rigid joints.</p>	<p>Rigid joints only.</p>
<p>Axial forces for each element; bending moments and shear forces for part of joints and elements in dependence of subtypes.</p>	<p>Axial forces for each element; bending moments and shear forces for part of joints and elements in dependence of type of neighboring joints (hinged / rigid)</p>	<p>Axial forces, bending moments and shear forces for each element and joint</p>

3 subtypes of ideal truss (model 1):

Photo: Author



1a. No bending moments.

1b. Bending moments (axial force · eccentricity) act on chord only.

1c. Bending moments (axial force · eccentricity) act on joint and chord.

Ideal truss: axial forces only for elements and joints.

Axial forces only for most part of elements;

Axial forces, bending moments and shear forces for part of chords;

Axial forces for joints.

Axial forces only for most part of elements;

Axial forces, bending moments and shear forces for part of chords;

Axial forces and bending moments for joints.

Algorithm

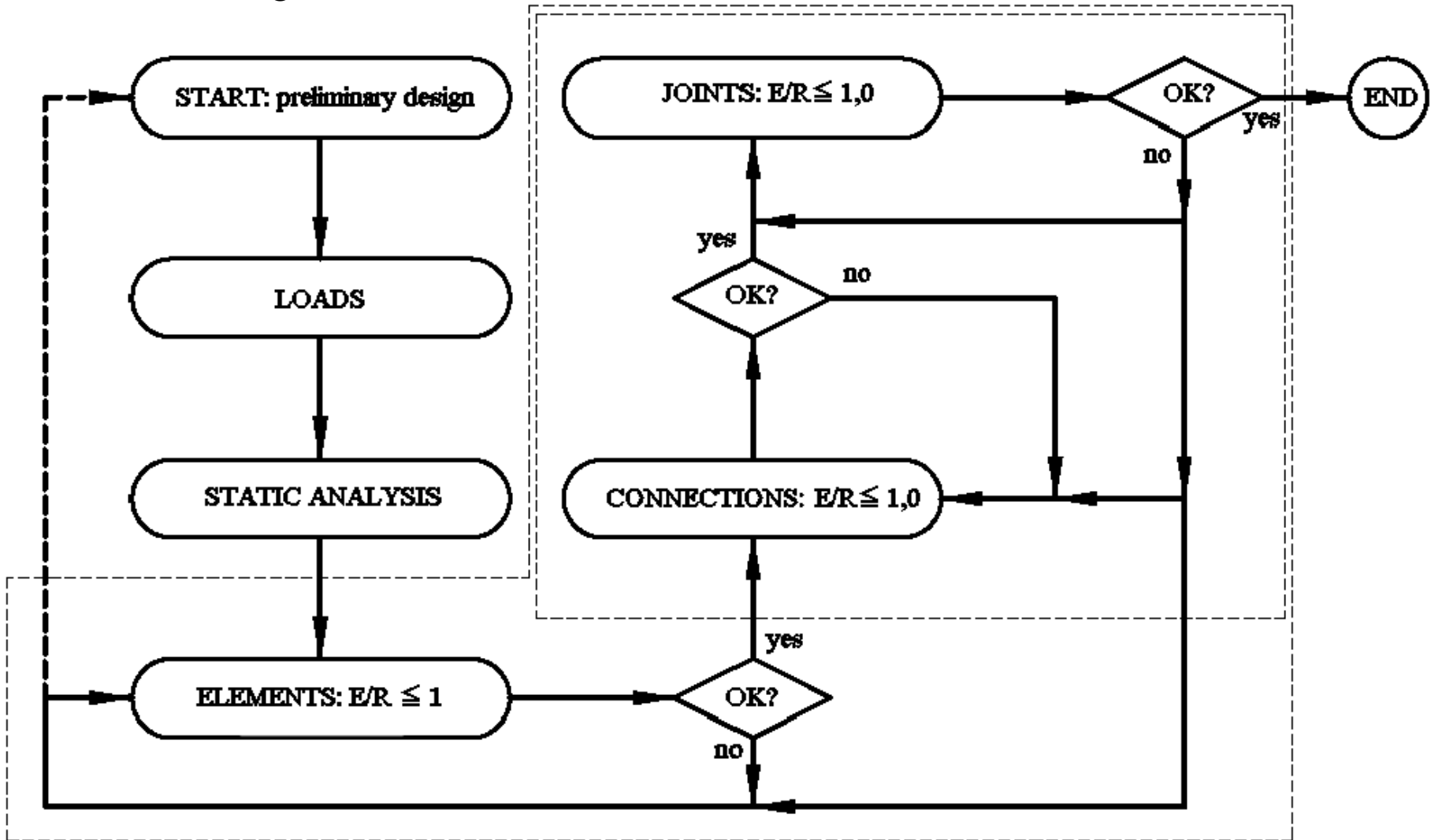


Photo: Author

Deformation ends of element are not accepted – what about systems for spatial trusses (Mero, SDC, Pyramitec, Unibat, Tesep, Tridimatec...)? In many cases ends of elements are deliberately deformed to connection with prefabricated nodes.

System solutions are tested in laboratories and manufacturer take guarantee, that such structure is safe.

The provision prohibiting deformation of elements concerns „normal" designers who do not have the opportunity to make full-test their solutions before erection.



Photo: altechsaudi.net



Photo: tatasteelconstruction.com

The same about system solutions of trusses, based on cold-formed sections. Manufacturers tested such structures – elements and joints – before application in construction sites. „Normal” designers do not have opportunity (lack of time, lack of money, lack of technical possibility) to make these tests.



Photo: kcse.com

Manufacturing details

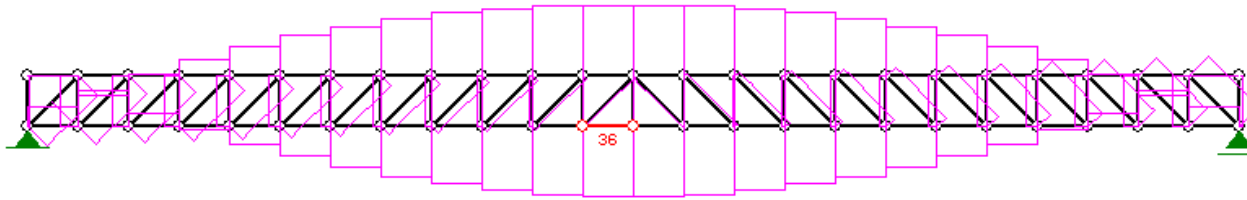


Photo: Author

Condition E / R should be as close 1,0 as possible because of economic reasons. Both cords could have smaller one cross-sections close to both ends of truss.

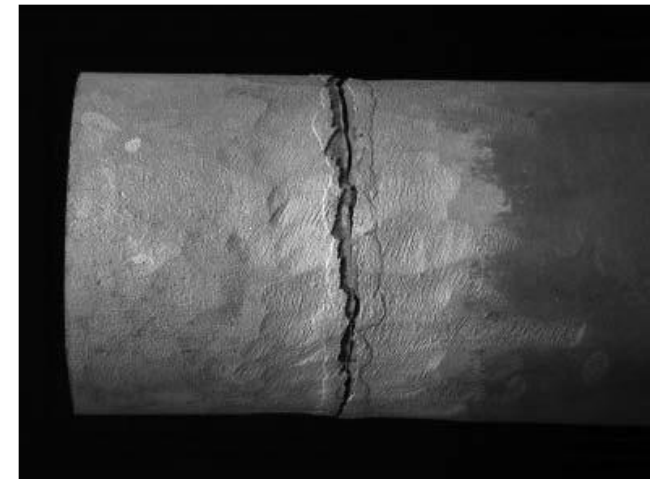
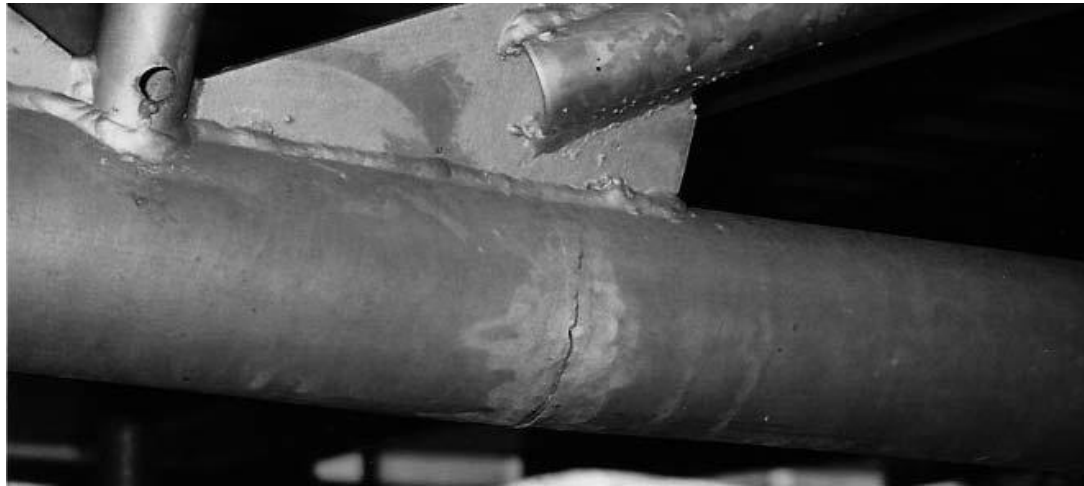


Photo: D. Kowalski, Problemy realizacji inwestycji z zakresu konstrukcji stalowych, Inżynieria Morska i Geotechnika 2013 / 5

But applying such solution can make cracking of welds, even if resistance of welds is enough.

Fatigue resistance

the amplitude or range of cyclic stress that can be applied to the material without causing fatigue failure

This is not the same as strength of material

Strength → static load

Fatigue resistance → cyclic (dynamic) load

This phenomenon is important for structures exposed to dynamic and cyclic loads. There is possible, that even little loads, but in many cycles ($> 10\ 000$) can destroy structure.

→ #2/ 78

On each elements exist imperfections and notches (holes, welds, micro-cracks). There is concentration of stress around each notches.

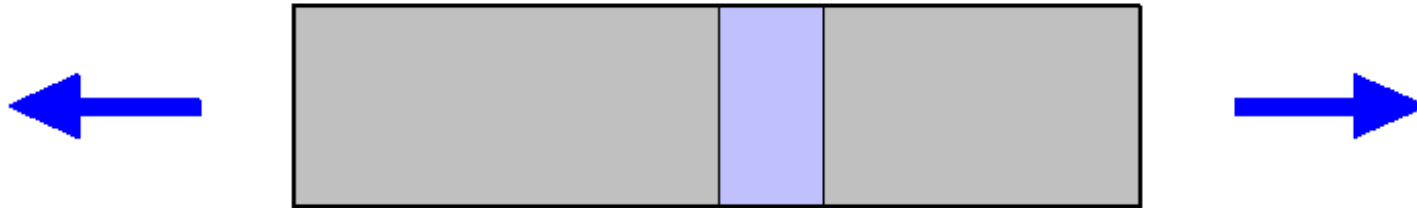
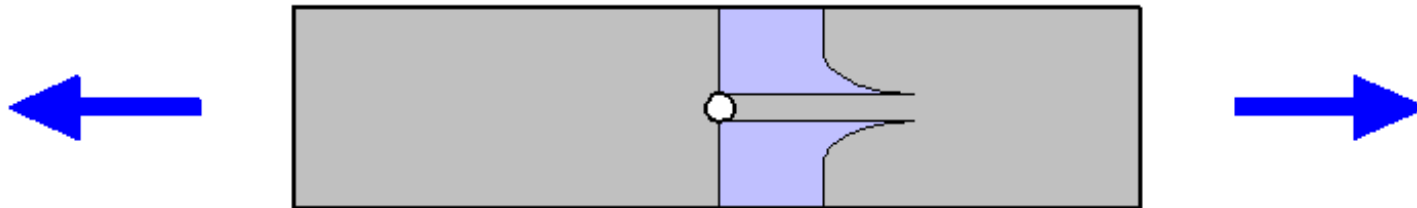


Photo: Author



$$\mu \sigma_{Ed} \leq f_y$$

→ #2/ 79

μ stress concentration factor
 σ_{Ed} stress as for ideal cross-section

Additional effects of welding process (#6 / 26 – 29): secondary pollutions (gas bubbles, ash) introduced during welding, local deformation of plates, cracks as effect of thermal shrinkage...

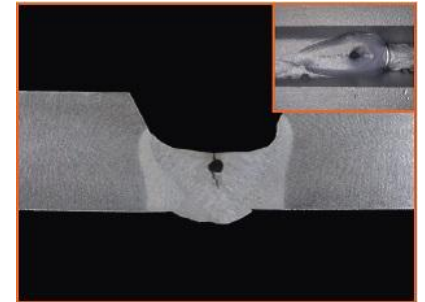
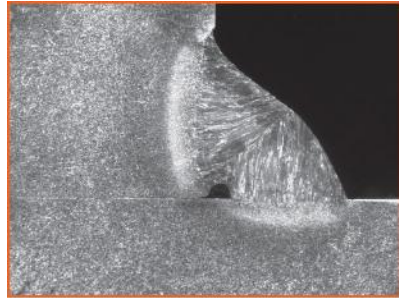


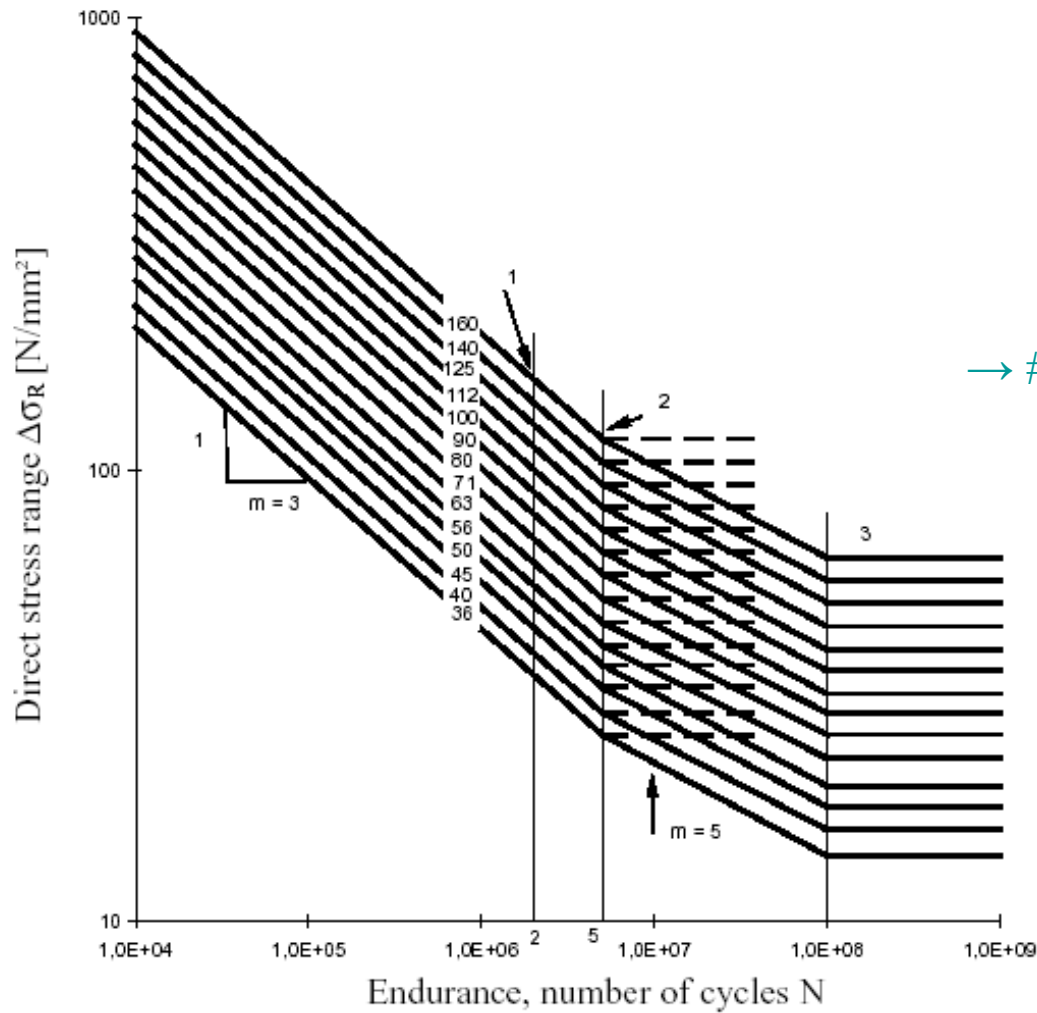
Photo: figel.pl

Ability and experience of worker ↓ → number of welding imperfection ↑ → quality level of welds ↓

→ #6 / 32

Accepted number of welding imperfection is limited by Execution Class. (→ #6 / 47).

Photo: EN 1993-1-9 fig 7.1



- 1 Detail category $\Delta\sigma_C$
- 2 Constant amplitude fatigue limit $\Delta\sigma_D$
- 3 Cut-off limit $\Delta\sigma_L$

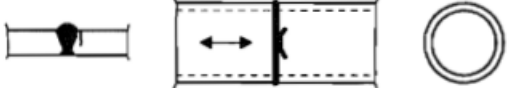
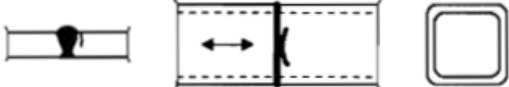
71	③		Transverse butt welds: 3) Butt-welded end-to-end connections between circular structural hollow sections.
56	④		4) Butt-welded end-to-end connections between rectangular structural hollow sections.

Photo: EN 1993-1-9 Tab. 8.6

Fatigue resistance for transverse welds for R/C HS is not very high.

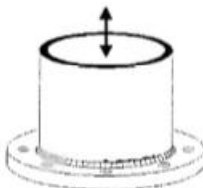
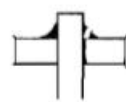

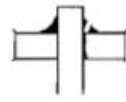

71		⑪	11) Tube socket joint with 80% full penetration butt welds.
40		⑫	12) Tube socket joint with fillet welds.

Photo: EN 1993-1-9 Tab. 8.6

Photo: EN 1993-1-9 Tab. 8.5

Fatigue resistance for welds between R/C HS and intermediate plate is even smaller.

40			8) Circular structural hollow sections, fillet-welded end-to-end with an intermediate plate.
36			9) Rectangular structural hollow sections, fillet-welded end-to-end with an intermediate plate.

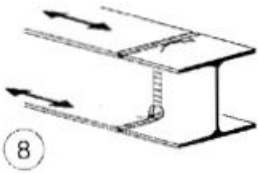
90	size effect for $t > 25\text{mm}$: $k_s = (25/t)^{0.2}$	
----	---	---

Photo: EN 1993-1-9 Tab. 8.3

Fatigue resistance for transverse welds I-beams is higher.

Conclusions:

Joint with longitudinal stiffeners is better (longer welds → smaller stresses → smaller amplitude of stresses) because of fatigue than joint without stiffeners.



Photo: en.wikiarquitectura.com

Photo: encrypted-tbn0.gstatic.com



Photo: tboake.com

Problems with fatigue for I-beams are smaller than for R/C HS.

Field splices connect two parts of truss. What about diagonal bar - to which part of the truss belongs, left or right?

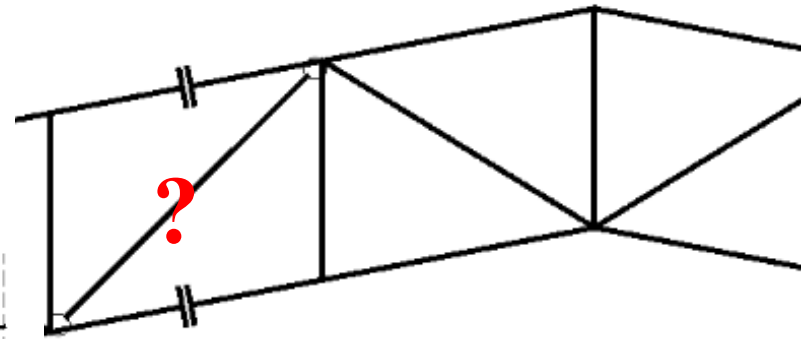
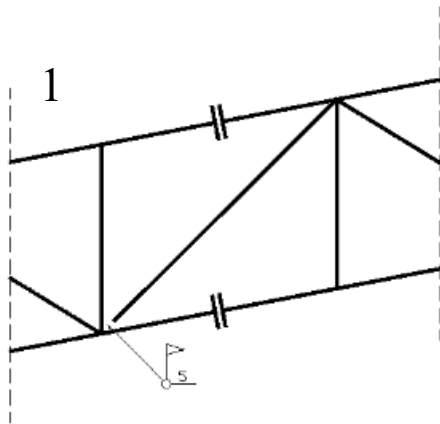
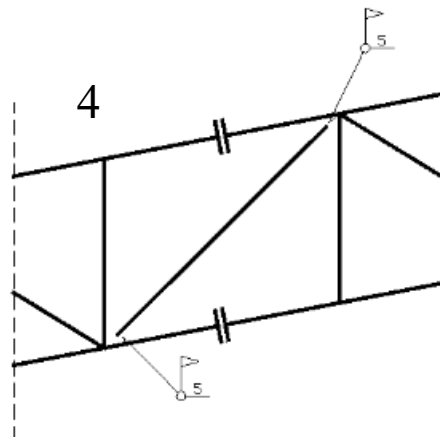
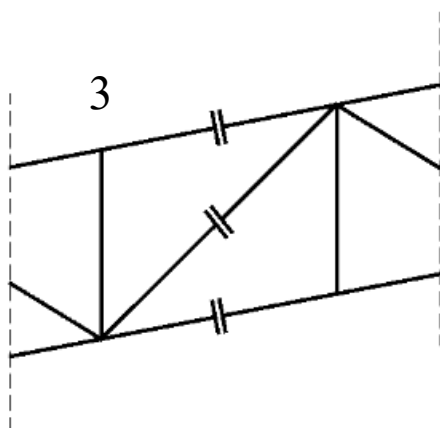
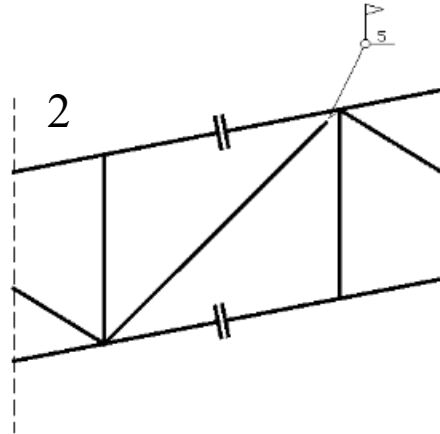


Photo: Author



→ Des #1/ 80



Four possibilities:

1. Belongs to right, on left assembly weld;
2. Belongs to left, on right assembly weld;
3. Divided by two parts, connected by bolted splice joint;
4. Separated element, assembly welds on both ends.

Problem for solutions 1, 2, 3 – diagonal bar is connected with the rest part of structure by weld only. Such a long element can behave as pendulum during transport. Vibrations during transport make cracks of welds.

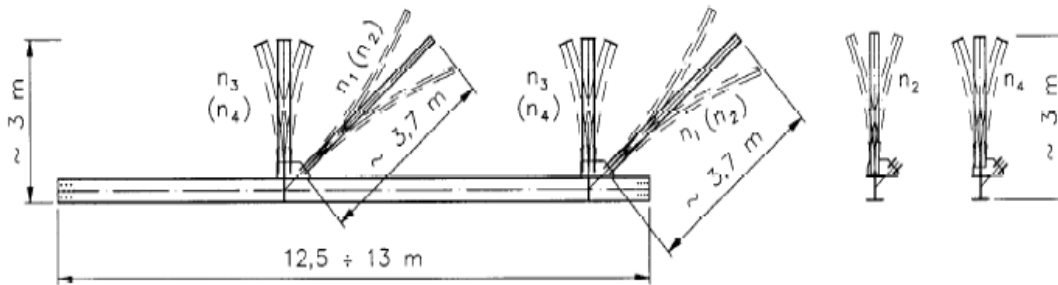
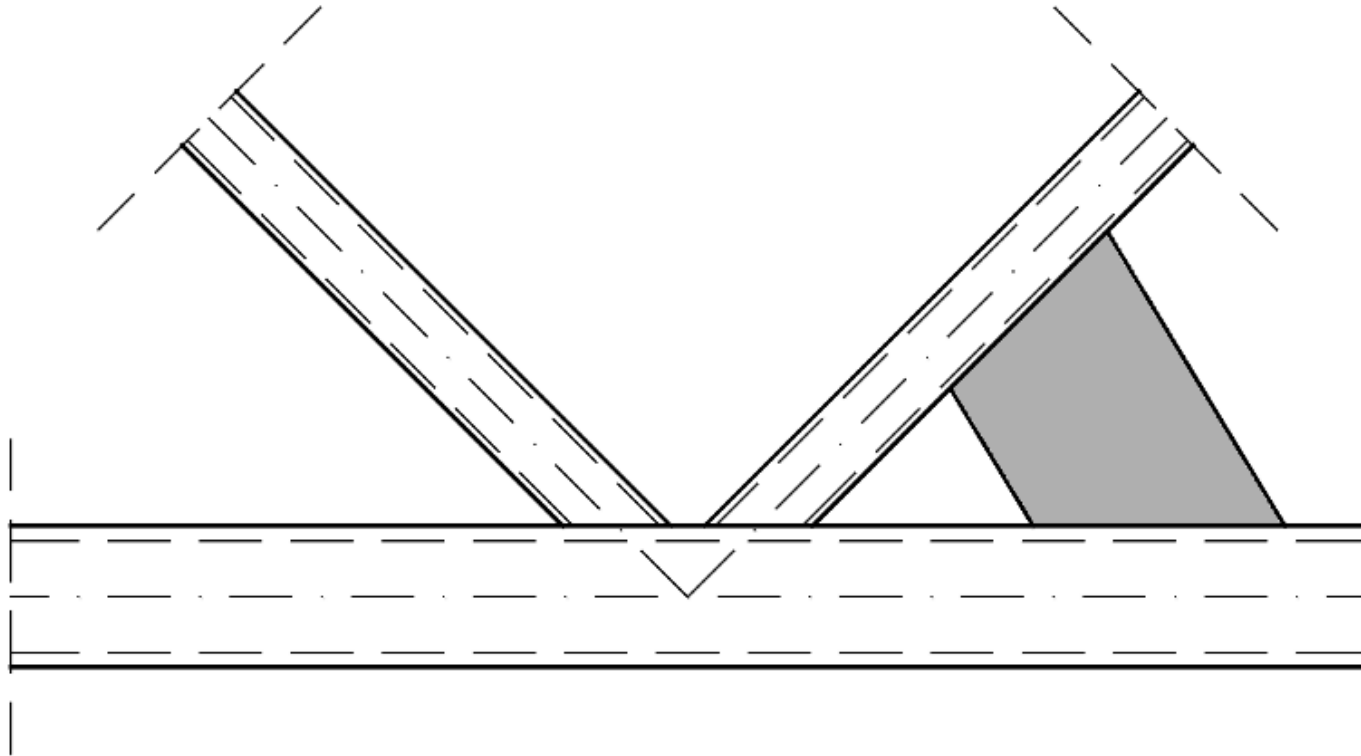


Photo: Awaryjne zagrożenie stalowej konstrukcji dachu hali widowiskowo-sportowej w Sopocie, E. Hotała et al, Awarie Budowlane 2009

→ Des #1/ 81

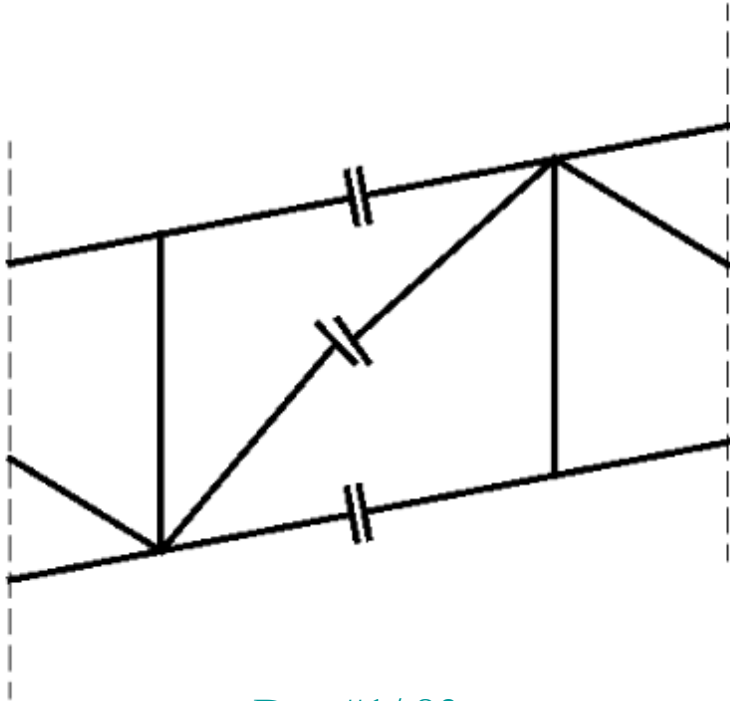
Prevention: temporary support for free-end bar. Steel plate welded to diagonal bar and chord of truss. The plate is removed during the assembly of the structure.



→ Des #1/ 82

Photo: Author

Side effect of welding the bar only at one end and use of temporary plate: imperfections. Even minor misalignment of the diagonal bar (rotation by $1^\circ - 2^\circ$) may make impossible to connect the truss.



→ Des #1/ 83

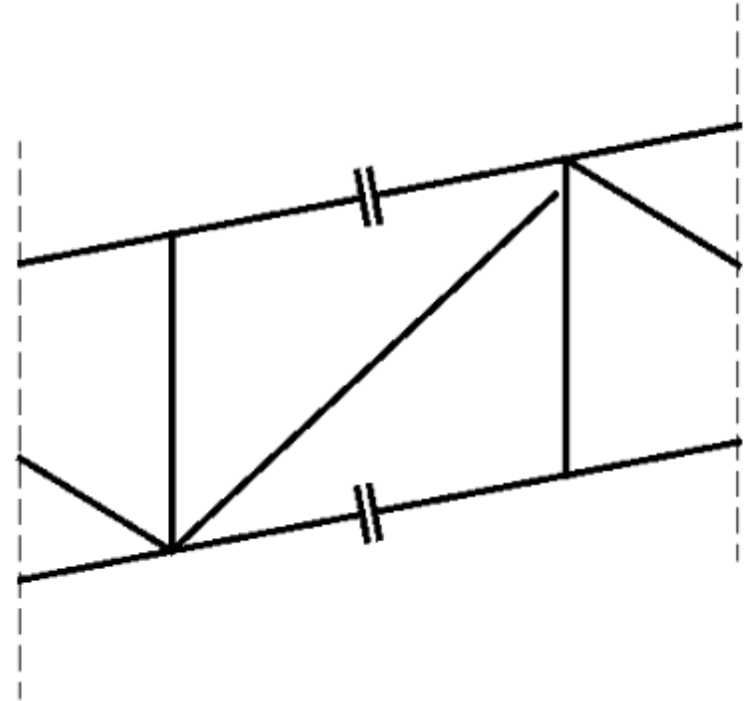
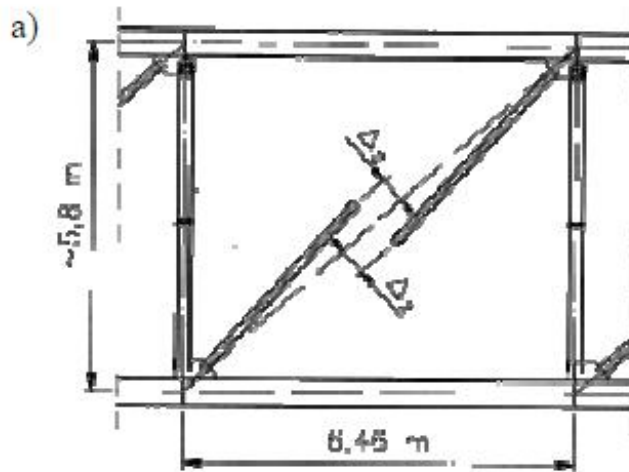


Photo: Author



Additional secondary effect of splice joint in half of web member: deformation after welding (\rightarrow #6 / 27). Both parts of member can have various position of axes and there will be problem with connection.

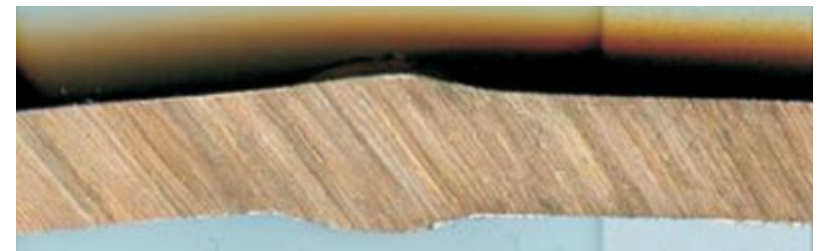


Photo: Analiza numeryczna odkształcenia i naprężenia w strefie złącza spawanego rurociągu przemysłowego, Gliha V Śnieżek L Zimmerman J, Biuletyn WAT Vol. LVIII, No. 2, 2009

Because of these problems, separated element with assembly welds on both ends is solution often used.

This is not fully in line with the principles of erecting steel structures. At the assembly, we prefer bolted joints, but sometimes it is not possible.

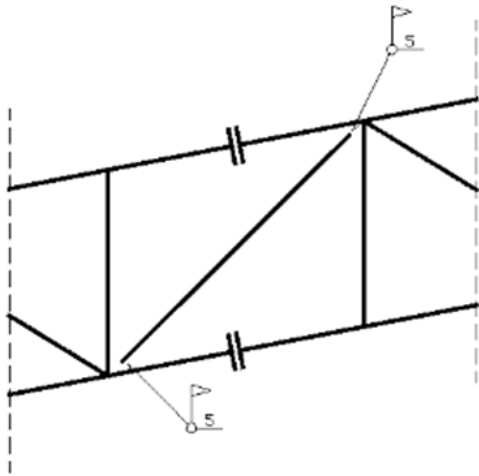


Photo: Author

→ Des #1/ 84



Photo: tboake.com



Photo: tboake.com

Similarities and differences for bolted and welded joints will be presented on Lec #8.

Recommendations from old Polish Standard:

$$\lambda = \max (L_{cr,y} / i_y ; L_{cr,z} / i_z)$$

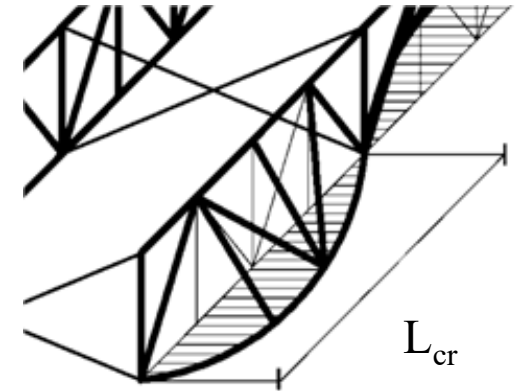
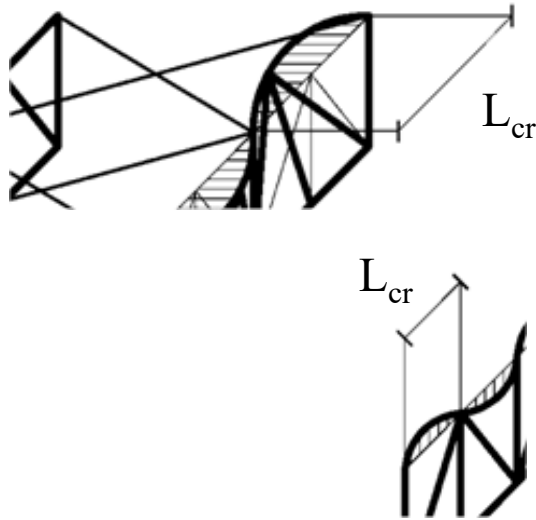
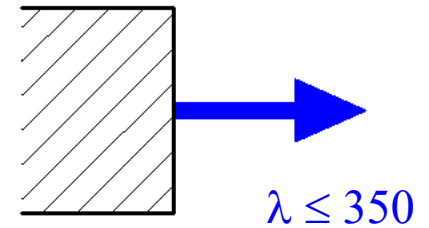
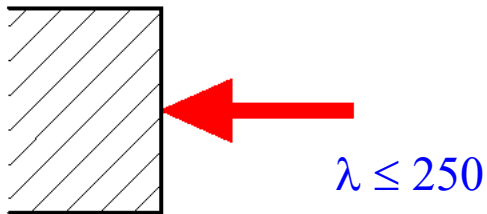


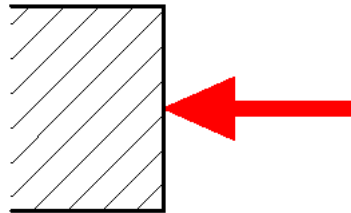
Photo: Author



Slenderness is important for a compressed member (it determines the buckling factor χ). It seems to be completely irrelevant for case of axial tension. Why did the old Standard impose restrictions? Why small χ with large λ cannot be ballanced by the large value of A? Why should you consider slenderness at axial tension at all?

$$N_{Ed} / N_{b,Rd} \leq 1,0$$

$$N_{b,Rd} = \chi A f_y / \gamma_{M0}$$



$$N_{Ed} / N_{t,Rd} \leq 1,0$$

$$N_{t,Rd} = A f_y / \gamma_{M1}$$

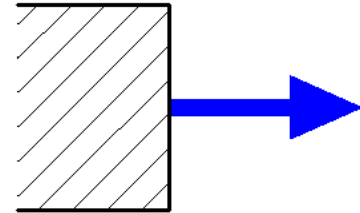


Photo: Author

In real world no ideal members. Not existed ideally straight members, without initial curvature.



Photo: prikol.ru

Photo: hourdose.com



Photo: PN 1090-2 tab. D1.10

No	Criterion	Parameter	Permitted deviation Δ
1	Straightness and camber:		$\Delta = \pm L/500$ But $ \Delta \geq 12 \text{ mm}$
NOTE Deviations measured after welding, with the component lying flat on its side. Key a actual camber b intended camber c actual line d intended line		Deviation at each panel point, relative to a straight line - or to the intended camber or curvature.	
2	Straightness of bracing components:	Deviation of bracing length L_1 from straightness:	$\Delta = \pm L_1/750$ but $ \Delta \geq 6 \text{ mm}$
NOTE Notations such as $\Delta = \pm L/500$ but $ \Delta \geq 12 \text{ mm}$ mean that $ \Delta $ is the larger of $L/500$ and 6 mm.			

In theory, on truss bar acts axial force only:

$$P / A \leq f_y$$

but, because of bow imperfection, bending moment on initial eccentricity e must be taken into consideration:

$$P / A + P e / W \leq f_y$$

Sectional modulus W , moment of inertia J , depth of cross-section h for bi-symmetrical cross-section:

$$W = J / (h/2) = 2J / h$$

Moment of inertia J , area of cross-section A , radius of gyration:

$$i = \sqrt{J / A} \rightarrow J = A i^2$$

Finally sectional modulus can be presented as:

$$W = 2 A i^2 / h$$

This means:

$$P / A + P e / (2 A i^2 / h) \leq f_y \rightarrow (P / A) [1 + e h / (2 i^2)] \leq f_y$$

Part of formula $[1 + e h / (2 i^2)]$ presents proportion between effect of axial force only (1) and effect of bow imperfection: secondary bending moment: $e h / (2 i^2)$.

Slenderness: proportion between length of member and radius of gyration:

$$\lambda = L / i$$

According to EN 1090-2:

$$e / L \leq 1 / 750$$

According to old Polish Standard:

$$\lambda \leq 250$$

For example:

$$e / L = 1 / 1000 \rightarrow e = L / 1000$$

and

$$\lambda = 100 \rightarrow i = L / 100$$

Lets take into consideration proportion between depth of cross-section and length of truss bat the same as for I-beam (it's not completey correct):

$$h = L / 25$$

Then:

$$\begin{aligned} (P / A) [1 + e h / (2 i^2)] &= (P / A) \{1 + (L / 1000) (L / 25) / [2 (L / 100)^2]\} = \\ &= (P / A) \{1 + [(L^2 / 25\,000) / (L^2 / 5\,000)]\} = (P / A) (1 + 0,2) = 1,2 P / A \end{aligned}$$

Taking into account bending imperfection for a bar with moderate slenderness gives a bending contribution of 20% effort from the axial force. Total effort for characteristic value of forces is 1,20. According to Eurocode, axial force only is taken into consideration, but multiplied safety factors. Even for CC1 it will give about 1,35 P / A (for CC2 and CC3 even bigger value), so 1,20 P / A is in the safe range.

What about slenderness out of recommended range?

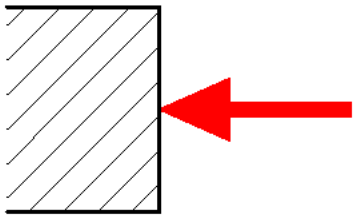
$$\lambda = 400 \rightarrow i = L / 400$$

And rest parameters (e, h) the same as in previous case:

$$\begin{aligned} (P / A) [1 + e h / (2 i^2)] &= (P / A) \{1 + (L / 1000) (L / 25) / [2 (L / 400)^2]\} = \\ &= (P / A) \{1 + (L^2 / 25\,000) / (L^2 / 80\,000)\} = (P / A) (1 + 3,2) = \mathbf{4,20 P / A (!!!)} \end{aligned}$$

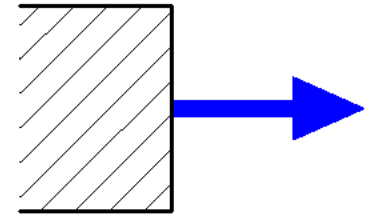
The effort from secondary bending from imperfection is now more than 4 times than the effort from axial force (for the same height of cross-sectional levels, the same surface areas and the same initial imperfection). We completely lose control over the behavior of the bar.

In analysed case stresses only were taken into consideration – this means, way of calculation is the same for compressive force and for tensile force. For too slender members secondary effects of imperfection can completely destruct cross-section, even if axial force is in safe range.



$$\lambda \leq 250$$

Photo: Author



$$\lambda \leq 350$$

More restrictive condition for compressive force are recommended (more compact elements are recommended) due to the possibility of loss of stability. Instability for more slender member is more possible.

Conclusion

Design according to	Limitations for imperfections	Slenderness of members	Impact of imperfections into calculations
PN / B 3200	PN 1090-2	Limitation	No impact
EN 1993-1-1		No limits	Schemes of equivalent actions

In formula

$$(P / A) [1 + e h / (2 i^2)]$$

part from secondary bending $[e h / (2 i^2)]$ can be limited in two ways:

- avoiding too big slenderness (too small i) as in old Polish Standard;
- analysis of effects of secondary bending moment from imperfection e (equivalent schemes of loads) as in Eurocodes. For too slender members (too small moment of inertia \rightarrow too small sectional modulus \rightarrow too small resistance for bending moment) secondary effects from imperfections make exceeding of resistance.

Numerical model

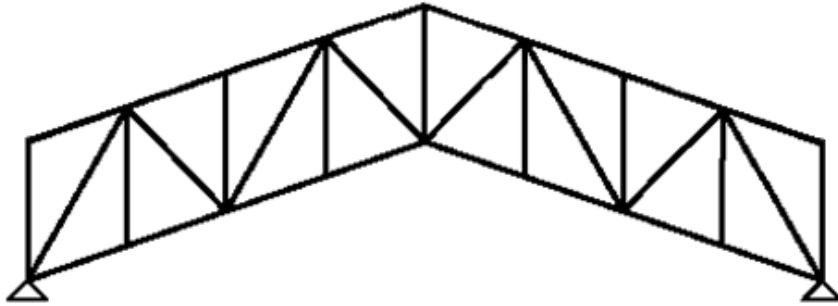
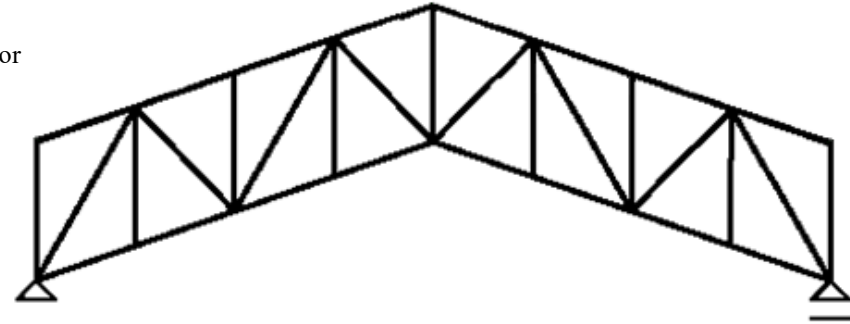


Photo: Author



Distribution of forces in top and bottom chords is completely different in case of symmetrical supports (pinned-pinned) and unsymmetrical (pinned-roller). The most important question is, which one model is closer to real behaviour of truss.

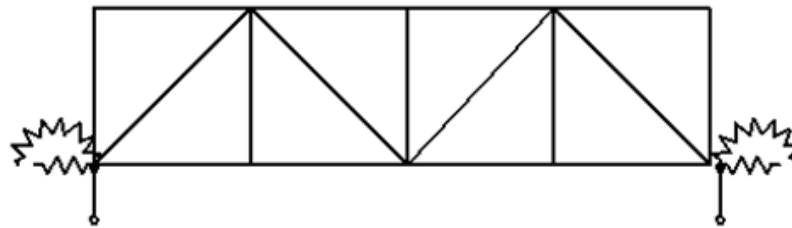




Photo: omniblock.com



Photo: aminds.com

Generally, support for truss is not completely rigid in each case.



Photo: structuralsteelen.com

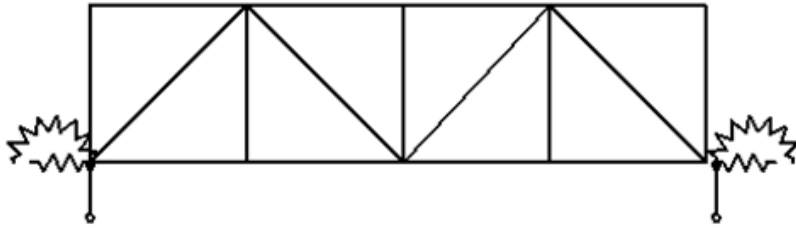
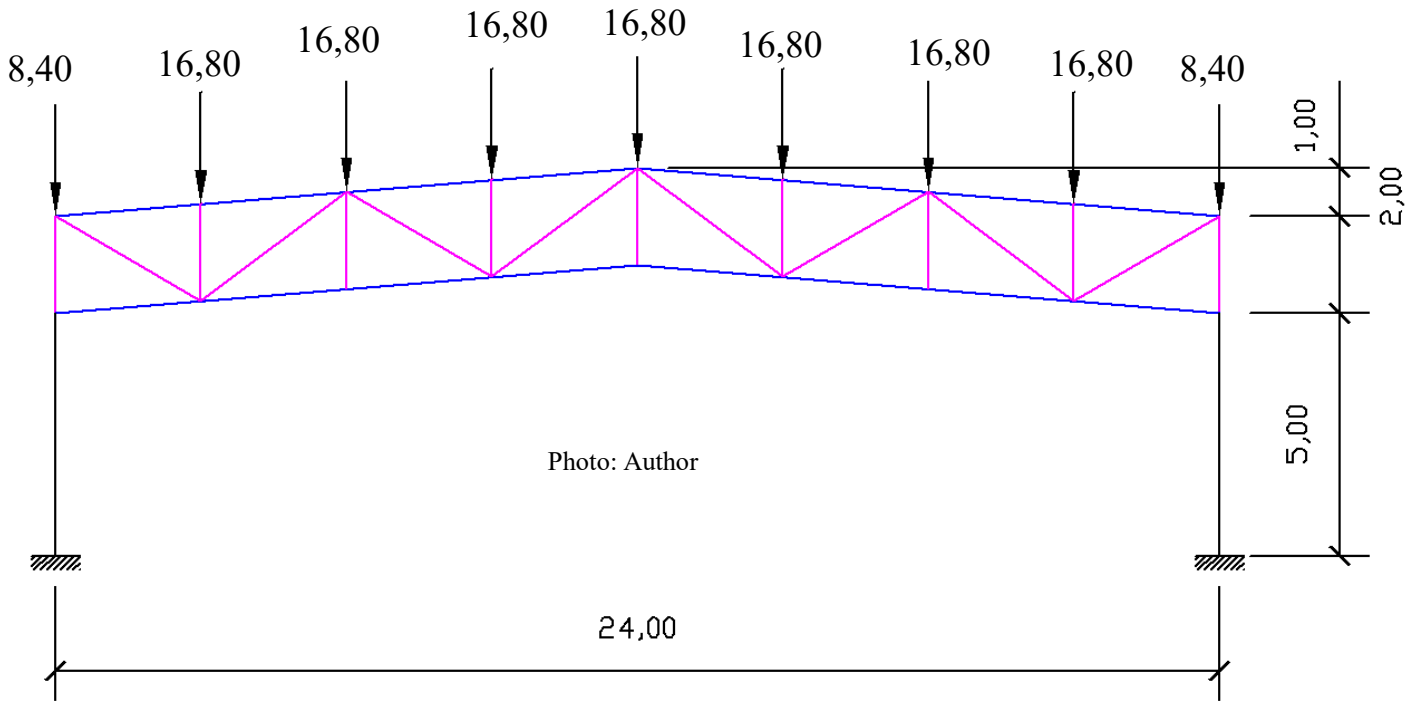


Photo: Author

The best static scheme is as above:

- vertical direction – very big stiffness of column / wall, so it could be presented as full support;
- horizontal direction – susceptibility of support (column / wall) for deformation make spring support of specific value of elasticity
- rotation - we purposely try to ensure rotation for support, but real behaviour is rather spring support because of problems with non-zero rigidity of rockers.

If rockers under truss are applied in right way, It could be taken into consideration as ideal hinge. Technical solutions will be presented in Lec #20. The biggest problem is influence of horizontal spring supports

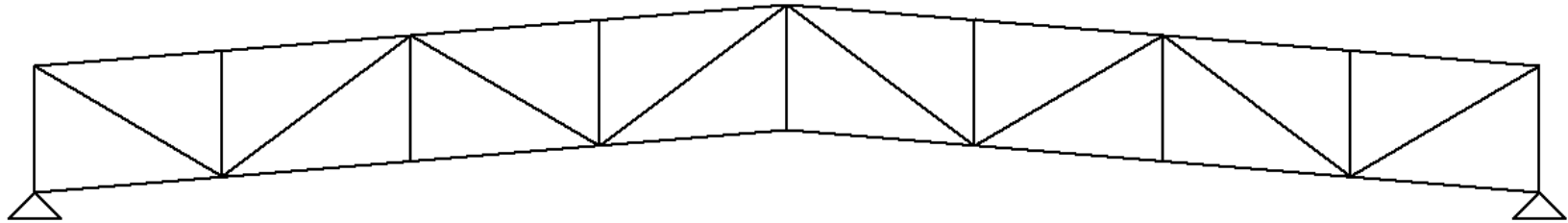
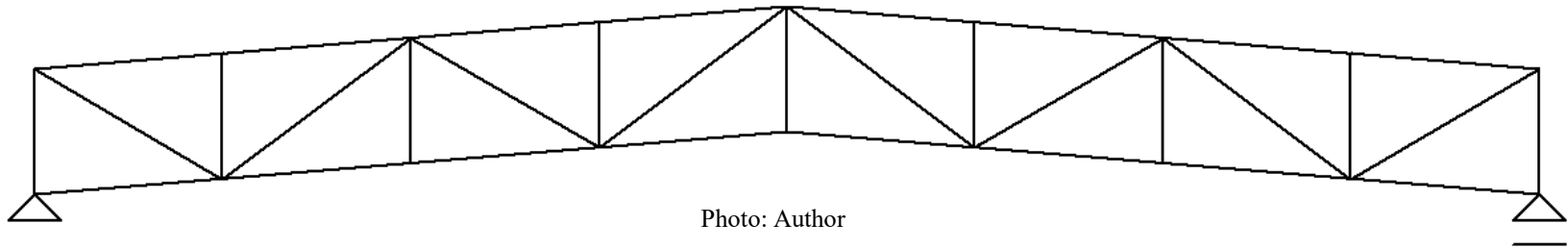


We take into account truss as above:

CHS 139,0 / 5,0

CHS 76,1 / 5,0

Concrete column 30x30 cm C 30/37 or steel column HEB 300



Two various models will taken for simplification:

1. Unsymmetrical supports (pinnd-rolled);
2. Symmetrical supports (pinned-pinned).

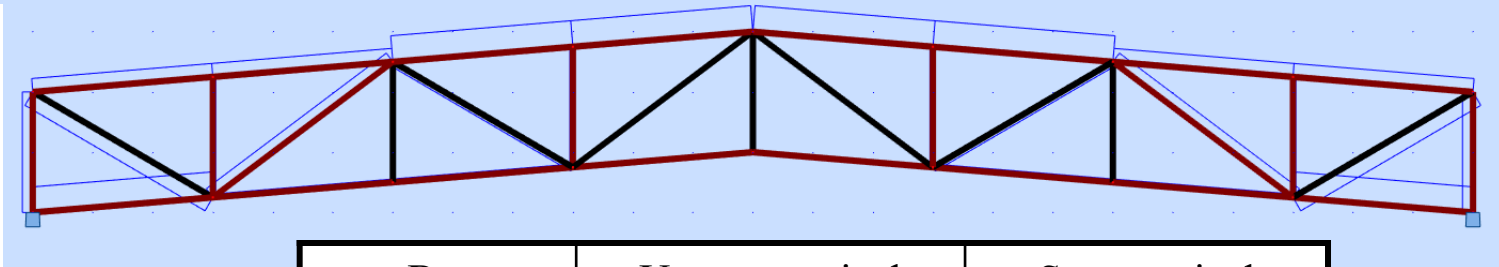
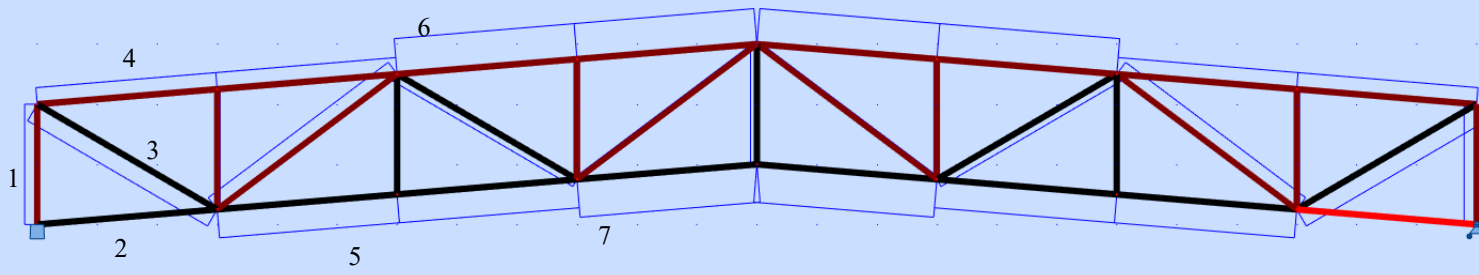


Photo: Author

Compression
Tension

Bar	Unsymmetrical	Symmetrical
1	72,82	60,59
2	0,00	147,30
3	111,04	89,79
4	96,24	77,83
5	164,99	19,14
6	206,24	151,00
7	220,00	0,95

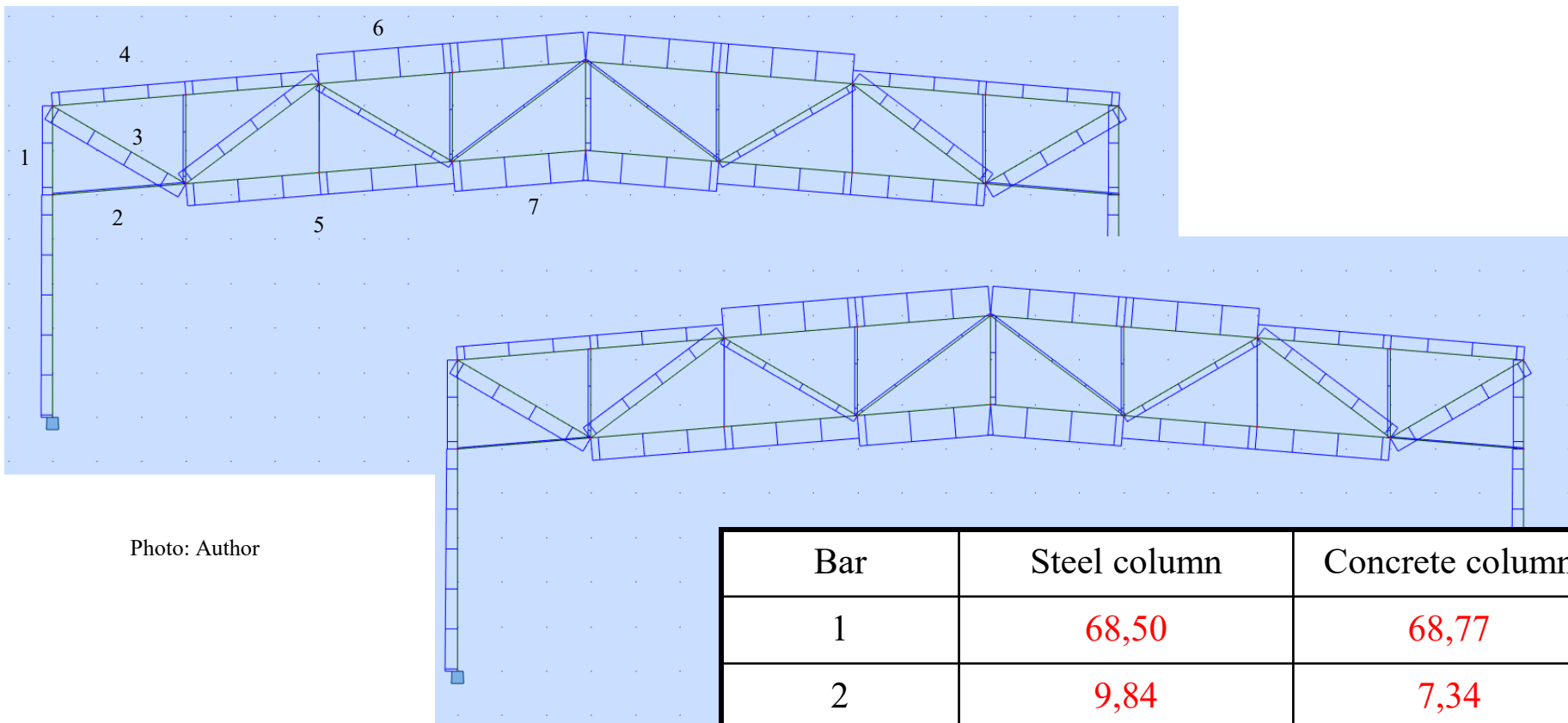


Photo: Author

Compression
Tension

Bar	Steel column	Concrete column
1	68,50	68,77
2	9,84	7,34
3	103,75	104,22
4	89,11	89,54
5	149,18	152,33
6	197,78	198,61
7	201,79	205,56

Bar	Steel column	Concrete column	Average	Unsymmetrical	Symmetrical
1	68,50	68,77	68,64	72,82	60,59 (-12%)
2	9,84	7,34	8,59	0,00	147,30 (+1 715 %)
3	103,75	104,22	103,99	111,04	89,79 (-14%)
4	89,11	89,54	89,33	96,24	77,83 (-13%)
5	149,18	152,33	150,76	164,99	19,14 (+113%)
6	197,78	198,61	198,20	206,24	151,00 (-24%)
7	201,79	205,56	203,68	220,00	0,95 (+100%)

Differences between unsymmetrical model of truss and full model (truss + column) are small; generally cross-sectional forces are a little bigger for unsymmetrical model (safe side results).

Between full model and symmetrical model are big differences, very big differences or even completely opposite direction of cross-sectional forces.

Bar	Steel column	Concrete column	Unsymmetrical	Symmetrical
Top chord	197,78	198,61	206,24	151,00
Bottom chord	201,79	205,56	220,00	147,30

Max values of axial forces in top and bottom chords show problems, which occurs after application wrong stati model: symmetrical supports.

Top chord – compression – will be designed for force about 75% only of real force: it means big probability of destruction of cord and truss.

Bottom chord – tension – will be designed for opposite (compressive, not tensile) force: it means big probability of oversizing for chord.

Conclusion: elastic supports in horizontal direction are simplified by pinned-rolled supports with very big accuracy.

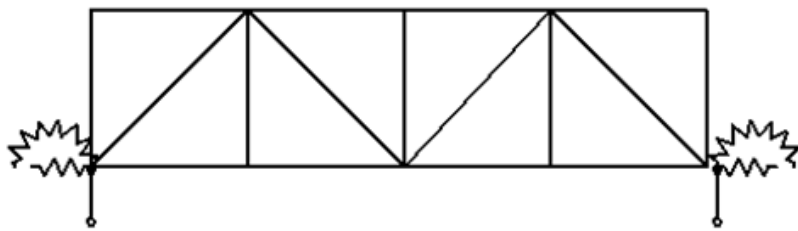
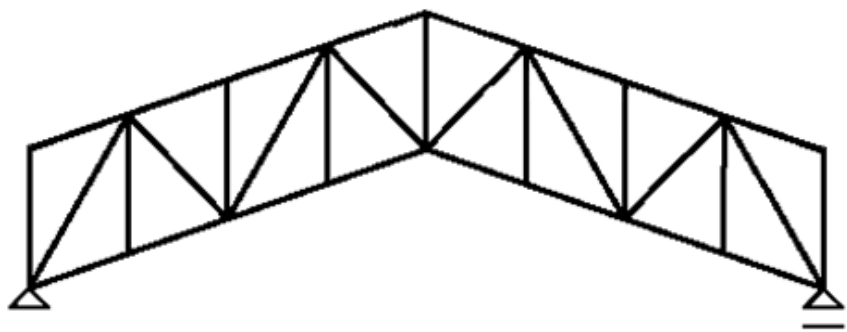
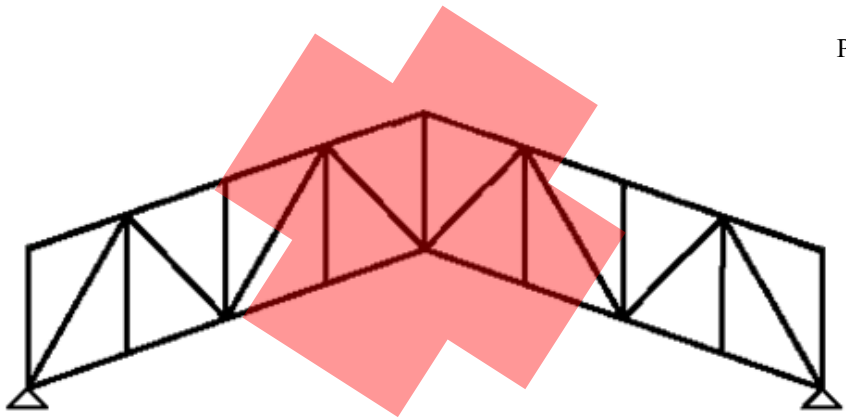
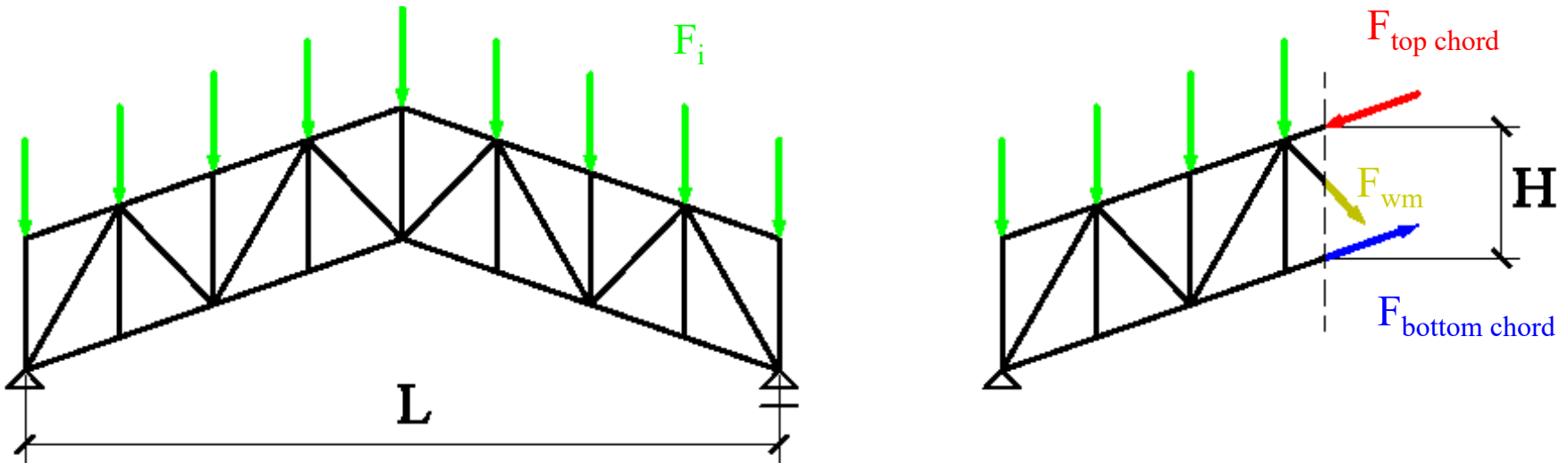


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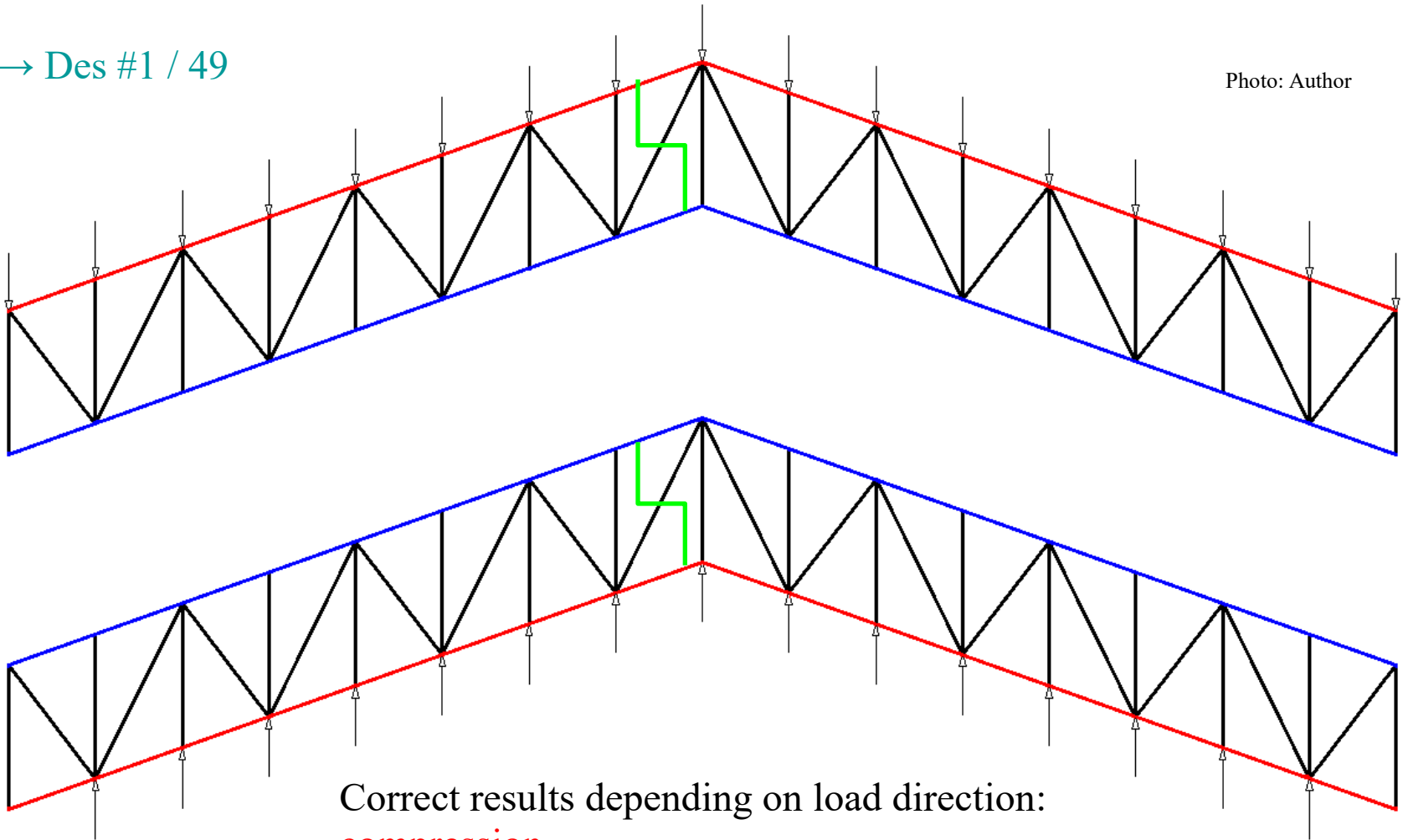
Results of calculations - verification



$$\Sigma X = 0 \quad ; \quad F_{\text{wm}} / F_{\text{bottom chord}} \approx 0 \rightarrow$$

Photo: Author

$$|F_{\text{bottom chord}}| \approx |F_{\text{top chord}}|$$



Correct results depending on load direction:

compression
tension

Additionally:

$$|\text{top tension}| \approx |\text{bottom compression}|$$

$$|\text{top compression}| \approx |\text{bottom tension}|$$

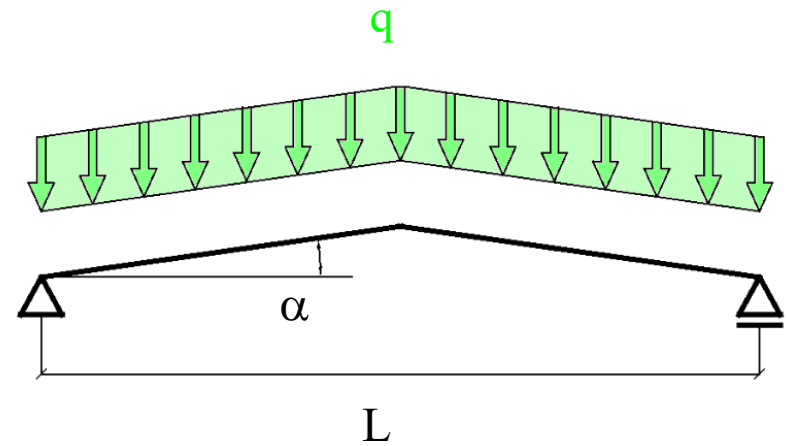
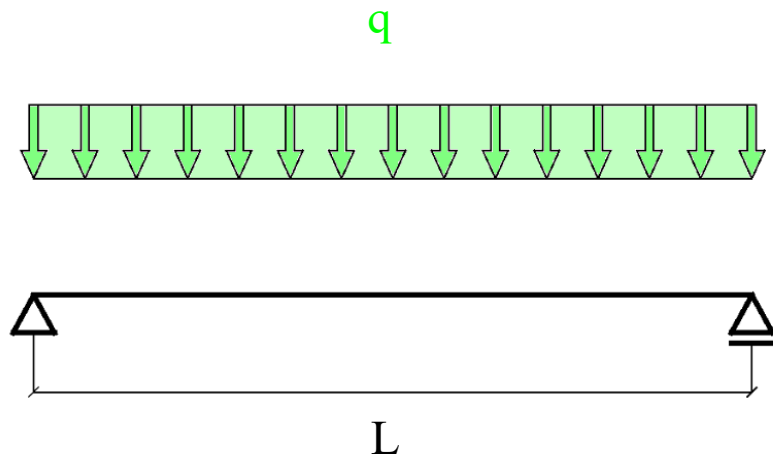


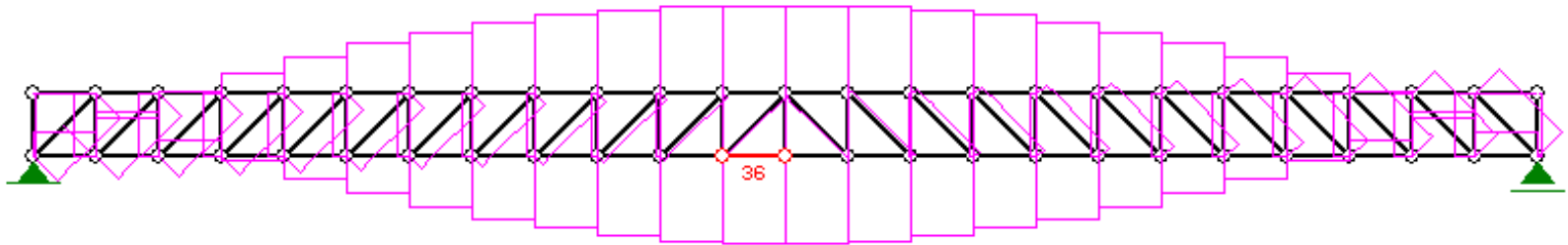
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It is possible to estimate force in top and bottom chords; we use beam-analogy:

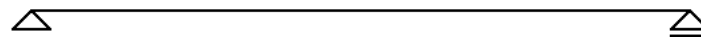
$$q = \Sigma F_i / L$$

$$M_{\max} = q L^2 / 8 \quad \text{or} \quad q L^2 / (8 \cos \alpha)$$

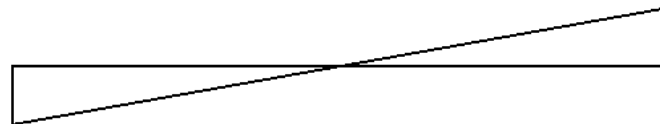
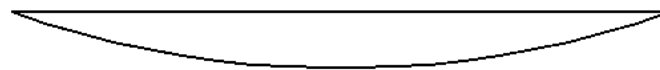
$$|F_{\text{top chord}}| \approx |F_{\text{bottom chord}}| \approx M_{\max} / H$$



Similar shape of axial forces in chord and bending moment for single-span beam;
the same for axial forces in web members and shear force in single-span beam.

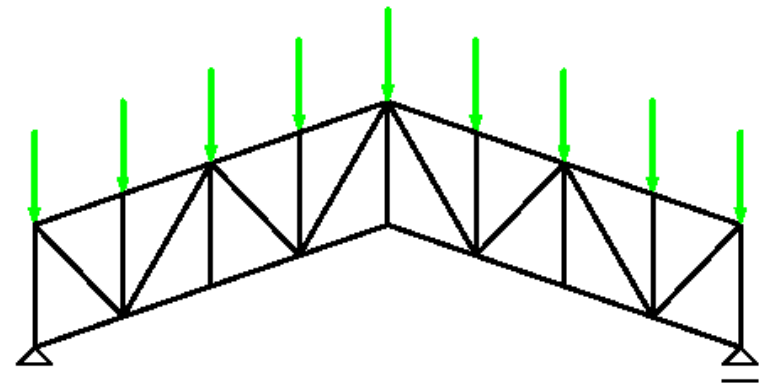
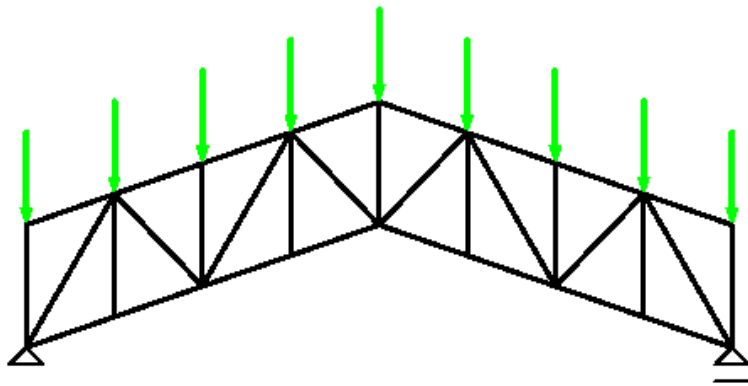


Forces in chords



Forces in web members

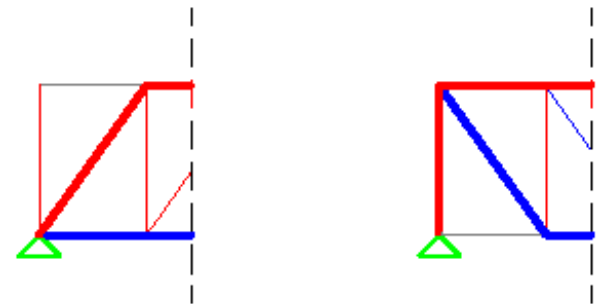
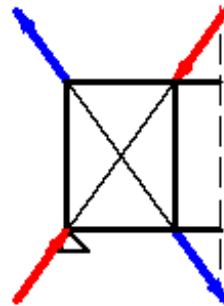
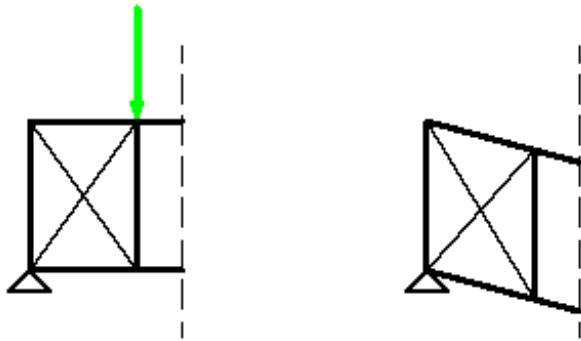
Difference between two directions of web members



Deformations: elongation (**tensile force**) and abridgement (**compressive force**):

Photo: Author

Transmission of forces from chords to supports and zero force members:



Examination issues

Types of truss structures

Models of truss – algorithm of identification

Impact of slenderness and imperfections

Laced members - słup wielogałęziowy skratowany
Battened members - słup wielogałęziowy z przewiązkami
Closely spaced build-up members - pręt wielogałęziowy

Thank you for attention

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