Metal Structures

Lecture V

Stability

Contents

Introduction

Popular experiment with a **compressed** ruler:

instability = buckling

Instability: always result of compression; never result of tension.

Photo: Author

Ultimate Limit States, ULS (EN 1990 6.4):

EQU (equilibrium) - loss of static equilibrium of the structure or any part of it, considered as a rigid body;

STR (strength) - internal failure or excessive deformaton of the structure or structural member;

GEO (geotechnics) - failure or excessive deformaton of the ground;

FAT (fatigue) - fatigue failure.

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Serviceability Limit States, SLS

What is the meaning of various types of Limit States?

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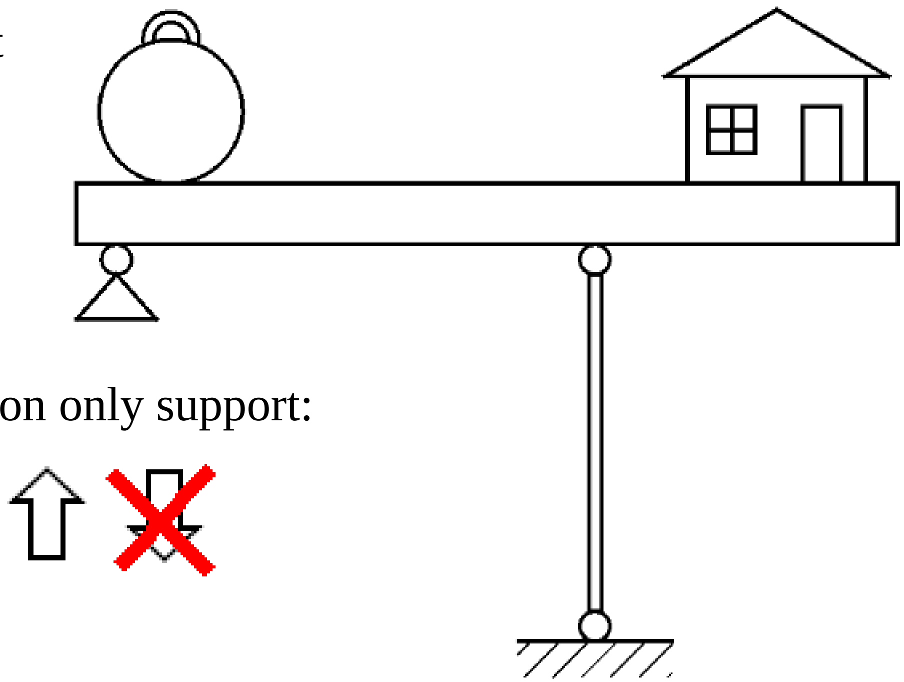
Counterweight

Most important part



Compression only support:

reaction

Photo: Author

No destruction and no deformation of structure, but dangerous situation for people or structure : EQU LS

Photo: Author

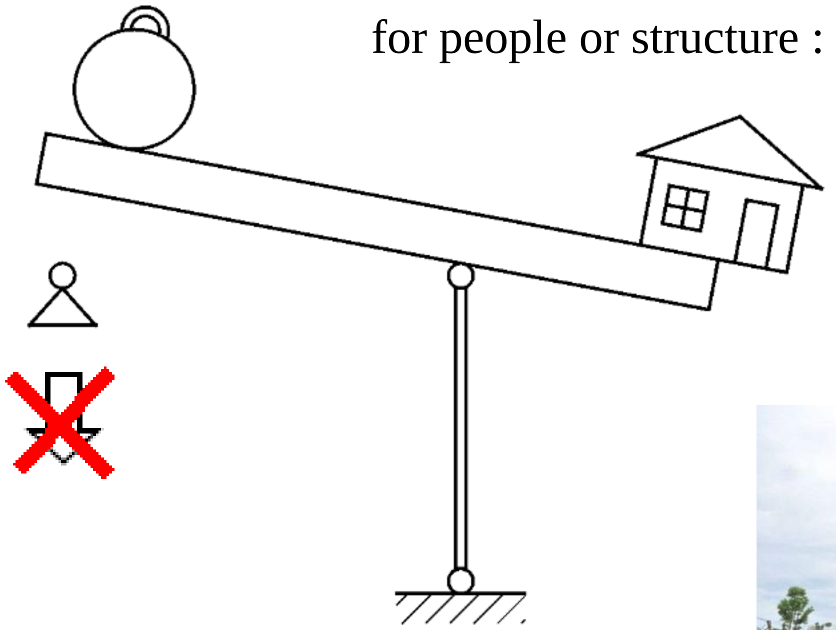


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Photo: craneaccidents.com

Photo: malaysiaconstructionservices.com

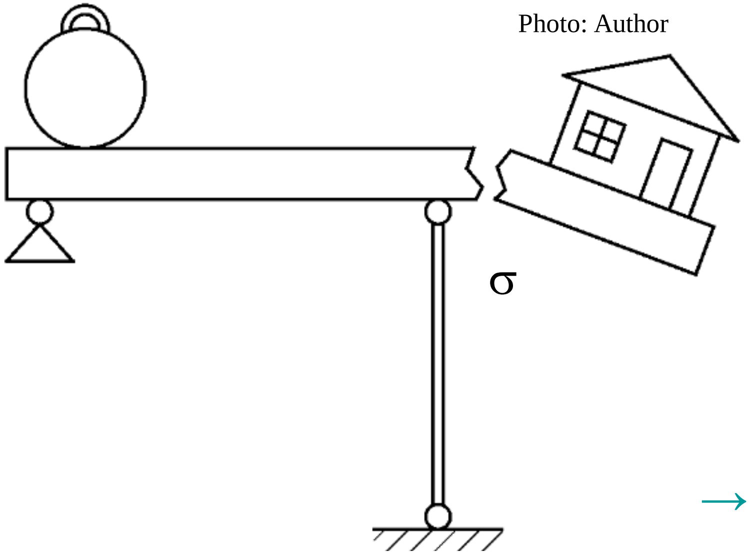
Displacement, rotation, lifting of tank by wind;

Stability of retaining wall;

Stability of crane;

Photo: link.springer.com

Photo: Author



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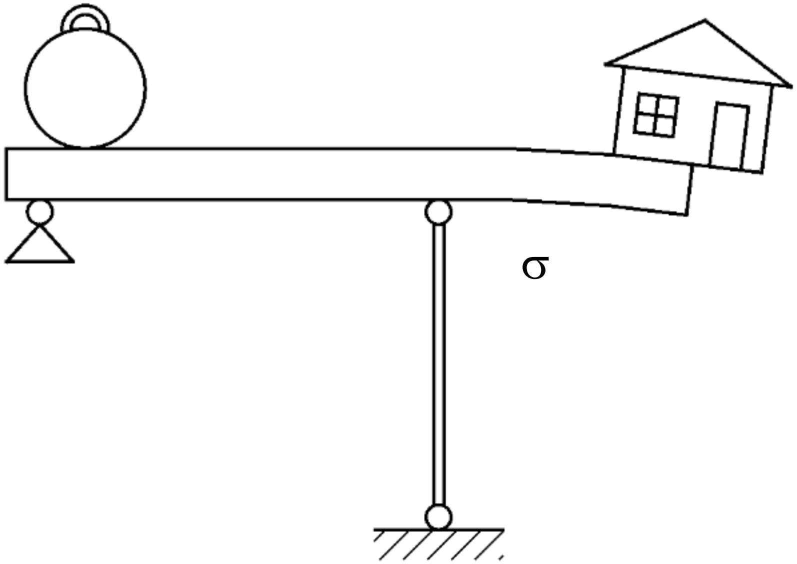

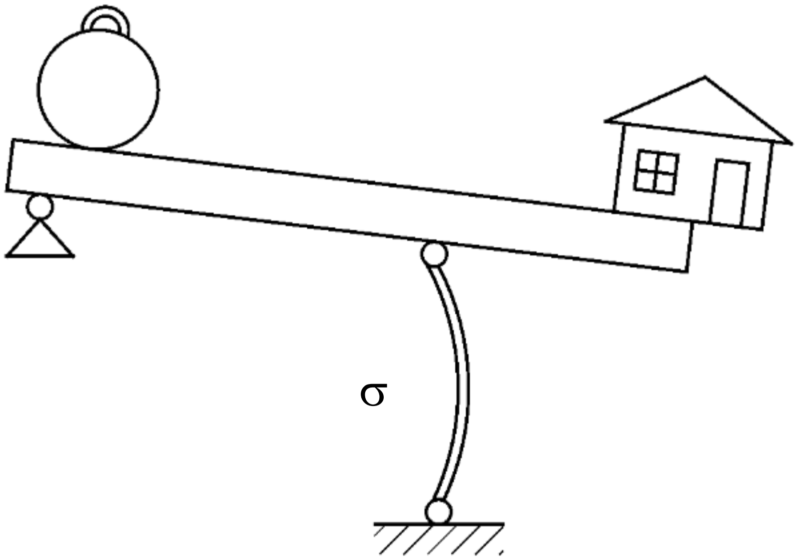

A dangerous situation for people or structure: STR LS

- exceeding strength ($\sigma > f_y$)
- excessive deformations ($\sigma < f_y$)
- buckling (instability; $\sigma < f_y$)

Types of formulas - different for different level of structure

Level of point:

$$T_\sigma = \begin{matrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_{33} \end{matrix}$$

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$$\sigma_{HMH} = \sqrt{[\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 - \sigma_{11}\,\sigma_{22} - \sigma_{11}\,\sigma_{33} - \sigma_{22}\,\sigma_{33} + 3(\tau_{12}^2 + \tau_{23}^2 + \tau_{13}^2)]}$$

$$\sigma_{HMH} / f_y \leq 1,0$$

$$\sigma_{HMH} = \sqrt{[\sigma^2 + 3(\tau_1^2 + \tau_2^2)]}$$

Welds (Ist step of study)
Shells, fatigue calculations, crane supporting structures (IInd step of study)

(~ 10% of calculation's conditions)

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Level of cross-sections:

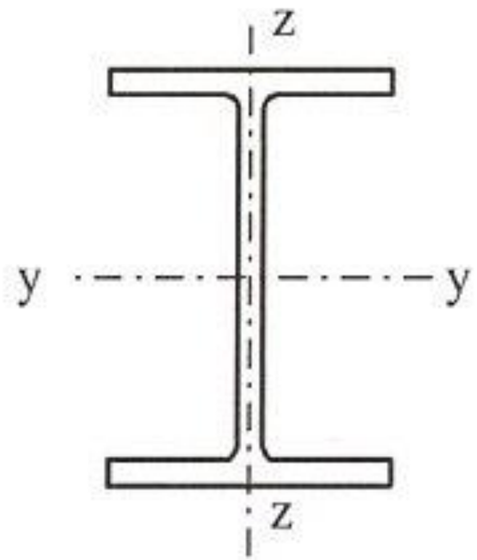

Photo: Author

F - geometry of cross-section

$$R = F\ f_y$$

$$E / R \leq 1{,}0$$

Elements, nodes - when instability is not important, bolts, rivets, pins

(~ 40% of calculation's conditions)

Level of elements:

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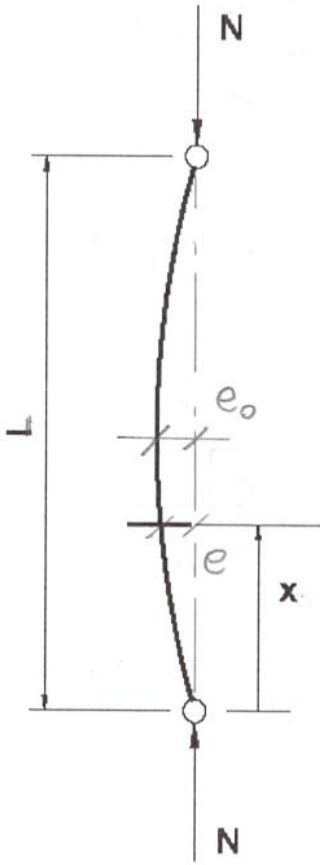
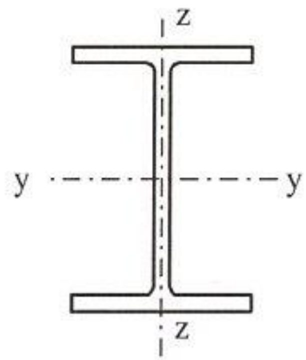
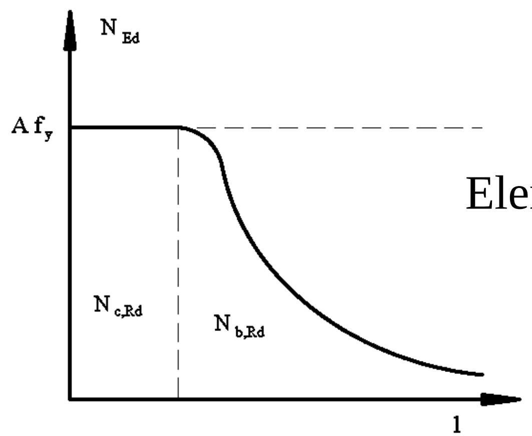


Photo: Author





F - geometry of cross-section

χ - instability coefficient (depends on element geometry)

$$R = \chi \, F \, f_y$$

$$E / R \leq 1{,}0$$

Elements, nodes - when instability is important

(~ 60% of calculation's conditions)

Formulas of resistance

Steel - different formulas for different class of cross-section

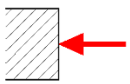
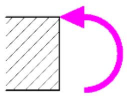
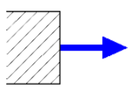
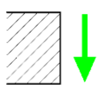
LOAD	I^{st} class	II^{nd} class	III^{rd} class	IV^{th} class
	$N_{Ed} / N_{c,Rd\,(1\text{-}3)} \leq 1{,}0$			$N_{Ed} / N_{c,Rd\,(4)} \leq 1{,}0$
	$M_{Ed\,(1)} / M_{Rd\,(1\text{-}2)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(1\text{-}2)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(3)} \leq 1{,}0$	$M_{Ed} / M_{Rd\,(4)} \leq 1{,}0$
	$N_{Ed} / N_{t,Rd} \leq 1{,}0$			
	$V_{Ed} / V_{Rd\,(1\text{-}3)} \leq 1{,}0$			$V_{Ed} / V_{Rd\,(4)} \leq 1{,}0$

Photo: Author

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$N_{c,Rd\,(1\text{-}3)} = A\ f_y / \gamma_{M0}$

$N_{c,Rd\,(4)} = A_{eff}\ f_y / \gamma_{M0}$

$M_{Rd\,(1\text{-}2)} = W_{pl}\ f_y / \gamma_{M0}$

$M_{Rd\,(3)} = W_{el}\ f_y / \gamma_{M0}$

$M_{Rd\,(4)} = W_{eff}\ f_y / \gamma_{M0}$

$V_{Rd\,(1\text{-}3)} = A_v\ f_y / (\gamma_{M0} \sqrt{3})$

$V_{Rd\,(4)}$ = impact of local instability + nonlinear relations with $M_{Rd\,(4)}$, $F_{Rd\,(4)}$ and $N_{c,Rd\,(4)}$

$N_{t,Rd} = A\ f_y / \gamma_{M0}$

Instability in real world

Flexural buckling of rails because of thermal expansion (STR LS)

Photo: tti.tamu.edu

Flexural buckling of steel trusses (STR LS)

Photo: ascelibrary.org

Photo: failuremechanisms.wordpress.com

Flexural-torsional buckling of bracings (STR LS)

Lateral buckling of beam (STR LS)

Photo: civildigital.com

Distortion of cold-formed section (STR LS)

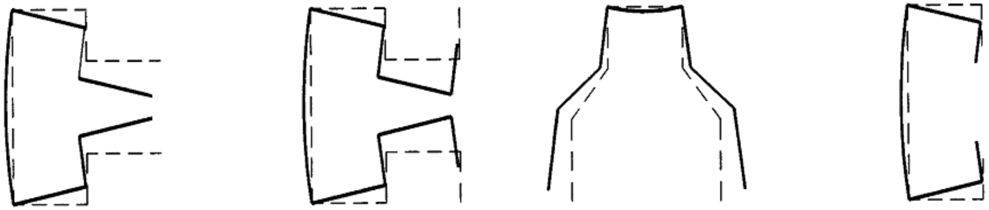
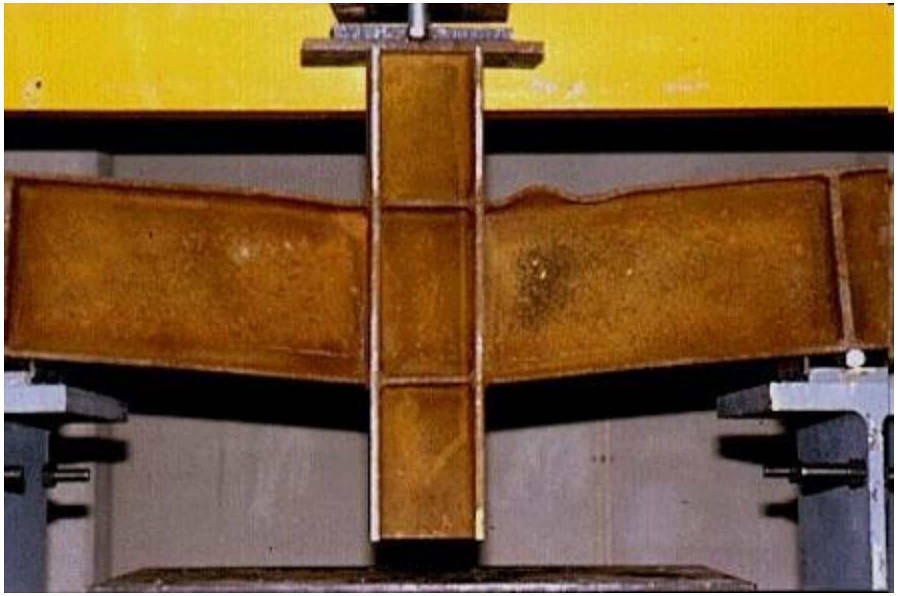


Photo: helpstud2.norod.ru



Local buckling of flanges of steel beam (STR LS)

Photo: tatasteelconstruction.com

Instability of shell structure (STR LS; silo; IInd step of study; specific name for shell structures: LS3)

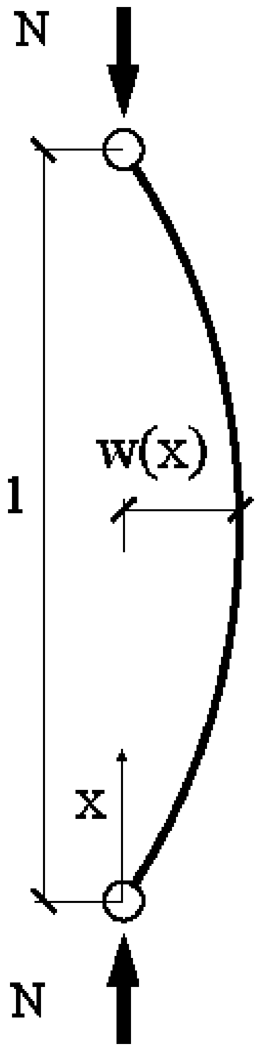
Photo: publish.ucc.ie

Equilibrium of rigid body (EQU LS)

Photo: craneaccidents.com

Flexural buckling of reinforced concrete columns (STR LS)

Photo: scedc.caltech.edu



Flexural buckling

According to Mechanics of Materials:

$$M(x) = N\,w(x)$$

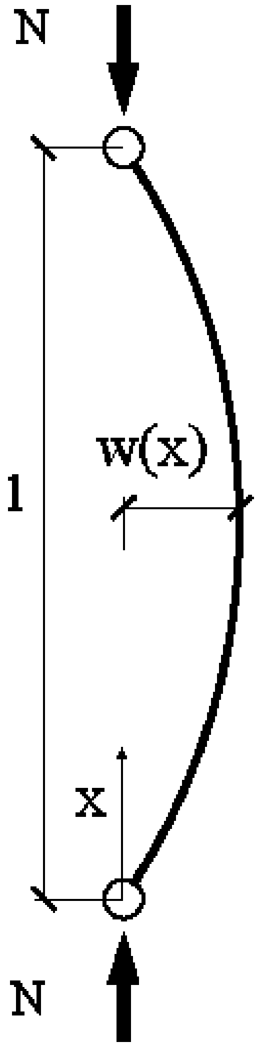
$$d[w(x)]^2 / dx^2 = -M(x) / EJ \rightarrow M(x) = -w''(x)\,E\,J$$

$$-w''(x)\,E\,J = N\,w(x)$$

$$w''(x) = -\,k^2\,w(x)$$

$$k = \sqrt{(N / EJ)}$$

Photo: Author



$$w''(x) = - k^2 \, w(x)$$

$$w(x) = W_1 \sin(k\,x) + W_2 \cos(k\,x)$$

$$w(0) = 0 \rightarrow W_2 = 0$$

$$w(l) = 0 \rightarrow W_1 = 0 \quad \text{or} \quad \sin(k\,l) = 0$$

$$\sin(k\,l) = 0 \rightarrow k\,l = n\,\pi$$

$$k = \sqrt{(N / EJ)}$$

$$l \sqrt{(N / EJ)} = n\,\pi$$

$$N / EJ = (n\,\pi / l)^2$$

$$N_{cr} = (n\,\pi / l)^2 \, EJ$$

$$w(x) = W_1 \sin(k\,x)$$

W_1 = ? (not important – buckling is important, not its amplitude)

Photo: Author

Experiment: the same geometrical characteristics of cross-section, the same values of forces, but different length of bars

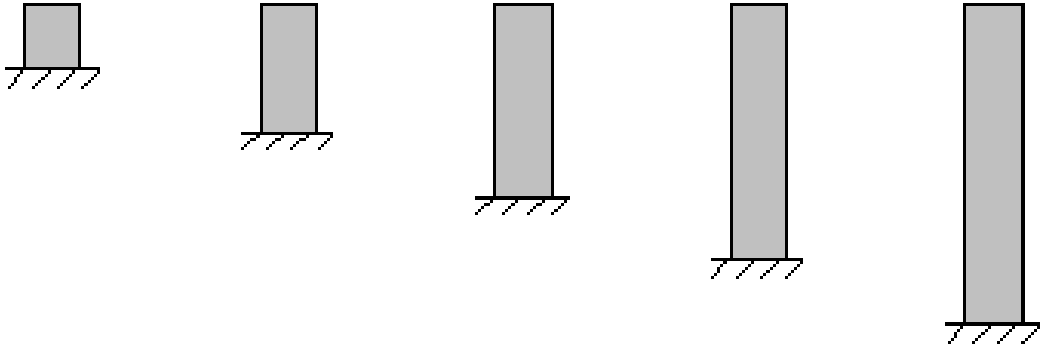


Photo: Author

$P_0 = 0$

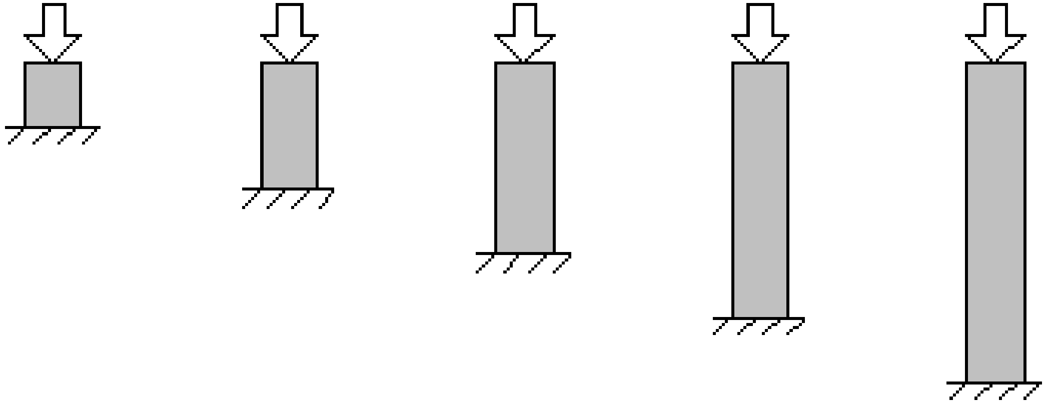
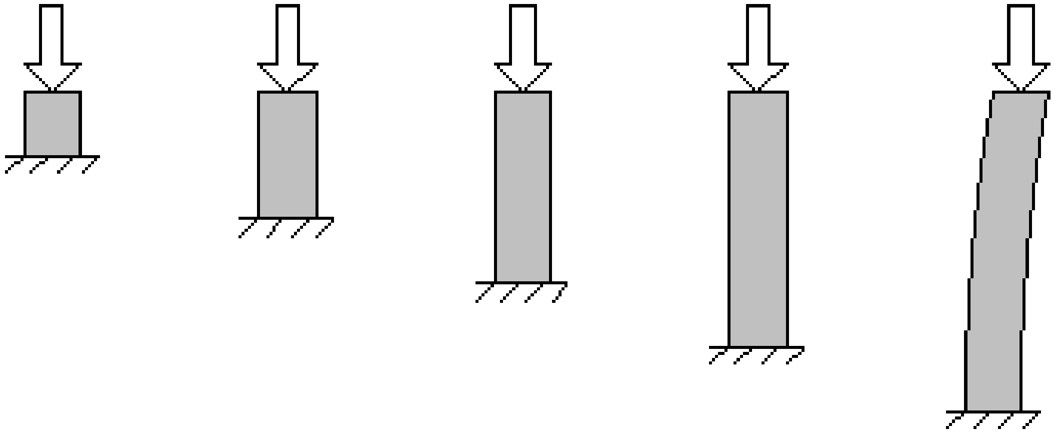


Photo: Author

$$P_1 \neq 0$$



Buckling

Photo: Author

$$P_2 = P_1 + \Delta P$$

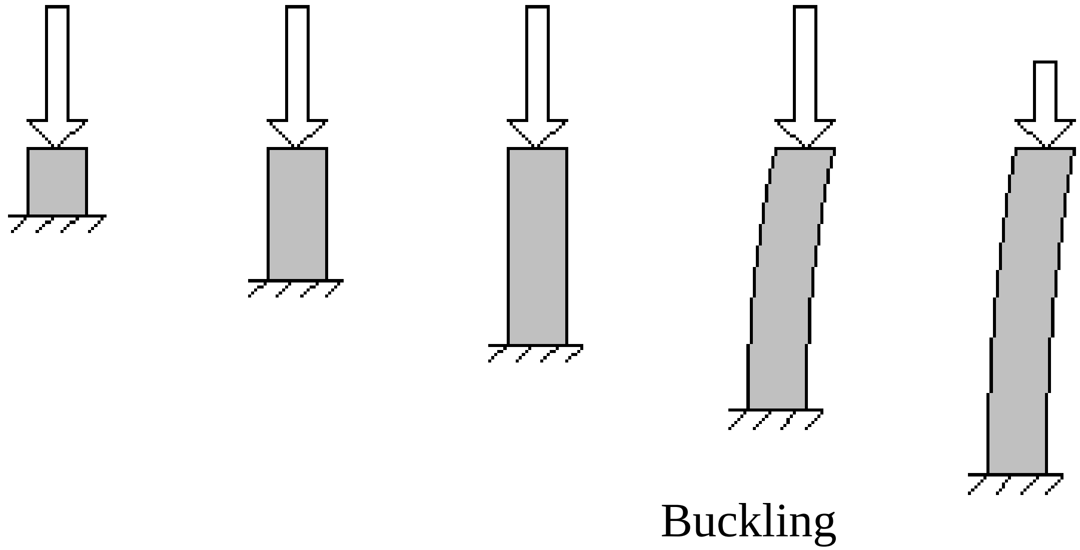
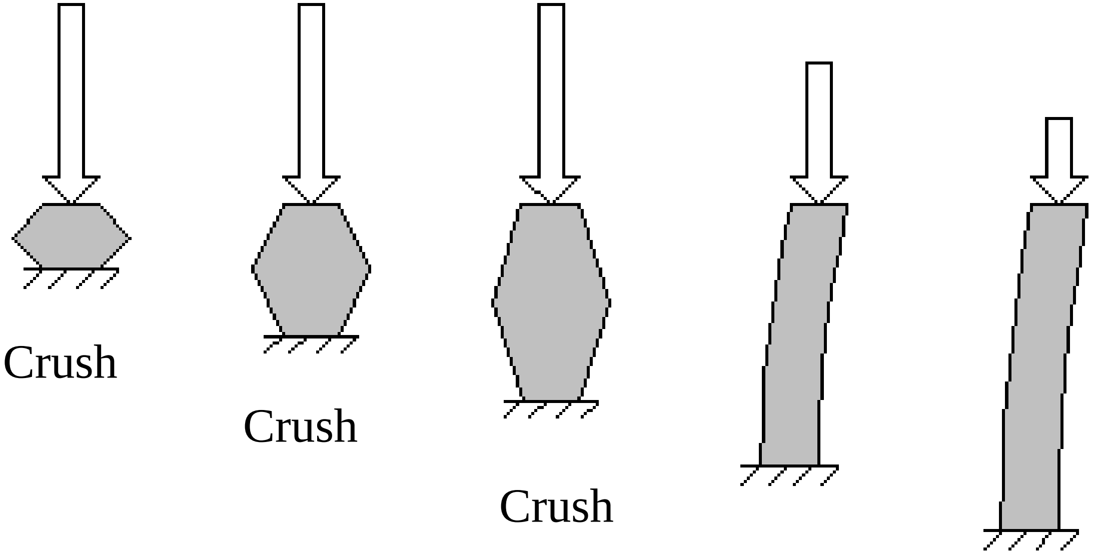


Photo: Author

$$P_3 = P_2 + \Delta P$$

$$P_4 = P_3 + \Delta P$$



Conclusion:

Photo: Author

Long bars: $N_{max} = N_{cr} = \theta / l^2$

Short bars: $N_{max} = A\, f_y$

$$N_{max} = \min (N_{cr} \; ; \; A\, f_y)$$

$$\chi = \min (1{,}0 \; ; \; N_{cr} / A\, f_y)$$

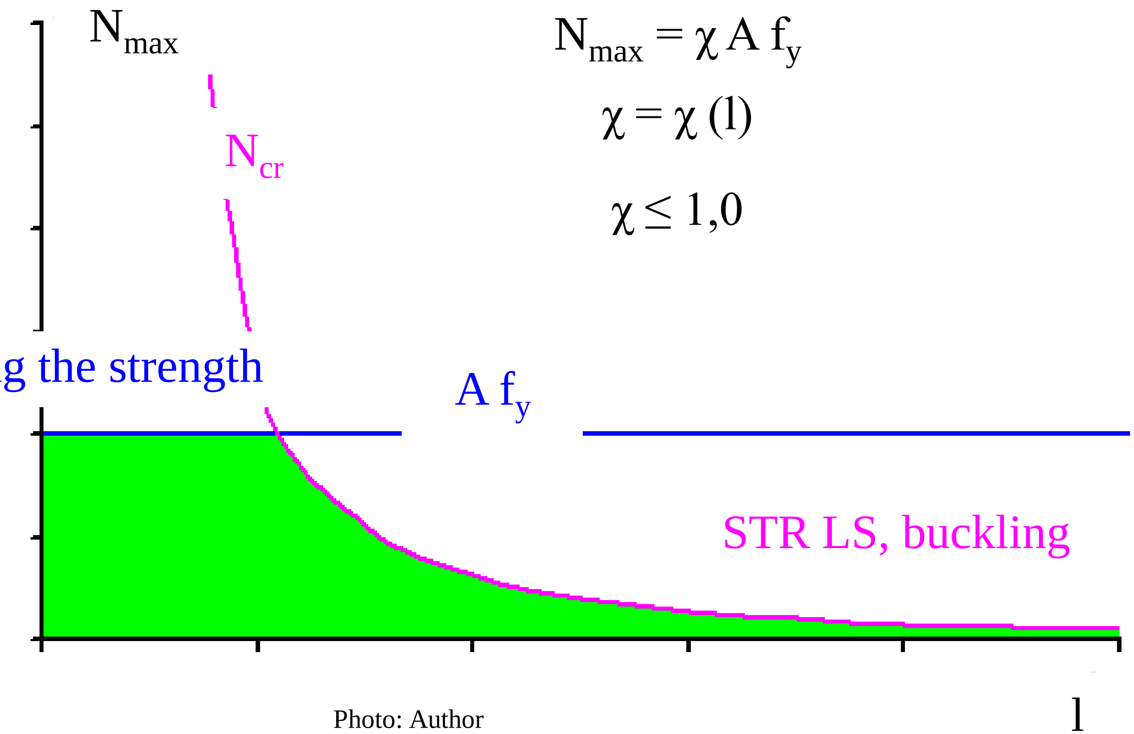
$$N_{max} = \chi\, A\, f_y$$

$$\chi = \chi (l)$$

$$\chi \leq 1{,}0$$

STR LS, exceeding the strength

Resistance of element the same as resistance of cross-section



Critical resistance of element smaller than resistance of cross-section

Photo: Author

Theoretically: we have only two conditions; for **crush** and for **buckling**.

For real structures, because of imperfections – described by buckling curves - we have **additional conditions**.

Influence of imperfections will be detaily presented on Lec #6.

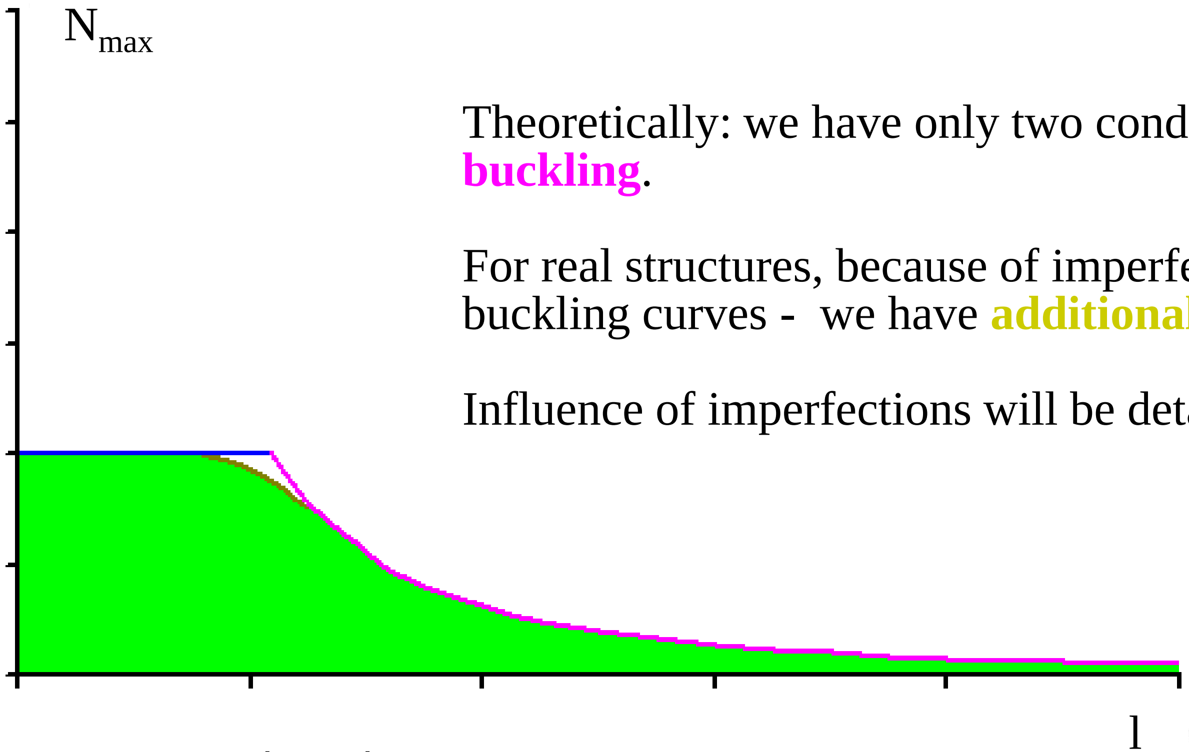
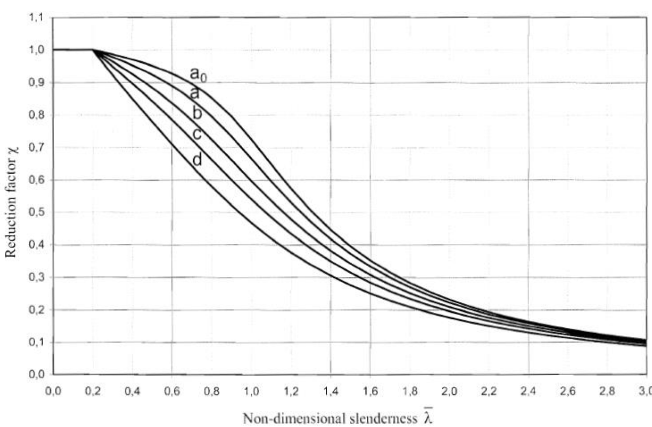


Photo: Author

Photo: EN 1993-1-1 fig. 6.4

Five different curves in Eurocode for flexural buckling:



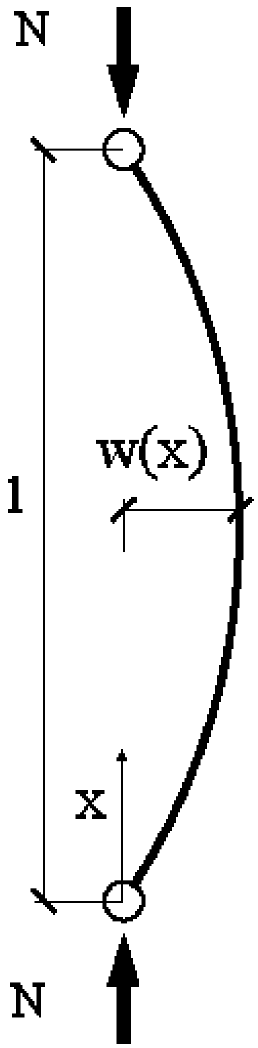


Photo: Author

Generalization:

Formula was elaborated for five assumptions as follow:

1. EJ = const (but what if not? → #t / 27)
2. N = const (but what if not? → #t / 28)
3. Two hinges (but what if not? → #t / 29-31)
4. Force applied in gravity center (but what if not? → #t / 32)
5. Bar has straight axis (but what if not? → #t / 32)

1. Generally, EJ = const is the most often case. Differences are negligible, if $\alpha \leq 10^o$; for calculations EJ = min (EJ_1 ; EJ_2). There are different rules for $\alpha > 10^o$; these rules will be presented on lecture #12

Photo: borga.pl

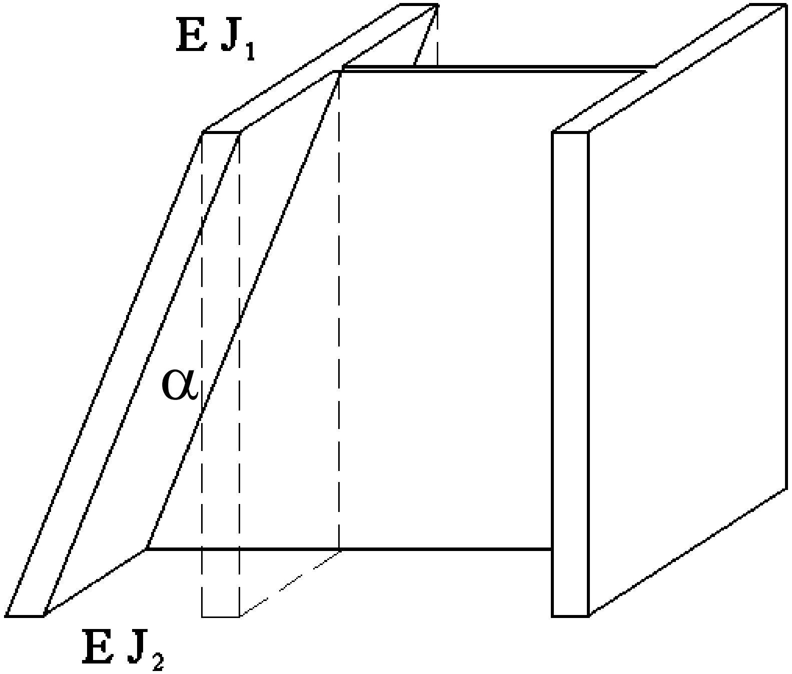


Photo: Author

2. Generally, changes of the axial force N_{Ed} along bars are very small. We should adopted to calculations $N_{Ed} = \max(N_{Ed1}\ ;\ N_{Ed2})$.

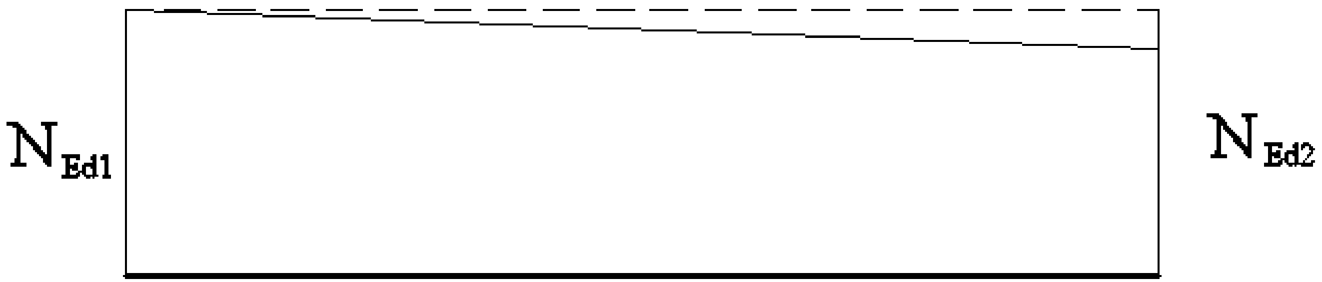


Photo: Author

3. When we have other supports (not two hinges), we have different modes of instability:

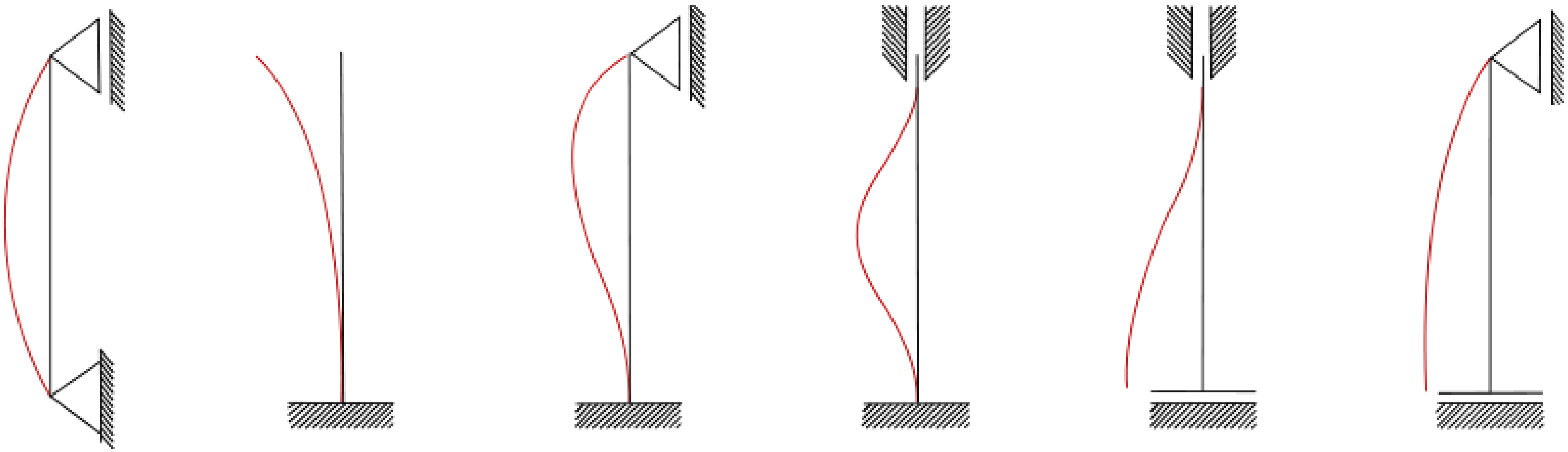


Photo: wikipedia

Critical length is a way to compare different modes of instability.

Critical length l_{cr} – theoretical length of one sinusoidal wave, which occurs in shape of bar after buckling.

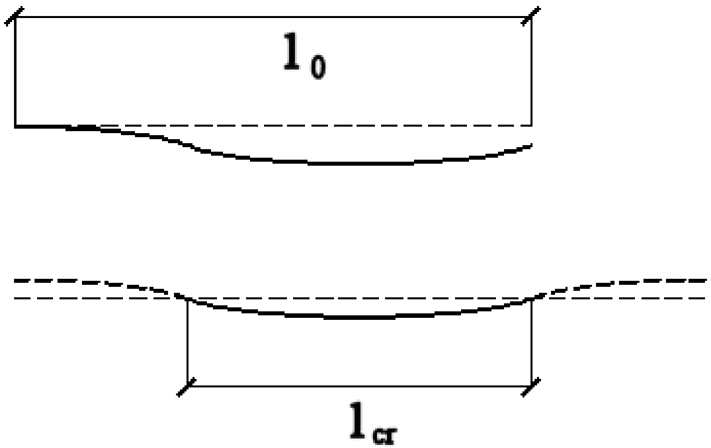
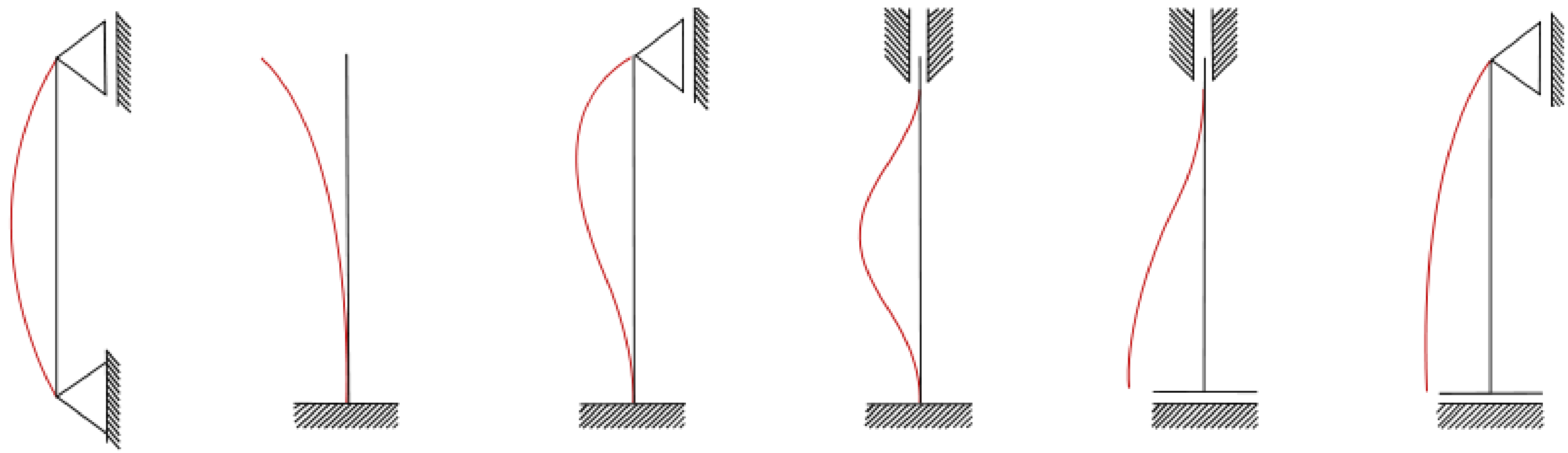


Photo: Author

Buckling length factor $\mu = l_{cr} / l_0$

There are different factors for different types of support

Photo: wikipedia



μ	1,0	2,0	0,7	0,5	1,0	2,0
l_{cr}	$1,0\ l_0$	$2,0\ l_0$	$0,7\ l_0$	$0,5\ l_0$	$1,0\ l_0$	$2,0\ l_0$

Conclusions: different types of support is important factor affecting the calculation:

$$N_{cr} = \pi^2 EJ / (\mu\ l_0)^2$$

4. Force applied out of gravity center → eccentricities between gravity center and point of force → axial force and bending moment simulateneously → such type of interaction during calculation of resistance and stability will be presented on Lec #13

5.a. Curve axis of bar → influence of imperfections → various buckling curves → #t / 25

5.b. Curve axis of bar → arch → stability of arches → Mechanics of Materials

Modes of bucklings under compressive force

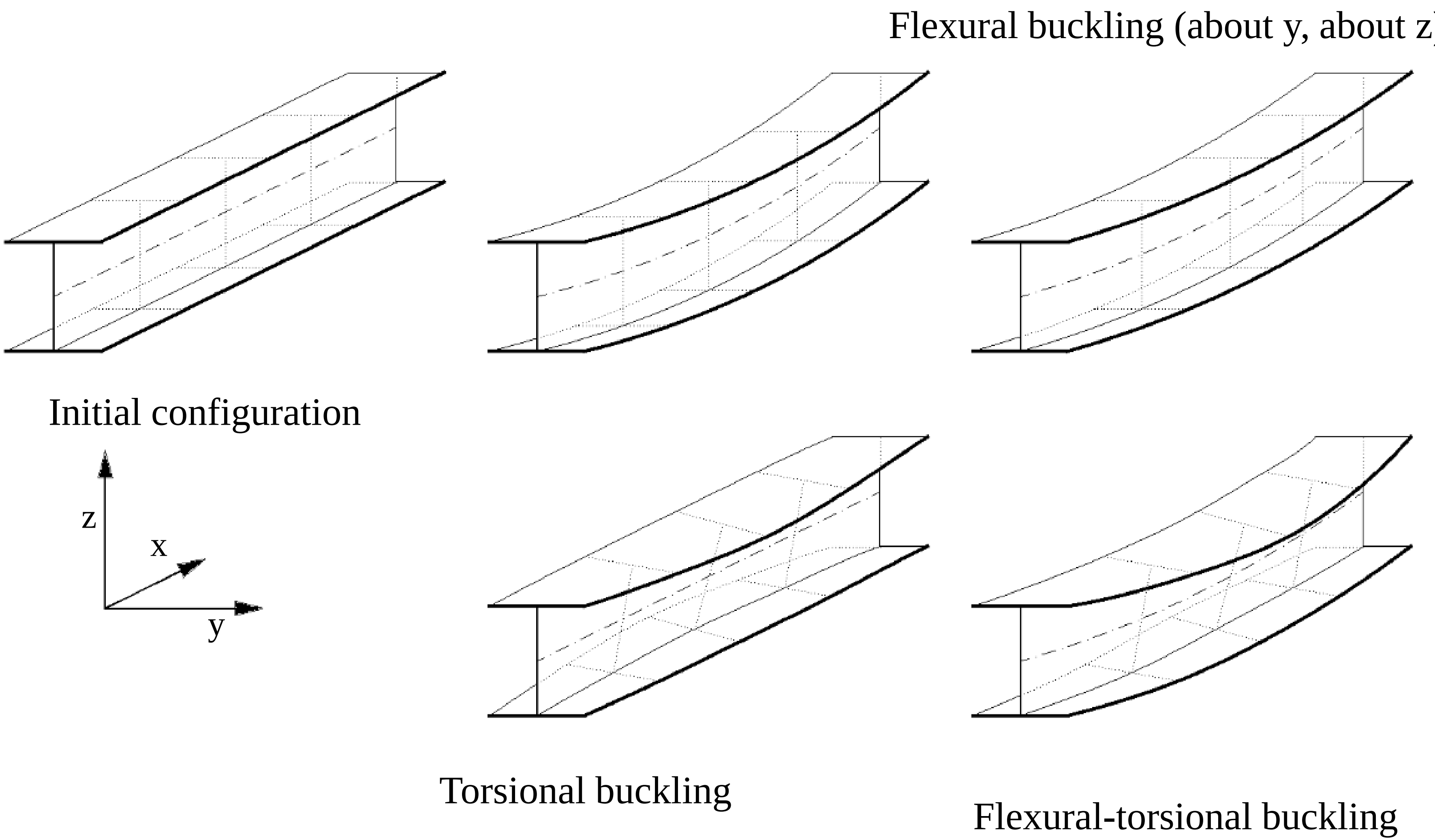
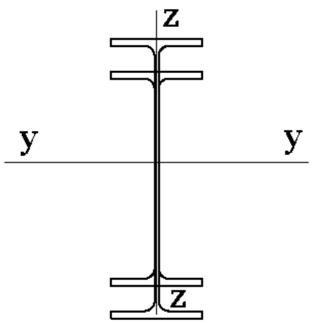
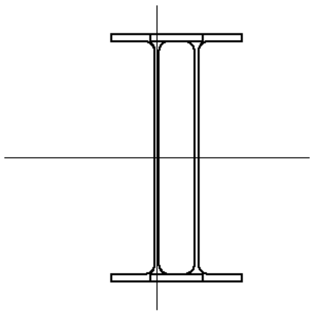
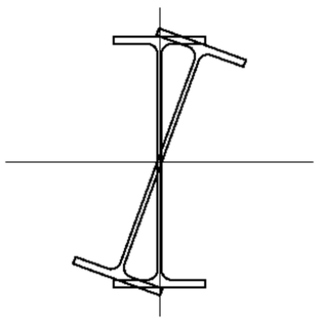
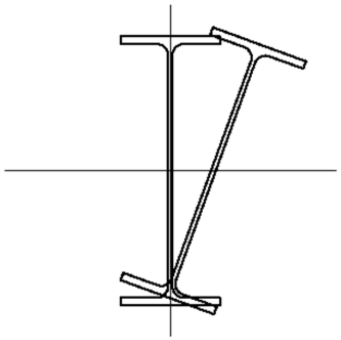


Photo: Author

Modes of cross-section's deformation for various types of buckling

Photo: Author

Buckling: initial and after-buckling position of cross-section A-A			
Flexural		Torsional	Flexural-torsional
			
J_y	J_z	J_w J_t	J_z J_w J_t

Buckling about y axis → translation parallel to z axis

Buckling about z axis → translation parallel to y axis

Formulas (according to Mechanics of Materials):

Flexural buckling, axis y $N_{cr,\,y} = \pi^2 EJ_y / (\mu_y\, l_{0y})^2$

Flexural buckling, axis z $N_{cr,\,z} = \pi^2 EJ_z / (\mu_z\, l_{0z})^2$

Torsional buckling $N_{cr,\,T} = [\pi^2 EJ_w / (\mu_T\, l_{0T})^2 + GJ_t] / i_s^2$

Flexural-torsional buckling $N_{cr,\,z\text{-}T} = \{N_{cr,\,z} + N_{cr,\,T} - \sqrt{[(N_{cr,\,z} + N_{cr,\,T})^2 - 4\, N_{cr,\,z}\, N_{cr,\,T}\, \xi]}\,\} / (2\,\xi)$

$$\xi = 1 - (\mu\, z_s^2 / i_s^2)$$

$$\mu = \min[\sqrt{(\mu_z / \mu_T)}\ ;\ \sqrt{(\mu_T / \mu_z)}]$$

$$i_0 = \sqrt{(i_y^2 + i_z^2)}$$

$$i_s = \sqrt{(i_0^2 + z_s^2)}$$

z_s - distance between centre of gravity and shear centre ($z_s \geq 0$)

J_y , J_z – moments of inertia

i_y , i_z – radius of inertia

E, G – Young and Kirchhoff modulus

J_t - torsion constant

J_w - warping constant

Here are values of these geometrical characteristics:

Designation Bezeichnung		strong axis y-y starke Achse y-y					weak axis z-z schwache Achse z-z							pure bending y-y			pure compression			HISTAR
	G kg/m	I_y cm⁴	$W_{el.y}$ cm³	$W_{pl.y}$♦ cm³	i_y cm	A_{vz} cm²	I_z cm⁴	$W_{el.z}$ cm³	$W_{pl.z}$♦ cm³	i_z cm	s_s mm	I_t cm⁴	$I_w \times 10^{-3}$ cm⁶	S 235	S 355	S 460	S 235	S 355	S 460	
IPE A 100	6.9	141.2	28.81	32.98	4.01	4.44	13.12	4.77	7.54	1.22	21.20	0.77	0.28	1	1	-	1	1	-	
IPE 100	8.1	171.0	34.20	39.41	4.07	5.08	15.92	5.79	9.15	1.24	23.70	1.2	0.35	1	1	-	1	1	-	
IPE A 120	8.7	257.4	43.77	49.87	4.83	5.41	22.39	7.00	10.98	1.42	22.20	1.04	0.71	1	1	-	1	1	-	
IPE 120	10.4	317.8	52.96	60.73	4.90	6.31	27.67	8.65	13.58	1.45	25.20	1.74	0.89	1	1	-	1	1	-	
IPE A 140	10.5	434.9	63.30	71.60	5.70	6.21	36.42	9.98	15.52	1.65	23.20	1.36	1.58	1	1	-	1	2	-	
IPE 140	12.9	541.2	77.32	88.34	5.74	7.64	44.92	12.31	19.25	1.65	26.70	2.45	1.98	1	1	-	1	1	-	

Photo: europrofil.lu

If we have cross-section, which not exists in tables:

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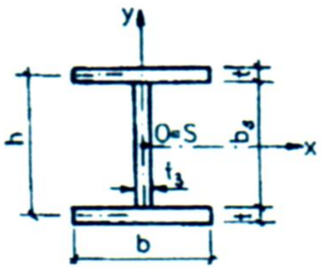
<tr><td>
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$y_s = 0$

$I_\omega = \frac{I_y h^2}{4}$

$I_T = \frac{1}{3}(2\, b t^3 + b_3\, t_3^3)$

$r_x = 0$

→ Lab #1 / 62
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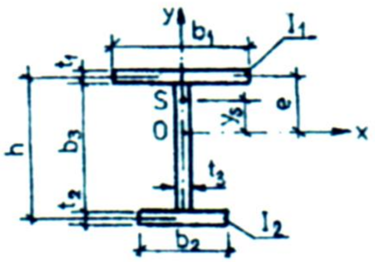
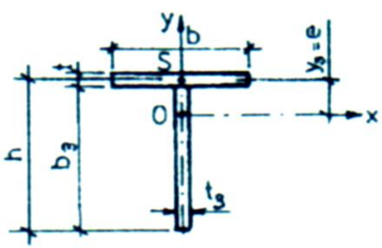
<tr><td>
</td><td>

$y_s = \frac{1}{I_y}[e\, I_1 - (h - e)\, I_2] = e - \frac{I_2}{I_y} h$

$I_\omega = \frac{I_1\, I_2\, h^2}{I_1 + I_2}$

$I_T = \frac{1}{3}(b_1\, t_1^3 + b_2\, t_2^3 + b_3\, t_3^3)$

$r_x = \frac{1}{I_x}\left\{y_s\, I_y + b_1\, t_1\, e^3 - b_2\, t_2\,(h - e)^3 + \frac{t_3}{4}[e^4 - (h - e)^4]\right\}$
</td></tr>
<tr><td>
</td><td>

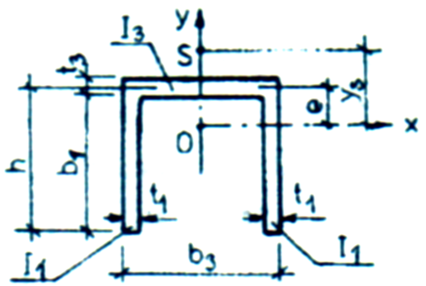
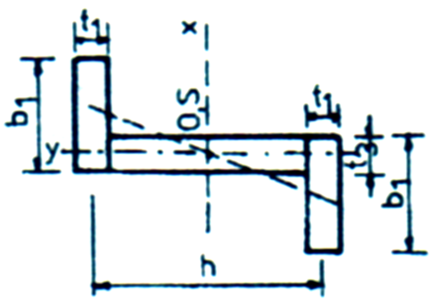
$y_s = e$

$I_\omega \cong 0$

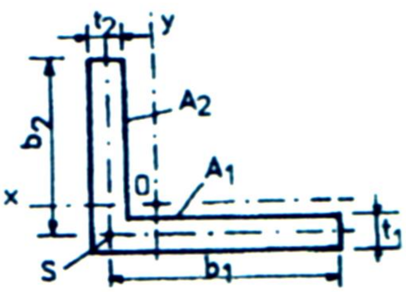
$I_T = \frac{1}{3}(b\, t^3 + b_3\, t_3^3)$

$r_x = \frac{1}{I_x}\left\{e\, I_y + b\, t\, e^3 + \frac{t_3}{4}[e^4 - (h - e)^4]\right\}$
</td></tr>
</table>

→ Lab #1 / 63

	$y_s = e + \frac{I_1 h}{I_y}$ $I_\omega = \frac{h^2}{3} \cdot \frac{I_1^2 + 2 I_1 I_3}{I_y}$ $I_T = \frac{1}{3}(2 b_1 t_1^3 + b_3 t_3^3)$
	$y_s = 0$ $I_\omega = \frac{h^2}{4} I_y$ $I_r = \frac{1}{3}(2 b_1 t_1 saup2 + h t_3^2)$

→ Lab #1 / 64

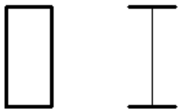
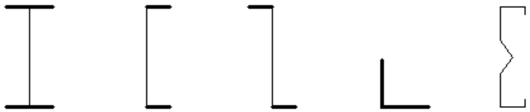
Przekrój	Cechy geometryczne
	$y_s = e_y - \frac{t}{2}$ $I_\omega = \frac{A_1^3 + A_2^3}{36}$ $I_r = \frac{1}{3}(b_1 t_1^3 + b_2 t_2^3)$
Oznaczenia:	
0 — środek ciężkości S — środek ścinania I_1 I_2 (I_3) — momenty bezwładności półek (środnika) względem osi symetrii I_y — moment bezwładności figury względem osi symetrii	

J. Żmuda, „Podstawy projektowania konstrukcji metalowych”, TiT Opole 1992

Photo: Author

The result of calculations is buckling factor χ.

It is calculated in different way for different cross-sections.

Buckling		
Flexural		Flexural, torsional, flexural-torsional
	 (hot rolled I)	 (welded I)
$\chi = \chi_y = \chi_z$ (only if $l_{cr,\,y} = l_{cr,\,z}$)	$\chi = \min(\chi_y \;;\; \chi_z)$	$\chi = \min(\chi_y \;;\; \chi_z \;;\; \chi_T \;;\; \chi_{z,\,T})$

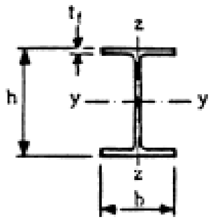
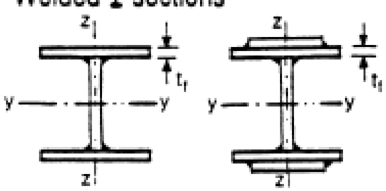
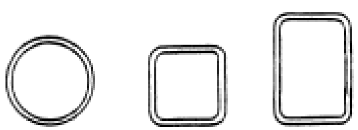
Algorithm

EN 1993-1-1   6.3.1

Flexural buckling (I, II, III class of cross-section)	$\lambda = (l_{cr} / i) / \lambda_1$ $\lambda_1 = 93{,}9\ \varepsilon$	$\Phi = [1 + \alpha\ (\lambda - 0{,}2) + \lambda^2] / 2$ $\alpha \rightarrow$ EN 1993-1-1, tab. 6.1, 6.2	$\chi = \min\{1/[\Phi + \sqrt{(\Phi^2 - \lambda^2)}]\ ;\ 1{,}0\}$
Flexural, torsional, flexural-torsional buckling,	$\lambda = \sqrt{(A_{(eff)}\ f_y / N_{cr})}$		

$$\lambda \leq 0{,}2 \rightarrow \chi = 1{,}0$$

Photo: EN 1993-1-1, tab. 6.2

Cross section	Limits	Buckling about axis	Buckling curve
Rolled I-sections 	h/b > 1,2: $t_f \le 40$ mm	y – y z – z	a b
	40 mm < $t_f \le 100$ mm	y – y z – z	b c
	h/b ≤ 1,2: $t_f \le 100$ mm	y – y z – z	a b
	$t_f > 100$ mm	y – y z – z	d d
Welded I-sections 	$t_f \le 40$ mm	y – y z – z	b c
	$t_f > 40$ mm	y – y z – z	c d
Hollow sections 	hot rolled	any	a
	cold formed — using f_{yb}*	any	b
	cold formed — using f_{ya}*	any	c

Bucking curve	a	b	c	d
Imperfection factor α	0,21	0,34	0,49	0,76

Photo: EN 1993-1-1, tab. 6.1

Example 1

C 300p
S235 → $f_y = 235$ MPa
$L = 3{,}00$ m
$E = 210$ GPa
$G = 81$ GPa
$A = 52{,}5\ cm^2$
$J_y = 7640\ cm^4$
$J_z = 473\ cm^4$
$J_w = 66\,500\ cm^6$
$J_T = 33{,}9\ cm^4$
$a = 3{,}12$ cm
$e = 2{,}89$ cm
$i_y = 12{,}1$ cm
$i_z = 3{,}01$ cm
$y_s = a + e = 6{,}01$ cm
In this case:
$z_s = y_s = 6{,}01$ cm

$N_{Ed} = 650$ kN

I class of cross-section

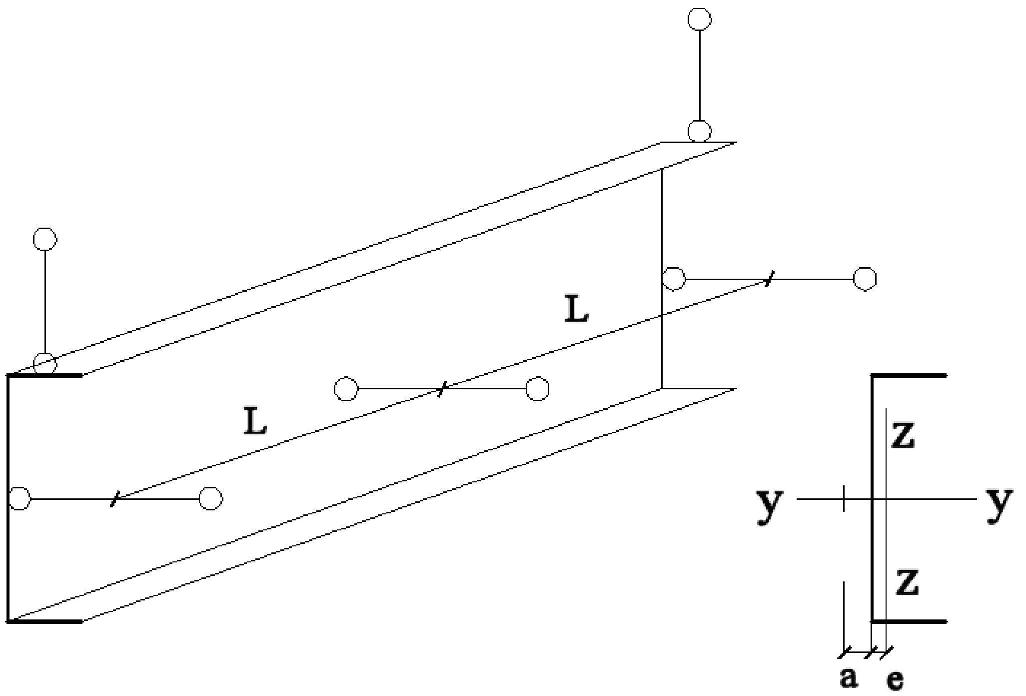
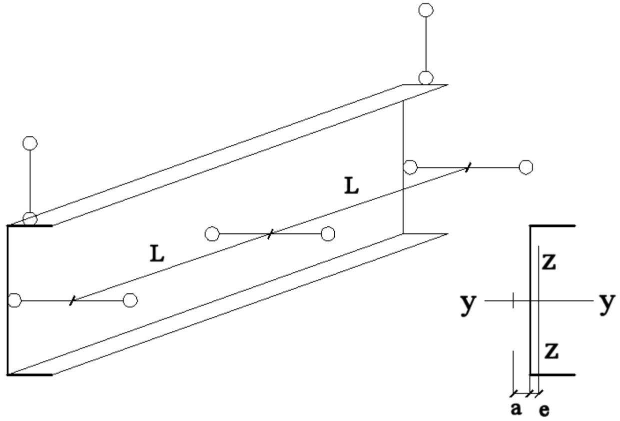
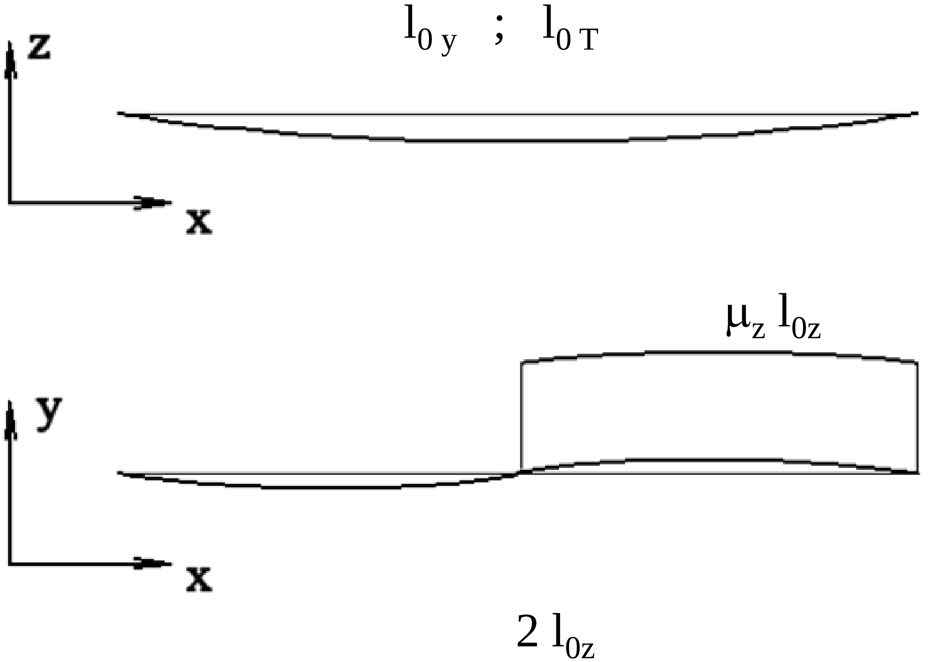


Photo: Author

Photo: Author

Supports and modes of instability:





$l_{0\,y}$ = 6,00 m
$l_{0\,z}$ = 3,00 m
$l_{0\,T}$ = 6,00 m

μ_y = 1,00
μ_z = 0,90 – 1,00 (in calculation 1,00)
μ_T = 1,00

$$N_{cr,\,y} = \pi^2 EJ_y / (\mu_y\, l_{0y})^2 = \pi^2\, 210\text{ GPa} \cdot 7640\text{ cm}^4 / (1{,}0 \cdot 6{,}00\text{ m})^2 = \underline{4\,398{,}554\text{ kN}}$$

$$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z\, l_{0z})^2 = \pi^2\, 210\text{ GPa} \cdot 473\text{ cm}^4 / (1{,}0 \cdot 3{,}00\text{ m})^2 = \underline{1\,089{,}211\text{ kN}}$$

$$i_0 = \sqrt{}\,(i_y^2 + i_z^2) = 12{,}47\text{ cm}$$

$$i_s = \sqrt{}\,(i_0^2 + z_s^2) = 13{,}84\text{ cm}$$

$$N_{cr,\,T} = [\pi^2 EJ_w / (\mu_T\, l_{0T})^2 + GJ_t] / i_s^2 =$$

$$= [\pi^2\, 210\text{ GPa} \cdot 66\,500\text{ cm}^6 / (1{,}0 \cdot 6{,}00\text{ m})^2 + 81\text{ GPa} \cdot 33{,}9\text{ cm}^4] / (13{,}84\text{ cm})^2 = \underline{1\,633{,}427\text{ kN}}$$

$$\mu = \min[\sqrt{}\,(\mu_z / \mu_T) \;\; ; \;\; \sqrt{}\,(\mu_T / \mu_z)] = 0{,}975$$

$$\xi = 1 - (\mu\, z_s^2 / i_s^2) = 0{,}816$$

$$N_{cr,\,zT} = \{N_{cr,\,z} + N_{cr,\,T} - \sqrt{}\,[(N_{cr,\,z} + N_{cr,\,T})^2 - 4\, N_{cr,\,z}\, N_{cr,\,T}\, \xi]\,\} / (2\,\xi) =$$

$$= \{1\,206{,}953\text{ kN} + 1\,633{,}427\text{ kN} +$$

$$- \sqrt{}\,[(1\,087{,}211\text{ kN} + 1\,633{,}427\text{ kN})^2 - 4 \cdot 1\,087{,}211\text{kN} \cdot 1\,633{,}427\text{ kN} \cdot 0{,}816]\,\} / (2 \cdot 0{,}816) =$$

$$= \underline{888{,}974\text{ kN}}$$

$$A f_y = 1\,233{,}750 \text{ kN}$$

$$\lambda_y = \sqrt{(A f_y / N_{cr,\,y})} = 0{,}530$$

$$\lambda_z = \sqrt{(A f_y / N_{cr,\,z})} = 1{,}064$$

$$\lambda_T = \sqrt{(A f_y / N_{cr,\,T})} = 0{,}869$$

$$\lambda_{zT} = \sqrt{(A f_y / N_{cr,\,zT})} = 1{,}178$$

C 300p → tab. 6.1, 6.2, EN 1993-1-1 → $\alpha_y = \alpha_z = \alpha_T = \alpha_{zT} = 0{,}49$

$$\Phi_y = [1 + \alpha_y (\lambda_y - 0,2) + \lambda_y^2] / 2 = 0,721$$

$$\Phi_z = [1 + \alpha_z (\lambda_z - 0,2) + \lambda_z^2] / 2 = 1,278$$

$$\Phi_T = [1 + \alpha_T (\lambda_T - 0,2) + \lambda_T^2] / 2 = 1,041$$

$$\Phi_{zT} = [1 + \alpha_{zT} (\lambda_{zT} - 0,2) + \lambda_{zT}^2] / 2 = 1,435$$

$$\chi_y = \min\{1/[\Phi_y + \sqrt{(\Phi_y^2 - \lambda_y^2)}] ; 1,0\} = 0,827$$

$$\chi_z = \min\{1/[\Phi_z + \sqrt{(\Phi_z^2 - \lambda_z^2)}] ; 1,0\} = 0,504$$

$$\chi_T = \min\{1/[\Phi_T + \sqrt{(\Phi_T^2 - \lambda_T^2)}] ; 1,0\} = 0,619$$

$$\chi_{zT} = \min\{1/[\Phi_{zT} + \sqrt{(\Phi_{zT}^2 - \lambda_{zT}^2)}] ; 1,0\} = 0,444$$

$$\chi = \min(\chi_y ; \chi_z ; \chi_T ; \chi_{zT}) = 0,444$$

$$A\ f_y = 1\ 233{,}750\ kN$$

$$\chi\ A\ f_y = 548{,}241\ kN$$

$$N_{Ed} = 650\ kN$$

$$N_{Ed} / (A\ f_y / \gamma_{M0}) = 0{,}527$$

OK.

$$N_{Ed} / (\chi\ A\ f_y / \gamma_{M0}) = 1{,}186$$

Wrong, buckling, destruction!

$L_{0z} = 2{,}00$ m

Proposition: other distance between supports on y-direction → change of critical length for z-buckling

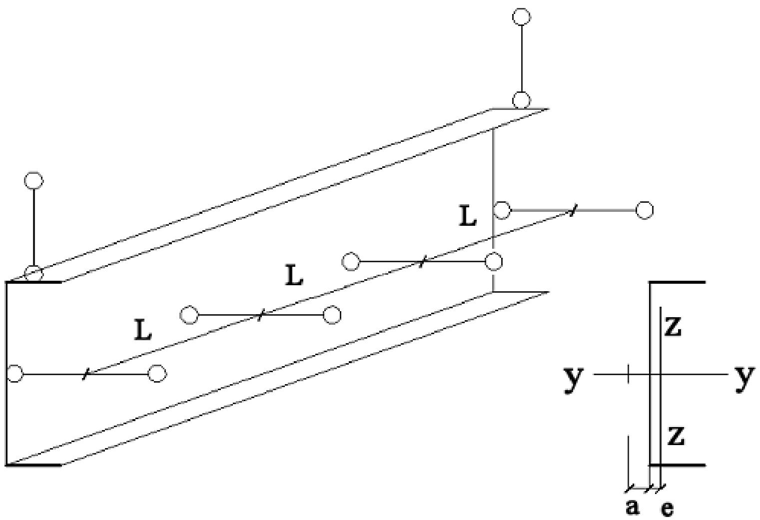


Photo: Author

$N_{cr,\,y} = \underline{4\,398{,}554 \text{ kN}}$

$N_{cr,\,z} = \underline{2\,450{,}725 \text{ kN}}$

$N_{cr,\,T} = \underline{1\,633{,}427 \text{ kN}}$

$N_{cr,\,zT} = \underline{1\,333{,}190 \text{ kN}}$

$\lambda_y = \sqrt{(A\, f_y / N_{cr,\,y})} = 0{,}530$

$\lambda_z = \sqrt{(A\, f_y / N_{cr,\,z})} = 0{,}710$

$\lambda_T = \sqrt{(A\, f_y / N_{cr,\,T})} = 0{,}869$

$\lambda_{zT} = \sqrt{(A\, f_y / N_{cr,\,zT})} = 0{,}962$

$\chi = \min(\chi_y\,;\, \chi_z\,;\, \chi_T\,;\, \chi_{zT}) = 0{,}562$

$$A\ f_y = 1\ 233{,}750\ kN$$

$$\chi\ A\ f_y = 693{,}930\ kN$$

$$N_{Ed} = 650\ kN$$

$$N_{Ed} / (A\ f_y / \gamma_{M0}) = 0{,}527$$

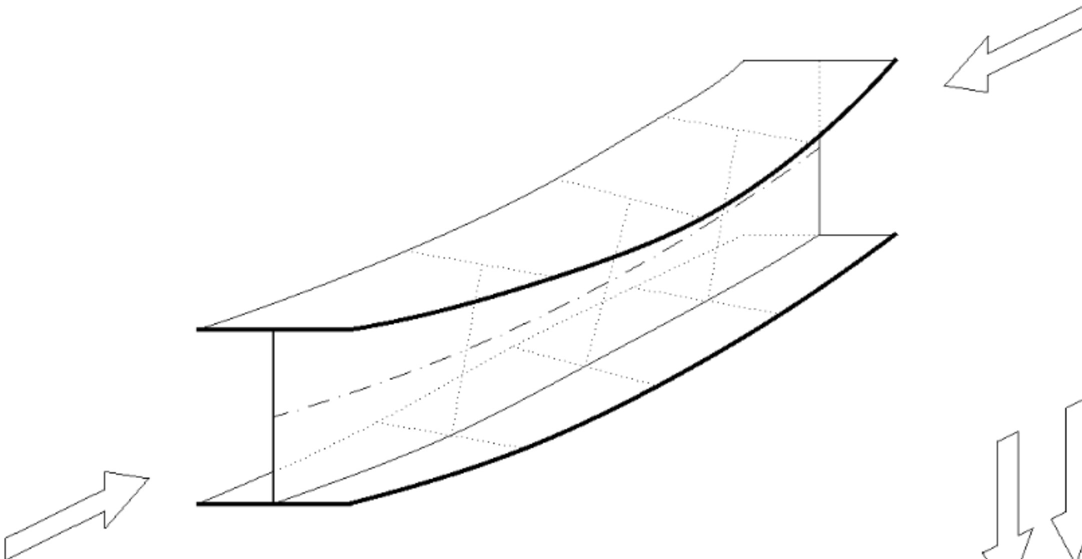
OK.

$$N_{Ed} / (\chi\ A\ f_y / \gamma_{M0}) = 0{,}944$$

OK.

Lateral buckling

Flexural-torsional buckling versus lateral buckling



Flexural-torsional buckling

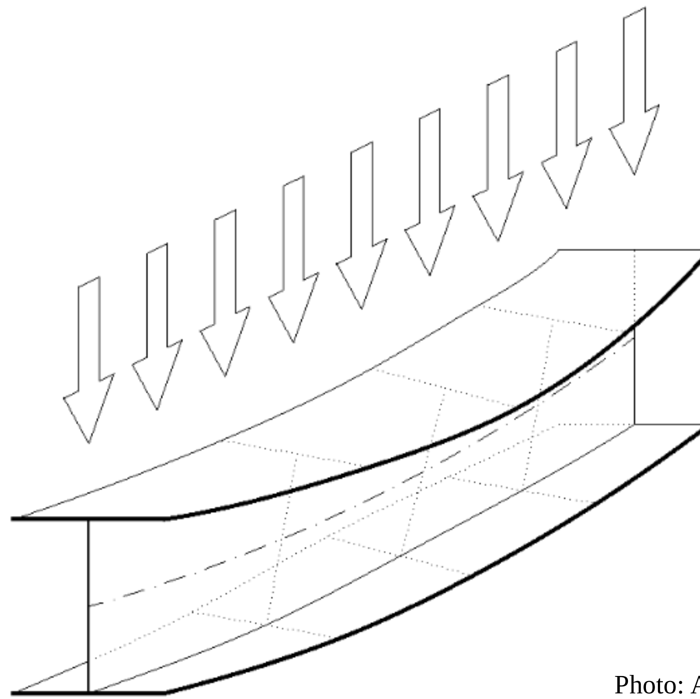


Photo: Author

The same shape of beam ansver,
but other reasons

Lateral buckling

There are two possibilities of destruction for axial force.

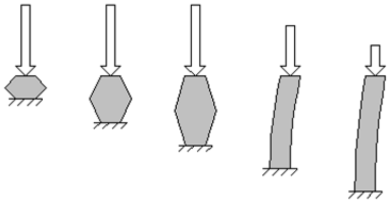


Photo: Author

Experiment: what's happend during bending cantilever about strong and weak axis?

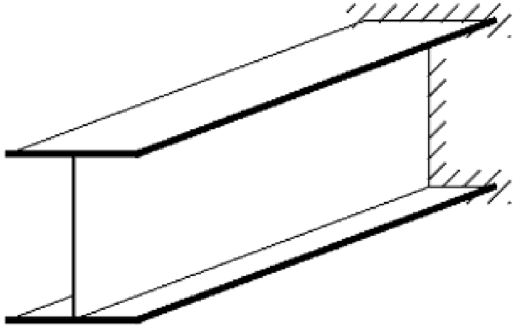
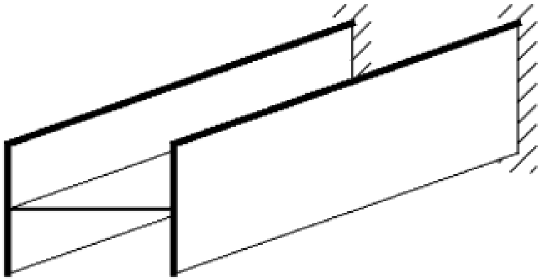


Photo: Author



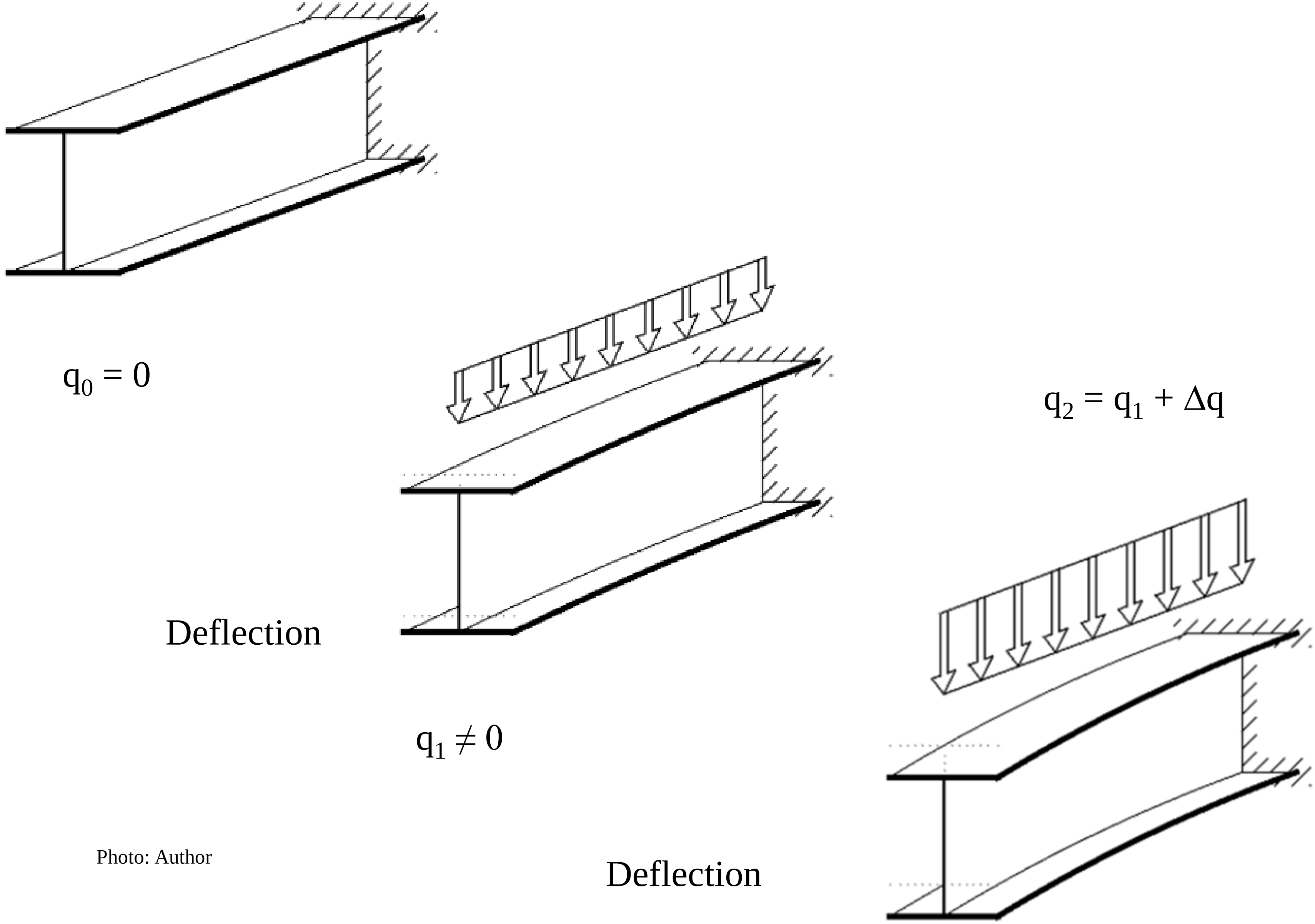
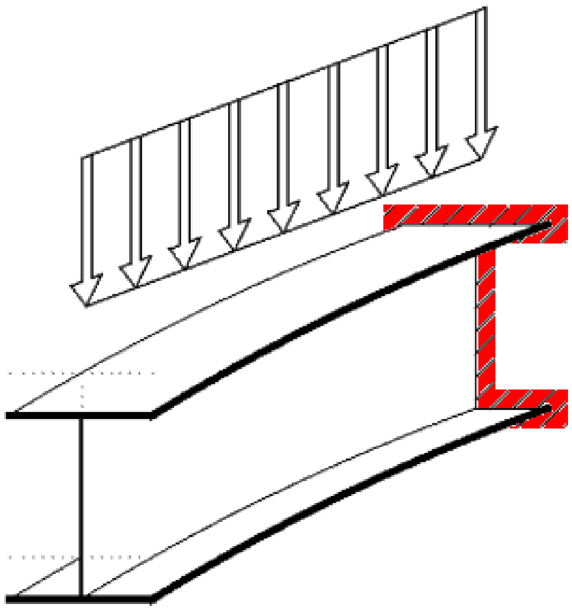


Photo: Author

Two possibilities of destruction:

1. stresses on support from bending > resistance → destruction of support → destruction of structure;

2. lateral buckling;



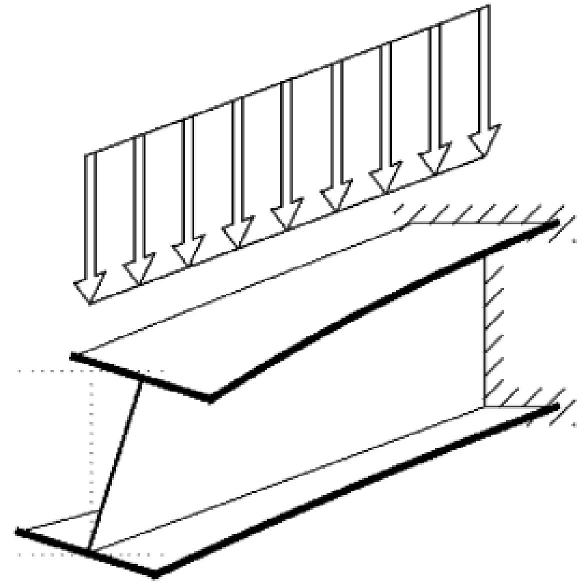
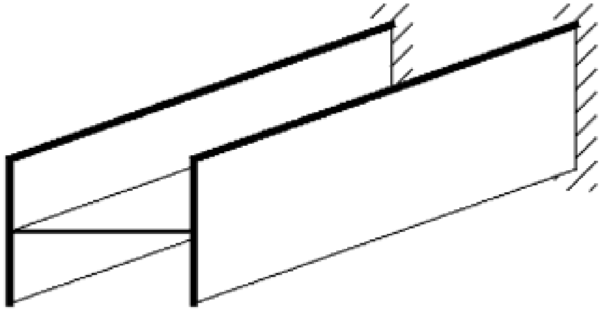


Photo: Author

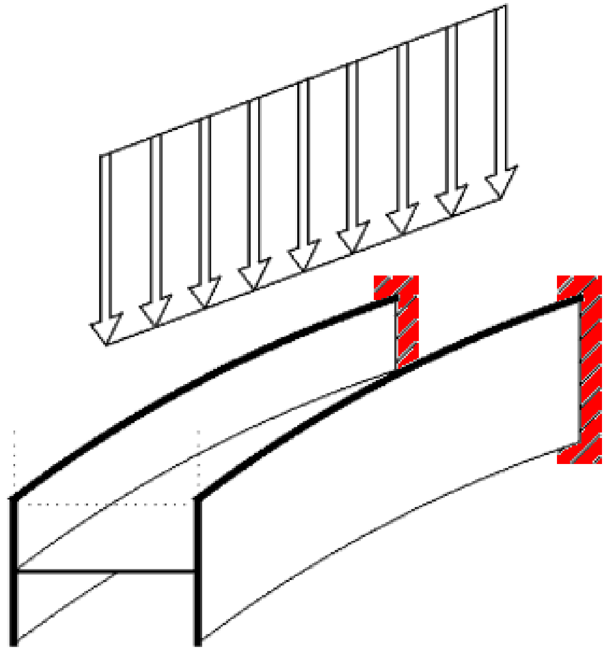
$$q_3 = q_2 + \Delta q$$

The same experiment for bending about weak axis.



$q_0 = 0$

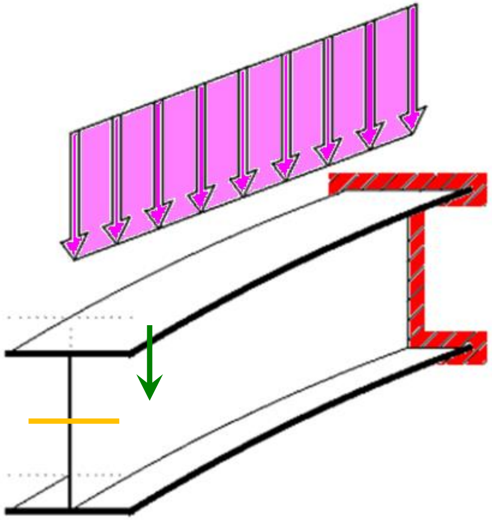
Photo: Author



$q_i \neq 0$

But for this situation only one way of destruction is possible – by destruction of support.

Deformation increases in the same direction as **load**. Bending about **strong axis**, which requires a high effort



Buckling occurs perpendicular to **load**, about **weak axis**, which may require a **lower effort** than before.

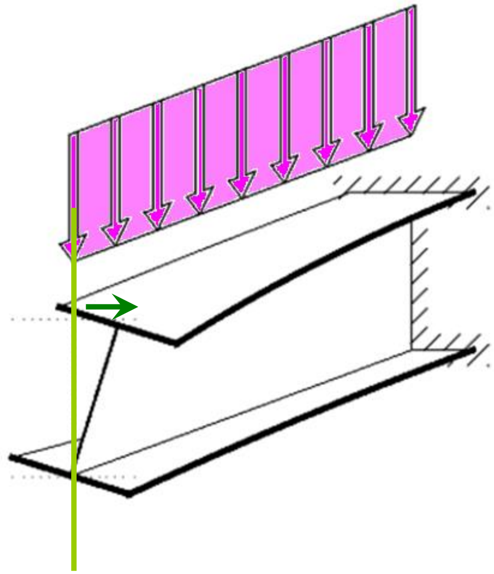
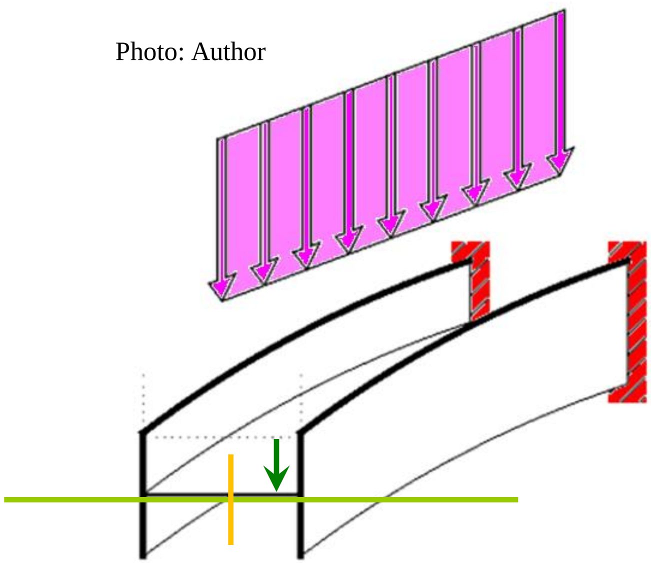
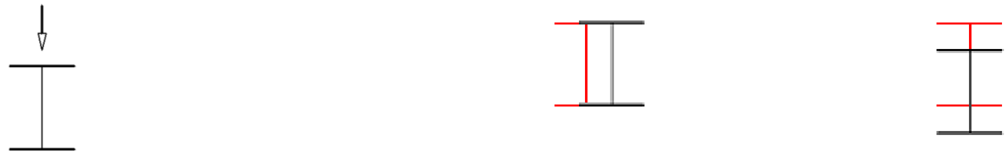


Photo: Author

Beam „didn't want to work" against big stiffness in case, when it can work against small stiffness.



Deformation increases in the same direction as **load**. Bending about **weak axis**; for sure effort is much more smaller, than bending about **strong axis**. So, bending about **strong axis** is, in this case, impossibe.

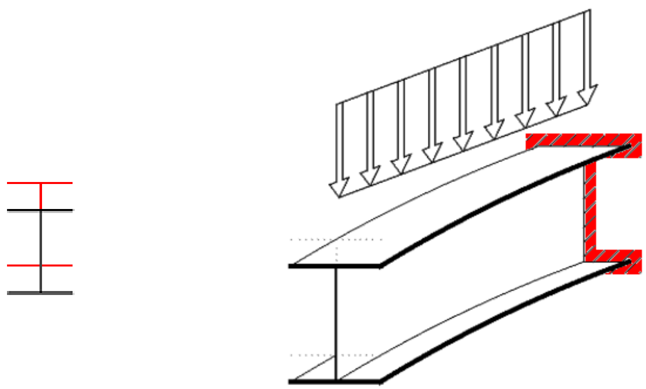


In first case, for big value of loads, both situations:

- parallel to load (deflection)
- perpendicular to load (lateral buckling)

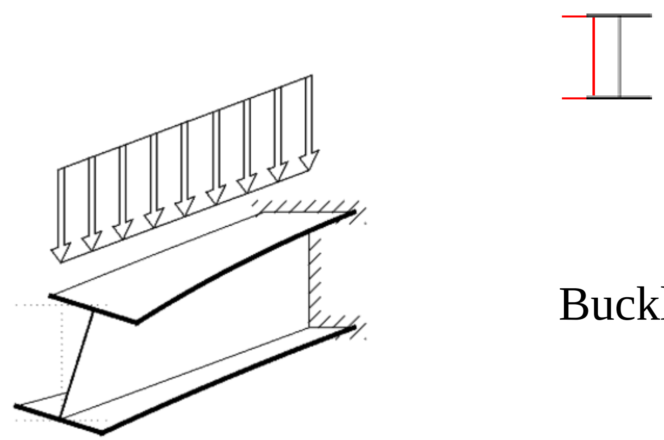
could have similar probability.

Final mode of destruction - buckling or destruction of support - depends on geometrical characteristics of beam, length of beam and type of supports.



Deflection and breaking

Photo: Author



Buckling

In second case, effort for sure is smaller for deformation paralell to load. Only deflection and breaking is possible.

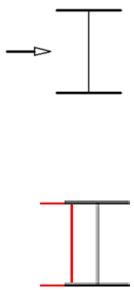
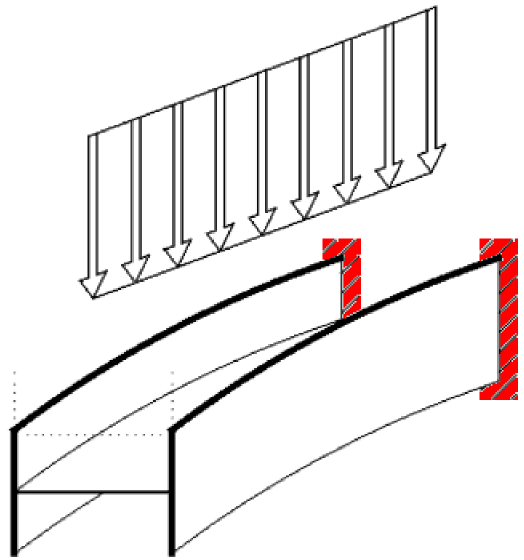
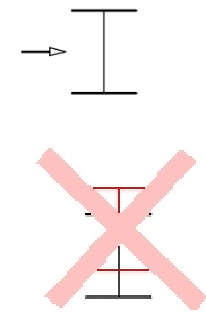


Photo: Author



Deflection and breaking



Impossible situation

Conclusions:

- lateral buckling must be analysed **only for bending about strong axis** (structure "has choice" between working in direction of load with high stiffness - or perpendicular to direction of load with lower stiffness);
- there is **no** need to analysis lateral buckling for bending about **weak axis** ("no option to choose" lower stiffness);
- for the same reason, flexural-torsional buckling is **always** interaction between torsional buckling and flexural buckling about weak axis, **not** torsional buckling and flexural buckling about strong axis;
- there is **no** lateral buckling and **no** torsional-flexural buckling for cross-section with $J_y = J_z$ ("no option to choose" lower stiffness);
- there is no completely clear, if rectangular hollow section are susceptible to lateral buckling (EN 1993-1-1 6.3.2.1 (2), EN 1993-1-1 tab. B1 – susceptible; many position of literature – not susceptibe).

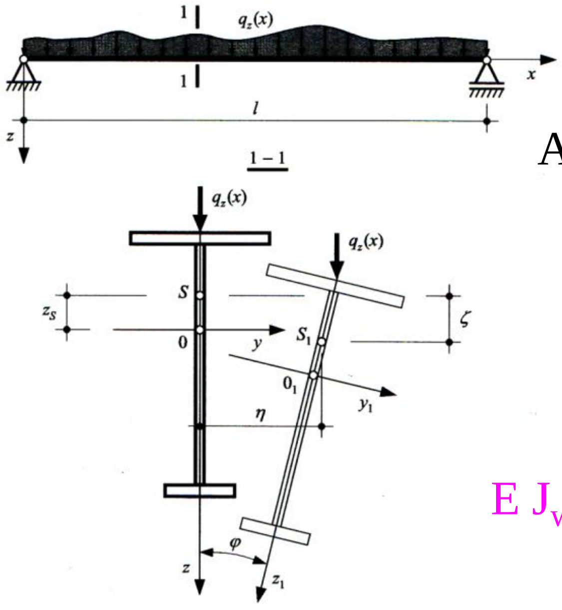


Photo: K. Rykaluk, Zagadnienia stateczności konstrukcji metalowych, DWE 2012

Analysis of lateral buckling in general case is very complicated. Phenomenon is described by three derivative formulas:

$$E\,J_z\,\eta'''' + (M_y\,\varphi)'' = 0 \qquad [1]$$

$$E\,J_w\,\varphi'''' - [(2\,\beta_z\,M_y + G\,J_T)\,\varphi']' + q_z\,(e_z - z_s)\,\varphi + M_y\,\eta'' = 0 \qquad [2]$$

$$\beta_z = \{ \int_A [\,z\,(y^2 + z^2)\,dA] - z_s \} / (2\,J_y) \qquad [3]$$

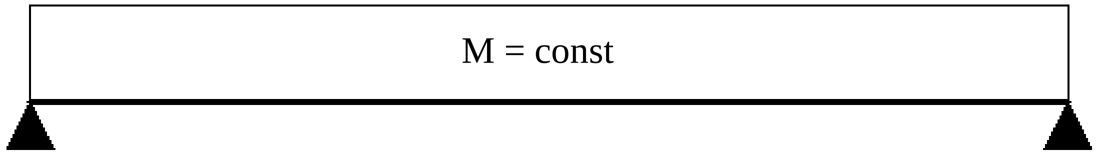
Solution of general problem is possible only by numerical way.

Analytical solution („by paper and pencil”) is possible only for long compex of simplifications:

$E\,J_z = const$; $E\,J_w = const$; $G\,J_T = const$; $M_y = const$; $q_z = 0$; $z_s = 0$; …

The most often applicated formulas are elaborated for six assumptions (similarly to formula for flexural buckling):

Photo: Author



1. M = const, 2. bi-symmetrical cross-section,

3. EJ = const, 4. two hinges, 5. load applied to shear center , 6. straight axis of bar

$$N_{cr,\,z} = \pi^2\, EJ_z \,/\, (\mu_z\, l_{0z})^2$$

$$N_{cr,\,T} = [\pi^2\, EJ_w \,/\, (\mu_T\, l_{0T})^2 \;+\; GJ_t] \,/\, i_s^2$$

$$M_{cr} = i_s \sqrt{(N_{cr,\,z}\, N_{cr,\,T})}$$

For M = const, M_{cr} is the lowest, which means the highest probability of loss of stability. For each realistic bending moment system, critical resistance to loss of stability is higher.

The same as in case of flexural buckling, we can ask whas happend, if assumprions are not satisfied:

1. M = const (but what if not? → #t / 63-75)

2. Bi-symmetrical cross-section (but what if not? → #t / 64-75)

3. EJ = const (but what if not? → #t / 79)

4. Two hinges (but what if not? → #t / 80)

5. Load applied to shear center (but what if not? → #t / 76-78)

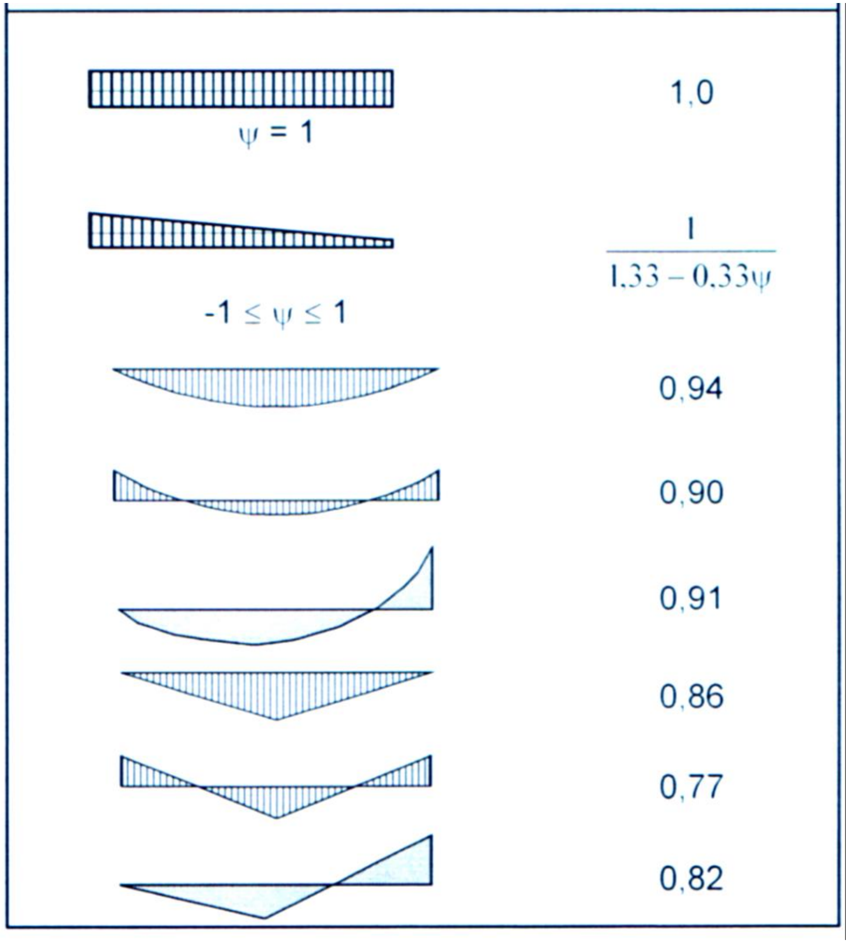
6. Bar has straight axis (but what if not? → #t / 79)

1. M ≠ const → other shape of bending moment k_c:

$\chi_{LT,\,mod} = \chi_{LT} / f$

$f = \min \{ 1 - 0{,}5(1-k_c)[1 - 2(\lambda_{LT} - 0{,}8)^2];\quad 1{,}0\}$

Photo: EN 1993-1-1 tab 6.6



This question is not completely clear in Eurocode. According to EN 1993-1-1 6.3.2.2 (2), critical moment M_{cr} depends on, amount others, shape of bending moment. But there is no other information about this influence, except recalculation as above (recalculation for χ_{LT} no M_{cr}). **Maybe** this means, that we **additionally** should use other formula for M_{cr} than formula on #t / 61.

1. $M \neq const$ (alternative way)

or

2. Not bi-symmetrical cross-section

or

5. Load applied out of shear center:

Generally, bi-symmetrical cross-section is the most often case for steel structures.

If not, we use other formula for M_{cr}

For example: old Polish Standard PN B 03200, appendix 3

3. WZORY I TABLICE DO OBLICZANIA OBCIĄŻENIA KRYTYCZNEGO PRZY NIESTATECZNOŚCI GIĘTNO-SKRĘTNEJ

3.1. Cechy geometryczne przekroju (tabl. Z1-1).

$J_z \quad J_T \quad J_W$

I_y - moment bezwładności względem osi Y,

I_T - moment bezwładności przy skręcaniu,

$x \rightarrow z \; ; \; y \rightarrow x \; ; \; z \rightarrow x$

$I\omega$ - wycinkowy moment bezwładności,

Shear center (#t / 37-39, point S)

y_s - współrzędna środka ścinania ($y_s \geq 0$),

a_0 - współrzędna punktu przyłożenia obciążenia względem środka ciężkości,

Point of load application

a_s - różnica współrzędnych środka ścinania i punktu przyłożenia obciążenia:

Distance: shear center – load point

$$a_s = y_s - a_0 ,$$

r_x - ramię asymetrii ($r_x \leq 0$);

$$r_x = \frac{1}{I_x} \int_A y\left(x^2 + y^2\right) dA$$

Arm of cross-section's assymetry

Bending parameter

b_y - parametr zginania ($b_y \geq 0$):

$$b_y = y_s - \frac{1}{2} r_x$$

Photo: PN B-3200

Table Z1-1 → extended version of this table is presented #t / 37 - 39.

Values of r_x and y_s are presented in this table.

2. For not bi-symmetrical cross-section: $a_s \neq 0 \; ; \; r_x \neq 0 \; ; \; b_y \neq 0$

3.3. Momenty krytyczne przy zwichrzeniu można obliczać wg poniższych wzorów, przyjmując znak (-), gdy środek ścinania znajduje się w strefie rozciąganej przekroju lub znak (+), w pozostałych przypadkach, przy czym w przypadku przekrojów bisymetrycznych zwrot osi *Y* należy przyjmować przeciwnie do kierunku obciążenia poprzecznego, a przy jego braku - w stronę pasa ściskanego.

a) belka jednoprzęsłowa podparta widełkowo ($\mu_x = \mu_y = \mu_\omega = 1$) i zginania stałym momentem

$$M_{cr} = \pm b_y N_y + \sqrt{\left(b_y N_y\right)^2 + i_s^2 \, N_y N_z} \quad \text{(Z1-7)}$$

One-span beam, M = const, hinge on both ends

b) belka jak w pozycji a) o przekroju bisymetrycznym ($b_y = 0$)

$$M_{cr} = i_s \sqrt{N_y N_z} \quad \text{(Z1-8)}$$

As previous, and, additionally, bi-symmetrical cross-section

c) belka jednoprzęsłowa - rozwiązanie ogólne

$$M_{cr} = \pm A_0 N_y + \sqrt{\left(A_0 N_y\right)^2 + B^2 \, i_s^2 \, N_y N_z} \quad \text{(Z1-9)}$$

General formula for one-span beam

gdzie: $A_0 = A_1 b_y + A_2 a_s$; A_1, A_2, B - wg tabl. Z1-2;

Photo: PN B-3200

Symbols: $N_y \rightarrow N_{cr,\,z}$; $N_z \rightarrow N_{cr,\,T}$

Cantilever

w przypadku belki wspornikowej o przekroju bisymetrycznym przyjmuje się $\mu_y = \mu_\omega = 2$, $A_1 = 0$, a ponadto:

- przy zginaniu stałym momentem;

  $A_2 = 0$; $B = 1$,

M = const

- przy obciążeniu równomiernie rozłożonym:

  $A_2 = 3{,}40$; $B = 4{,}10$,

q = const

- przy sile skupionej na końcu wspornika:

  $A_2 = 1{,}10$; $B = 2{,}56$;

Force in point at the end of cantilever

One-span I-beam with lateral plate-type bracings, which can change axis of rotation (case of cooperation with rigid roofing)

d) belka jednoprzęsłowa o przekroju dwuteowym usztywniona bocznym stężeniem podłużnym, które wymusza położenie osi obrotu

$$M_{cr} = \frac{i_s^2 N_z + c_y^2 N_y}{C_1\left(c_y - b_y\right) + C_2\left(c_y - a_s\right)} \qquad \text{(Z1-10)}$$

Photo: PN B-3200

gdzie:

c_y - różnica współrzędnych środka ścinania i punktu przecięcia śladu płaszczyzny stężenia z osią środnika; $c_y = y_s - y_c$;

C_1, C_2 - wg tabl. Z1-2;

N_z, N_y - siły krytyczne obliczone jak dla pręta bez stężenia.

Distance: shear center – bracing point

P - hinge

U - rigid support

$x \to z$; $y \to x$; $z \to x$

Tablica Z1-2

M linear or constant

q = const

Force applied on the half of span

Obciążenie belki (w płaszczyźnie symetrii przekroju *YZ*)	Warunki podparcia[1)] w płaszczyźnie *YZ*	*XZ*	μ_y	μ_ω	Współczynniki A_1	A_2	B	C_1	C_2
Moment stały (β = 1) lub zmienny liniowo[2)]	P	P	1	1	1/β	0	1/β	2	0
	P	P	1	0,5	1,33/β	0	1,15/β	-	-
	P	U	0,5	0,5	1/β	0	1/β	2	0
Obciążenie równomiernie rozłożone	P	P	1	1	0,61	0,53	1,14	0,93	0,81
	P	P	1	0,5	1,23	0,52	1,31	-	-
	P	U	0,5	0,5	0,68	0,29	0,97	1,43	0,61
	U	U	0,5	0,5	0,27	1,61	1,88	0,15	0,91
Siła skupiona w środku rozpiętości	P	P	1	1	0,55	0,76	1,37	0,60	0,81
	P	P	1	0,5	1,07	0,87	1,46	-	-
	P	U	0,5	0,5	0,62	0,50	1,12	1	0,81
	U	U	0,5	0,5	0	1,23	1,23	0	1,62

[1)] P - podparcie obustronnie przegubowe (swobodne); U - obustronne utwierdzenie;
μ_y, μ_ω - współczynniki długości wyboczeniowej w płaszczyźnie *XY* i przy skręcaniu.
[2)] Współczynnik β należy przyjmować wg tabl. 12 - poz. a).

Photo: PN B-3200

Photo: Author

P - hinge

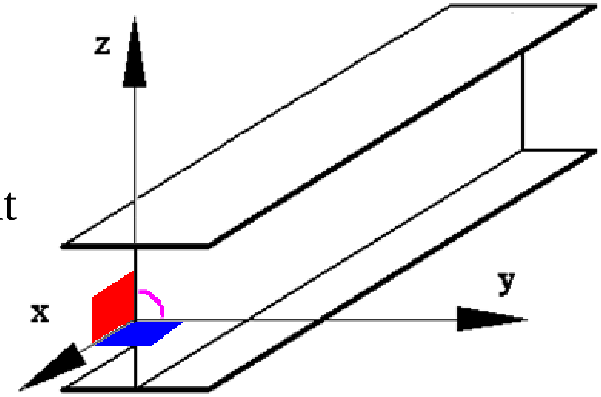
U - rigid support

x → y ; y → z ; z → x

Warunki podparcia[1)]			
w płaszczyźnie		μ_y	μ_ω
YZ	XZ		
P	P	1	1
P	P	1	0,5
P	U	0,5	0,5
P	P	1	1
P	P	1	0,5
P	U	0,5	0,5
U	U	0,5	0,5
P	P	1	1
P	P	1	0,5
P	U	0,5	0,5
U	U	0,5	0,5

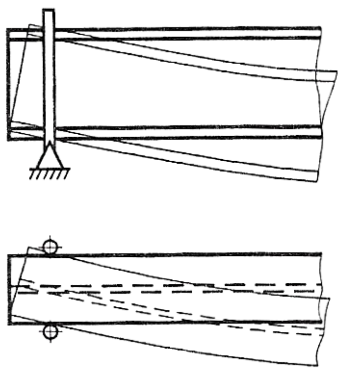
Photo: PN B-3200

Support conditions are the same at both ends of beam



Support		Critical length factor	
about y	about z	about z	rotation
P = Hinge	P = Hinge	1,0	1,0
P = Hinge	P = Hinge	1,0	0,5
P = Hinge	U = Rigid	0,5	0,5
U = Rigid	U = Rigid	0,5	0,5

Support		Critical length factor		Situation	Comments
about y	about z	about z	rotation		
P = Hinge	P = Hinge	1,0	1,0	A	Hinge support for beams
P = Hinge	P = Hinge	1,0	0,5	B	Hinge support for columns
P = Hinge	U = Rigid	0,5	0,5	C	Technically difficult to perform
U = Rigid	U = Rigid	0,5	0,5	D	Rigid joints / supports



A

B

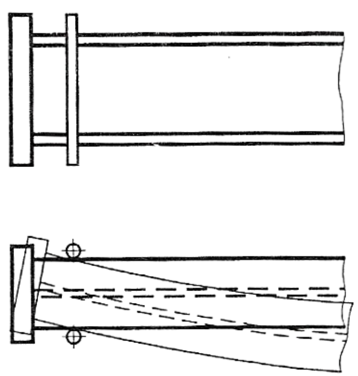
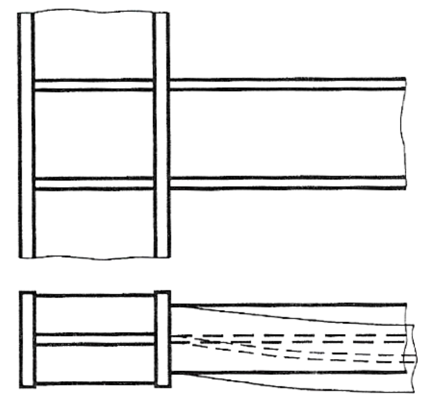


Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001

D



Other proposal: Access Steel

Support at both ends:

- no rotation of beam;
- no lateral deflection of beam;

Photo: eurocodes.jrc.ec.europa.eu

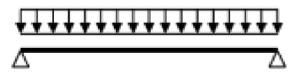
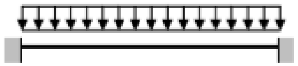
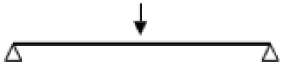
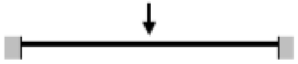
$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + (C_2 z_g)^2} - C_2 z_g \right\}$$

$$k = \mu_z$$

$$k_w = \mu_T$$

Various suport condition

Table 3.2 Values of factors C_1 and C_2 for cases with transverse loading (for k = 1)

Loading and support conditions	Bending moment diagram	C_1	C_2
		1,127	0,454
		2,578	1,554
		1,348	0,630
		1,683	1,645

Note : the critical moment M_{cr} is calculated for the section with the maximal moment along the member

Other proposal: literature (many-many-many various proposals), example:

$$M_{cr} = M_{cr,\,0} / m$$

$$M_{cr,\,0} = i_s \sqrt{(N_{cr,\,z} N_{cr,\,T})}$$

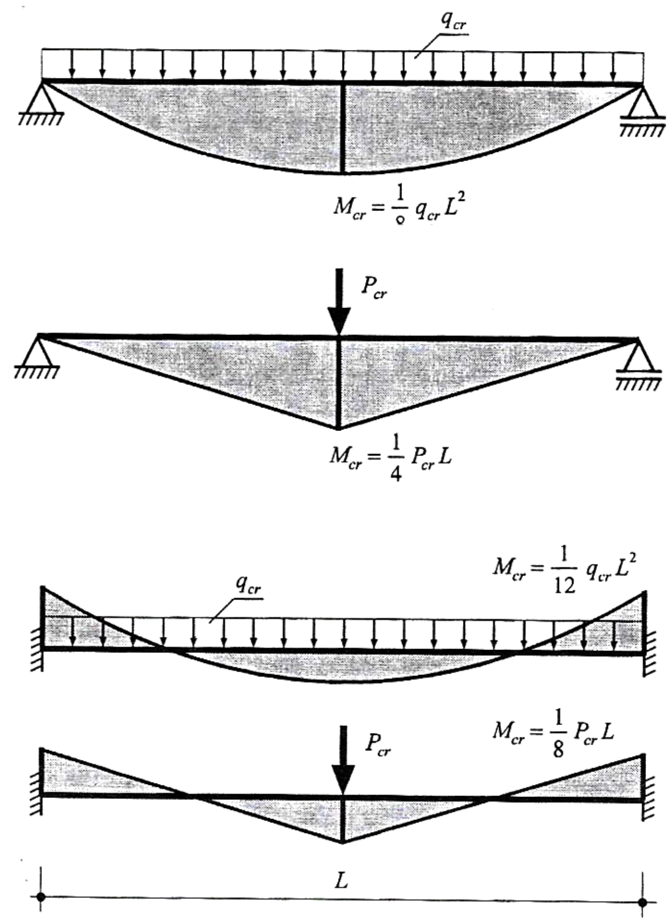
Various suport condition

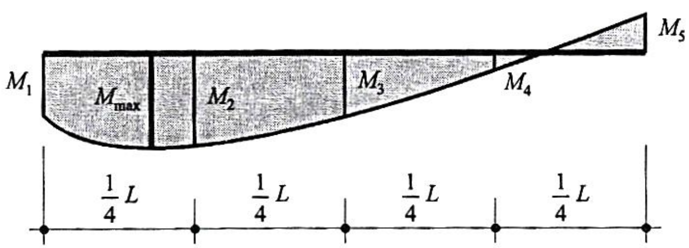
m = 0,88

m = 0,74

m = 0,39

m = 0,59



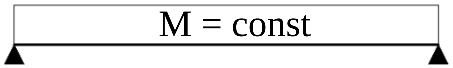


$$m = (3M_2 + 4M_3 + 3M_4 + 2M_{max}) / 12M_{max}$$

Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001

Summation

Photo: Author



$$M_{cr,\,0} = i_s \sqrt{(N_{cr,\,z}\, N_{cr,\,T})}$$

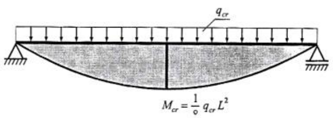
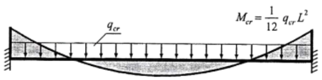
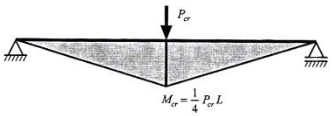
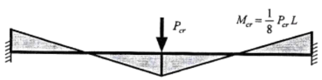
Bending moment	M_{cr}		
	Old Polish Standard	Access Steeel	Literature
	$1{,}14\ M_{cr,\,0}$	**$\underline{1{,}13\ M_{cr,\,0}}$**	$1{,}14\ M_{cr,\,0}$
	**$\underline{1{,}88\ M_{cr,\,0}}$**	$2{,}58\ M_{cr,\,0}$	$2{,}56\ M_{cr,\,0}$
	$1{,}37\ M_{cr,\,0}$	**$\underline{1{,}35\ M_{cr,\,0}}$**	**$\underline{1{,}35\ M_{cr,\,0}}$**
	**$\underline{1{,}23\ M_{cr,\,0}}$**	$1{,}63\ M_{cr,\,0}$	$1{,}69\ M_{cr,\,0}$

Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001

Full complex of three derivative formulas (→ #t / 60) is simplified in various way in Old Polish Standard, Access Steel and in many other sources of literatue. **The smallest evaluation** is the safest: the biggest probability of lost of stability.

General idea:

M_{cr} = function { [parameters (support conditions)] ; [critical forces (support conditions)]}

PN B 3200: A_1, A_2, B, C_1, C_2 (→ #t / 68);

Access Steel: C_1, C_2 (→ #t / 71);

Literature: m (→ #t / 72);

Rather „global shape” of supports

$$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z l_{0z})^2$$

$$N_{cr,\,T} = [\pi^2 EJ_w / (\mu_T l_{0T})^2 + GJ_t] / i_s^2$$

Critical lenghts (→ #t / 80);

Rather „local shape” of supports (i.e. secondary effects of bracings, gusset plates etc.)

Example: various technical solutions for rigid joint beam-column

Additional stiffeners or gusset plates

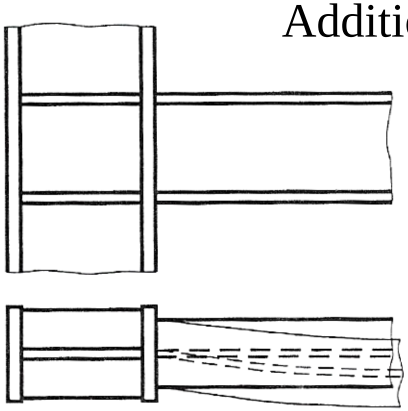


Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001

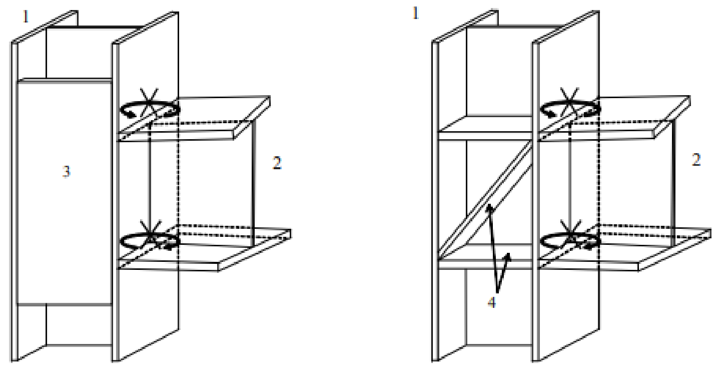


Photo: eurocodes.jrc.ec.europa.eu

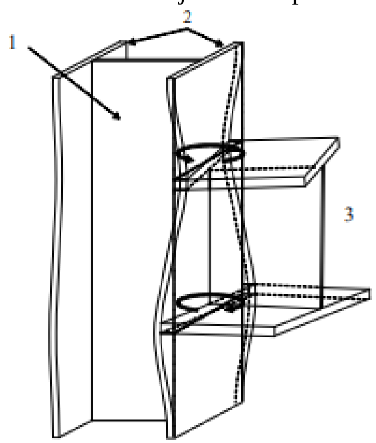
For critical forces: $\mu_z = 0{,}5$; $\mu_z = 0{,}5$

For factors, both cases:

Support		Critical length factor	
about y	about z	μ_z	μ_T
U = Rigid	U = Rigid	0,5	0,5

No additional elements

Photo: eurocodes.jrc.ec.europa.eu



For critical forces (safe evaluation):
$\mu_z = 1{,}0$; $\mu_z = 1{,}0$

5. Point of application of load

Value of M_{cr} depends on point of application of load. In formulas for general solution of lateral buckling important is distance between shear centerand point of application z_g or a_s.

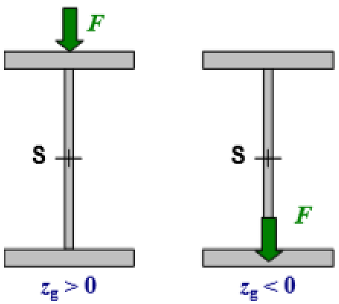


Photo: eurocodes.jrc.ec.europa.eu

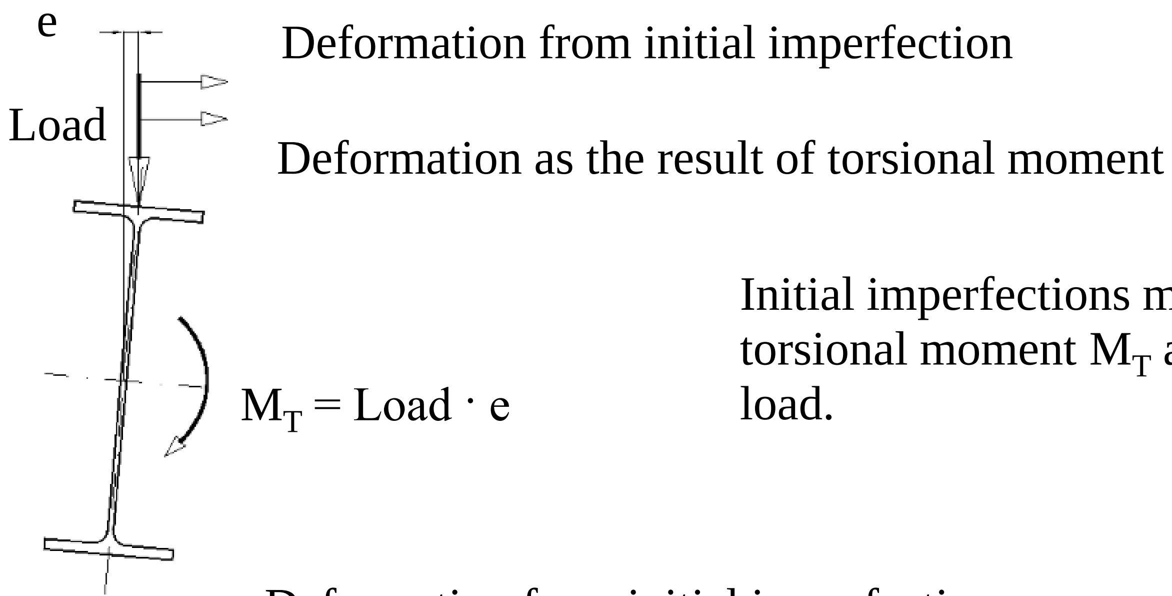
$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + (C_2 z_g)^2} - C_2 z_g \right\} \qquad \textbf{(1)}$$

$$M_{cr} = \pm A_0 N_y + \sqrt{(A_0 N_y)^2 + B^2 i_s^2 N_y N_z}$$

gdzie: $A_0 = A_1 b_y + A_2 a_s$; A_1, A_2, B - wg tabl. Z1-2;

Photo: PN B-3200

Value of M_{cr} is smaller for load on top flange, bigger fot load on bottom flange (bigger probability of instability for load on top flange).



Initial imperfections make eccentricity e and torsional moment M_T as the secondary effect of load.

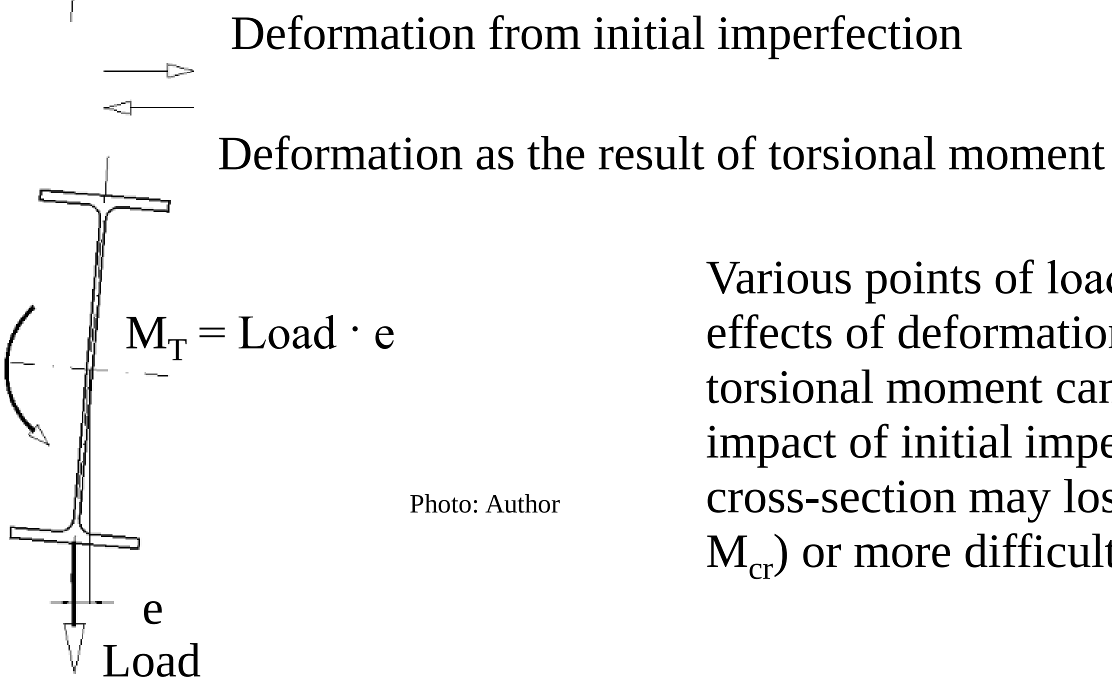


Photo: Author

Various points of load's application makes various effects of deformations. These deformations from torsional moment can intensify or weaken the impact of initial imperfections. As a result, the cross-section may lose stability more easily (smaller M_{cr}) or more difficult (larger M_{cr})

Example: M_{cr} (according formulas presented in PN B 3200 and Access steel) for I-beam IPE 300, 12 m span, simple supported at both ends, continous load along beam, no additional bracings

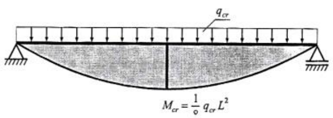
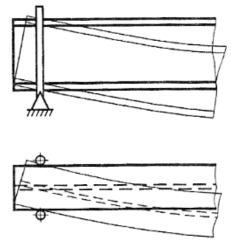


Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001



Application of load	M_{cr} [kNm]	
	PN B 3200	Access Steel
to top flange (150 mm over centre of gravity)	39,829	39,512
to centre of shear (= centre of gravity for bi-symmetrical I-beam)	46,224	45,697
to bottom flange (150 mm below centre of gravity)	53,646	52,851

Small M_{cr} → small buckling factor → small critical resistance under bending

3. The same situation as for flexural buckling.

EJ = const is the most often case. Differences are negligible, if $\alpha \leq 10^o$; for calculations $EJ = \min(EJ_1 ; EJ_2)$. There are different rules for $\alpha > 10^o$; these rules will be presented on lecture #12

6. The same situation as for flexural buckling.

Curve axis of bar → influence of imperfections → various buckling curves → #t / 82

4. Other types of support → μ_T

Generally, rules are the same as for flexural buckling. At now we must analysed not only translation but additionally change of rotation of cross-section and types of support for rotation.

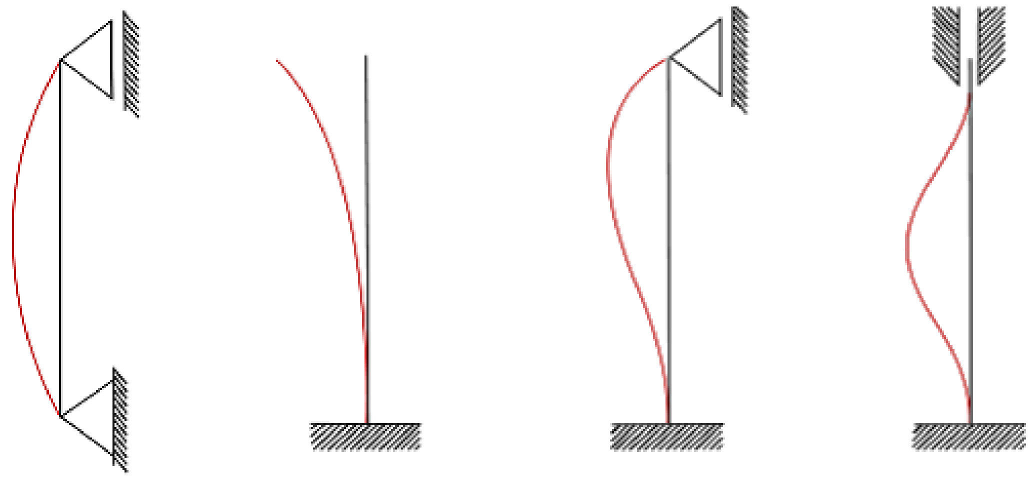


Photo: wikipedia

μ	1,0	2,0	0,7	0,5
l_{cr}	1,0 l_0	2,0 l_0	0,7 l_0	0,5 l_0

Algorithm

EN 1993-1-1 6.3.2

$\lambda_{LT} = \sqrt{(W_y f_y / M_{cr})}$	EJ = const	$\Phi_{LT} = [1 + \alpha_{LT} (\lambda_{LT} -0,2) + \lambda_{LT}^2] / 2$ $\alpha_{LT} \rightarrow$ tab. 6.3, 6.4, EN 1993-1-1	$\chi_{LT} = \min\{$ $1/[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}]$; $1,0\}$
	Hot-rolled and welded I-beam	$\Phi_{LT} =$ $= [1 + \alpha_{LT} (\lambda_{LT} -0,4) + 0,75\, \lambda_{LT}^2] / 2$ $\alpha_{LT} \rightarrow$ tab. 6.4, 6.5, EN 1993-1-1	$\chi_{LT} = \min\{$ $1/[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}]$; $1/ \lambda_{LT}^2$; $1,0\}$

$\lambda_{LT} \leq 0,4 \rightarrow \chi_{LT} = 1,0$

For lateral buckling, various buckling curves the same are taken into consideration

General case (EJ = const):

Cross-section	Limits	Buckling curve
Rolled I-sections	h/b ≤ 2 h/b > 2	a b
Welded I-sections	h/b ≤ 2 h/b > 2	c d
Other cross-sections	-	d

Photo: EN 1993-1-1 tab. 6.4

⇧

Distinction not cleary explained

⇩

Hot-roled and welded I-sections:

Cross-section	Limits	Buckling curve
Rolled I-sections	h/b ≤ 2 h/b > 2	b c
Welded I-sections	h/b ≤ 2 h/b > 2	c d

Photo: EN 1993-1-1 tab. 6.5

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

Photo: EN 1993-1-1 tab. 6.3

Example 2

IPE 300

h_{IPE300} = 300 mm

S235 → f_y = 235 MPa

L = 6,00 m

E = 210 GPa

G = 81 GPa

J_y = 8 356 cm^4

J_z = 603,8 cm^4

W_y = 557,1 cm^3

$W_{pl,\,y}$ = 628,4 cm^3

J_w = 125 900 cm^6

J_T = 20,12 cm^4

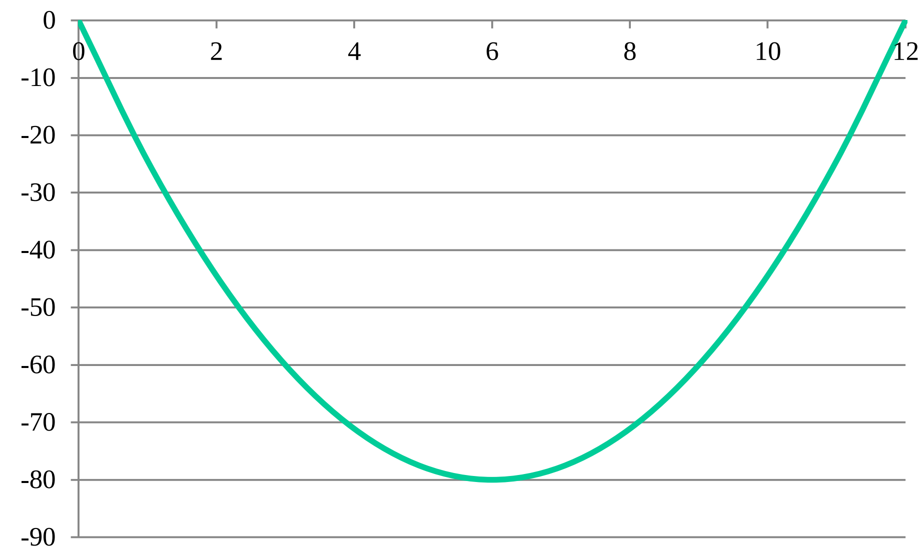
i_y = 12,46 cm

i_z = 3,35 cm

y_s = 0,0 cm

$M_{Ed,\,y}$ = 80,0 kNm

I class of cross-section



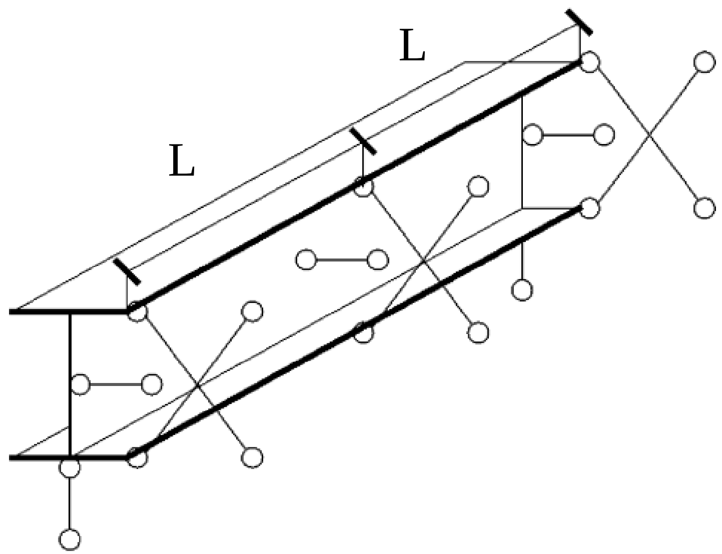


Photo: Author

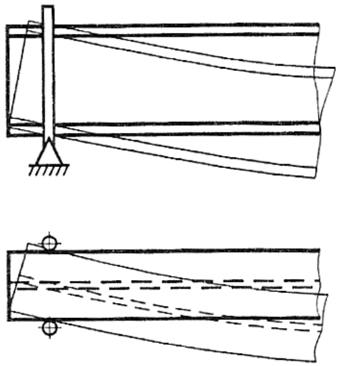
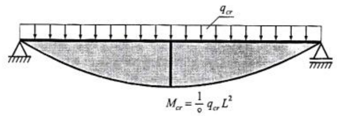
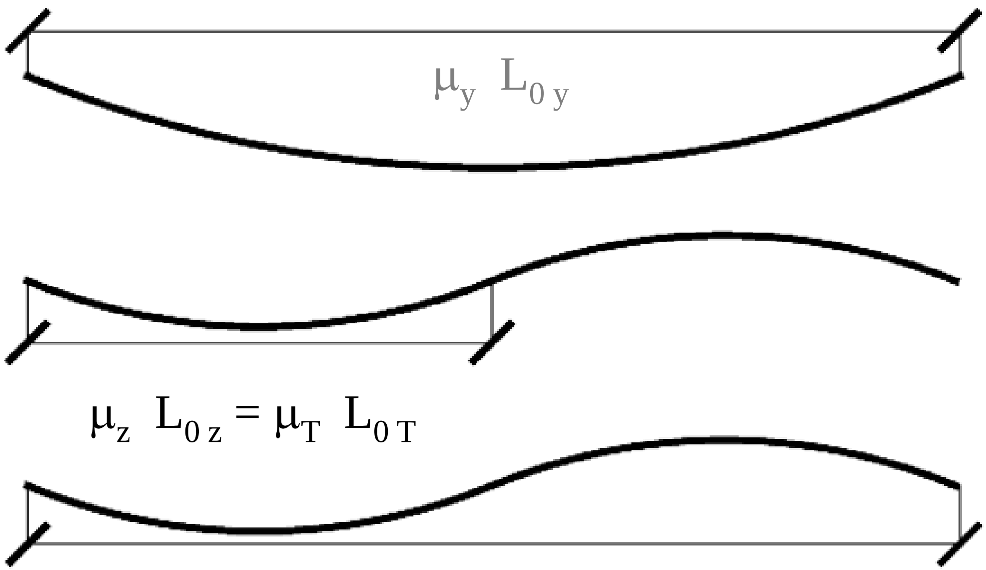


Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001



„Global shape" of suport conditions: factors as for one-span simple supported beam, L = 12,00 m



$\mu_z \; L_{0\,z} = \mu_T \; L_{0\,T}$

$L_{0\,y} = 2\,L_{0\,z} = 2\,L_{0\,T}$

Photo: Author

„Local shape" of suport conditions: critical lengths for critical forces

$L_{0\,y} = 12{,}00$ m

$L_{0\,z} = 6{,}00$ m

$L_{0\,T} = 6{,}00$ m

$\mu_y = 1{,}00$

$\mu_z = 0{,}90 – 1{,}00$ (in calculation 1,00)

$\mu_T = 0{,}90 – 1{,}00$ (in calculation 1,00)

General formula (loads in centre of gravity):

$$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z l_{0z})^2 = \underline{347{,}603 \text{ kN}} \; (L_{cr} = 6{,}00 \text{ m})$$

$$N_{cr,\,T} = [\pi^2 EJ_w / (\mu_T l_{0T})^2 + GJ_t] / i_s^2 = \underline{1\,463{,}001 \text{ kN}} \; (L_{cr} = 6{,}00 \text{ m})$$

$$i_0 = i_s = 12{,}90 \text{ cm}$$

$$M_{cr,\,0} = i_s \sqrt{(N_{cr,\,z} N_{cr,\,T})} = \underline{92{,}011 \text{ kNm}}$$

$$M_{cr} = (\rightarrow \#t / 74) = 1{,}13 \, M_{cr,\,0} = 103{,}972 \text{ kNm}$$

Formulas with analysis of point of loads application (→ #t / 76):

Formula according to PN:

$$M_{cr} = \sqrt{[B^2 N_{cr,\,z} N_{cr,\,T} i_s^2 + (A_0 N_{cr,\,z})^2]} - A_0 N_{cr,\,z}$$

$$A_0 = A_1 b_y + A_2 z_g$$

Formula according to AS (after transformation):

$$M_{cr} = \sqrt{[C_1^2 N_{cr,\,z} N_{cr,\,T} i_s^2 + (C_1 C_2 N_{cr,\,z} z_g)^2]} - C_1 C_2 N_{cr,\,z} z_g$$

Obciążenie belki (w płaszczyźnie symetrii przekroju YZ)	Warunki podparcia[1) w płaszczyźnie YZ	XZ	μ_y	μ_ω	Współczynniki A_1	A_2	B	C_1	C_2
Obciążenie równomiernie rozłożone	P	P	1	1	0,61	0,53	1,14	0,93	0,81
	P	P	1	0,5	1,23	0,52	1,31	-	-
	P	U	0,5	0,5	0,68	0,29	0,97	1,43	0,61
	U	U	0,5	0,5	0,27	1,61	1,88	0,15	0,91

Photo: PN B-3200

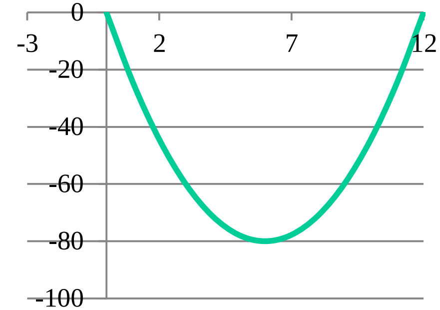


Photo: Author

Photo: eurocodes.jrc.ec.europa.eu

$A_0 = A_1\, b_y + A_2\, z_g$

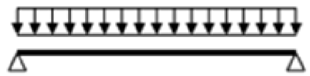
$b_y = y_s - r_x / 2$

y_s = shear center relative to center of gravity; for bi-symmetrical cross-section = 0

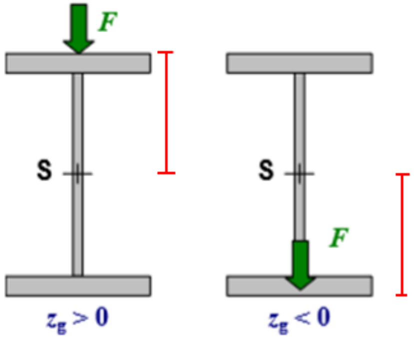
r_x = arm of assymetry (→ #t / 37-39); for bi-symmetrical I-beam = 0

$A_0 = A_2\, z_g$

Photo: eurocodes.jrc.ec.europa.eu

Loading and support conditions	Bending moment diagram	C_1	C_2
		1,127	0,454

$z_g = h / 2$



$z_g = -h / 2$

Formula according to PN:

$$M_{cr} = \sqrt{[1{,}14^2 N_{cr,z} N_{cr,T} i_s^2 + (0{,}53 N_{cr,z} z_g)^2]} - 0{,}53 N_{cr,z} z_g$$

Formula according to AS:

$$M_{cr} = \sqrt{[1{,}127^2 N_{cr,z} N_{cr,T} i_s^2 + (1{,}127 \cdot 0{,}454 N_{cr,z} z_g)^2]} - 1{,}127 \cdot 0{,}454 N_{cr,z} z_g$$

$$1{,}127 \cdot 0{,}454 = 0{,}512$$

Application of load	M_{cr} [kNm]	
	PN B 3200	**Access Steel**
to top flange (150 mm over centre of gravity; z_g = 150 mm)	**80,837**	**80,395**
to centre of shear (= centre of gravity for bi-symmetrical I-beam; z_g = 0 mm)	104,892	103,696

Conclusions

General formula (loads in centre of gravity): $M_{cr, 0} = i_s \sqrt{} (N_{cr, z} N_{cr, T})$ gives nearly the same results, as formula with analysis of point of loads application (→ #t / 76) when load is applicated in centrer of gravity.

General formula is good approximation of reality for columns: biggest past of bending moment comes from cooperated beams (so, could be taken into consideratio as effect of load applicated in graviry center), small part comes from wall girts on flange.

Formula with analysis of point of loads application is good for beams and roof girders: biggest part of load is applicated to flnage, small part (dead-weight of beam only) in center of gravity.

$$M_{cr} = \min(\text{PN} \;;\; \text{AS}) = \min(80{,}837 \;;\; 80{,}395) = \underline{80{,}395 \text{ kNm}}$$

$$W_{y,\,pl}\, f_y = 147{,}674 \text{ kNm}$$

$$\lambda_{LT} = \sqrt{(W_y\, f_y / M_{cr})} = 1{,}355$$

Two ways of analysis (→ #t / 82):

For EJ = const:	For hot-rolled and welded I-beam:
IPE 300: hot-rolled, $h/b \leq 2{,}0$	
Buckling curve a: $\alpha_{LT} = 0{,}21$	Buckling curve b: $\alpha_{LT} = 0{,}34$
$\Phi_{LT} = [1 + \alpha_{LT}(\lambda_{LT} - 0{,}2) + \lambda_{LT}^2] / 2 =$ $= 1{,}540$	$\Phi_{LT} = [1 + \alpha_{LT}(\lambda_{LT} - 0{,}4) + 0{,}75\, \lambda_{LT}^2] / 2 =$ $= 1{,}351$
$\chi_{LT} = \min\{1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] \;;\; 1{,}0\} =$ $= \min(0{,}440 \;;\; 1{,}0) = 0{,}440$	$\chi_{LT} = \min\{1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] \;;\; 1/\lambda_{LT}^2 \;;\; 1{,}0\}$ Impossible to application: $\sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)} =$ $= \sqrt{(1{,}351^2 - 1{,}355^2)} = \sqrt{(-0{,}011)}$

EN 1993-1-1 6.3.2.2.(2)

M_{cr} is based on gross cross sectional properties and takes into account the loading conditions, the real moment distribution and the lateral restraints.

So, shape of bending moment sould (?) be taken into consieration twice: in M_{cr} and in recalculation for $\chi_{LT, mod}$ → #t / 63

$$\chi_{LT, mod} = \chi_{LT} / f$$

$$f = \min \{ 1 - 0{,}5(1-k_c)[1 - 2(\lambda_{LT} - 0{,}8)^2]; \quad 1{,}0\}$$

$$k_c = 0{,}94$$

$$f = 0{,}988$$

$$\chi_{LT, mod} = 0{,}445$$

$$W_{pl,y}\, f_y = 147{,}674 \text{ kNm}$$

$$\chi_{LT,mod}\, W_{pl,y}\, f_y = 65{,}734 \text{ kNm}$$

$$M_{Ed} = 80{,}000 \text{ kNm}$$

$$M_{Ed} / (W_{pl,y}\, f_y / \gamma_{M0}) = 0{,}542$$

OK.

$$M_{Ed} / (\chi_{LT,mod}\, W_{pl,y}\, f_y / \gamma_{M0}) = 1{,}217$$

Wrong, buckling, destruction!

$L_{0\,y} = 12{,}00$ m

$L_{0\,z} = 4{,}00$ m

$L_{0\,T} = 4{,}00$ m

Proposition: other distance between supports → change of critical length for z-buckling and torisional buckling.

$N_{cr,\,z} = \underline{782{,}108\text{ kN}}$

$N_{cr,\,zT} = \underline{2\,007{,}228\text{ kN}}$

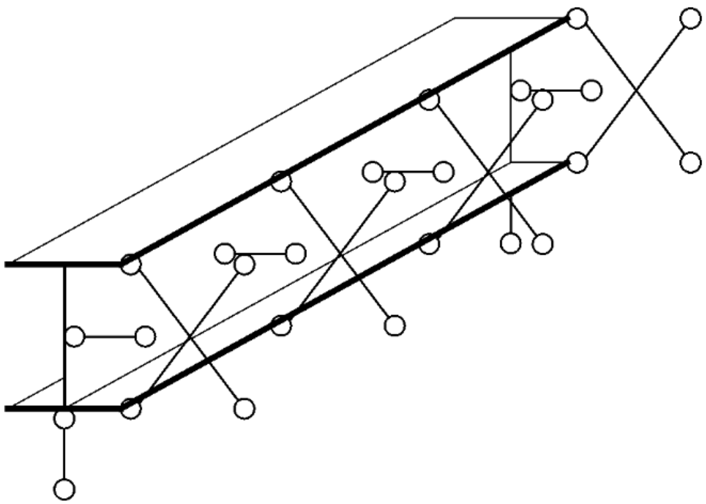


Photo: Author

$M_{cr} = \min(\text{ PN } ; \text{ AS}) = 131{,}800$ kNm

$\lambda_{LT} = 1{,}059$

$\Phi_{LT} = 1{,}150$

$\chi_{LT} = 0{,}625$

$\chi_{LT,\,mod} = 0{,}641$

$\chi_{LT,\,mod}\,W_{pl,\,y}\,f_y = 94{,}711$ kNm

$M_{Ed} / (\chi_{LT,\,mod}\,W_{pl,\,y}\,f_y / \gamma_{M0}) = 0{,}845$ OK

Photo: craneaccidents.com

Summation

Instability

Equilibrium of rigid body
LS EQU

„Classical" instability
LS STR

Shell structures (IInd step of study)

Photo: publish.ucc.ie

Bar structures

Local instability (Lab #2)

Reason: compression (stresses / forces), sometimes shear, **never tension**

Global instability (Lec #5)

Four types of local instability for slender cross-section:

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Two types effects of compressive axial stresses.

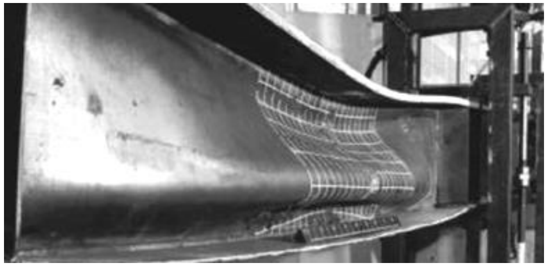
Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Effect of shear stresses.

→ Lab 2 / 6

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Effect of tranversal force applied in point.



Rys: Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

Global instability

Type:	Buckling			Distortion (EN 1993-1-3)	Lateral buckling
	Flexural	Torsional	Flexural-trosional		
Reason:	$N_{Ed, c}$				M_{Ed}
Axis of element:	curved	straight	curved	straight	curved
Rotation of cross-sections:	no	yes	yes	no	yes
Deplanation of cross-section	no	yes	yes	no	yes
Deformation of cross-sections	no	no	no	yes	no

The same reason, various answers of member

The same answers of member, various reasons

We can limit or eliminate global instability by using bracings. Different type of bracings can eliminate different types of buckling. More information will be presented on lecture #10.

We can limit or eliminate local instability by using stiffeners. More information will be presented on lecture #21.

Photo: steelconstruction.info

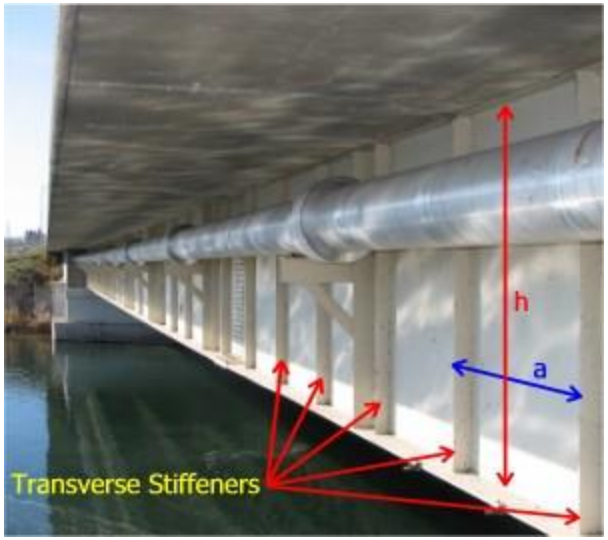


Photo: bgstructuralengineering.com

Examination issues

Difference between resistance of cross-section and stability of element

The cause of instability

Types of instability

Similiarities and differences of various modes of instability

Stability - stateczność
Buckling - wyboczenie / utrata stateczności
Flexural buckling - wyboczenie giętne
Flexural-torsional buckling - wyboczenie gietno-skrętne
Torsional buckling - wyboczenie skrętne
Lateral buckling - zwichrzenie
Local buckling - niestateczność lokalna
Shear centre - środek ścinania
Torsion constant - moment bezwładności przy skręcaniu
Warping constant - wycinkowy moment bezwładności
Stiffener - żebro
Laced members - słup wielogałęziowy skratowany
Battened members - słup wielogałęziowy z przewiązkami
Closely spaced build-up members - pręt wielogałęziowy
Bracing - stężenie

Thank you for attention

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