Metal Structures

Lecture IV

Classes of cross-section

Contents

Introduction

I-beam cross-section is the most often used cross-section. Flanges and web can be more or less slender.

There are different classes of cross-section depending on diferent slenderness, but the same A, J_y, $W_{el,\,y}$.

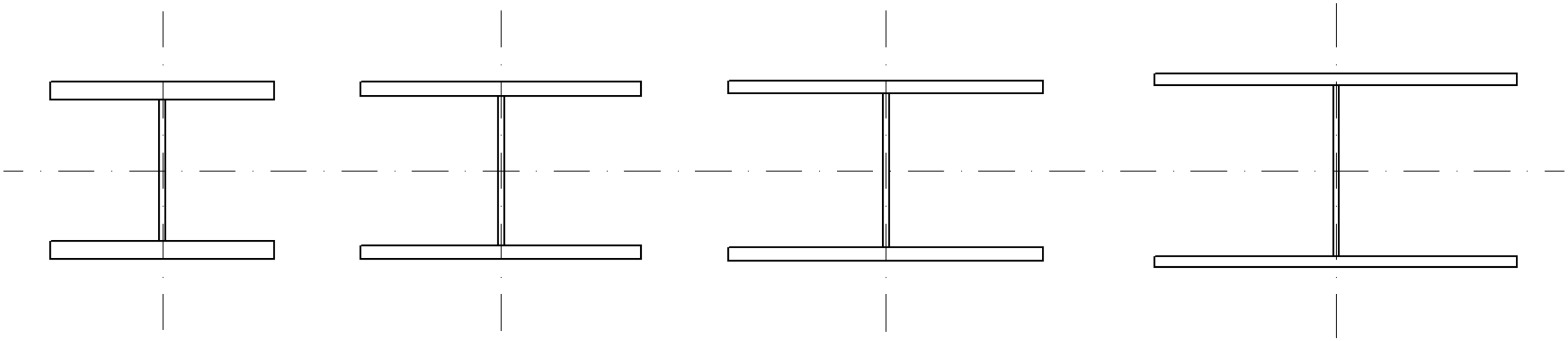


Photo: Author

Classes of cross-section - different resistance for local instabilities

Local instability occurs in compressed part only.

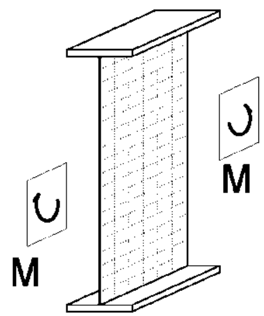
Class of cross-section is important for compressed member only.

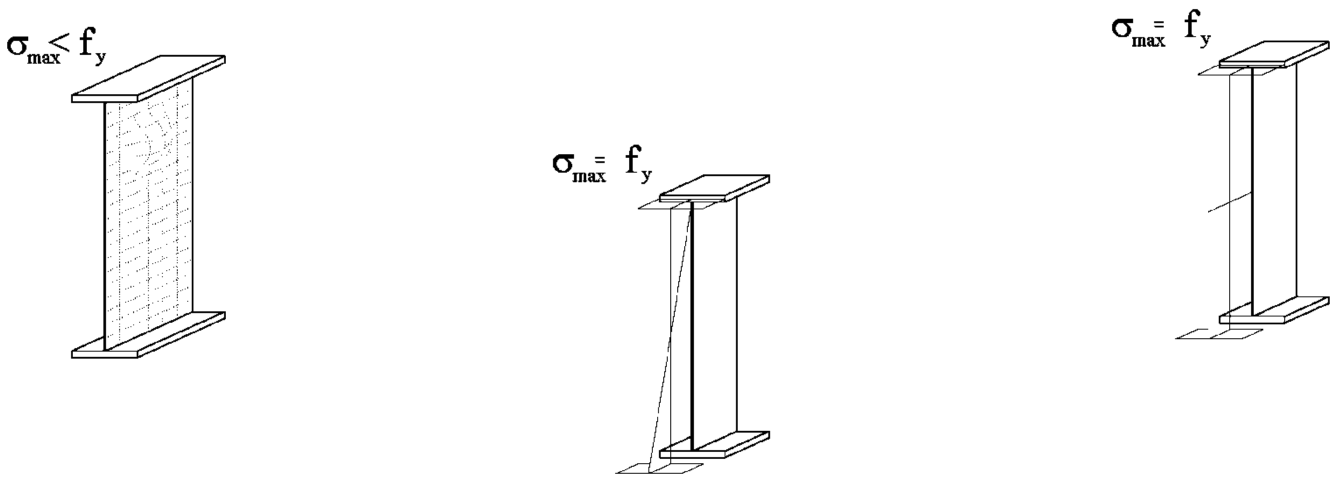
For these cases we have different formulas of resistance.

Shear or tension - not important for determine class of cross-section (the same type of formulas).

Classes of cross-section - different resistance for local instabilities

→ #3 / 85





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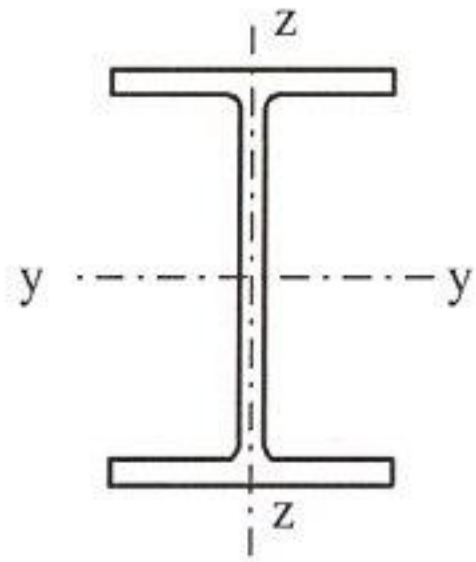
Different formulas of R

→ #3 / 75

Level of cross-sections:



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F – geometrical characteristic of cross-section

$$R = F f_y$$

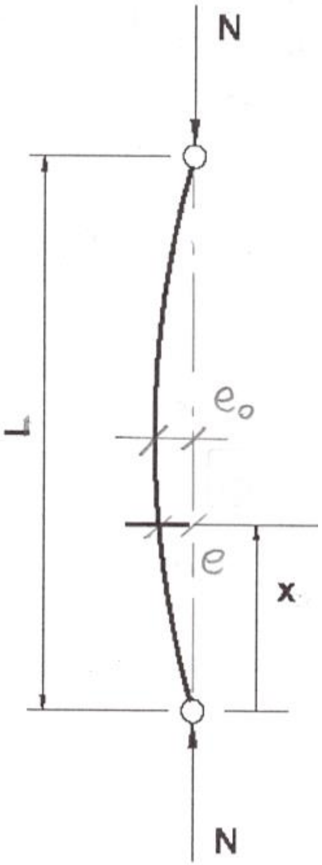
$$E / R \leq 1{,}0$$

Elements, nodes - when instability is not important; bolts, rivets, pins

(~ 40% of calculation's conditions)

Level of elements:

→ #3 / 76



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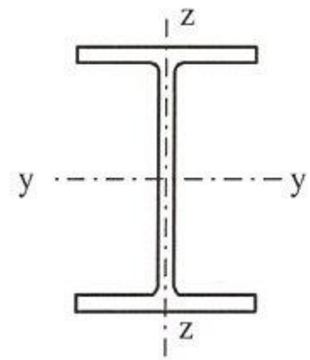
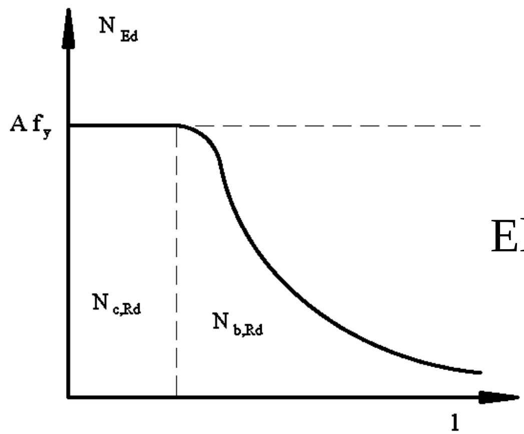




F – geometrical characteristic of cross-section
χ - instability coefficient (depends on element geometry)

$$R = \chi F f_y$$

$$E / R \leq 1{,}0$$

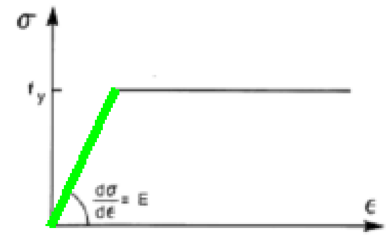
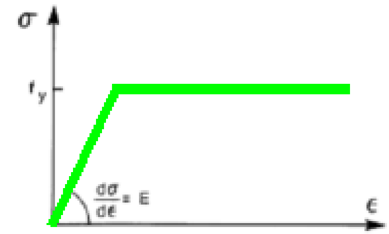
Elements, nodes - when instability is important

(~ 60% of calculation's conditions)

→ #3 / 87

Elastic and plastic analysis

Photo: Author

Analysis	Class of corss-section → #3 / 85	Stress-strain relationship EN 1993-1-1 fig. 5.3
Elastic	I, II, III, IV	
Plastic	I	

Different formulas for resistance for elastic and plastic analysis

Different classes → differnt types of formulas for calculation of resistance;

What is class? → definition of class;

Which class? → calculations of limits between classes;

Which resistance? → calculations of resistance for different classes;

• Class 1 cross-sections are those which can form a plastic hinge with the rotation capacity required for plastic analysis.

• Class 2 cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity.

EN 1993-1-1 5.2.2

Is everything clear?

– Klasa 1: przekroje, które osiągają nośność przegubu plastycznego i wykazują przy tym zdolność do obrotu niezbędną do plastycznej redystrybucji momentów.

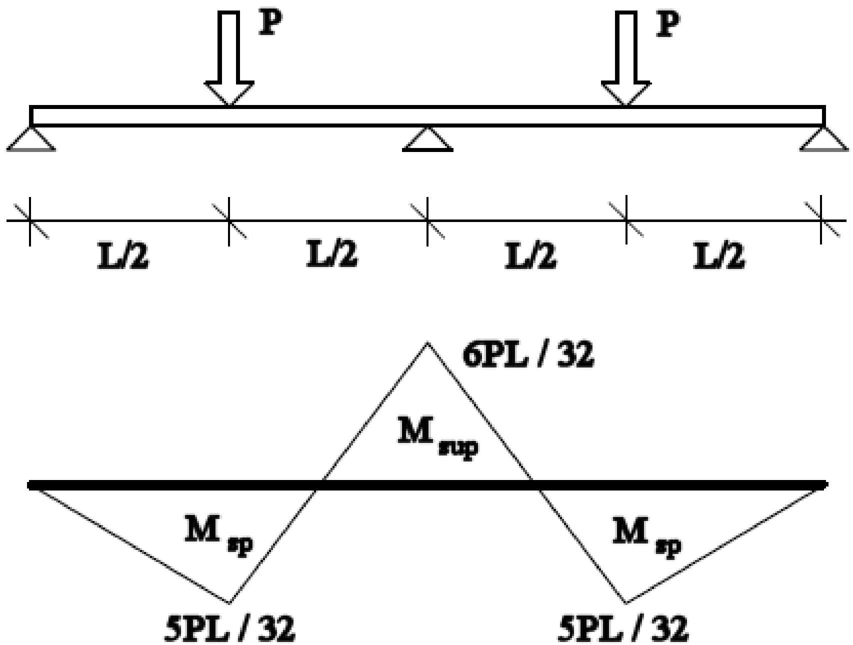
– Klasa 2: przekroje, które osiągają nośność przegubu plastycznego, lecz wskutek niestateczności miejscowej (w stanie plastycznym) wykazują ograniczoną zdolność do obrotu.

PN EN 1993-1-1 5.2.2

Is everything clear?

Experiment

Let's make an experiment: two-span continous I-beam; forces P can change its value.



$$M_{sup} = 6\ PL / 32$$

$$M_{sp} = 5\ PL / 32$$

$$M_{max} = M_{sup}$$

$$\boldsymbol{\sigma}_{max} = \boldsymbol{\sigma}(M_{sup})$$

Photo: Author

The same geometrical characteristics of cross-sections, but four various classes of cross-ection.

What will happened with beams of different cclasses?

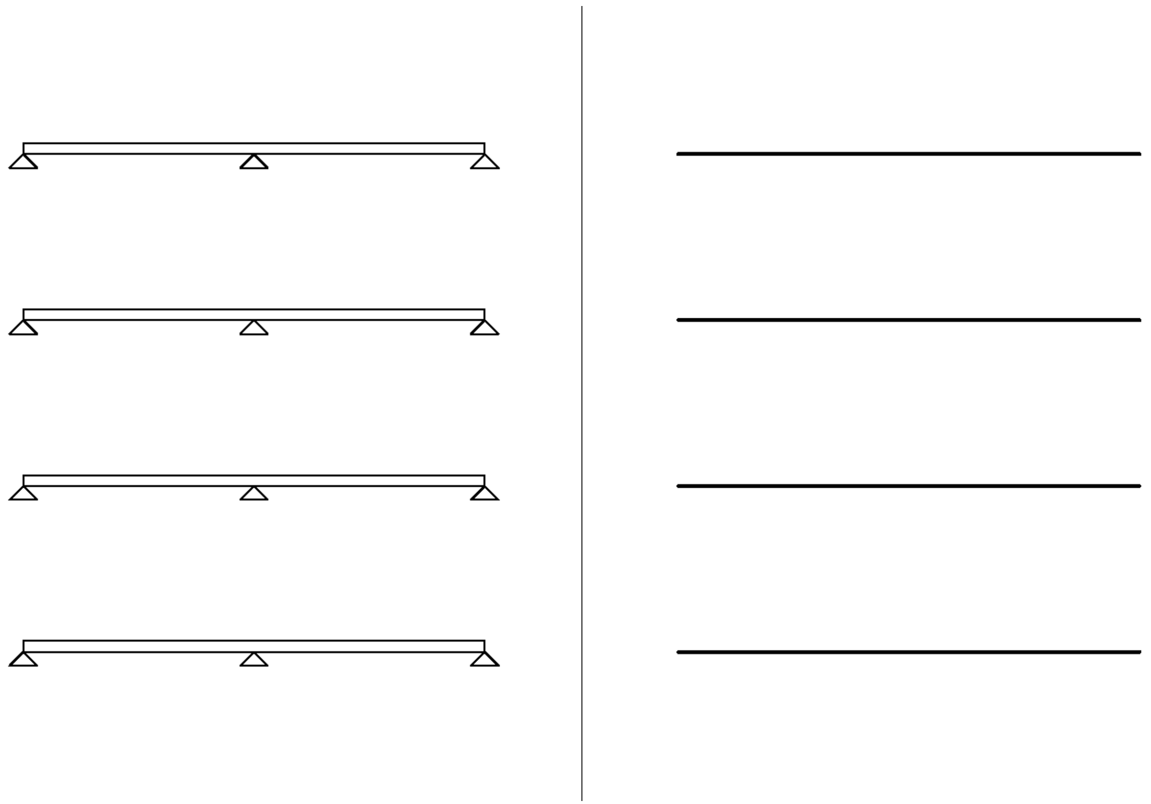
Class:

IV

III

II

I



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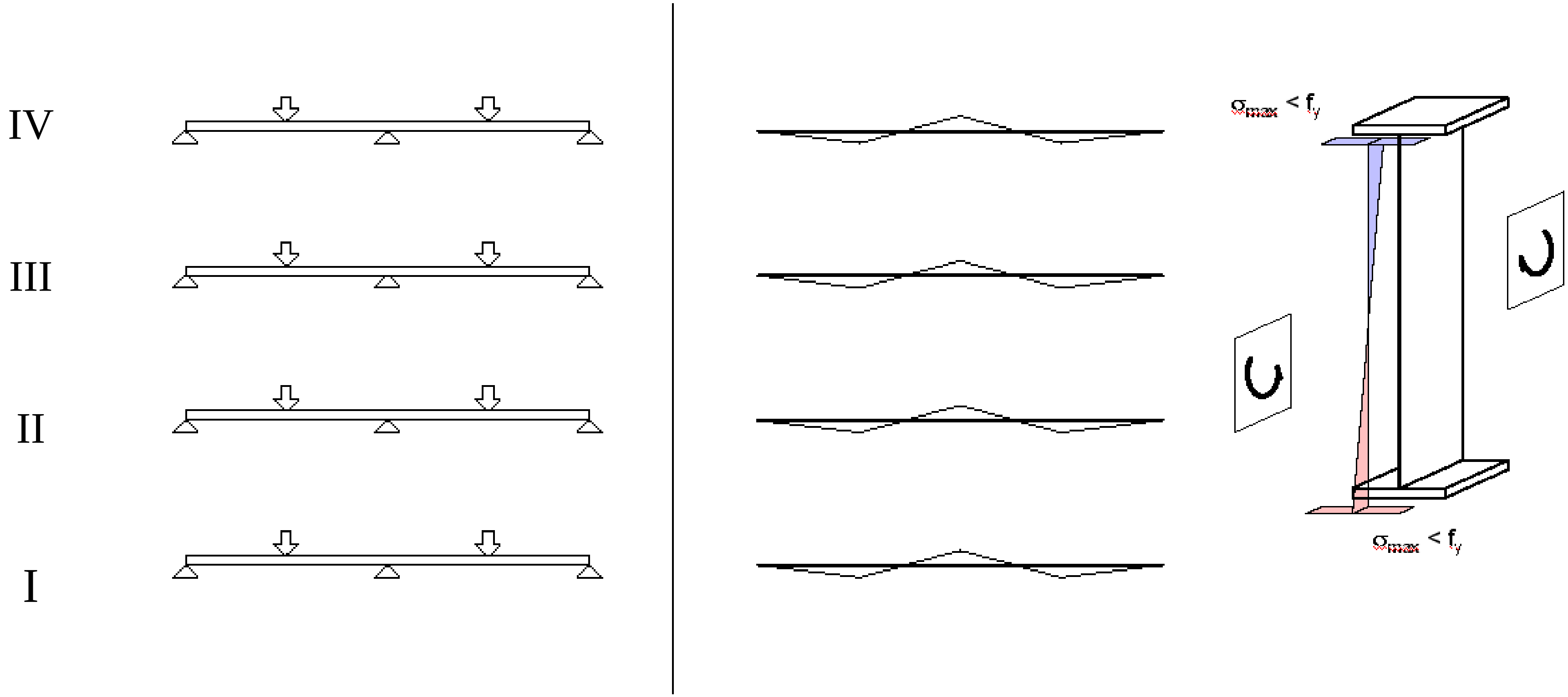
$P_0 = 0 \quad M = 0$



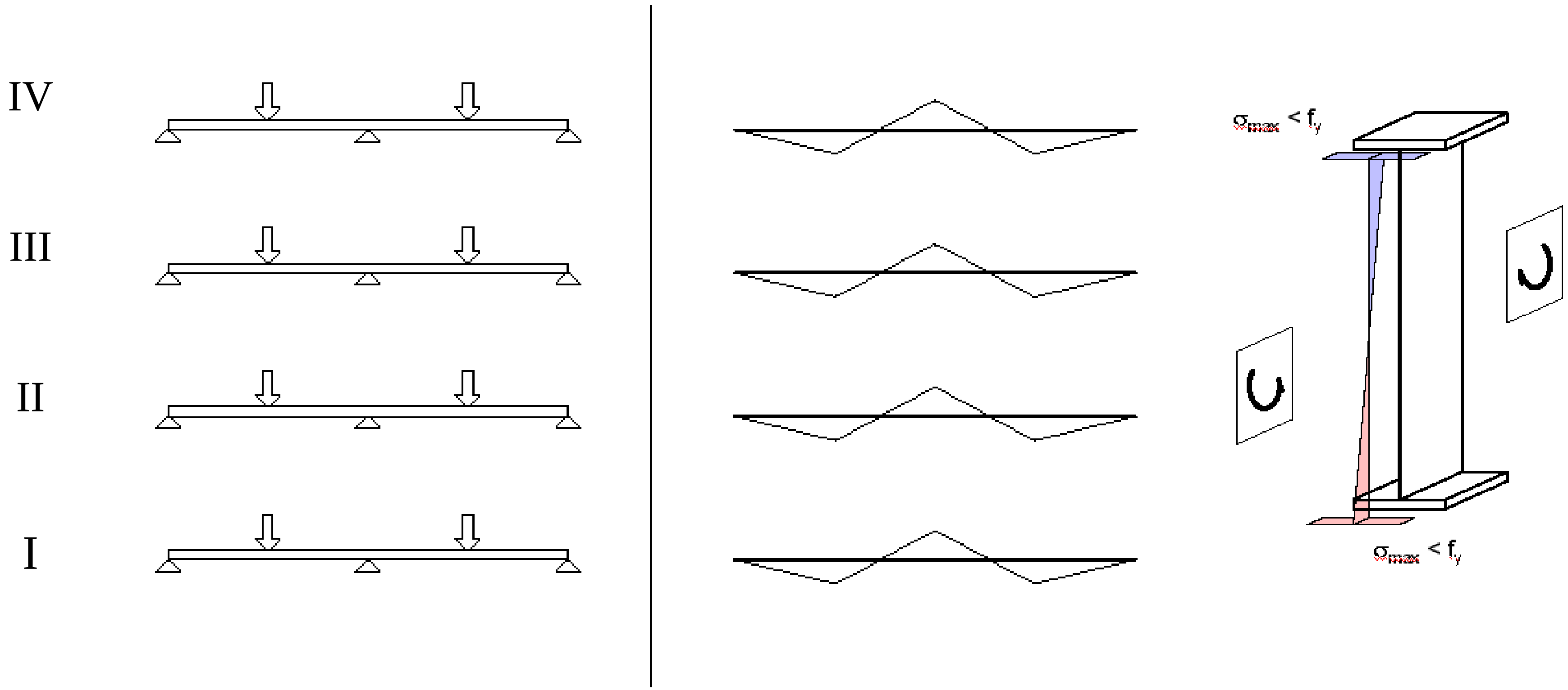
$$P_1 \neq 0$$

$$M_{sup} = 6\ P_1\ L\ /\ 32$$

Photo: Author

$$M_{sp} = 5\ P_1\ L\ /\ 32$$

$$M_{sup}\ /\ M_{sp} = 1{,}2 = \text{constant for each beam}$$



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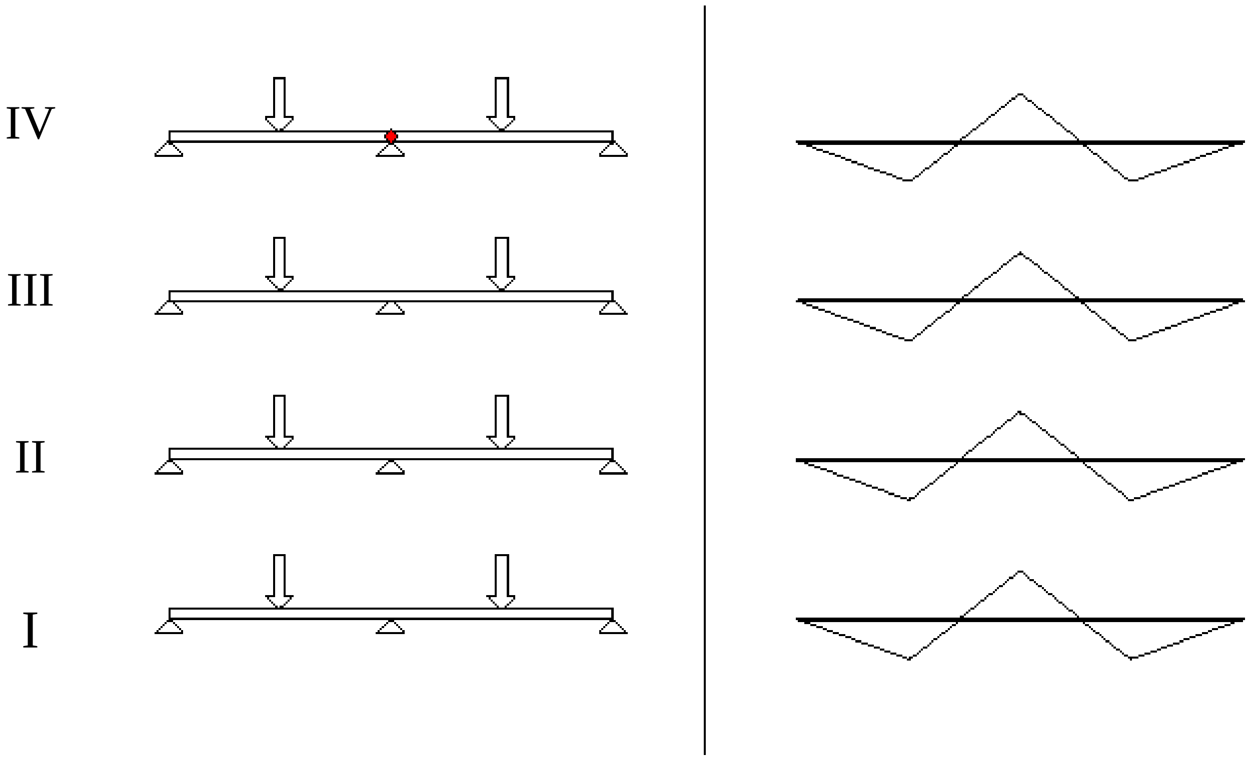
$$P_2 = P_1 + \Delta P$$

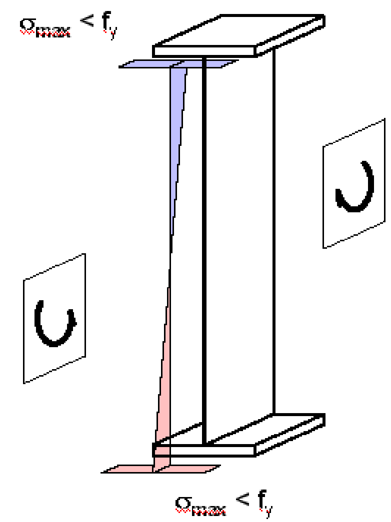
$$M_{sup} = 6\,P_2\,L / 32$$

$$M_{sp} = 5\,P_2\,L / 32$$

$$M_{sup} / M_{sp} = 1{,}2 = \text{constant for each beam}$$

Photo: Author





$$P_3 = P_2 + \Delta P$$

There is local instability for IV class I-beam in compressed part of cross-section. Local instability occurs for cross-section of the max value of bending moment.

End of resistance for IV class I-beam.

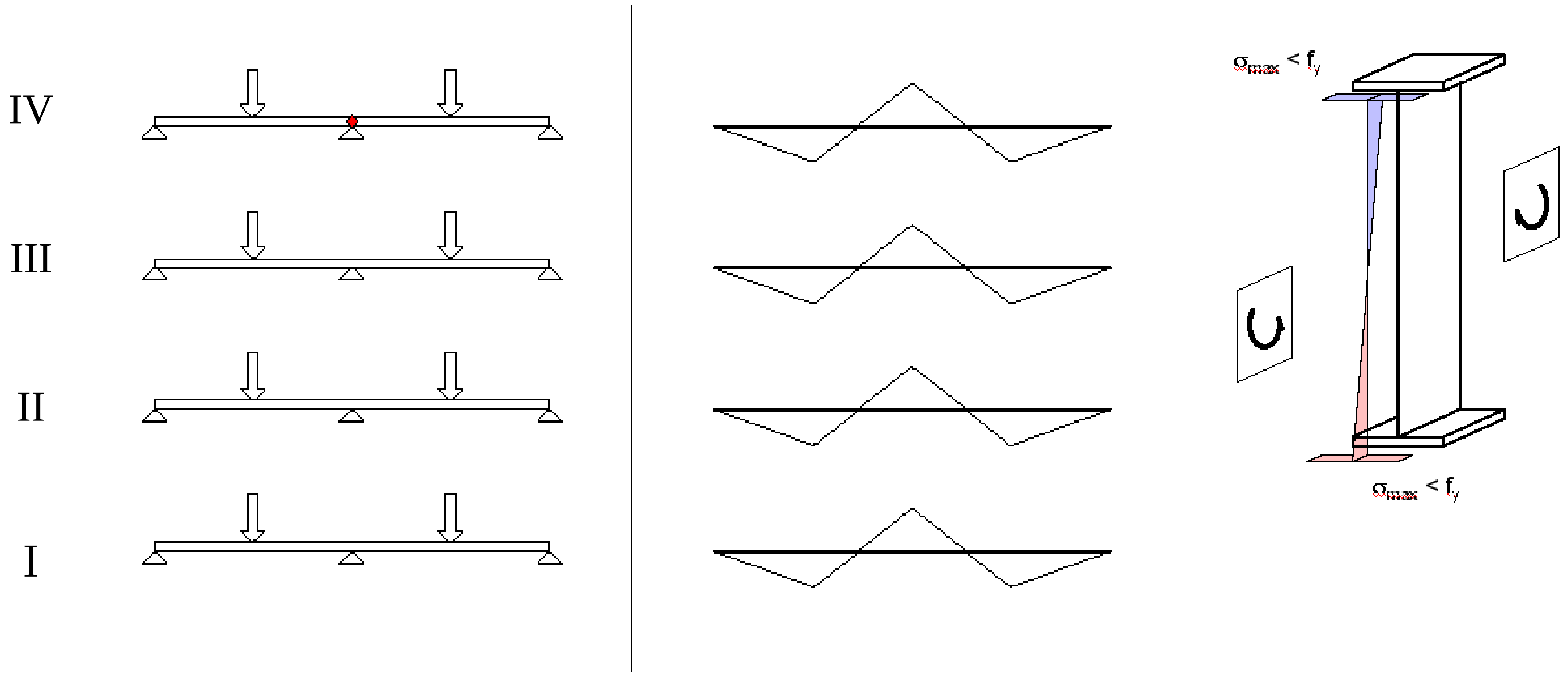


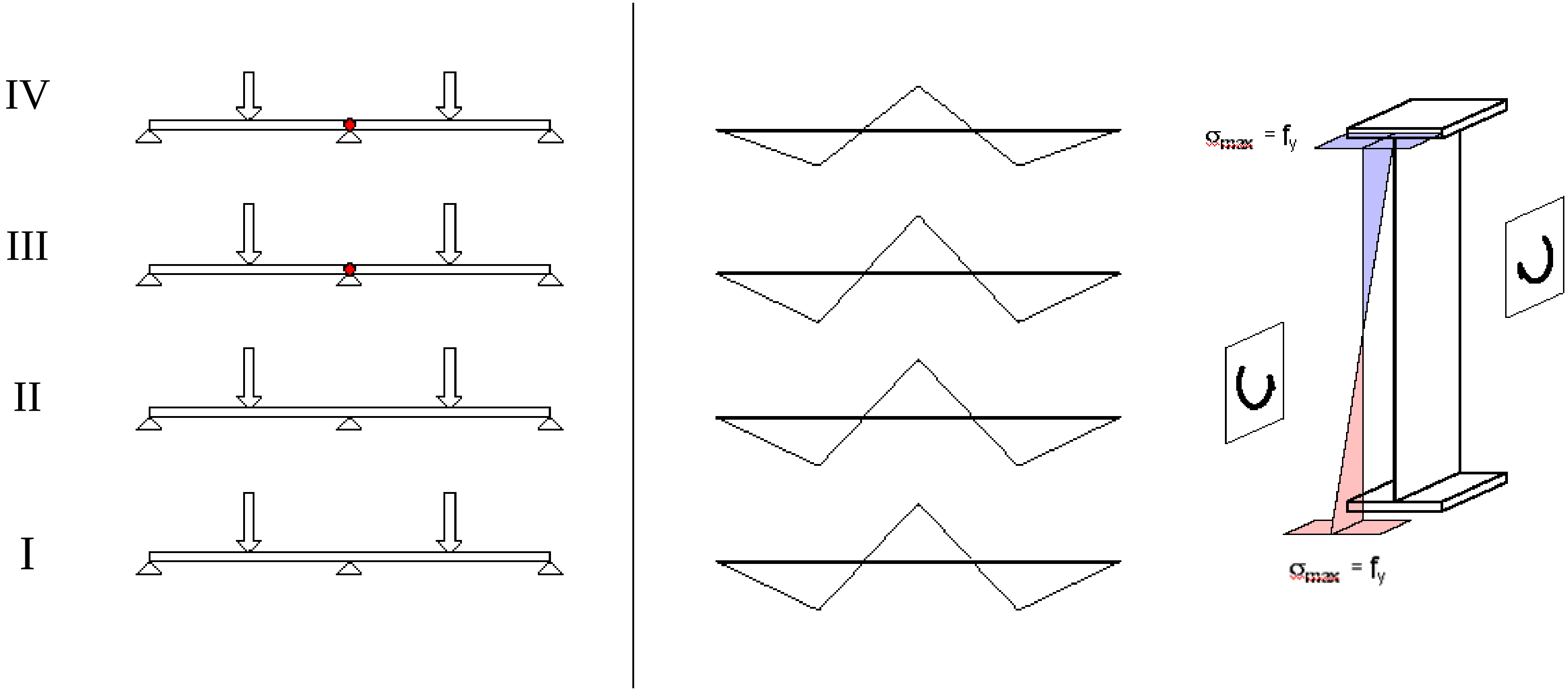
Photo: Author

$$P_4 = P_3 + \Delta P$$

$$M_{sup} = 6\ P_4\ L\ /\ 32$$

$$M_{sp} = 5\ P_4\ L\ /\ 32$$

$M_{sup}\ /\ M_{sp} = 1{,}2$ = constant for I, II, III beams



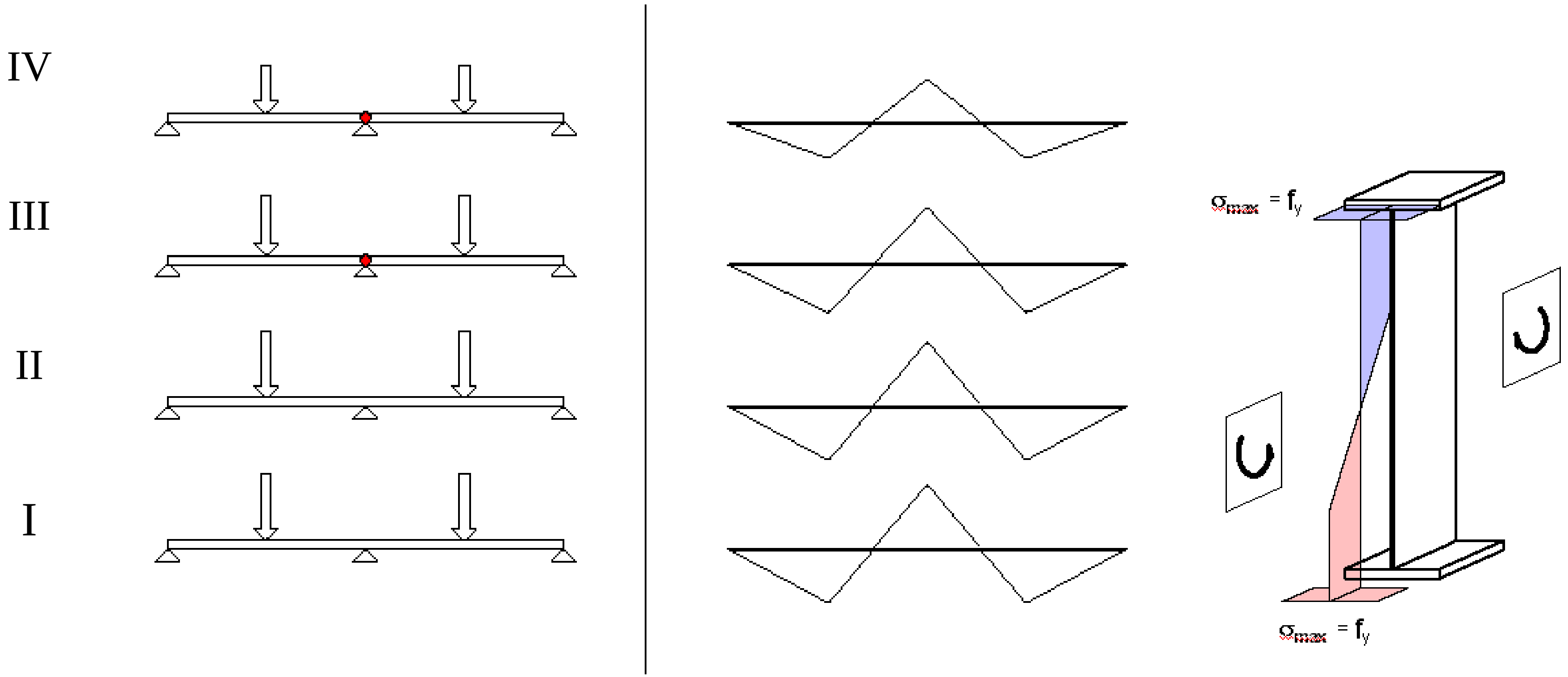
$$P_5 = P_4 + \Delta P$$

Photo: Author

$$\boldsymbol{\sigma}_{max,\ comp} = \sigma(M_{sup}) = f_y$$

End of resistance for III class I-beam.

End of elastic work of cross-section for I and II beams.



$$P_6 = P_5 + \Delta P$$

Photo: Author

$$M_{sup} = 6\ P_6\ L / 32$$

$$M_{sp} = 5\ P_6\ L / 32$$

$$M_{sup} / M_{sp} = 1{,}2 = \text{constant for I, II beams}$$

Elasto-plastic work of cross-section for I and II beams

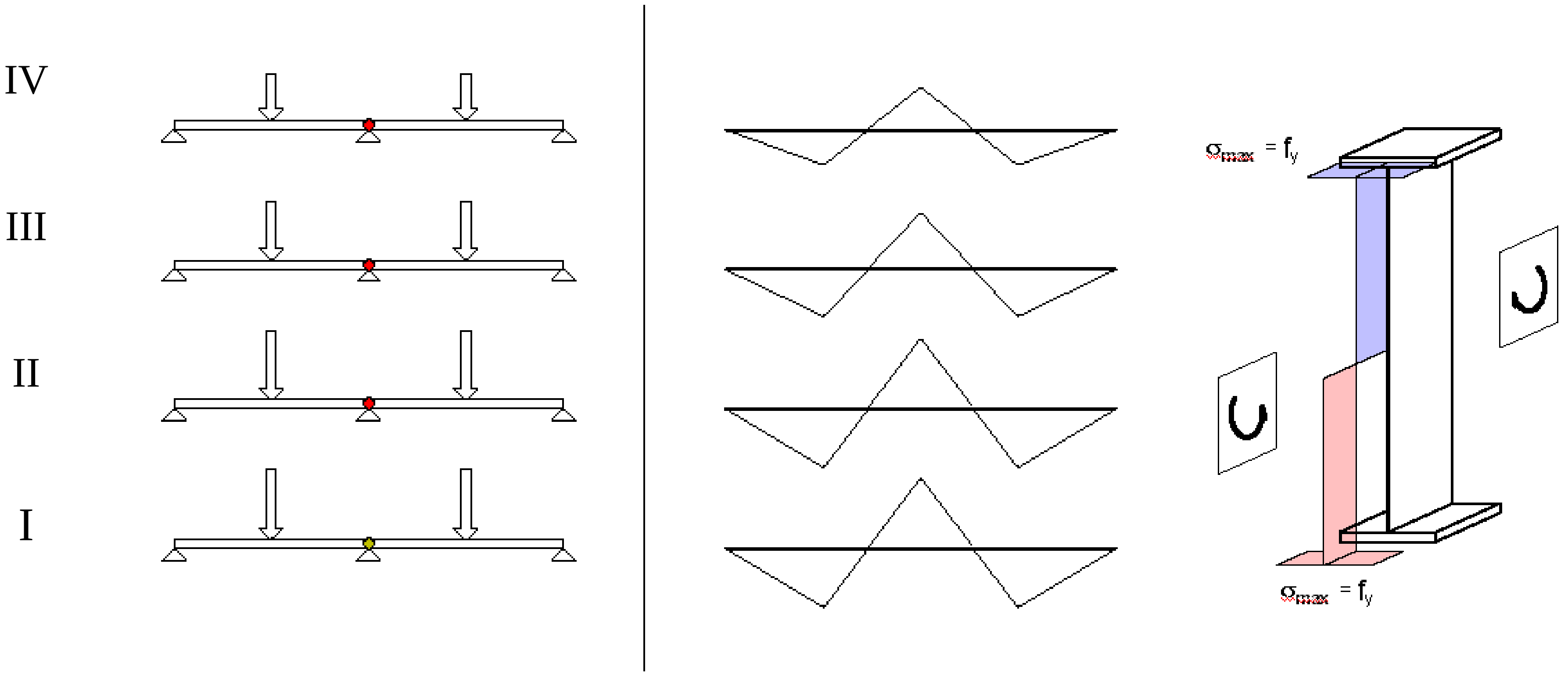
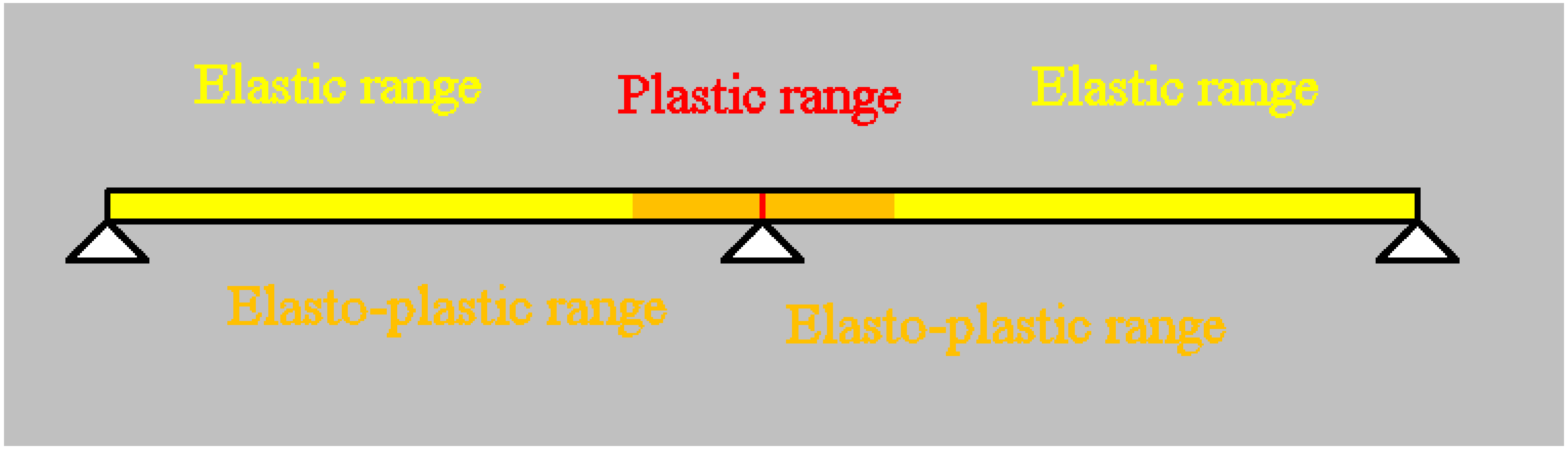


Photo: Author

$$P_7 = P_6 + \Delta P$$

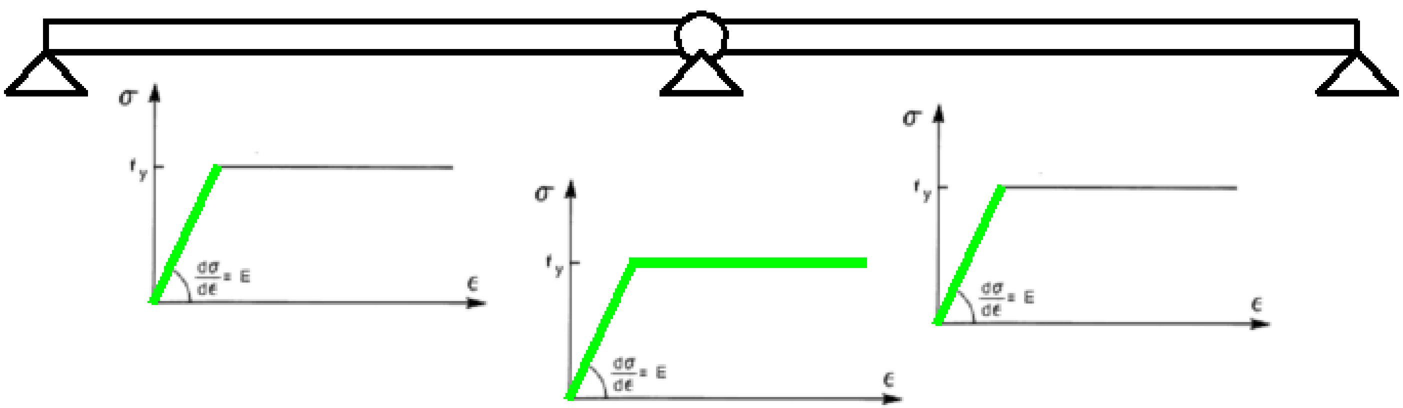
Whole cross-section with maximum bending moment in plastic range.

End of resistance for II class I-beam.



Cross-section in plastic range behaves the same way, as hinge. Plastic shelf = no limit for deformations (rotation).

Photo: Author

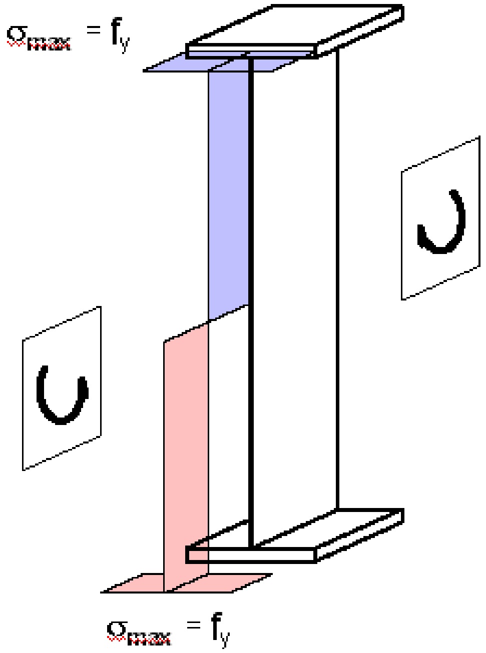


Cross-section in plastic range = plastic hinge

"Normal" hinge → $M = 0$

Plastic hinge → $M = M_{pl} \neq 0$

Stresses for cross-section with M_{pl} are as follow:



M_{pl} is maximum bending moment, which can be carried by cross-section

Photo: Author

For $P = P_7 - dP$:

Bending moments are calculated as for statically indeterminate structure

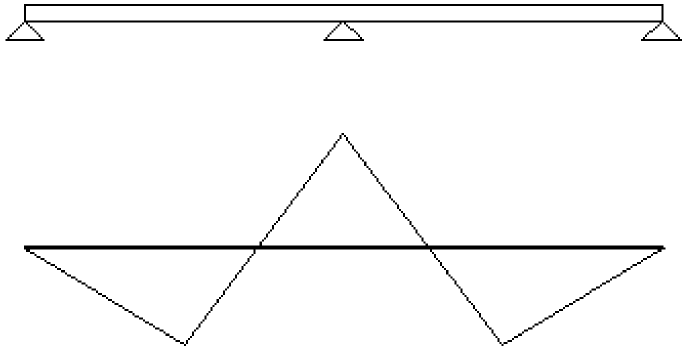


Photo: Author

For $P = P_7$:

Static scheme changes:

Statically indeterminate two span continous beam → two single-span beams supported by common support

Loads change:

Pair of identical forces P → pair of identical forces P and bending moment M_{pl} in plastic hinge

For $P = P_7$:

Bending moments are sum from P and M_{pl}

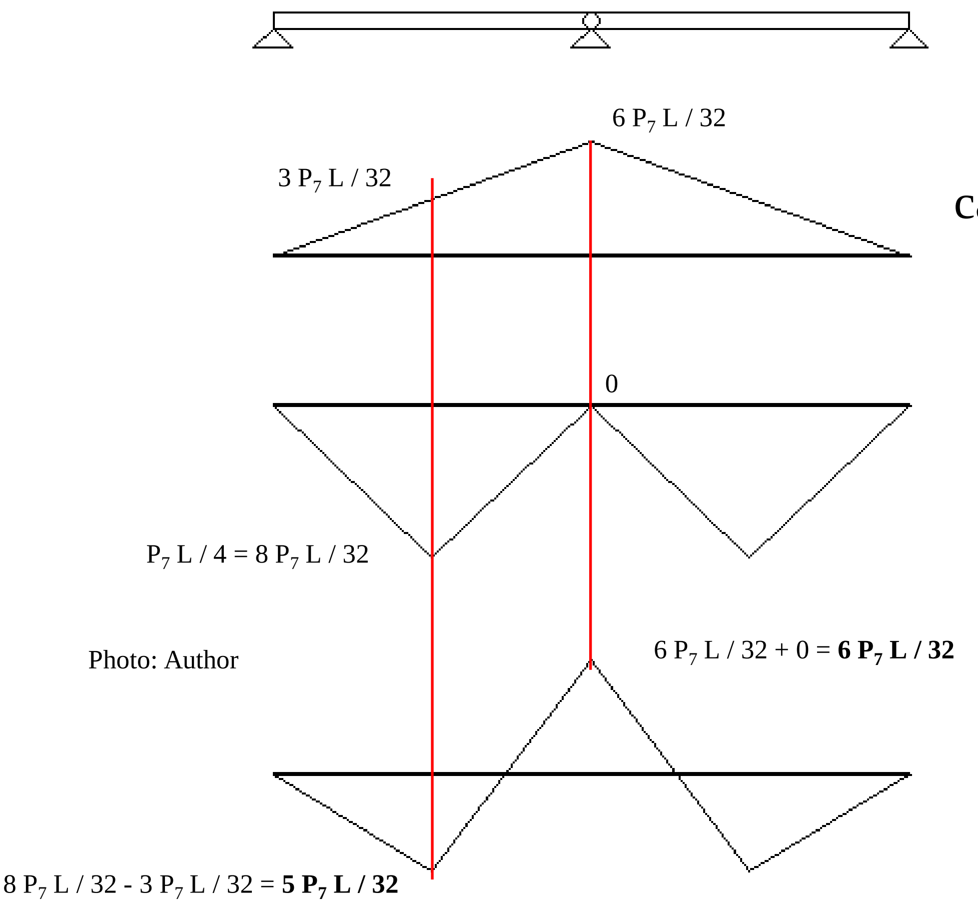


Photo: Author

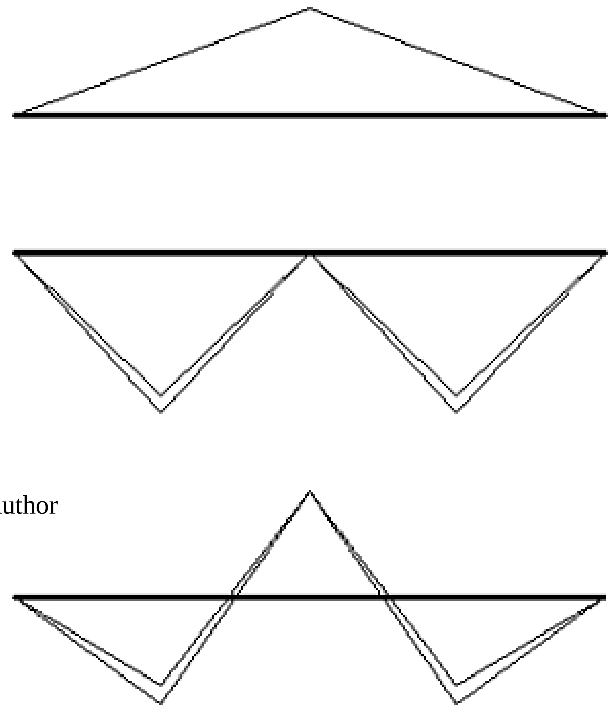
Bending moments from M_{pl} calculated for two single-span beams

Bending moments from P calculated for two single-span beams

Sum: the same as for $P < P_7$

For $P > P_7$:

There is still possible to increase value of P. But part of bending moment from M_{pl} will be still the same. Part of bending moment from P will increases only.



Bending moments from M_{pl} calculated for two single-span beams

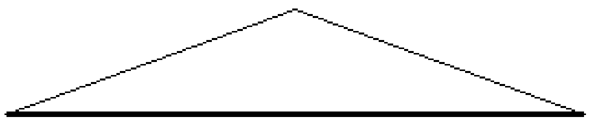
Bending moments from P calculated for two single-span beams

Sum

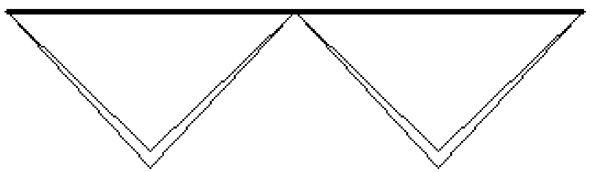
Photo: Author

For $P > P_7$:

End of increasing possibility will be, when summarized bending moment reaches the maximum value of bending moment, which can be carried by the cross section. If cross-section is the same for all beam, $M_{max} = M_{pl} = const$

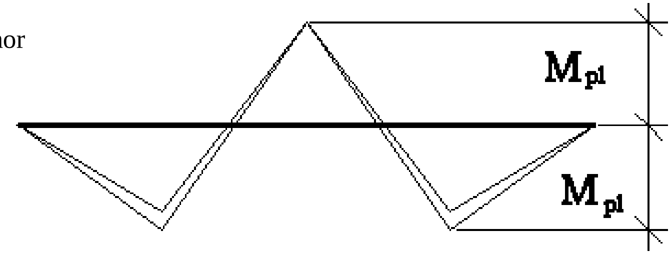


Bending moments from M_{pl} calculated for two single-span beams



Bending moments from P calculated for two single-span beams

Photo: Author



Sum

For P > P_7 :

For each cross-section, where $M_{max} = M_{pl}$, forms plastic hinge. End of resistance for I beam comes, when our structure transform into mechanism.

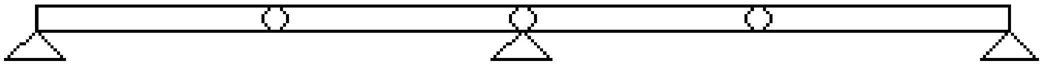
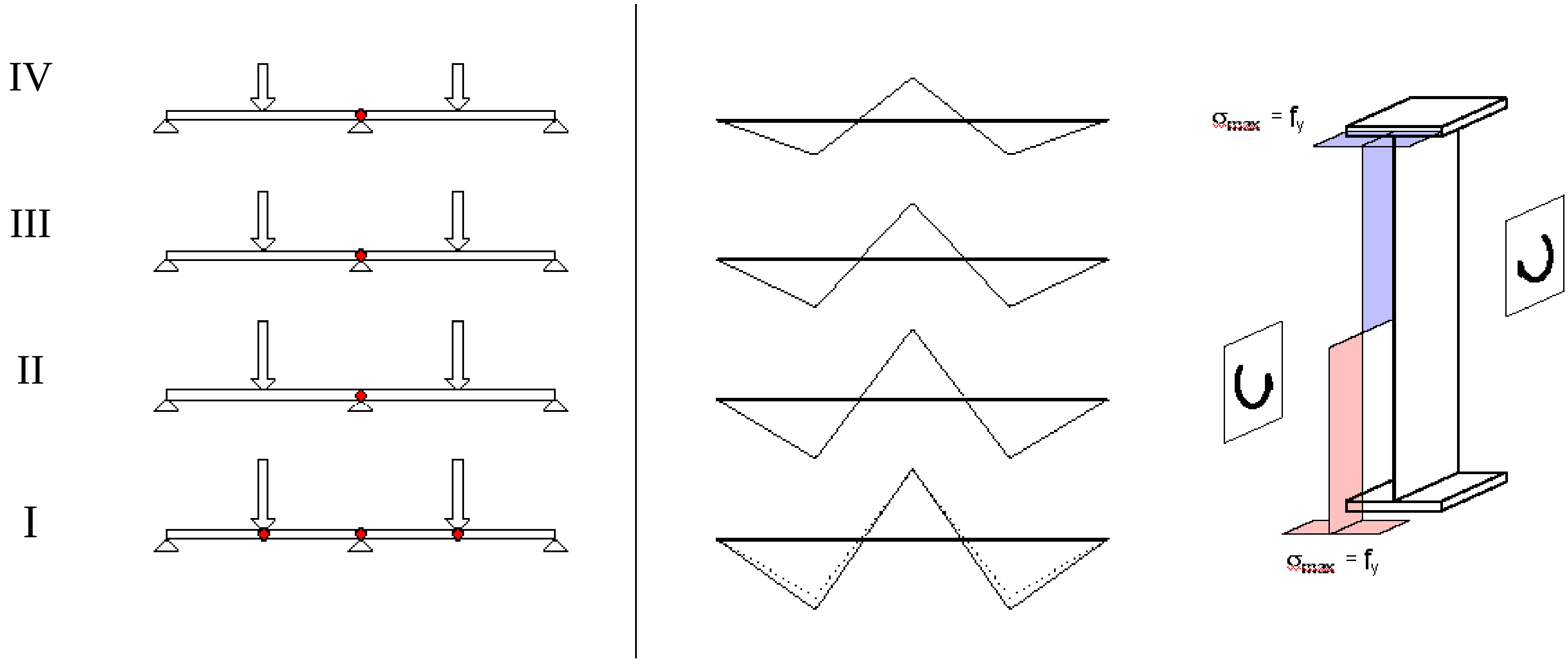


Photo: Author



$$P_8 = P_7 + \Delta P$$

$$M_{sup} = 6\ P_8\ L\ /\ 32$$

$$M_{sp} = 6\ P_8\ L\ /\ 32$$

$$M_{sup}\ /\ M_{sp} = 1{,}0 \text{ I beam}$$

Photo: Author

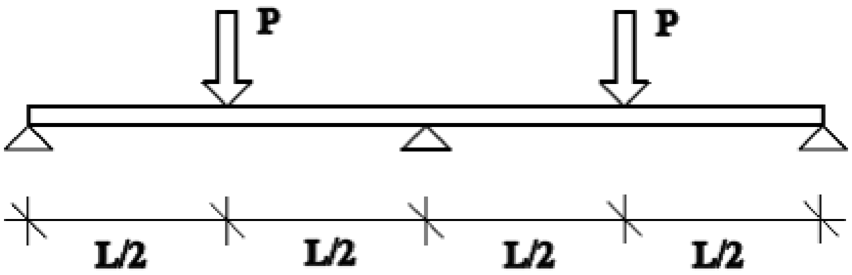
Summary

Class of cross-section	Destruction by / end of resistance	Statical calculations
IV	Local instability for compressed part of cross-section	"Normal" statical calculations (The Force Method, The Displacement Method, computer calculations...)
III	$\sigma_{max,\ comp} = f_y$	
II	First plastic hinge	
I	Transform structure into mechanism	Recalculation according to change of static scheme → plastic redistribution of bending moment

Real behaviour vs. idealisation

Class of cross-section	Real behaviour	Idealisation
IV	End of resistance = local instability for $M_{Ed} \leq M_{Rd,\ el}$	
III	End of resistance = local instability for $M_{Rd,\ pl} > M_{Ed} > M_{Rd,\ el}$	End of resistance = local instability for $M_{Ed} = M_{Rd,\ el}$
II	End of resistance = local instability for $M_{Ed} \approx M_{Rd,\ pl}$ but without reduistribution of bending moments	End of resistance = local instability for $M_{Ed} = M_{Rd,\ pl}$
I	End of resistance = full redistribution of bending moments, change structure into mechanizm	

Photo: Author



For analysed structure (if geometrical characteristisc are the same for each beams):

$P_3 < P_5$ - destruction of IV beam

P_5 - destruction of III beam

$P_7 \approx (1,1 \div 1,2)\ P_5$ - destruction of II beam

$P_8 \approx (1,25 \div 1,35)\ P_5$ - destruction of I beam

Proportions for I-beam in bending about stron axis

Way of classification

EN 1993-1-1 5.5 5.6

Class = Class [strength of material (→ #t / 36) ; shape of cross-section (→ #t / 37) ; shape of compressive stress (→ #t / 38) ; slenderness of sub-parts of cross-section (→ #t / 39)]

The most often used types of cross-section:

- Hot-rolled
- Welded
- Cold formed

Photo: tradekorea.com

Photo: cnzjbs.en.made-in-china.com

Photo: cedricbodeengineering.com

According to

EN 1993-1-1 5.5 5.6

we can calculate hot-rolled and welded cross-section.

For cold-formed we must used EN 1993-1-3 additionally.

Thin-walled structures

Strength of material

We must calculate non-dimensional strength of material:

$$\varepsilon = \sqrt{(235 / f_y)}$$

f_y - yield strength [MPa]

235 [MPa] - comparative yield strength

Shape of cross-section

According to EN 1993-1-1, tab. 5.2, each cross-section (except L and O) must be divided into sub-parts. Each sub-parts or each type of cross-section belongs to one of three gropus:

- quasi-web (tab. 5.2 part 1)
- quasi-flange (one free end; tab. 5.2 part 2)
- L-section and CHS (circular hollow section ≡ round pipe; tab. 5.2 part 3)

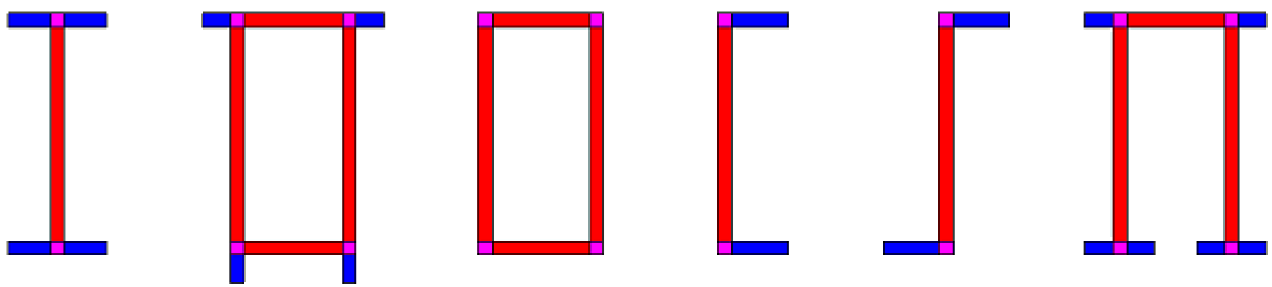


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Table 5.2 defines four values for limits for each sub-parts / cross-sections:

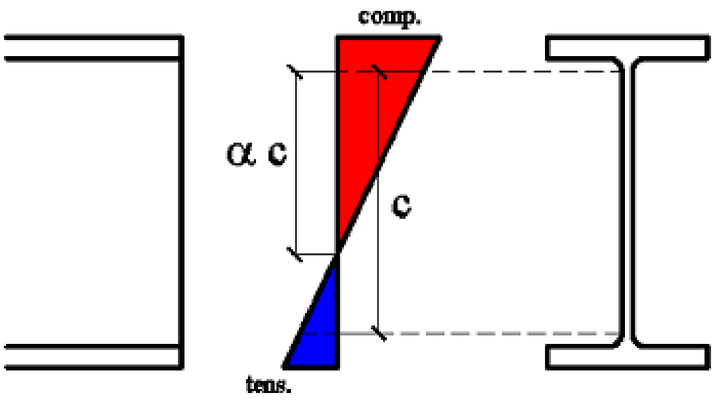
0 - default value, not disclosed in table;

$A = A(\sigma_c)$

$B = B(\sigma_c)$

$C = C(\sigma_c)$

σ_c - shape of compressive stress



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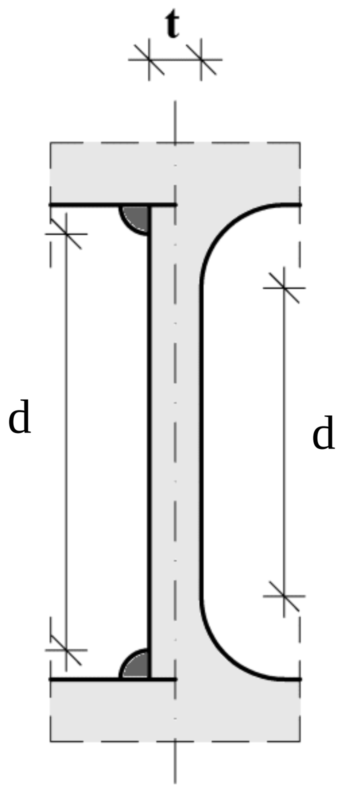
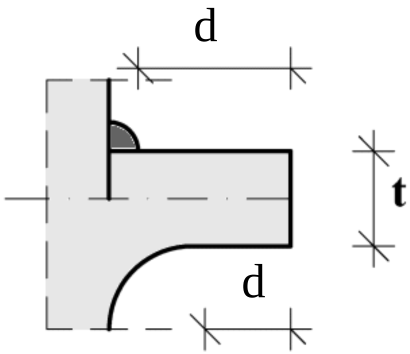
Slenderness of sub-parts of cross-section

For each sub-parts we calculate slenderness λ: width or length of sub-part (out of welds and round parts) divided by thickness of sub-part



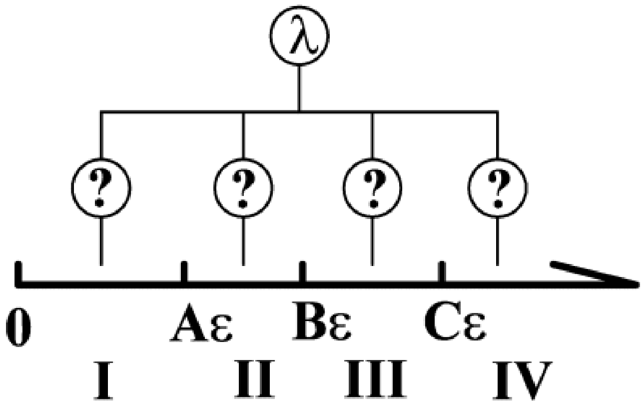


$$\lambda = d / t$$

Photo: Author

Class of cross-section for sub-part

Photo: Author



$0 < \lambda \leq A\varepsilon \rightarrow$ I class

$A\varepsilon < \lambda \leq B\varepsilon \rightarrow$ II class

$B\varepsilon < \lambda \leq C\varepsilon \rightarrow$ III class

$C\varepsilon < \lambda \rightarrow$ IV class

Class of cross-section for total cross-section

Class of cross-section = max (class for Ist sub-part ; class for IInd sub-part ; class for IIIrd sub-part ; ...)

Example:

Flange - Ist class

Web - IVth class

Class of cross-section = max (1, 4) = IVth class

Classification according to class of cross-section is important for compressive stresses only.

This means, it is important for bending moment, compressive axial force, interaction of both, sometimes interaction between bending moment and tensile axial force.

It is not important for tensile axial force and shear force.

Different procedures for calculations of A, B, C

EN 1993-1-1, tab. 5.2

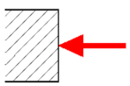
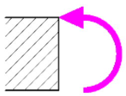
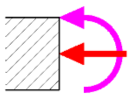
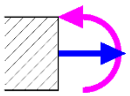
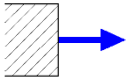
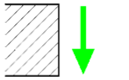
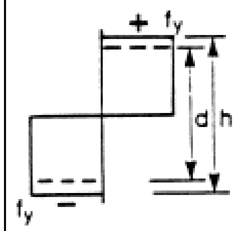
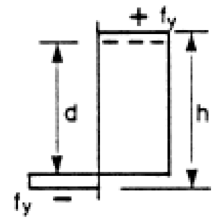
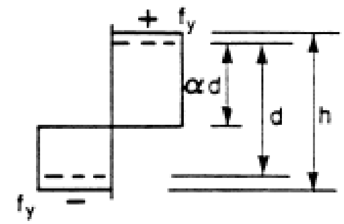
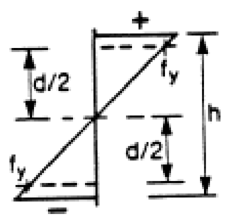
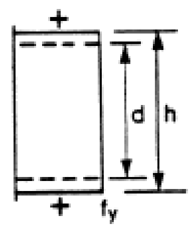
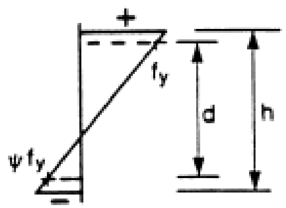
<table>
<tr><th>LOAD</th><th>PART 1
„web”</th><th>PART 2
„flange”</th><th>PART 3
O</th><th>PART 3
L</th><th>other
● |</th><th>cold-formed</th></tr>
<tr><td></td><td>Proc. 1</td><td>Proc. 4</td><td rowspan="4">Proc. 7</td><td>Proc. 8</td><td rowspan="4">Not recommended</td><td rowspan="6">EN 1993-1-3
(another rules)</td></tr>
<tr><td></td><td>Proc. 2</td><td rowspan="3">Proc. 4 or 5 or 6</td><td rowspan="3">Not recommended</td></tr>
<tr><td></td><td>Proc. 3a</td></tr>
<tr><td></td><td>Proc. 3b</td></tr>
<tr><td></td><td colspan="3" rowspan="2">no σ_c</td><td colspan="2">no σ_c</td></tr>
<tr><td></td><td>Not recommended</td><td>Not recommended</td></tr>
</table>

Photo: Author

EN 1993-1-1 tab 5.2 part 1

Class	Web subject to bending	Web subject to compression	Web subject to bending and compression
Stress distribution in element (compression positive)			
1	$d/t_w \leq 72\varepsilon$	$d/t_w \leq 33\varepsilon$	when $\alpha > 0,5$: $d/t_w \leq 396\varepsilon/(13\alpha - 1)$ when $\alpha \leq 0,5$: $d/t_w \leq 36\varepsilon/\alpha$
2	$d/t_w \leq 83\varepsilon$	$d/t_w \leq 38\varepsilon$	when $\alpha > 0,5$: $d/t_w \leq 456\varepsilon/(13\alpha - 1)$ when $\alpha \leq 0,5$: $d/t_w \leq 41,5\varepsilon/\alpha$
Stress distribution in element (compression positive)			
3	$d/t_w \leq 124\varepsilon$	$d/t_w \leq 42\varepsilon$	when $\psi > -1$: $d/t_w \leq 42\varepsilon/(0,67 + 0,33\psi)$ when $\psi \leq -1$: $d/t_w \leq 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

Examples of calculations - steel

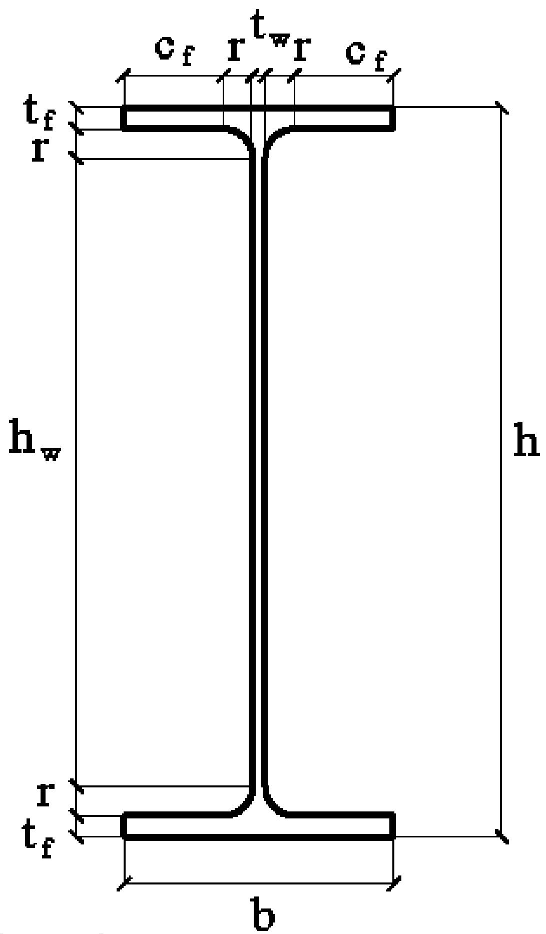


Photo: Author

IPE A 600

h	b	t_f	t_w	r
597	220	17,5	9,8	24

$h_w = h - 2\ t_f - 2r$

$c_f = (b - t_w - 2r) / 2$

$\lambda_w = h_w / t_w = (h - 2t_f - 2r) / t_w = (597 - 2 \cdot 17{,}5 - 2 \cdot 24) / 9{,}8 = 52{,}449$

$\lambda_f = c_f / t_f = [(b - t_w - 2r) / 2] / t_f = (220 - 9{,}8 - 2 \cdot 24) / (2 \cdot 17{,}5) = 4{,}634$

S235

$f_y = 235$ MPa

$\varepsilon_{235} = \sqrt{(235 / f_y)} = \sqrt{(235 / 235)} = 1{,}000$

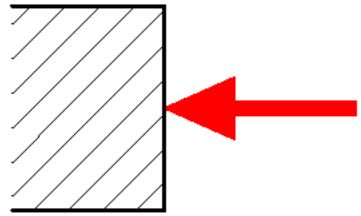
S355

$f_y = 355$ MPa

$\varepsilon_{355} = \sqrt{(235 / f_y)} = \sqrt{(235 / 355)} = 0{,}814$

Photo: Author

Web, compressive axial force



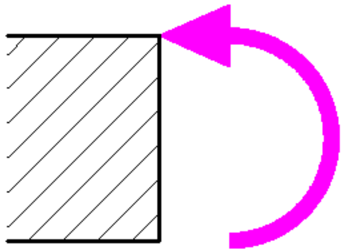
Proc. 1

f_y = 235 MPa	f_y = 355 MPa
◆ Aε = 33ε = 33,000 ◆ Bε = 38ε = 38,000 ◆ Cε = 42ε = 42,000	◆ Aε = 33ε = 26,862 ◆ Bε = 38ε = 30,932 ◆ Cε = 42ε = 34,188
λ_w = 52,449 > Cε → IVth class	λ_w = 52,449 > Cε → IVth class

Imagine, what's happened, when λ_w = 35,0?

Photo: Author

Web, bending moment

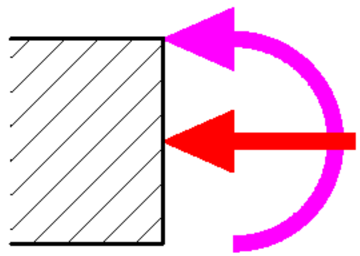


Proc. 2

f_y = 235 MPa	f_y = 355 MPa
◆ Aε = 72ε = 72,000 ◆ Bε = 83ε = 83,000 ◆ Cε = 124ε = 124,000	◆ Aε = 72ε = 58,608 ◆ Bε = 83ε = 67,562 ◆ Cε = 124ε = 100,936
λ_w = 52,449 < Aε → I^{st} class	λ_w = 52,449 < Aε → I^{st} class

Imagine, what's happened, when λ_w = 70,0?

Web, compressive axial force and bending moment



Compressive axial force and bending moment $\rightarrow \alpha \geq 0{,}5$

- $A\varepsilon = 396\,\varepsilon / (13\,\alpha - 1)$
- $B\varepsilon = 456\,\varepsilon / (13\,\alpha - 1)$
- $C\varepsilon = 42\,\varepsilon / (0{,}66 + 0{,}33\,\Psi)$

Proc. 3a

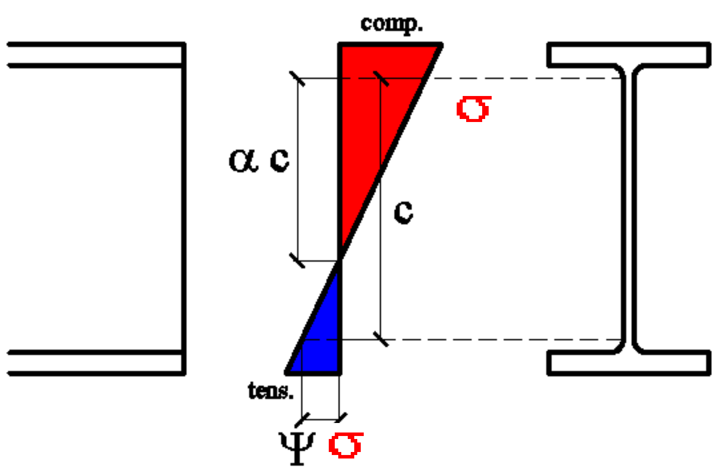


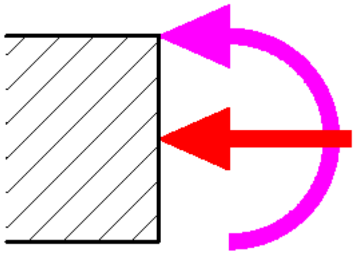
Photo: Author

Def:

$\Psi = \sigma / \sigma$

Ψ (tens.) < 0

Ψ (comp.) > 0

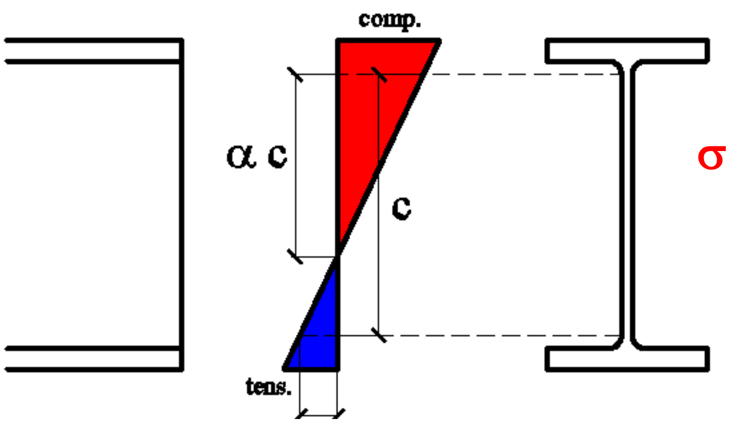


$S235 \rightarrow f_y = 235\ MPa \rightarrow \varepsilon = 1{,}000$

$N_{Ed} = 164{,}829\ kN \rightarrow \boldsymbol{\sigma}_{max} = 0{,}1\ f_y$

$M_{Ed} = 221{,}441\ kNm \rightarrow \boldsymbol{\sigma}_{max} = 0{,}3\ f_y$

$\boldsymbol{\sigma}_{max} = 0{,}400\ f_y$



$\sigma = 0{,}358\ f_y$

$c = 514\ mm$

$\boldsymbol{\alpha}c = 357\ mm \rightarrow \boldsymbol{\alpha} = 0{,}694$

$\boldsymbol{\sigma}_{max} = -0{,}200\ f_y$

$\psi\ \sigma = -0{,}158\ f_y \rightarrow \psi = -0{,}441$

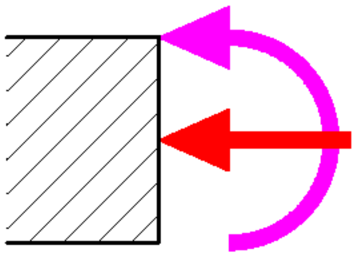
Photo: Author

- $A\varepsilon = 396\ \varepsilon / (13\ \alpha - 1) = 49{,}364$
- $B\varepsilon = 456\ \varepsilon / (13\ \alpha - 1) = 56{,}843$
- $C\varepsilon = 42\ \varepsilon / (0{,}66 + 0{,}33\ \Psi) = 81{,}637$

$\lambda_w = 52{,}449$

$$A\varepsilon < \lambda_w < B\varepsilon$$

Second class of cross-section



Compressive axial force and bending moment $\rightarrow \alpha \geq 0,5$

$N_{Ed} \neq 0 \quad M_{Ed} \rightarrow 0$

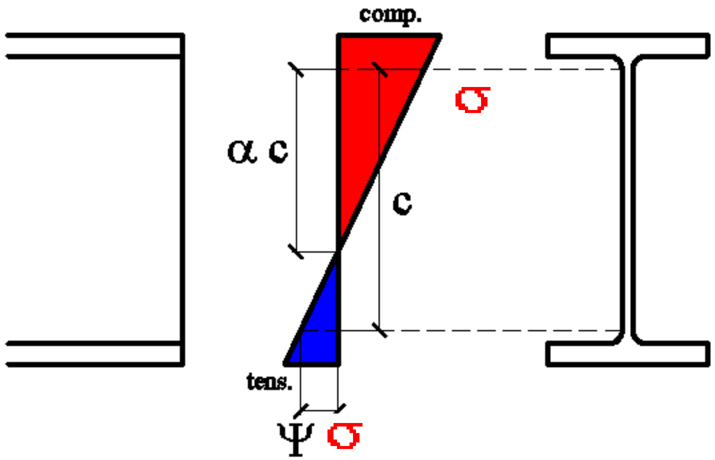
$\Psi \rightarrow 1,0 \quad \alpha \rightarrow 1,0$

$A\varepsilon \rightarrow 33\ \varepsilon$

$B\varepsilon \rightarrow 38\ \varepsilon$

$C\varepsilon \rightarrow 42\ \varepsilon$

The same as for compressive axial force only



$N_{Ed} \rightarrow 0 \quad M_{Ed} \neq 0$

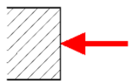
$\Psi \rightarrow -1,0 \quad \alpha \rightarrow 0,5$

$A\varepsilon \rightarrow 72\ \varepsilon$

$B\varepsilon \rightarrow 82,9\ \varepsilon \approx 83\ \varepsilon$

$C\varepsilon \rightarrow 127,3\ \varepsilon \approx 124\ \varepsilon$

Nearly the same as for bending moment only



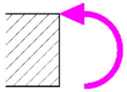
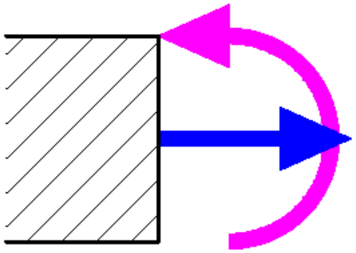


Photo: Author

Web, tensile axial force and
bending moment



Tensile axial force and
bending moment → $\alpha \leq 0{,}5$

- $A\varepsilon = 36\ \varepsilon / \alpha$
- $B\varepsilon = 41{,}5\ \varepsilon / \alpha$
- $C\varepsilon = 62\ \varepsilon\ (1 - \Psi)\ \sqrt{(-\Psi)}$

Proc. 3b

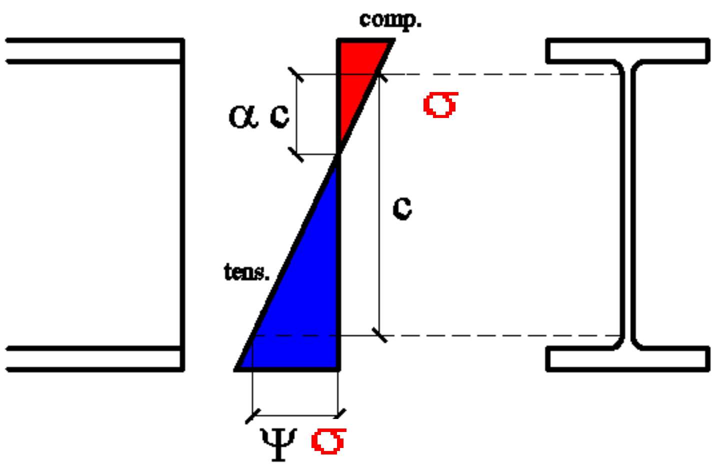


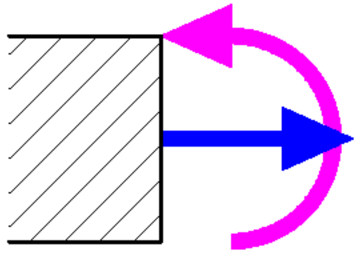
Photo: Author

Def:

$\Psi = \sigma / \sigma$

Ψ (tens.) < 0

Ψ (comp.) > 0

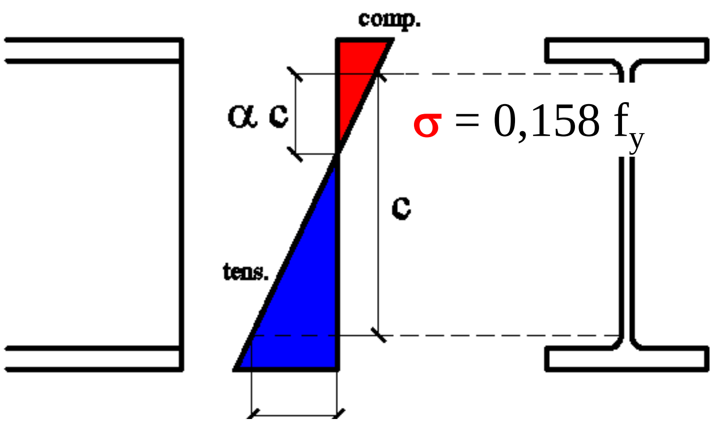


$S235 \rightarrow f_y = 235\ MPa \rightarrow \varepsilon = 1{,}000$

$N_{Ed} = 164{,}829\ kN \rightarrow \boldsymbol{\sigma}_{max} = 0{,}1\ f_y$

$M_{Ed} = 221{,}441\ kNm \rightarrow \boldsymbol{\sigma}_{max} = 0{,}3\ f_y$

$\boldsymbol{\sigma}_{max} = 0{,}200\ f_y$



$c = 514\ mm$

$\boldsymbol{\alpha} c = 157\ mm \rightarrow \boldsymbol{\alpha} = 0{,}306$

$\boldsymbol{\sigma}_{max} = -0{,}400\ f_y$

$\boldsymbol{\psi}\ \boldsymbol{\sigma} = -0{,}358\ f_y \rightarrow \boldsymbol{\psi} = -2{,}266$

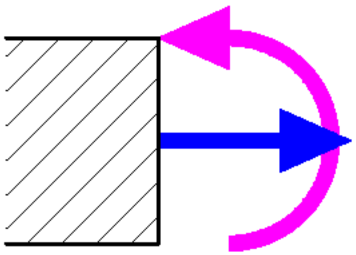
Photo: Author

- $A\varepsilon = 36\,\varepsilon / \alpha = 117{,}647$
- $B\varepsilon = 41{,}5\,\varepsilon / \alpha = 135{,}621$
- $C\varepsilon = 62\,\varepsilon\,(1 - \Psi)\,\sqrt{(-\Psi)} = 306{,}816$

$\lambda_w = 52{,}449$

$$\lambda_w < A\varepsilon$$

First class of cross-section



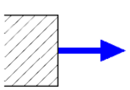
Tensile axial force and bending moment → $\alpha \leq 0{,}5$

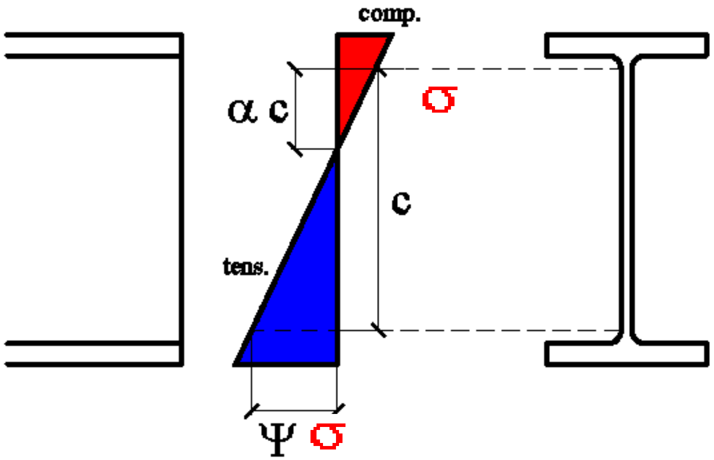
$N_{Ed} \neq 0 \quad M_{Ed} \to 0$

$\Psi \to 1{,}0 \quad \alpha \to 0$

$A\varepsilon \to \infty$

The same as for
tensile axial force only





$N_{Ed} \to 0 \quad M_{Ed} \neq 0$

$\Psi \to -1{,}0 \quad \alpha \to 0{,}5$

$A\varepsilon \to 72\ \varepsilon$

$B\varepsilon \to 83\ \varepsilon$

$C\varepsilon \to 124\ \varepsilon$

The same as for
bending moment only

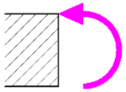


Photo: Author

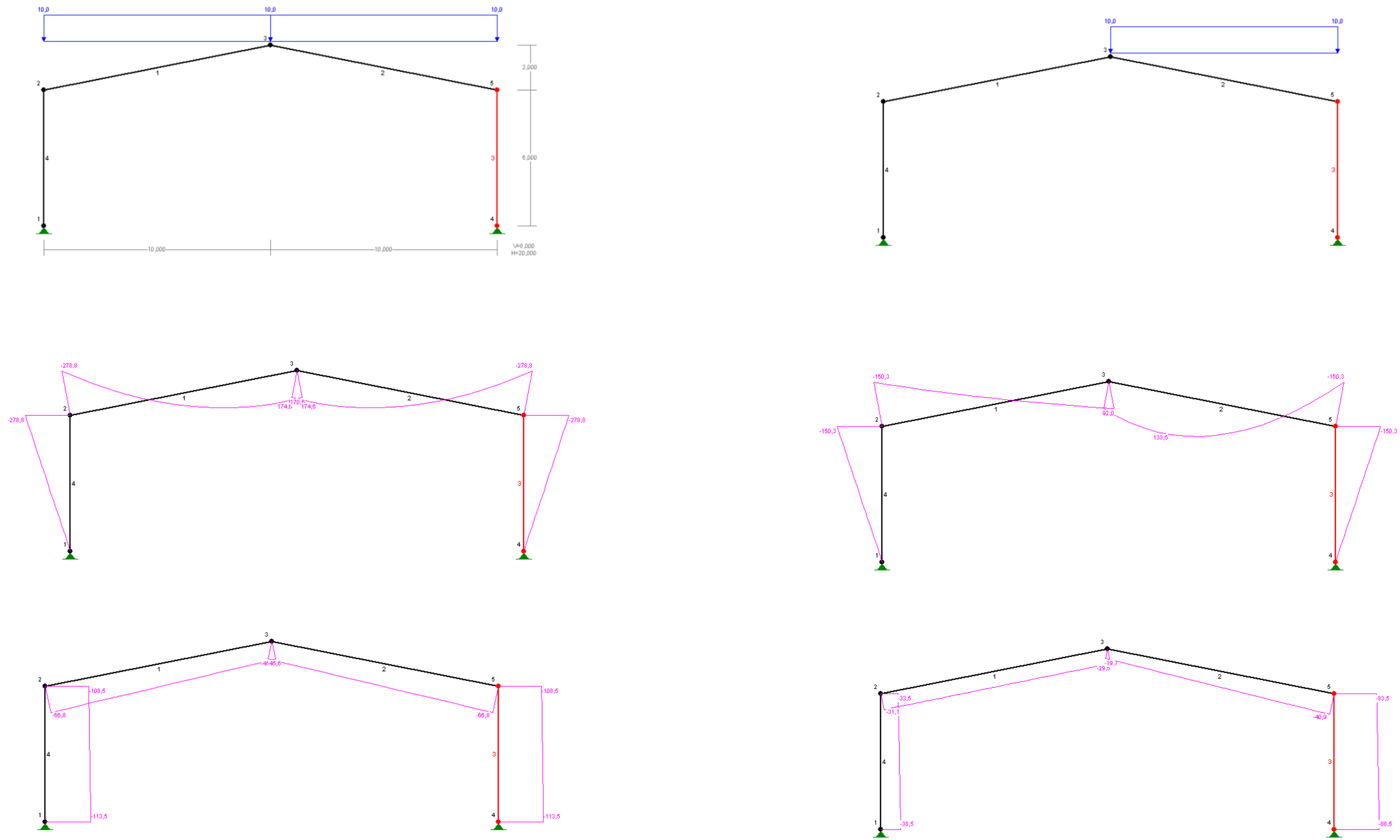


Photo: Author

Conclusion from examples #t / 45 - 56: class of cross-section strongly depends on proportion between M_{Ed} and N_{Ed}. There is different proportion for analysed frame for different points and for different combinations of loads.

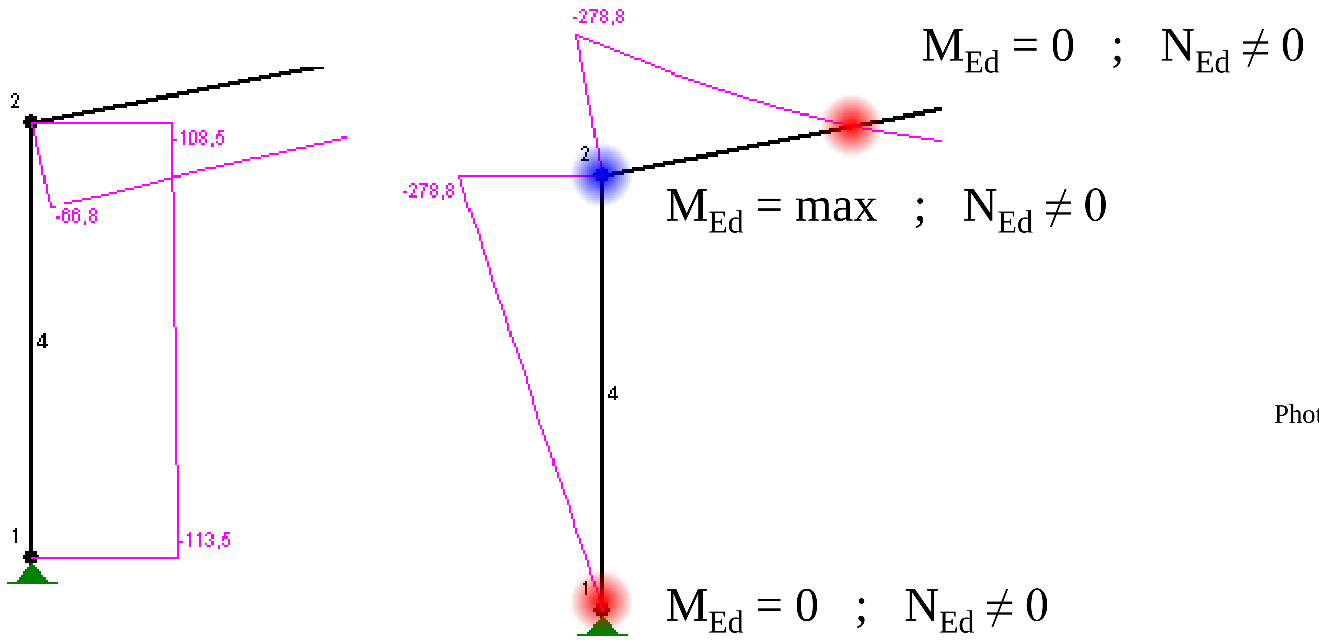
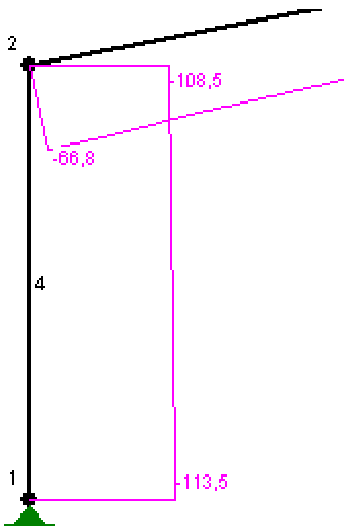
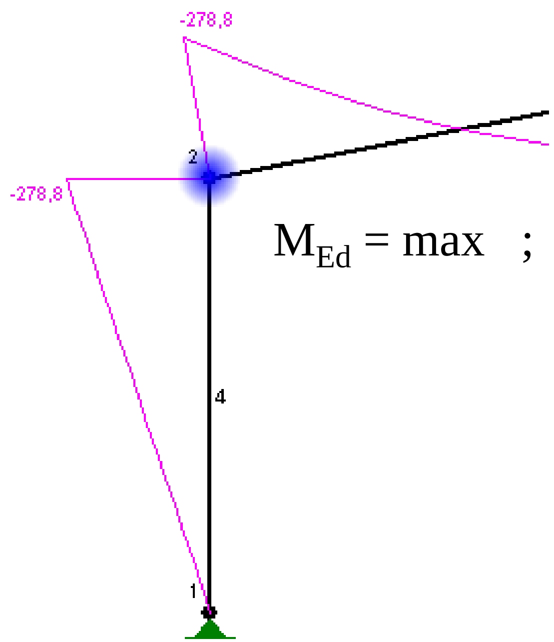


Photo: Author

There is possible, that for $M_{Ed} = 0$ $N_{Ed} \neq 0$, is IIIrd of IVth class of cross-section. There is possible, that for $M_{Ed} = \max$ $N_{Ed} \neq 0$, is Ist or IInd class of cross-section. Theoretically, there is change of class of cross-section along bar.

Generally, class of cross-section should be calculated for cross-section with the biggest effort (M_{Ed} = max for this situation) for each combination and is treated as constant along bar.



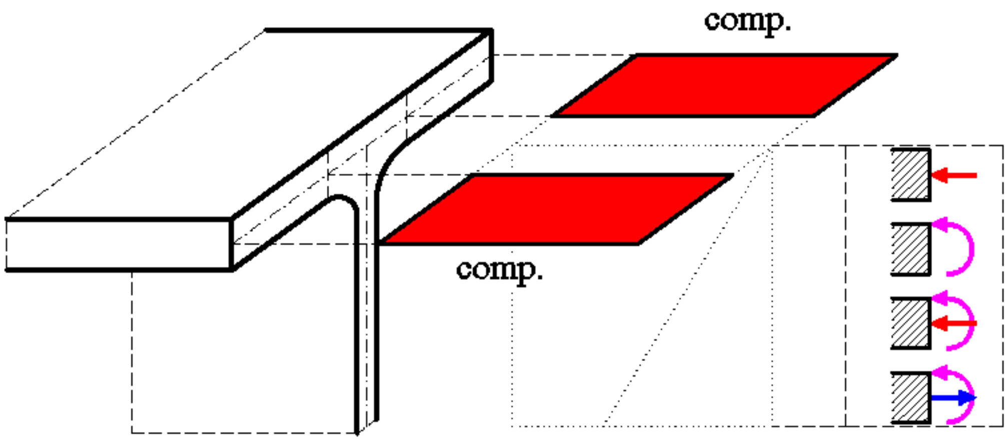


$M_{Ed} = \max \quad ; \quad N_{Ed} \neq 0$

Photo: Author

Flange, uniform compression

Photo: Author

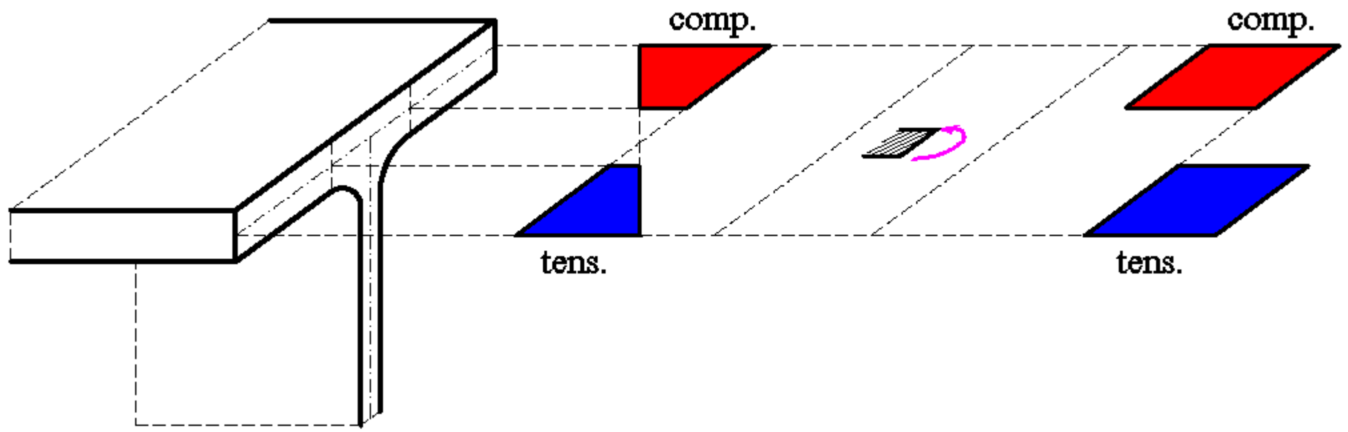


Proc. 4

f_y = 235 MPa	f_y = 355 MPa
◆ Aε = 9ε = 9,000	◆ Aε = 9ε = 7,326
◆ Bε = 10ε = 10,000	◆ Bε = 10ε = 8,140
◆ Cε = 14ε = 14,000	◆Cε = 14ε = 11,396
λ_f = 4,634 < Aε → I^{st} class	λ_f = 4,634 < Aε → I^{st} class

Flange, $M_z \neq 0$

Proc. 4



For tensed flange classes are not important.

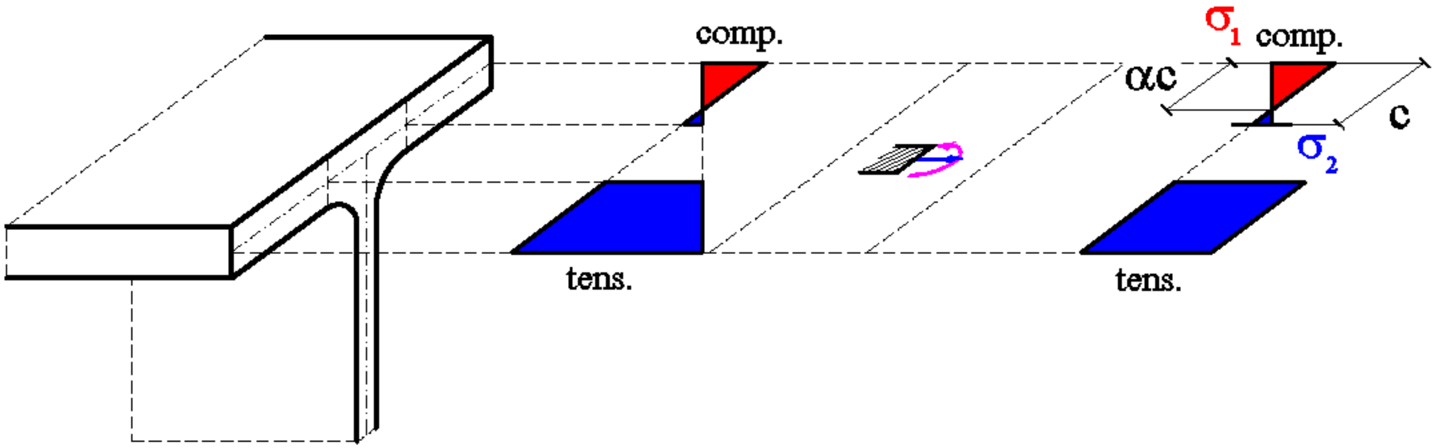
For compressed flange calculations are the same, as for uniform compression

Photo: Author

Flange, $M_z \neq 0$ and tensile axial force

Photo: Author

Proc. 5



- $A\varepsilon = 9\ \varepsilon / \alpha$
- $B\varepsilon = 10\ \varepsilon / \alpha$
- $C\varepsilon = 21\ \varepsilon \sqrt{k_\sigma}$

$$\Psi = \sigma_2 / \sigma_1 < 0$$

$$0 > \Psi > -1 \rightarrow k_\sigma = 0{,}57 - 0{,}28\ \Psi$$

$$\Psi < -1 \rightarrow k_\sigma = 0{,}57 - 0{,}21\ \Psi + 0{,}07\ \Psi^2$$

S235 → f_y = 235 MPa → ε = 1,000

N_{Ed} = 164,829 kN → $\boldsymbol{\sigma}_{max}$ = 0,1 f_y

M_{Ed} = 94,475 kNm → $\boldsymbol{\sigma}_{max}$ = 0,3 f_y

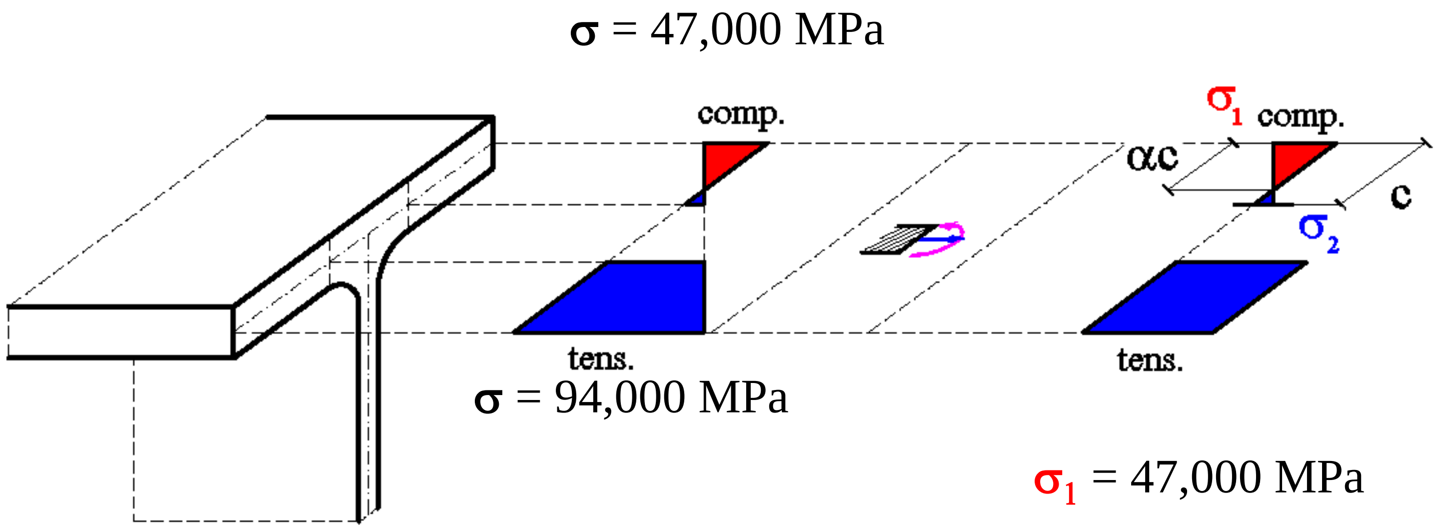


Photo: Author

σ_1 = 47,000 MPa

σ_2 = 4,978 MPa

α = 0,984

Ψ = σ_2 / σ_1 = -0,106

- $A\varepsilon = 9\ \varepsilon / \alpha = 9{,}146$
- $B\varepsilon = 10\ \varepsilon / \alpha = 10{,}163$

$k_\sigma = 0{,}57 - 0{,}28\ \Psi = 0{,}600$

- $C\varepsilon = 21\ \varepsilon \sqrt{k_\sigma} = 16{,}267$

$\lambda_f = 4{,}634$

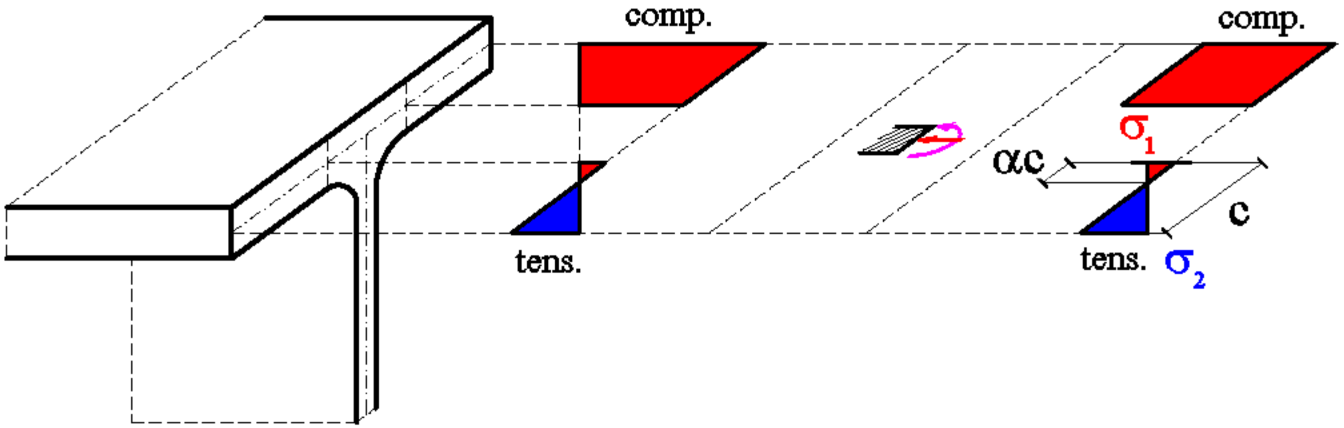
$$\lambda_w < A\varepsilon$$

First class of cross-section

Flange, $M_z \neq 0$ and compressive axial force

Photo: Author

Proc. 6



- $A\varepsilon = 9\ \varepsilon / (\alpha \sqrt{\alpha})$
- $B\varepsilon = 10\ \varepsilon / (\alpha \sqrt{\alpha})$
- $C\varepsilon = 21\ \varepsilon \sqrt{k_\sigma}$

$\Psi = \sigma_2 / \sigma_1 < 0$

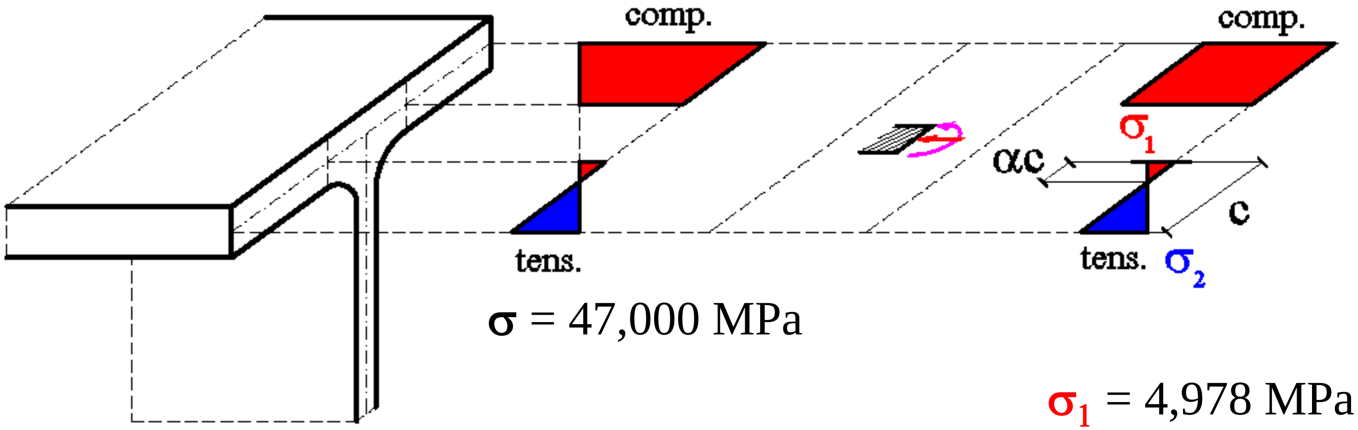
$k_\sigma = 1{,}7 - 5\ \Psi + 17{,}1\ \Psi^2$

$S235 \rightarrow f_y = 235$ MPa $\rightarrow \varepsilon = 1{,}000$

$N_{Ed} = 164{,}829$ kN $\rightarrow \boldsymbol{\sigma}_{max} = 0{,}1\ f_y$

$M_{Ed} = 94{,}475$ kNm $\rightarrow \boldsymbol{\sigma}_{max} = 0{,}3\ f_y$

$\boldsymbol{\sigma} = 94{,}000$ MPa



$\boldsymbol{\sigma} = 47{,}000$ MPa

Photo: Author

$\boldsymbol{\sigma}_1 = 4{,}978$ MPa

$\boldsymbol{\sigma}_2 = 47{,}000$ MPa

$\boldsymbol{\alpha} = 0{,}096$

$\Psi = \sigma_2 / \sigma_1 = -9{,}442$

- $A\varepsilon = 9\ \varepsilon / (\alpha \sqrt{\alpha}) = 302{,}577$
- $B\varepsilon = 10\ \varepsilon / (\alpha \sqrt{\alpha}) = 336{,}196$

$k_\sigma = 1{,}7 - 5\ \Psi + 17{,}1\ \Psi^2 = 1573{,}398$

- $C\varepsilon = 21\ \varepsilon \sqrt{k_\sigma} = 832{,}988$

$\lambda_f = 4{,}634$

$\lambda_w < A\varepsilon$

First class of cross-section

RHS

$$\lambda_{horizontal} = (b - 2t - 2r) / t$$

$$\lambda_{vertical} = (h - 2t - 2r) / t$$

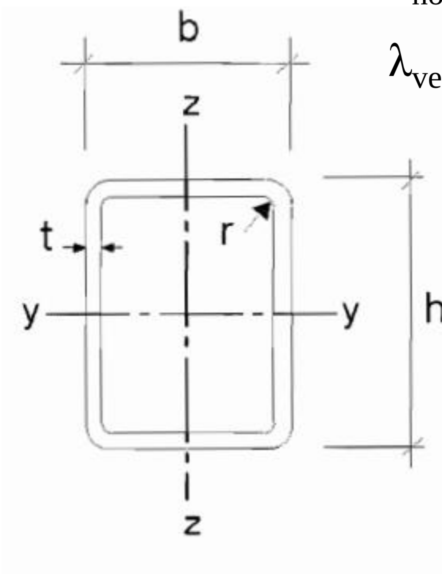
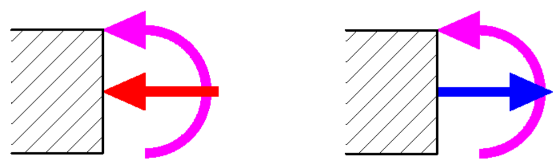
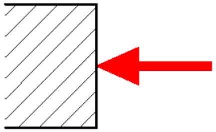


Photo: EN 1993-1-1 fig. 1.1

Calculation the same as for web (→ #t / 46-56) under various type of actions

Photo: Author





Total under compression, for horizontal and for vertical part → #t / 47

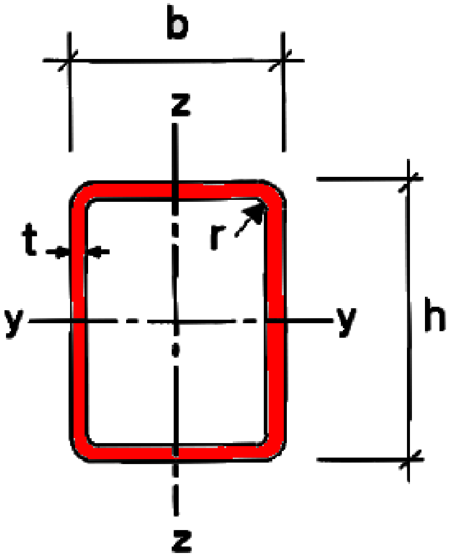


Photo: Author

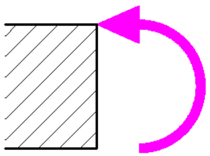
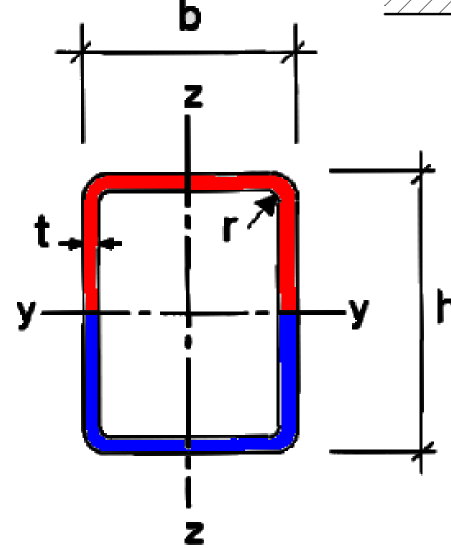
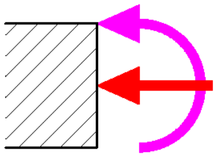


Photo: Author



Half under compression, for horizontal part → #t / 47,
for vertical part → #t / 48



More than half under compression, for horizontal part → #t / 47, for vertical part → #t / 49-52

Photo: Author

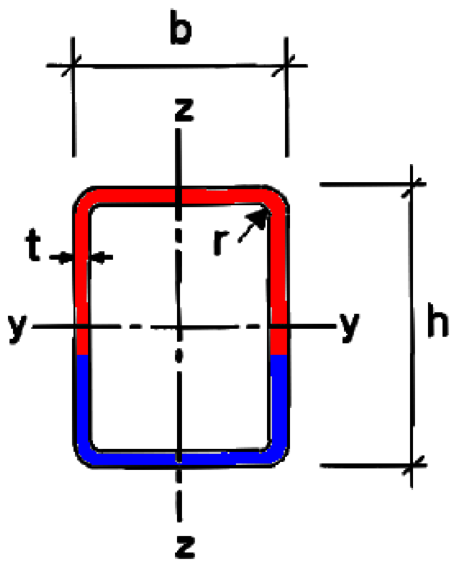
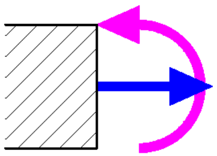
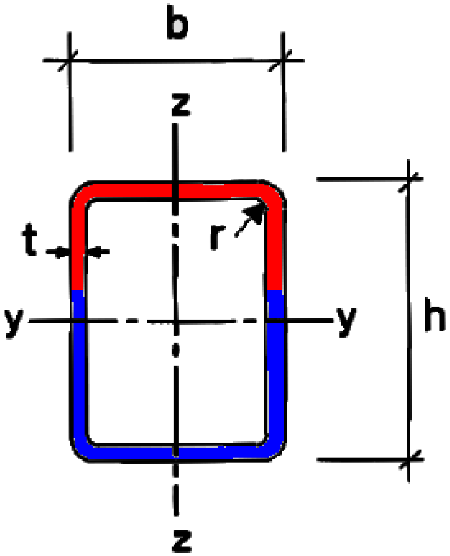


Photo: Author

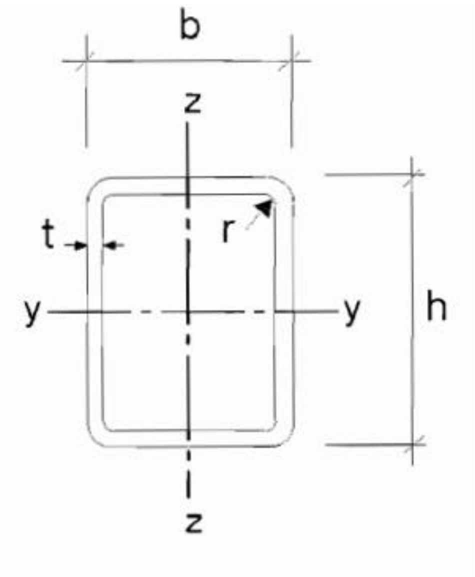




Less than half under compression, for horizontal part → #t / 47, for vertical part → #t / 53-56

RHS 100x50x3 (r = 4,5)

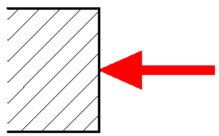
Photo: EN 1993-1-1 fig. 1.1



$\lambda_{horizontal} = (b - 2t - 2r) / t = (50 - 2 \cdot 3 - 2 \cdot 4{,}5) / 3 = 35 / 3 = 11{,}667$

$\lambda_{vertical} = (h - 2t - 2r) / t = (100 - 2 \cdot 3 - 2 \cdot 4{,}5) / 3 = 85 / 3 = 28{,}333$

S 355 $\rightarrow \varepsilon = 0{,}814$



Total under compression, for horizontal and for vertical part → #t / 47

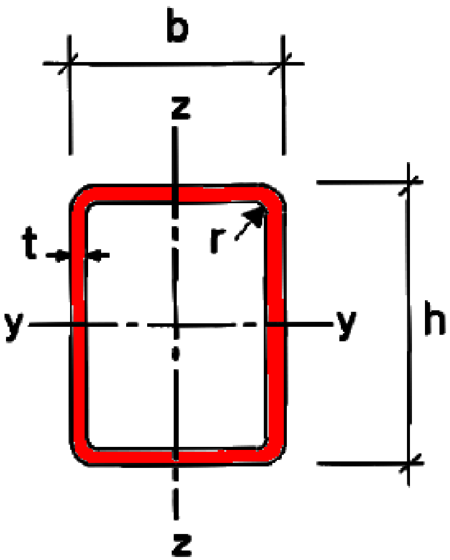


Photo: Author

f_y = 355 MPa
◆ $A\varepsilon = 33\varepsilon = 26{,}862$ ◆ $B\varepsilon = 38\varepsilon = 30{,}932$ ◆ $C\varepsilon = 42\varepsilon = 34{,}188$
$\lambda_{horizontal} = 11{,}667 < A\varepsilon \rightarrow$ I[st] class $A\varepsilon < \lambda_{vertical} = 28{,}333 < B\varepsilon \rightarrow$ II[nd] class

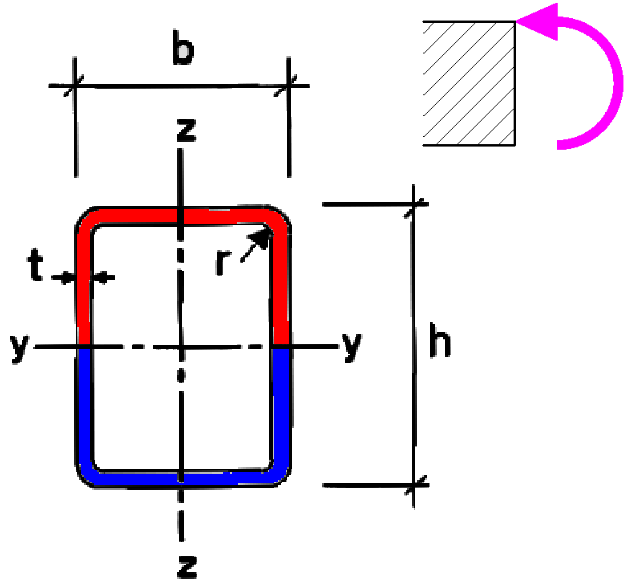


Photo: Author

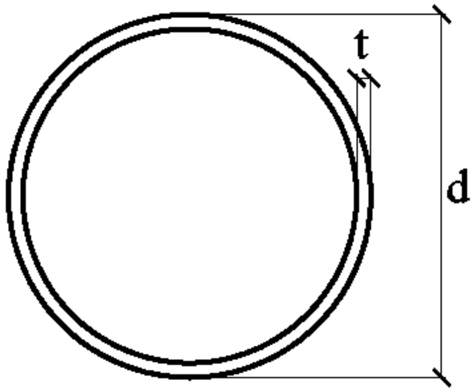
Half under compression, for horizontal part → #t / 47, for vertical part → #t / 48

f_y = 355 MPa
◆ $A\varepsilon = 33\varepsilon = 26{,}862$
◆ $B\varepsilon = 38\varepsilon = 30{,}932$
◆ $C\varepsilon = 42\varepsilon = 34{,}188$
$\lambda_{horizontal} = 11{,}667 < A\varepsilon \rightarrow$ I^{st} class

f_y = 355 MPa
◆ $A\varepsilon = 72\varepsilon = 58{,}608$
◆ $B\varepsilon = 83\varepsilon = 67{,}562$
◆ $C\varepsilon = 124\varepsilon = 100{,}936$
$\lambda_{vertical} = 28{,}333 < A\varepsilon \rightarrow$ I^{st} class

CHS - each type of load

Proc. 7



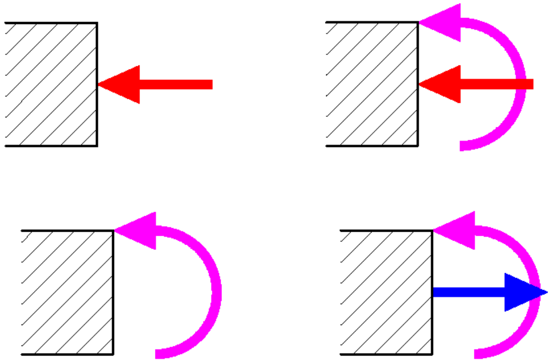


Photo: Author

$\lambda = d / t$

- $A\varepsilon = 50\ \varepsilon^2$
- $B\varepsilon = 70\ \varepsilon^2$
- $C\varepsilon = 90\ \varepsilon^2$

CHS; S 355 → $\varepsilon = 0,814$

- $A\varepsilon = 50\ \varepsilon^2 = 33,099$
- $B\varepsilon = 70\ \varepsilon^2 = 46,338$
- $C\varepsilon = 90\ \varepsilon^2 = 59,577$

ϕ 101,6 / 8,8 → d = 101,6 mm ; t = 8,8 mm

d / t = 11,545 < 33,099 → I^st class of cross-section

ϕ 244,5 / 7,1 → d = 244,5 mm ; t = 7,1 mm

d / t = 34,437 > 33,099 → II^nd class of cross-section

ϕ 508,0 / 11,00 → d = 508 mm ; t = 11 mm

d / t = 46,182 > 33,099 → II^nd class of cross-section

L-section, compressive axial force

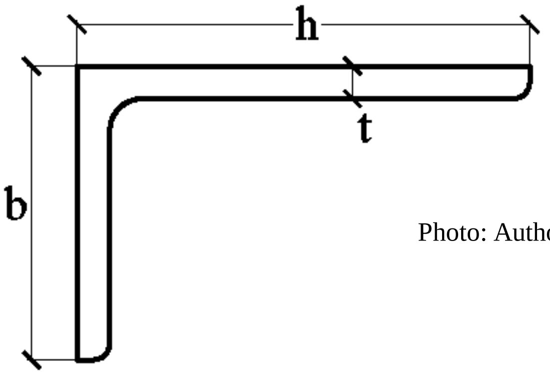
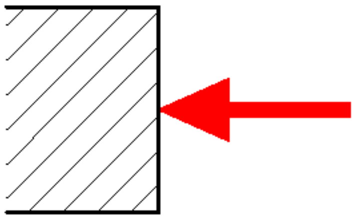


Photo: Author



Proc. 8

$h \geq b$

$\lambda = h / t$

$C\varepsilon = 15\varepsilon$

$h \geq b$

$\lambda = (h+b) / 2t$

$C\varepsilon = 11{,}5\varepsilon$

For <u>both</u> $\lambda > C\,\varepsilon \rightarrow IV^{th}$ class of cross-section

L-section;   S 235 $\rightarrow \varepsilon = 1{,}000$

100x100x10 $\rightarrow$ h = b = 100 mm; t = 10 mm

$h / t = \underline{10{,}0 < 15\varepsilon}$; $(h + b) / (2t) = \underline{10{,}0 < 11{,}5\varepsilon}$

Both conditions fulfilled $\rightarrow$ not IV^{th} class of cross-section

150x150x12 $\rightarrow$ h = b = 150 mm; t = 12 mm

$h / t = \underline{12{,}5 < 15\varepsilon}$; $(h + b) / (2t) = \underline{12{,}5 > 11{,}5\varepsilon}$

First condition fulfilled, second not fulfilled $\rightarrow$ not IV^{th} class of cross-section

200x100x10 $\rightarrow$ h = 200 mm; b = 100 mm; t = 10 mm

$h / t = \underline{20{,}0 > 15\varepsilon}$; $(h + b) / (2t) = \underline{15{,}0 > 11{,}5\varepsilon}$

Both conditions not fulfilled $\rightarrow$ IV^{th} class of cross-section

Sigma-beams, cold formed - calculations according to EN 1993-1-3, not EN 1993-1-1.

Photo: guardrailbarrier.net

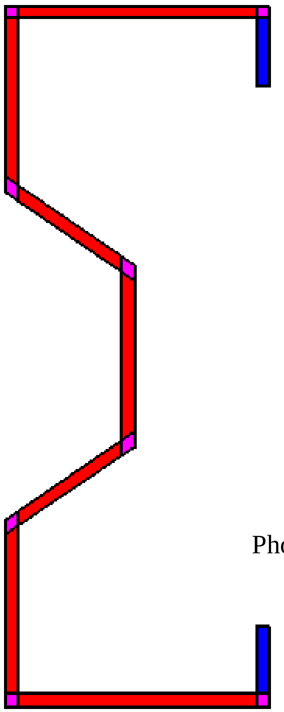


Photo: Author

If we want to calculate its class of cross-section according to EN 1993-1-1, we must remember that this cross-section consist on quasi-webs generally.

Examples of calculations - aluminum

EN 1999-1-1, 6.1.4

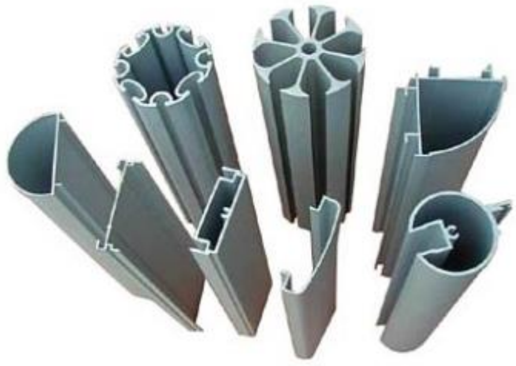


Photo: aluminum-profiles.com

Photo: ecvv.com

Photo: isel.com

<table>
<tr><td colspan="3">Flat outstand element</td><td colspan="2">Flat internal element</td><td rowspan="3">Curved internal element</td><td rowspan="3">Pipe</td></tr>
<tr><td rowspan="2">Symmetical</td><td colspan="2">Unsymmetrical</td><td rowspan="2">No reinforced</td><td rowspan="2">Reinforced</td></tr>
<tr><td>No reinforced</td><td>Reinforced</td></tr>
</table>

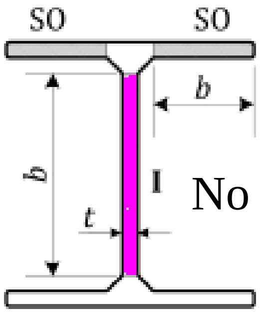
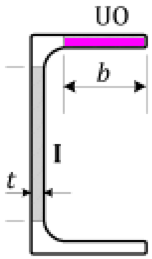


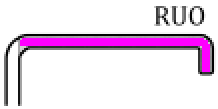
Photo: EN 19991-1-1 fig. 8.1

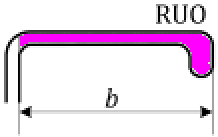
No reinforced flat internal

No reinforced

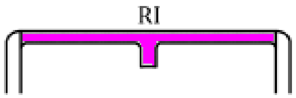


Standard reinforced

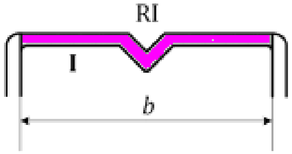




Non-standard reinforced



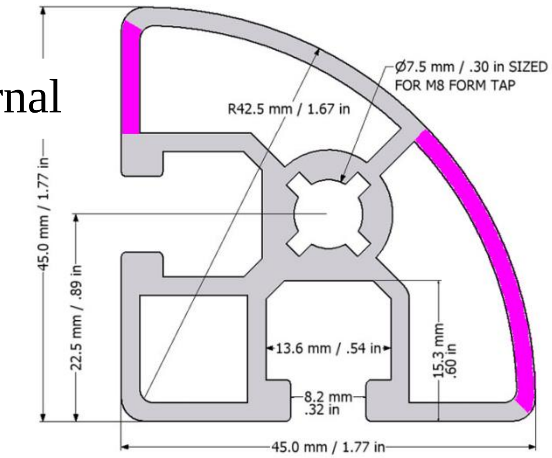
Standard reinforced flat internal (thickness of reinforcement the same as thickness of element)



Non- standard reinforced flat internal

Photo: EN 19991-1-1 fig. 8.1

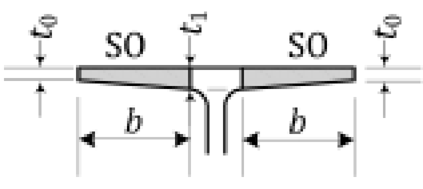
Flat internal



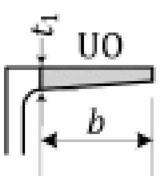
Curved internal

Photo: minitecframing.com

Photo: EN 19991-1-1 fig. 8.1



Symmetrical



Unsymmetrical

Additionally, important is type of local instability

Ist type: total, element + reinforcement

Flat internal and outstand reinforced:

3 types of instability

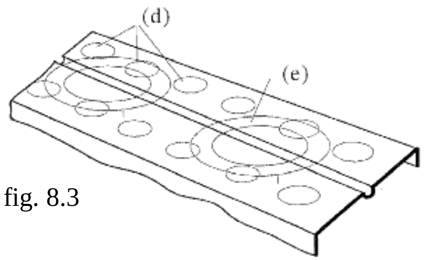


Photo: EN 1999-1-1 fig. 8.3

IIIrd type: complex, Ist + IInd

IInd type: element without reinforcement

Several different formulas to calculate slenderness

EN 1999-1-1 tab 8.1

Table 8.1 — Slenderness parameters β_2/ε and β_3/ε [a]

	Buckling Class according to Table 5.3 and Table 5.4	Internal part		Outstand part	
		β_2/ε	β_3/ε	β_2/ε	β_3/ε
Without welds	A	16	22	4,5	6
	B	16	20	4,5	5,5
	C	16	18	4,5	5
With welds	A	13	18	4,0	5
	B	13	16,5	3,8	4,5
	C	13	15	3,5	4

[a] with for (a): $\varepsilon = \sqrt{\frac{250}{f_o}}$ with f_o in MPa

Formulas of resistance

Steel - different formulas for different class of cross-section

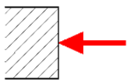
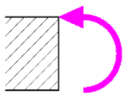
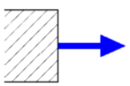
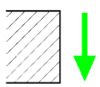
LOAD	I^{st} class	II^{nd} class	III^{rd} class	IV^{th} class
	$N_{Ed} / N_{c,Rd\,(1\text{-}3)} \leq 1,0$			$N_{Ed} / N_{c,Rd\,(4)} \leq 1,0$
	$M_{Ed\,(1)} / M_{Rd\,(1\text{-}2)} \leq 1,0$	$M_{Ed} / M_{Rd\,(1\text{-}2)} \leq 1,0$	$M_{Ed} / M_{Rd\,(3)} \leq 1,0$	$M_{Ed} / M_{Rd\,(4)} \leq 1,0$
	$N_{Ed} / N_{t,Rd} \leq 1,0$			
	$V_{Ed} / V_{Rd\,(1\text{-}3)} \leq 1,0$			$V_{Ed} / V_{Rd\,(4)} \leq 1,0$

Photo: Author

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

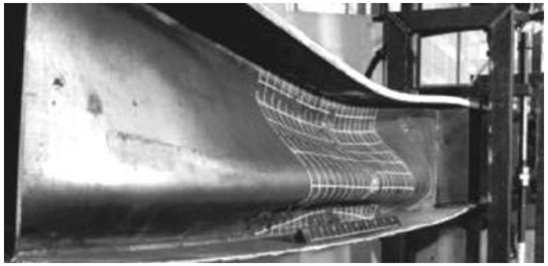
IVth class of cross-section:

Effective cross-section, local instability

Example of calculations: Laboratory #2

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013



Rys: Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

IIIrd class of cross-section:

Resistance for bending moment depends on elastic sectional modulus

$W_{el,\,y}$

Notations pages 104-108 / Bezeichnungen Seiten 104-108

Désignation Designation Bezeichnung		Valeurs statiques / Section properties / Statis							
		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse		
	G kg/m	I_y cm^4	$W_{el.y}$ cm^3	$W_{pl.y}$♦ cm^3	i_y cm	A_{vz} cm^2	I_z cm^4	$W_{el.z}$ cm^3	$W_{pl.z}$ cm^3
IPE A 100	6.9	141.2	28.81	32.98	4.01	4.44	13.12	4.77	7.
IPE 100	8.1	171.0	34.20	39.41	4.07	5.08	15.92	5.79	9.
IPE A 120	8.7	257.4	43.77	49.87	4.83	5.41	22.39	7.00	10.
IPE 120	10.4	317.8	52.96	60.73	4.90	6.31	27.67	8.65	13.

Photo: europrofil.lu

I[st] and II[nd] class of cross-section:

Resistance for bending moment depends on plastic sectional modulus

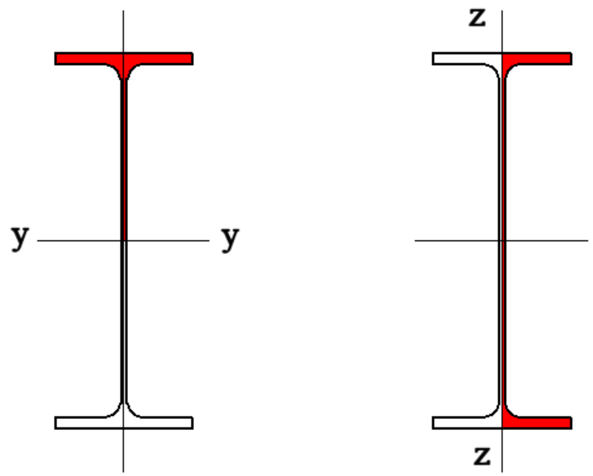
$W_{pl,\,y}$ (Laboratory #1)

Notations pages 104-108 / Bezeichnungen Seiten 104-108

Désignation Designation Bezeichnung	G kg/m	I_y cm⁴	$W_{el.y}$ cm³	$W_{pl.y}$♦ cm³	i_y cm	A_{vz} cm²	I_z cm⁴	$W_{el.z}$ cm³	$W_{pl.z}$ cm³
		Valeurs statiques / Section properties / Statis							
		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse		
IPE A 100	6.9	141.2	28.81	32.98	4.01	4.44	13.12	4.77	7.
IPE 100	8.1	171.0	34.20	39.41	4.07	5.08	15.92	5.79	9.
IPE A 120	8.7	257.4	43.77	49.87	4.83	5.41	22.39	7.00	10.
IPE 120	10.4	317.8	52.96	60.73	4.90	6.31	27.67	8.65	13.

Photo: europrofil.lu

Photo: Author



$$W_{pl,\,y} = 2\,S_y\,(1/2\ I)$$

The most often situation:

- VIth class: welded I-beams, cold-formed sections;
- IIIrd class: welded I-beams, hot-rolled I-beams (rare), cold-formed sections;
- IInd clas: hot-rolled I-beams, R / C HS;
- Ist class: hot-rolled I-beams, R / C HS.

$$N_{c,Rd\,(1\text{-}3)} = A\, f_y \,/\, \gamma_{M0}$$

$$N_{c,Rd\,(4)} = A_{eff}\, f_y \,/\, \gamma_{M0}$$

$$M_{Rd\,(1\text{-}2)} = W_{pl}\, f_y \,/\, \gamma_{M0}$$

$$M_{Rd\,(3)} = W_{el}\, f_y \,/\, \gamma_{M0}$$

$$M_{Rd\,(4)} = W_{eff}\, f_y \,/\, \gamma_{M0}$$

$$V_{Rd\,(1\text{-}3)} = A_v\, f_y \,/\, (\gamma_{M0}\, \sqrt{3})$$

$V_{Rd\,(4)}$ = impact of local instability + nonlinear relations with $M_{Rd\,(4)}$, $F_{Rd\,(4)}$ and $N_{c,Rd\,(4)}$

$$N_{t,Rd} = A\, f_y \,/\, \gamma_{M0}$$

Bending moment only:

$$M_{Ed} / M_{Rd} \leq 1,0$$

Class of cross-section	IVth	IIIrd	IInd	Ist
Distribution of σ across cross-section	Elastic		Plastic	
Effects	Local instability	Resistance of cross-section		
M_{Ed} =	From „normal" static calculations of structure			From special recalculation to new static scheme and new loads (**redistribution**)
M_{Rd} =	$W_{eff}\ f_y / \gamma_{M0}$	$W_{el}\ f_y / \gamma_{M0}$	$W_{pl}\ f_y / \gamma_{M0}$	

W_{eff} – IInd laboratory

W_{el} – tables for design

W_{pl} – Ist laboratory

Aluminum – analogously: four classes of cross-section, various resistancef for various classes:

- local instability for IVth class;
- elastic range for IIIrd class;
- plastic range for IInd class;
- plastic range and plastic redistribution of bending moment for Ist class.

Examination issues

Algorithm of calculations for cross-section class

Similiarities and differences for checking resistance of Ist and IInd class of cross-section

Similiarities and differences for checking resistance of IIIrd and IVth class of cross-section

Hot-rolled and welded I-beams, similiarities and differences

Plastic hinge - przegub plastyczny
Support - podpora
Span - przęsło
Local buckling - niestateczność miejscowa

Thank you for attention

tmichal@pk.edu.pl