Metal Structures

Laboratory II

Geometrical characteristics of welded I-beam

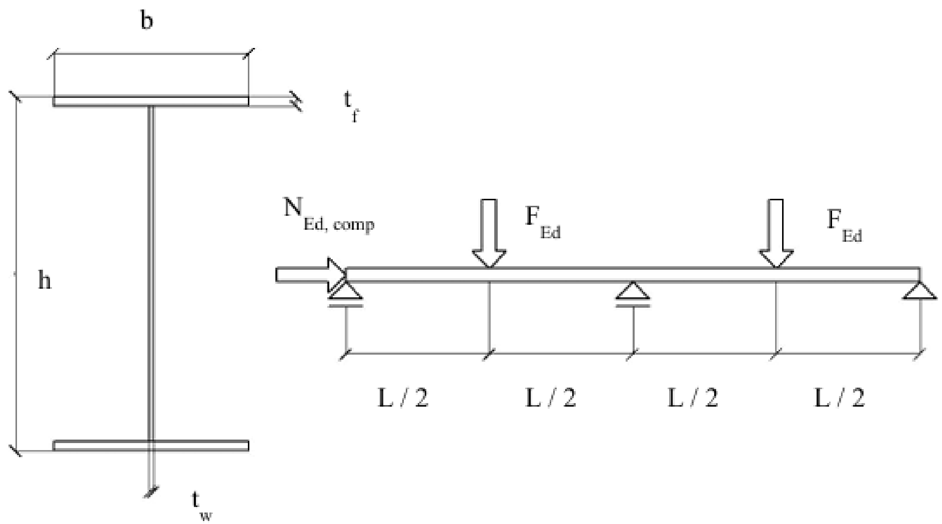
LABORATORY OBJECTIVE’S

Analysis of very slender cross-sections; steel and aluminium;

Student..

Topic:

1. Analyse effects of local instabilities for beam as follow; steel S 275

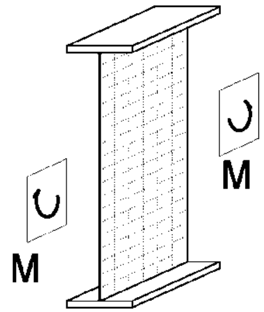


h [mm]	t_w [mm]	b [mm]	t_f [mm]	F_{Ed} [kN]	$N_{Ed, comp}$ [kN]	L [m]

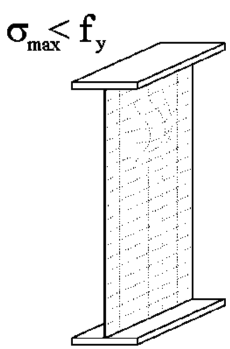
Grade, date ....

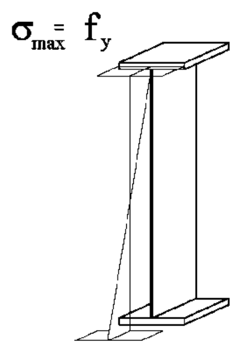
Photo: Author

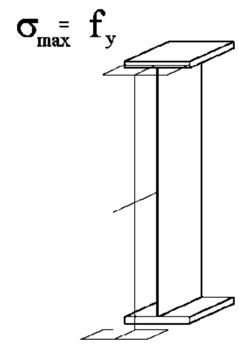
Classes of cross-section - different resistance for local instabilities



→ #3 / 85







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Different formulas of R

Geometrical characteristics, presented on previous Lab (A, J_y, $W_{y, el}$), could be the same for various shapes of cross-section:

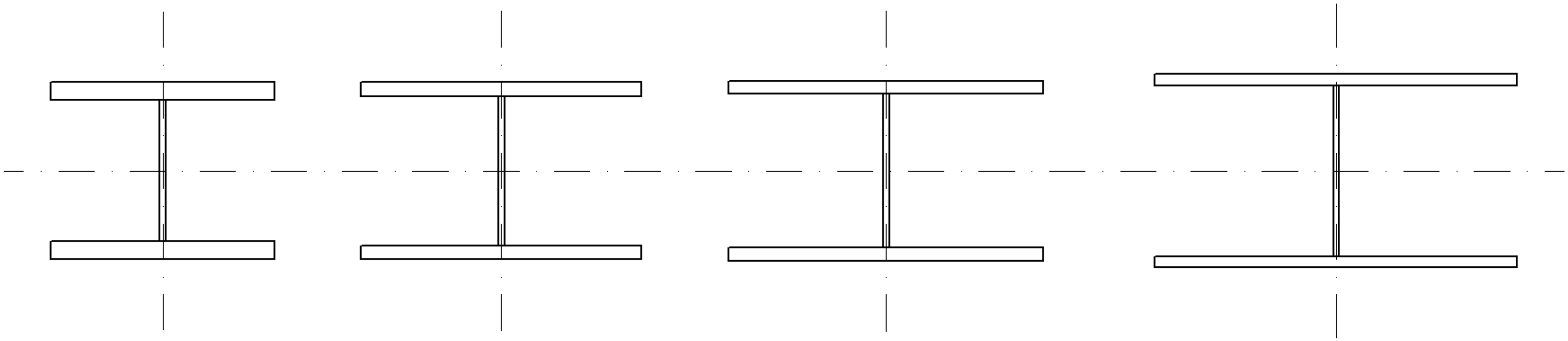


Photo: Autor

But for metal structures, real behavior of element under actions depends not only on geometrical characteristisc, mentioned above. Very important is slenderness of branches of cross-section.

First and second: massive cross-sections; $W_{y, pl}$ is taken into calculation (Lab #1).

Third: „average" cross-sections; $W_{y, el}$ is taken into calculation (Lab #1).

Last: slender cross-section; behaviour like as chart of paper under action. Many local instavilities must be analised.

Four types of local instability for slender cross-section:

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

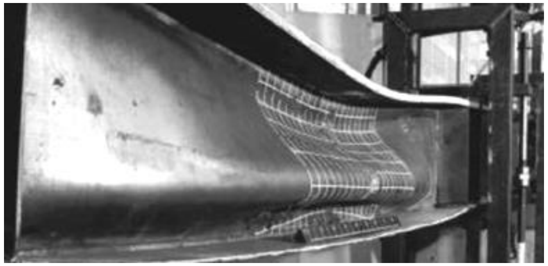
Two types effects of compressive axial stresses.

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Effect of shear stresses.

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Effect of tranversal force applied in point.



Rys: Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

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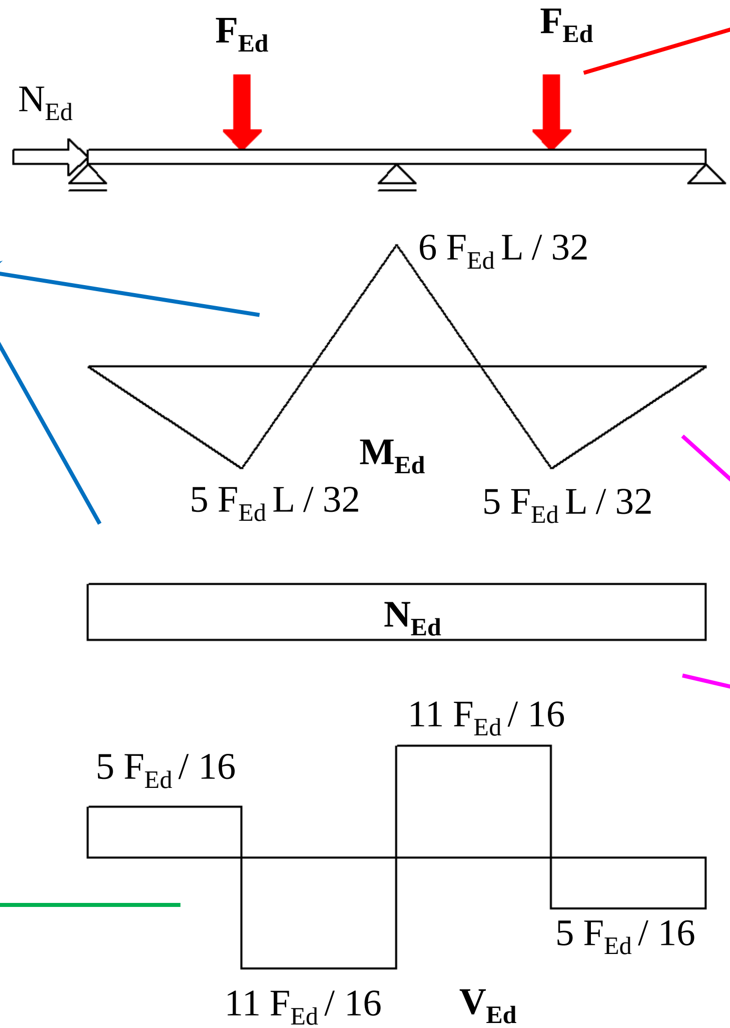

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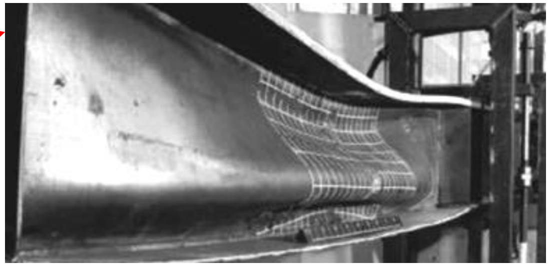

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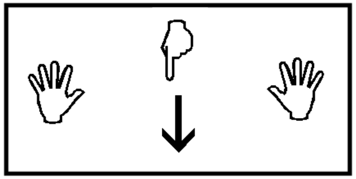

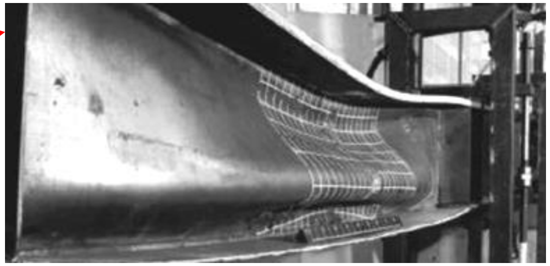

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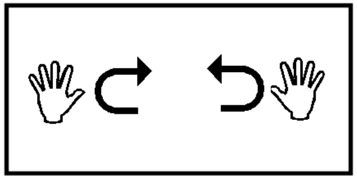
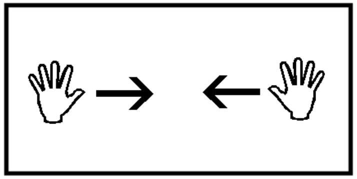




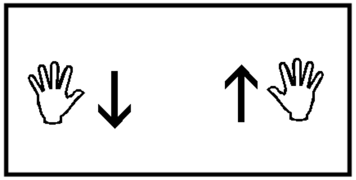


Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Photo: Autor

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Calculation of various slenderness of cross-sections (classes of cross-section) will be presented on Lec #4.

Elastic and plastic sectional moduluses were presented on Lab #1.

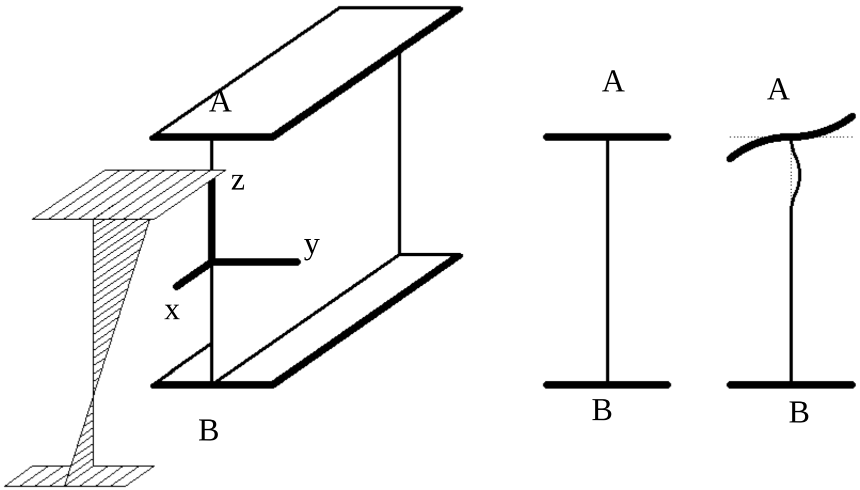
Detailing explanation about various types of local instabilities will be presented on Lec #12.

Today, only one case (the most complicated) of many local instabilities will be presented, this means first effect of axial stresses from compressive axial force and bending moment.

Instability of flange under compressive axial stresses, instability of web under compressive axial stresses

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Photo: Author



Reason: axial stresses σ_x (from bending moment and / or compressive axial force). Firts type flange-web: lost of stability for both sub-patrs (web and flange) is independent; behavior of flange doesn’t affected on web and behavior of web doesn’t affected on flange. Position of point A not changes after lost of stability.

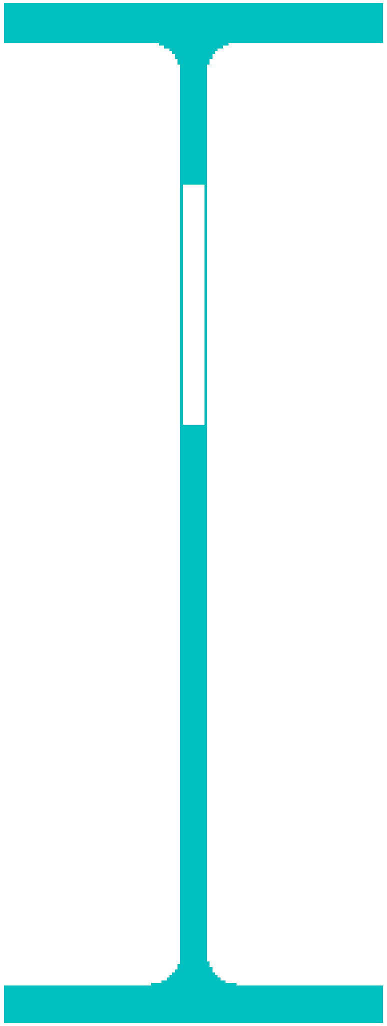


Photo: Autor

The part that is subject to local instability is ignored. Effective cross-section is a fictitious cross-section from which fragments of flanges / web (subject to instability) have been removed.

When considering the effective cross-section, it is necessary to find the effective cross-sectional area, effective center of gravity, effective moment of inertia and effective sectional modulus. More information will be presented on Lab #2.

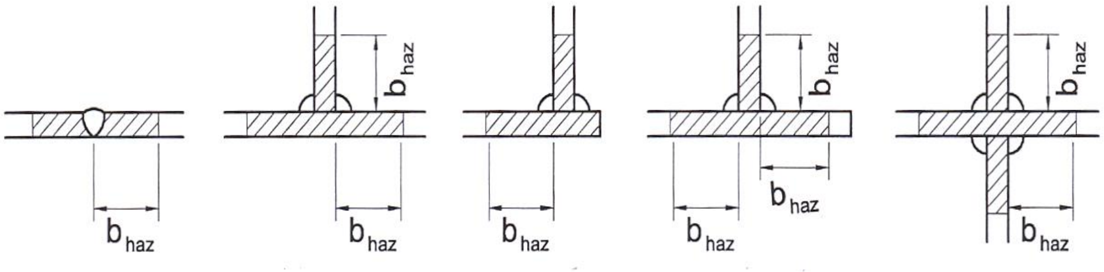
$$W_{eff} \leq W_{el} < W_{pl}$$

$$A_{eff} \leq A$$

→ Lab #1 / 47

Such phenomenon – effect of local instability – is analised for **steel** and **aluminum** crosss-sections. Of course, thid phenomenon existed on compressed part of cross-section only.

Photo: Autor



Another phenomenon for **aluminum** I-beam is reduction of strength of material through welding process. This phenomenon is important for **both** (compressed and tensed) part of the cross-section.

As in the case of steel, alloying elements are added to aluminum alloys.

Alloying elements → lec. #2

There is very important type of "heat treating" for aluminium: precipitation hardening (age hardening). The effect of process is increasing of strength and decreasing of plasticity. Increasing of aluminum strength can be up to few dozens %.

Precipitation hardening is devastated during welding. Because of this, heat affected zones (HAZ) must be analysed during calculation of aluminum welding.

HAZ → lab. #2

→ #1 / 93

Strength of aluminum alloys comes mainly from special heat treating. High temperatures - for example during welding - devastate this effect.

Heat affected zones (HAZ) for aluminum - reduction of strength parameters as a result of welding

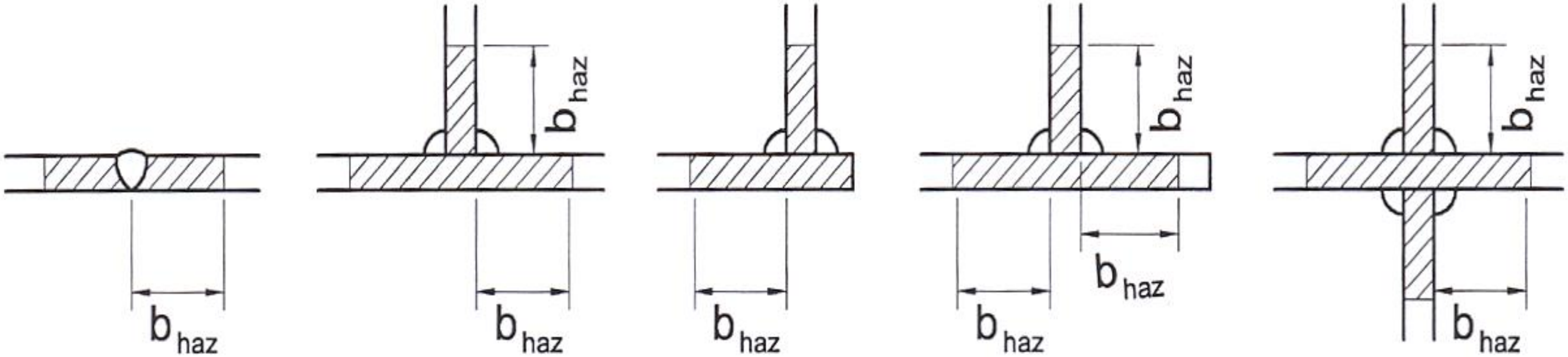


Photo: Author

Reduction factor

$1{,}0 \geq \rho_{haz} \geq 0{,}28$

Resistance R of cross-section is depend on geometrical characteristic X and strength f; ρ is reduction factor:

$$R = X \cdot f$$

For instability we must take into consideration reducted part of cross-section:

$$R = (X \cdot \rho_1) \cdot f$$

For welding joints in aluminum, strength of material locally decreases:

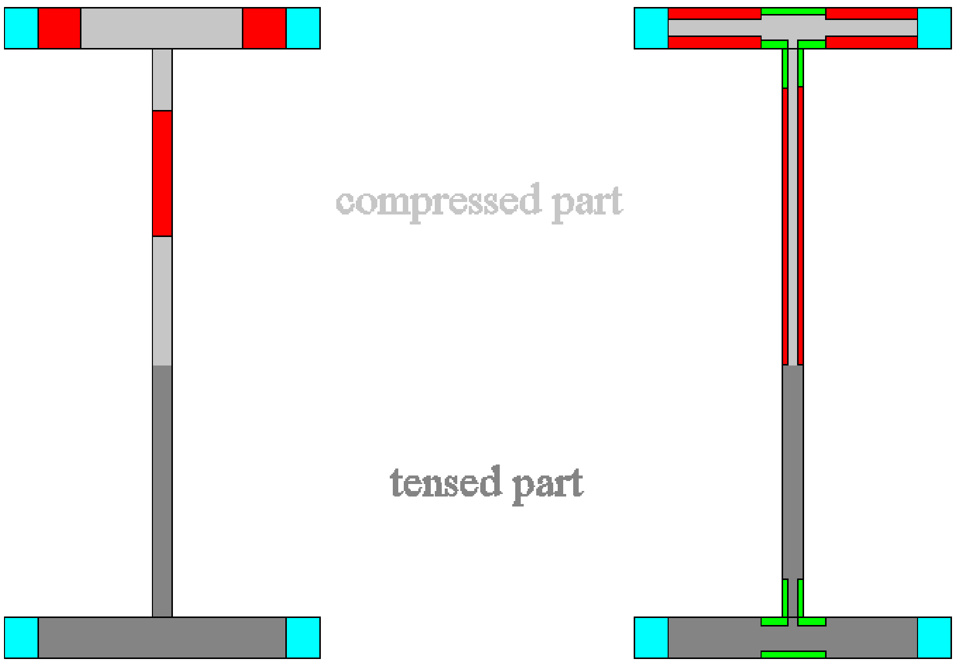
$$R = X \cdot (f \cdot \rho_2) = (X \cdot \rho_2) \cdot f$$

There is way of calculation for both phenomenon: **strength is the same** in each point of cross-section, but **geometry is reduced**:

$$R = (X \cdot \rho) \cdot f$$

We reduced geometry for different parts of cross-section in calculations:

- for steel welded I-beam we reduced width of elements, $d_{eff} = d_0\ \rho$
- for aluminum welded I-beam we reduced thickness of elements, $t_{eff} = t_0\ \rho$
- for cold-formed cross-sections we reduced thickness of elements, $t_{eff} = t_0\ \rho$



Steel Aluminum

There is effective geometry of cross-section after reduction

(without blue parts for ended part of too wide flanges, without red parts for instability and without green parts for welding):

$$A_0 \rightarrow A_{eff}$$

$$W_{y0} \rightarrow W_{y\ eff}$$

Photo: Autor

Aluminum

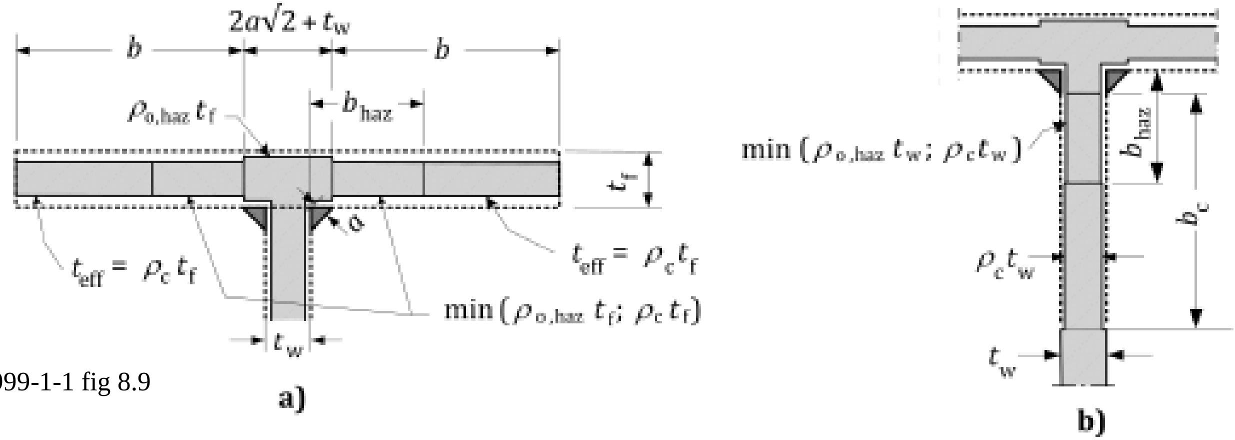


Photo: EN 1999-1-1 fig 8.9

Reduction factor for tensed welded zone - depends on type of alloy;

Reduction factor for non-welded compressed part - depends on values of stresses;

Reduction factor for compressed welded zone = min (both above) (EN 1999-1-1 8.2.5.2.(2)).

Steel; steps of reductions:

reduction of too wide flanges (both flanges; only one step; symmetrical cross-section, after reduction recalculations for geometrical characteristics)

reduction of compressed flange (only one step; asymmetrical cross-section; after reduction recalculations for centre of gravity and geometrical characteristics)

reduction of compressed part of web (few steps; asymmetrical cross-section; after reduction recalculations for centre of gravity and geometrical characteristics)

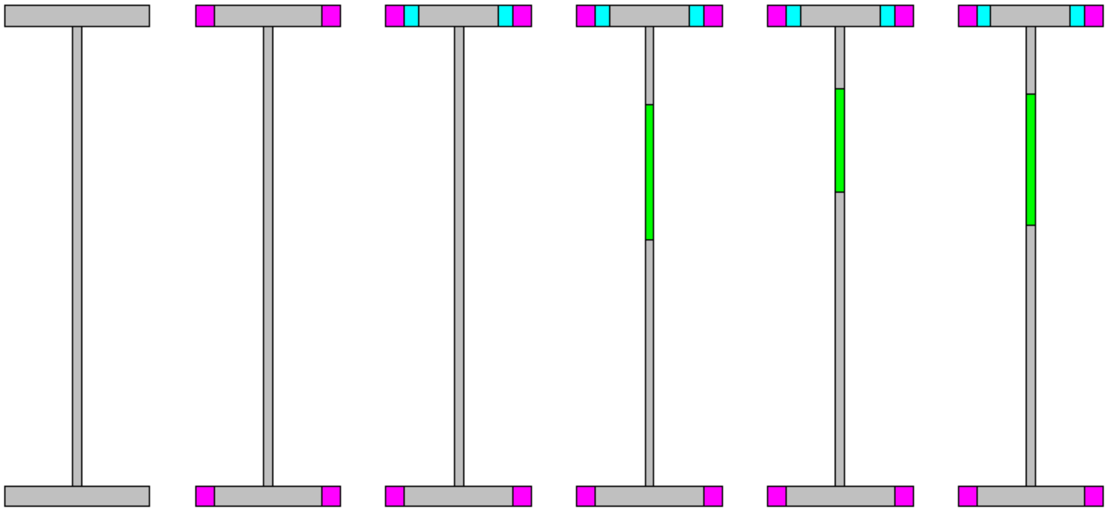


Photo: Autor

Alumnium; steps of reductions:

reduction of welded zones (both sides; only one step; symmetrical cross-section, after reduction recalculations for geometrical characteristics)

reduction of too wide flanges (both flanges; only one step; symmetrical cross-section, after reduction recalculations for geometrical characteristics)

reduction of compressed flange (only one step; asymmetrical cross-section; after reduction recalculation for centre of gravity and geometrical characteristics)

reduction of compressed part of web (few steps; asymmetrical cross-section; after reduction recalculation for centre of gravity and geometrical characteristics)

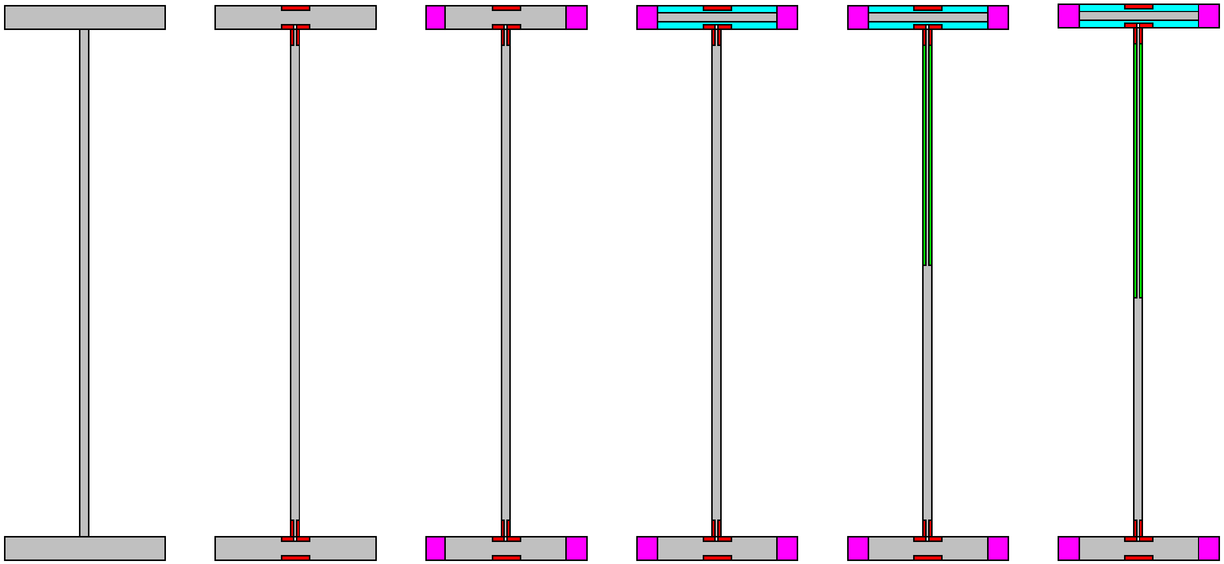


Photo: Autor

There is big inconsequency in Eurocode.

- According to EN 1993-1-5 4.3 (3) and (4), calculation for cases „bending and compressive force” or „bi-axial bending and compressive force” should be made separately, i.e:
  - A_{eff} based on $F_{Ed, comp}$ only;
  - $W_{y, eff}$ based on $M_{Ed, y}$ only;
  - $W_{z, eff}$ based on $M_{Ed, z}$ only.
- But in EN 1993-1-5 tab. 4.1 and tab. 4.2 are presented dictributions of stresses, which come from complex load ($M_{Ed, y} + F_{Ed, comp}$ or $M_{Ed, y} + M_{Ed, z}$).

So, there is not clear, which way of calculation should be applicated.

Example for steel cross-section is presented as for calculation of separated effects $M_{Ed, y}$ and $F_{Ed, comp}$.

Example for aluminum cross-section is presented as for calculation of complex effects $M_{Ed, y} + F_{Ed, comp}$.

Additional problem, concerns method „ $M_{Ed,y} + F_{Ed,comp}$”, is that values of cross-sectional forces change along bar. Values of $M_{Ed,y}$ and $F_{Ed,comp}$ are various in various points, so effective geometry should be various in various cross-sections.

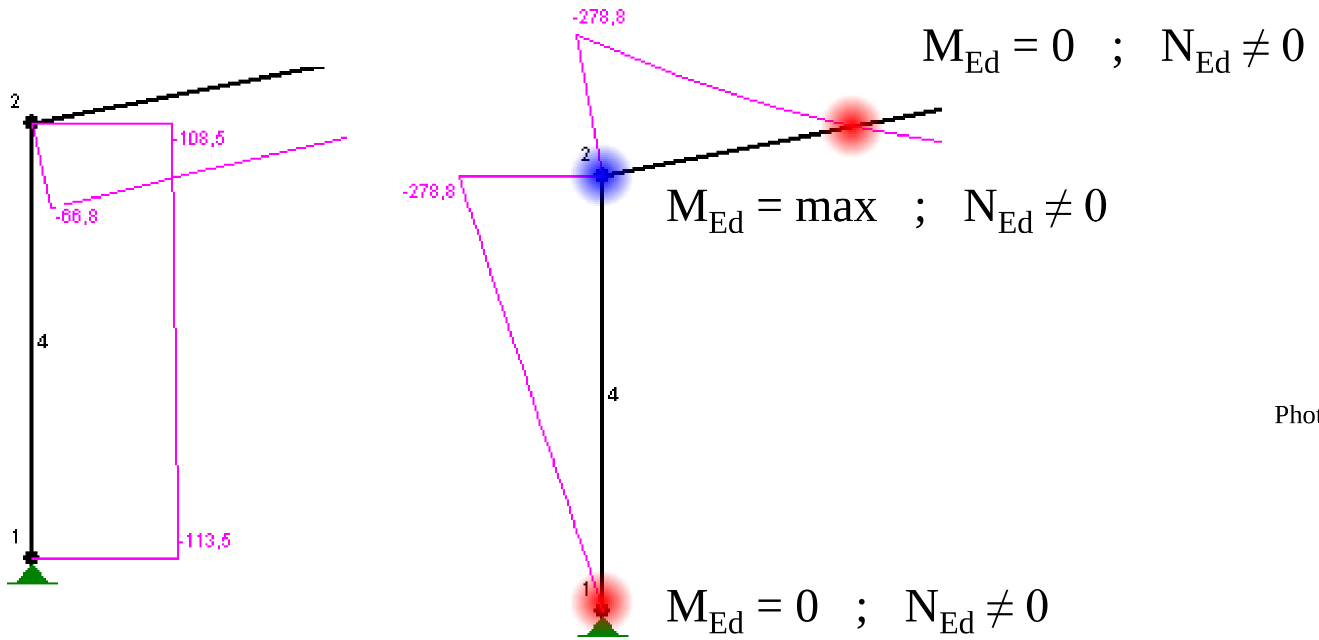


Photo: Author

Simplification:

A_{eff} is calculated for max $F_{Ed,comp}$ and applicated to total member;

W_{eff} is calculated for max $M_{Ed,comp}$ and applicated to total member.

h [mm]	t_w [mm]	b [mm]	t_f [mm]	F_{Ed} [kN]	$N_{Ed, comp}$ [kN]	L [m]
1 300	11	300	20	717,082	64,722	25,00

Example:

steel I-beam

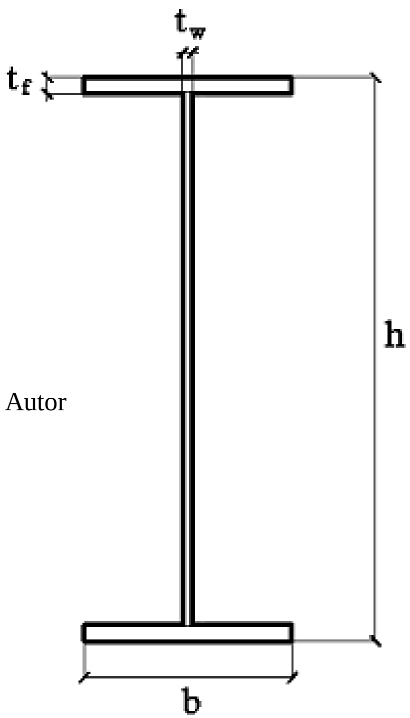


Photo: Autor

Steel S355 → f_y = 355 MPa

Length of beam: 25,0 m

Initial geometry:

A_0 = 258,600 cm^2

$J_{y,\,0}$ = 674 887,800 cm^4

$W_{y,\,0}$ = 10 382,889 cm^3

Point in border web-flange:

$W_{y,\,0,\,wf}$ = 10 712,505 cm^3

Thickness of welds web-flange:

a = 5 mm

F_{Ed} = 717,082 kN

For this statis scheme:

$M_{Ed,\,y} = 6\ F_{Ed}\ L / 32$

$M_{Ed,\,y}$ = 3 361,320 kNm

I^{st} **step:** shear lag effect

Flange – too wide or not?

L_e = length of the beam = 25 m; → $L_e / 50 = 500$ mm

b = width of the flange = 300 mm → half of flange $b_0 = b / 2 = 150$ mm

$b_0 < L_e / 50$ 150 < 500   EN 1993-1-5 p.3.1

Flange is not too wide → shear lag in flanges is not danger→ after I^{st} step geometry not must be recalculated

$b_{eff,\,1} = b_{initial}$

$A_{eff\,1} = A_0 = 258{,}600\ cm^2$

$W_{y\ eff\ 1} = W_{y,\,0} = 10\ 382{,}889\ cm^3$

Shear lag effect

Unfortunate name - it has nothing to do with shear force

Polish definition is better: „wide flange effect”

There is nonlinear distribution of stress diagram for wide flanges. This nonlinear shape can be recalculated to linear → effective width of flanges.

Wide flanges

Narrow flanges or effective width for wide flanges

$$b_0 \rightarrow b_{eff} \quad ; \quad b_{eff} \leq b_0$$

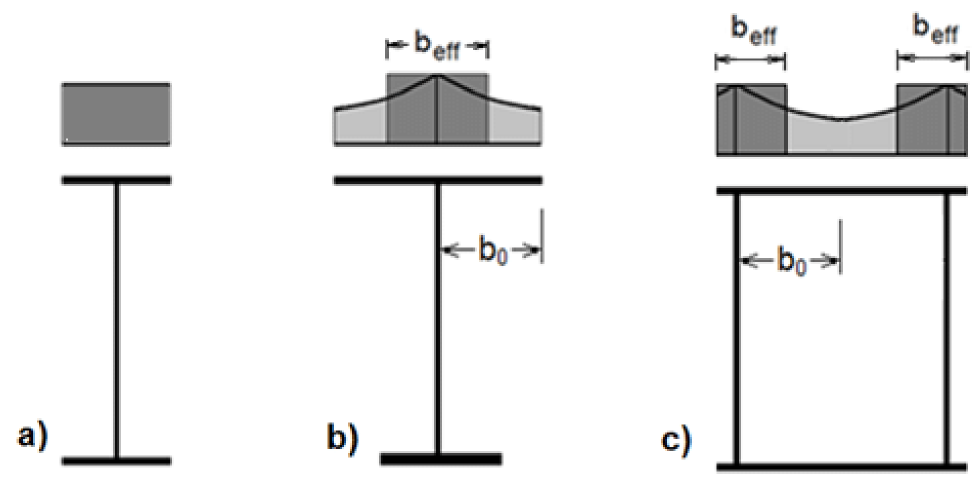


Photo: *Effects of Shear Lag in Steel Box Girders of a Crane Runway*
C. Moga, D. Dragan, R. Nerisanu

Additionally, effective width is different for different points along beam.

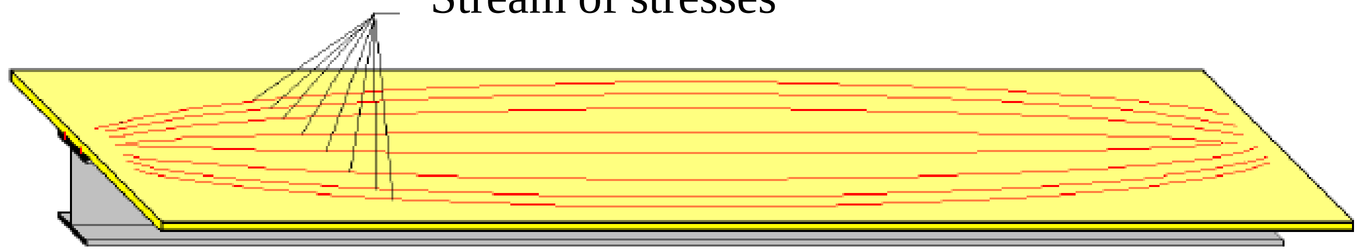
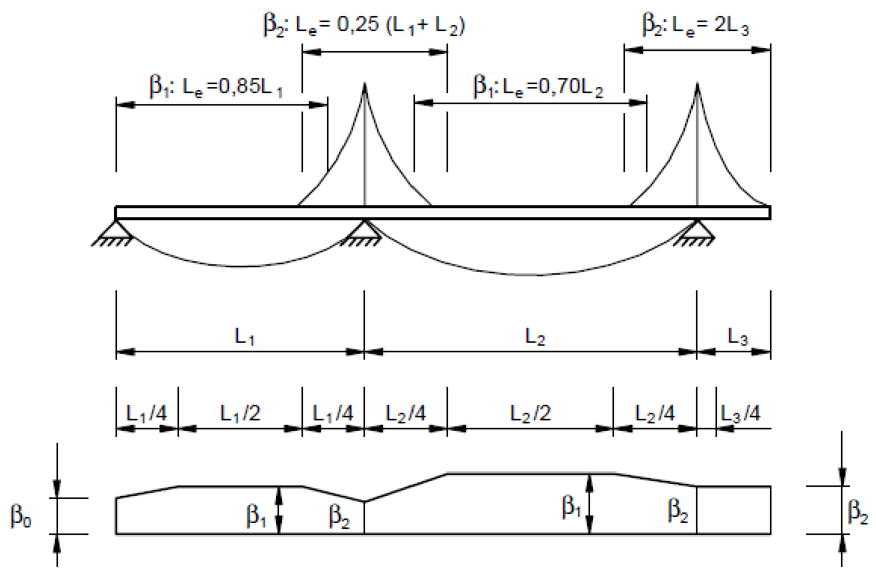


Photo:A Biegus, Projektowanie zespolonych konstrukcji stalowo-betonowych według Eurokodu 4, Politechnika Wrocławska



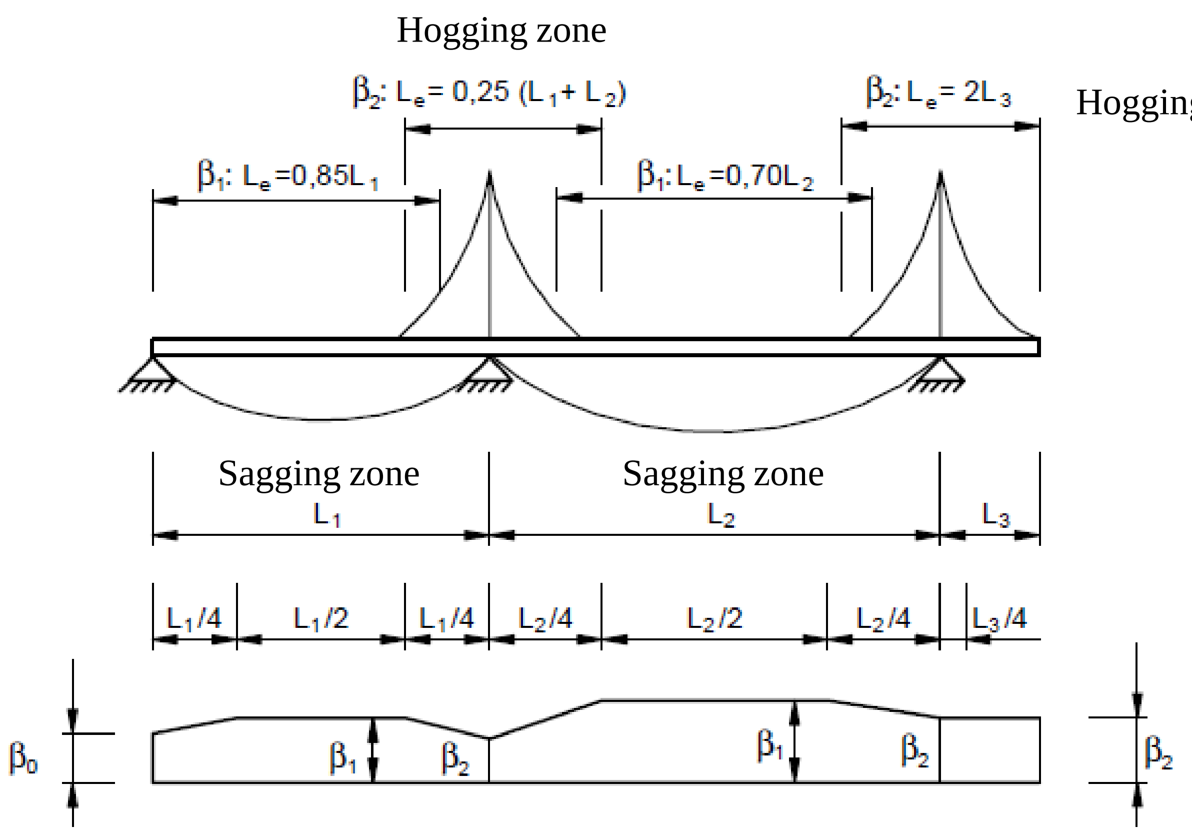
Value of reduction factor depends on static scheme of I-beam and point along beam.

Photo: EN 1993-1-5, fig 3.1

Shear lag effect id not danger in analysed case. But, if **will be**, reduction factor must be applicated.

$b_{eff1} = b_{initial}\,\beta$

β – reduction factor for wide flanges; different value for different part of beam:



For one-span-beam: $L_e = L$,

β for point of the biggest value of M_{Ed} , this means β_1 in sagging zone.

Photo: EN 1993-1-5, fig 3.1

κ	verification	β – value
$\kappa \leq 0{,}02$		$\beta = 1{,}0$
$0{,}02 < \kappa \leq 0{,}70$	sagging bending	$\beta = \beta_1 = \dfrac{1}{1 + 6{,}4\,\kappa^2}$
	hogging bending	$\beta = \beta_2 = \dfrac{1}{1 + 6{,}0\left(\kappa - \dfrac{1}{2500\,\kappa}\right) + 1{,}6\,\kappa^2}$
$> 0{,}70$	sagging bending	$\beta = \beta_1 = \dfrac{1}{5{,}9\,\kappa}$
	hogging bending	$\beta = \beta_2 = \dfrac{1}{8{,}6\,\kappa}$
all κ	end support	$\beta_0 = (0{,}55 + 0{,}025 / \kappa)\,\beta_1$, but $\beta_0 < \beta_1$
all κ	cantilever	$\beta = \beta_2$ at support and at the end

$\kappa = \alpha_0\, b_0 / L_e$ with $\alpha_0 = \sqrt{1 + \dfrac{A_{s\ell}}{b_0 t}}$

in which $A_{s\ell}$ is the area of all longitudinal stiffeners within the width b_0 and other symbols are as defined in Figure 3.1 and Figure 3.2.

EN 1993-1-5, tab 3.1

$L_e = 25\ m$

No longitudinal stiffeners $\rightarrow A_{sl} = 0 \rightarrow \alpha_0 = 1$

$b_{initial} = 300\ mm$

$b_0 = 150\ mm$

$\kappa = 1 \cdot 150 / 25\,000 = 0{,}006 \rightarrow \kappa < 0{,}02 \rightarrow \beta = 1{,}0$

$b_{eff,\,1} = b_{initial} \cdot \beta = b_{initial} \cdot 1{,}0 = b_{initial}$

No change of geometry in analysed case (the same conclusion as on #t / 23)

$A_{eff\ 1} = A_0 = 258{,}600\ cm^2$

$W_{y\ eff\ 1} = W_{y,\,0} = 10\,382{,}889\ cm^3$

Generally: it is possible that we need to reduce width of flanges after first step due to shear lag effect. This will entail necessity to recalculate effective geometry after first step. We will obtain new values of $A_{eff,1}$ and $W_{y\,eff,1}$. This reduction applies **equally to both flanges**, so initially bi-symmetric cross-section will still remain bi-symmetrical.

II step
Flange under compression

Stress distribution in flange is identical, regardless of whether we take into account compressive axial force or the bending moment: constant value of compression in compressed flange. One common analysis will be made. **Stress distribution is calculated for effective geometry after I step: $A_{eff,1}$, $W_{y,eff,1}$**

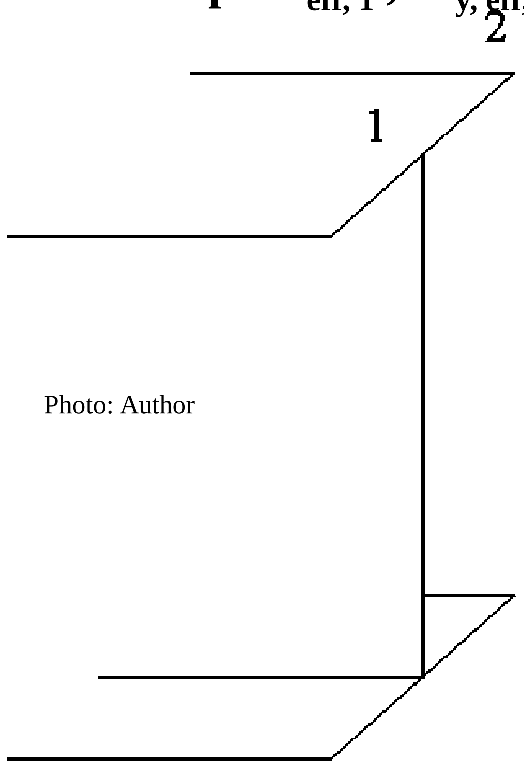
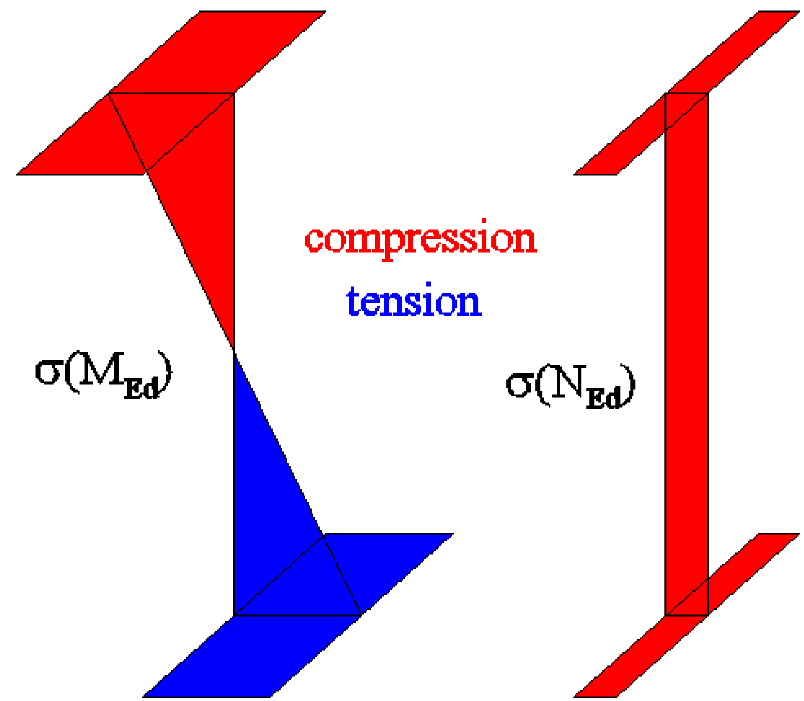
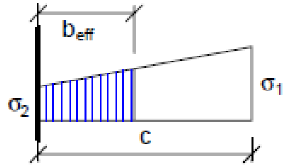
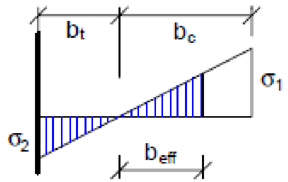


Photo: Author



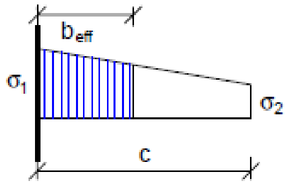
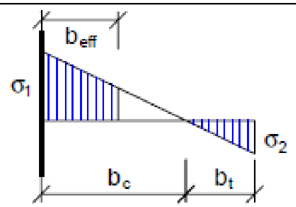
Stresses in flange – „blue part” remains after reduction aa effective geometry

Stress distribution (compression positive)	Effective^p width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\, c$
	$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, c\, /\, (1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	0	-1	$1 \geq \psi \geq -3$
Buckling factor k_σ	0,43	0,57	0,85	$0{,}57 - 0{,}21\psi + 0{,}07\psi^2$

Compression of flange; nex to the web ≤ at the one of flange

Compression and tension of flange; tension nex to the web

Stress distribution (compression positive)	Effective^p width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\, c$
	$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, c\, /\, (1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
Buckling factor k_σ	0,43	$0{,}578\, /\, (\psi + 0{,}34)$	1,70	$1{,}7 - 5\psi + 17{,}1\psi^2$	23,8

Compression of flange; nex to the web ≥ at the one of flange

Compression and tension of flange; tension at the end of flange

Photo: EN 1993-1-5, tab 4.2

$\psi = \sigma_2 / \sigma_1$ = tension / compression or **smaller compression / compression**

Flange $\rightarrow \sigma = const \rightarrow \sigma_1 = \sigma_2 \rightarrow \psi = \sigma_2 / \sigma_1 = 1{,}0$

$\psi = 1{,}0 \rightarrow$ table $\rightarrow k_\sigma = 0{,}43$

Steel S355 $\rightarrow f_y = 355$ MPa

$\varepsilon = \sqrt{(235 / f_y)} \rightarrow \varepsilon = \sqrt{(235 / 355)} = 0{,}814$

ρ – reduction factor for compression elements

– internal compression elements:

Internal ≡ web

$\rho = 1{,}0$ for $\bar{\lambda}_p \leq 0{,}673$

$$\rho = \frac{\bar{\lambda}_p - 0{,}055\,(3+\psi)}{\bar{\lambda}_p^2} \leq 1{,}0 \quad \text{for } \bar{\lambda}_p > 0{,}673\text{ , where } (3+\psi) \geq 0 \qquad (4.2)$$

– outstand compression elements:

Outstand ≡ flange

$\rho = 1{,}0$ for $\bar{\lambda}_p \leq 0{,}748$

$$\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^2} \leq 1{,}0 \quad \text{for } \bar{\lambda}_p > 0{,}748 \qquad (4.3)$$

where $\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28{,}4\,\varepsilon\,\sqrt{k_\sigma}}$

ψ is the stress ratio determined in accordance with 4.4(3) and 4.4(4)

$\bar{b}$ is the appropriate width to be taken as follows (for definitions, see Table 5.2 of EN 1993-1-1)

b_w	for webs;
b	for internal flange elements (except RHS);
$b - 3\,t$	for flanges of RHS;
c	for outstand flanges;
h	for equal-leg angles;
h	for unequal-leg angles;

EN 1993-1-5 p.4.4

b_{eff1} – width of flange; a – thickness of welds; t_w – thickness of web; t_f – thickness of flange;

$c = (b - 2\ a \sqrt{2} - t_w) / 2 = (300 - 14 - 11) / 2 = 137{,}5$ mm

$\lambda_p = (c / t_f) / (28{,}4\ \varepsilon \sqrt{k_\sigma}) = (137{,}5 / 20) / (28{,}4 \cdot 0{,}814 \cdot \sqrt{0{,}43}) = 0{,}454 \rightarrow$
$\rightarrow$ EN 1993-1-5 (4.3) $\rightarrow 0{,}454 < 0{,}748 \rightarrow \rho = 1{,}0$

$b_{eff,\ 2} = \rho\ b_{eff,\ 1} = b_{eff,\ 1} = b_{initial}$

No reduction of the compressed flange → after the second step, the geometry is the same as at the beginning:

$A_{eff\ 2} = A_0 = 258{,}600\ cm^2$

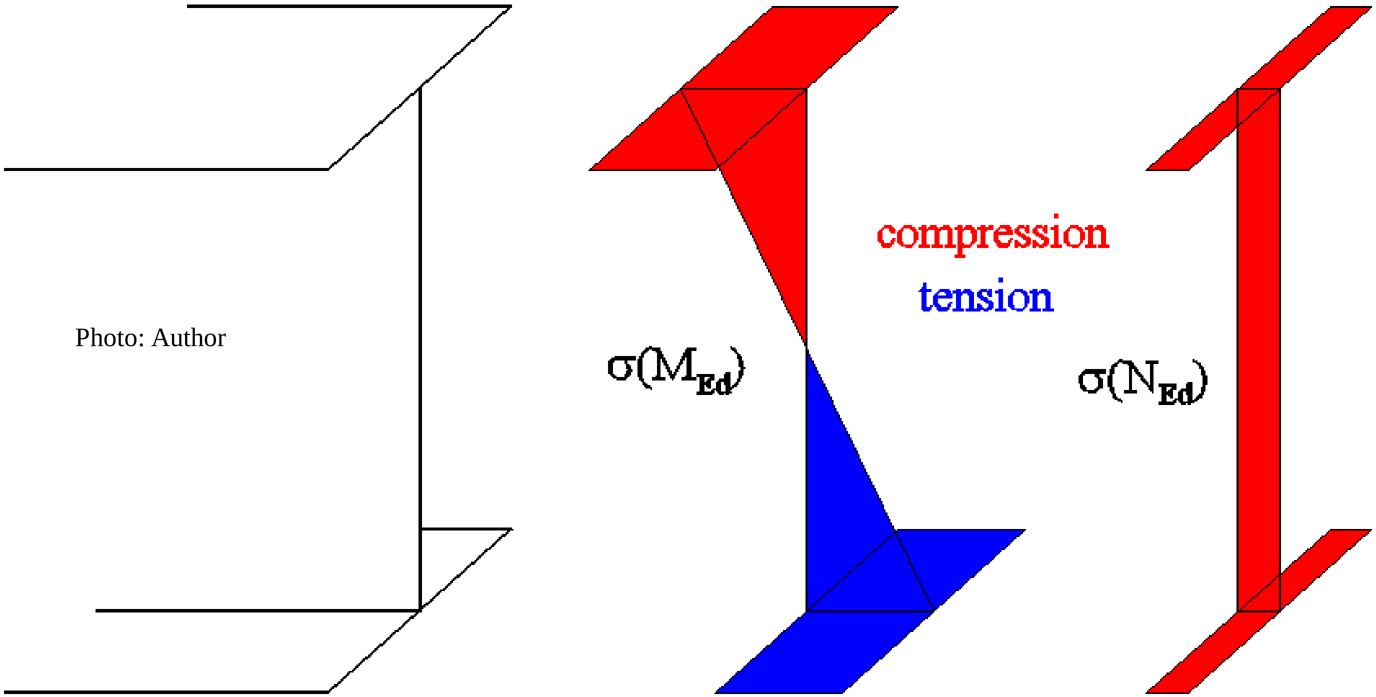
$W_{y\ eff\ 2} = W_{y,\ 0} = 10\ 382{,}889\ cm^3$

Generally: it is possible that we need to reduce width of flange after second step due to lost of stability under axian compressive stresses. This will entail necessity to recalculate effective geometry after second step. This reduction applies for **flange under compression only** (sometimes top, sometimes bottom, sometimes both), so initially bi-symmetric cross-section will lose symmetry with the horizontal axis. We will obtain new values for unsymmetrical cross-section: $A_{eff,2}$, $W_{y, top, eff, 2}$, $W_{y, bottom, eff, 2}$

III step

Web under compression

Stress distribution in web various for effects from bending moment and axial force. Calculation will be made separately, for axial force and for bending moment. **Stress distribution is calculated for effective geometry after II step: $A_{eff, 2}$, $W_{y, eff, 2}$**



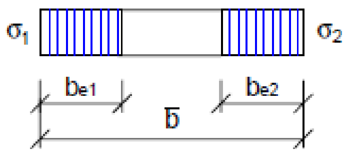
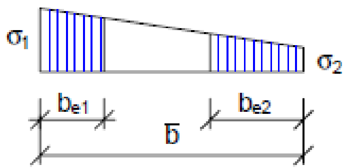
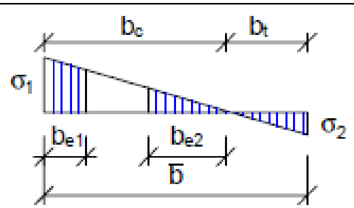
Axial force:

$\sigma_{max} = N_{Ed} / A_{eff,\,2} = 64{,}722\ kN / 258{,}600\ cm^2 = 2{,}503\ MPa = const$

Bending moment, max value of stress in web (on border between web and flange):

$\sigma_{max} = \pm M_{y,\,Ed} / W_{y,\,eff,\,2,\,wf} = \pm 3\ 361{,}320\ kNm / 10\ 712{,}505\ cm^3 =$
$= \pm 313{,}775\ MPa$

Stresses in flange – „blue part" remains after reduction aa effective geometry

Stress distribution (compression positive)	Effective^p width b_{eff}
	$\underline{\psi = 1}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = 0{,}5\ b_{eff}$ $\quad b_{e2} = 0{,}5\ b_{eff}$
	$\underline{1 > \psi \geq 0}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = \dfrac{2}{5-\psi} b_{eff}$ $\quad b_{e2} = b_{eff} - b_{e1}$
	$\underline{\psi < 0}$: $b_{eff} = \rho\ b_c = \rho\ \bar{b} / (1-\psi)$ $b_{e1} = 0{,}4\ b_{eff}$ $\quad b_{e2} = 0{,}6\ b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8{,}2 / (1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98\ (1 - \psi)^2$

Axial compression

Eccentric compression – whole web under compressed

Eccentric compression – part of web compressed, part tensed

EN 1993-1-5, tab 4.1

$\psi = \sigma_2 / \sigma_1$ = tension / compression or **smaller compression / compression**

Axial force $\rightarrow \sigma = const \rightarrow \sigma_1 = \sigma_2 \rightarrow \psi = \sigma_2 / \sigma_1 = 1{,}0$

$\Psi = 1{,}0 \rightarrow$ table $\rightarrow k_\sigma = 4{,}000$

Part of web under compression: total, $h_{wc,\,3} = 1\,260$ mm

Bending moment $\rightarrow \sigma_1 = -\sigma_2 \rightarrow \psi = \sigma_2 / \sigma_1 = -1{,}0$

$\Psi = -1{,}0 \rightarrow$ table $\rightarrow k_\sigma = 23{,}900$

Part of web under compression: half, $h_{wc,\,3} = 630$ mm

Steel S 355 $\rightarrow f_y = 355$ MPa

$\varepsilon = \sqrt{(235 / f_y)} \rightarrow \varepsilon = \sqrt{(235 / 355)} = 0{,}814$

Slenderness for axial force:

b – total height of web; a – thickness of weld; t_w – thickness of web;

$c = b - 2\, a \sqrt{2} = 1\,260 - 14 = 1\,246$ mm

$\lambda_p = (c / t_w) / (28{,}4\, \varepsilon \sqrt{k_\sigma}) = (1\,246 / 11) / (28{,}4 \cdot 0{,}814 \cdot \sqrt{4{,}000}) = 2{,}450$

Slenderness for bending moment:

b – total height of web; a – thickness of weld; t_w – thickness of web;

$c = b - 2\, a \sqrt{2} = 1\,260 - 14 = 1\,246$ mm

$\lambda_p = (c / t_w) / (28{,}4\, \varepsilon \sqrt{k_\sigma}) = (1\,246 / 11) / (28{,}4 \cdot 0{,}814 \cdot \sqrt{23{,}900}) = 1{,}002$

ρ – reduction factor for compression elements

Internal ≡ web

– internal compression elements:

$\rho = 1{,}0$ for $\bar{\lambda}_p \le 0{,}673$ *

$$\rho = \frac{\bar{\lambda}_p - 0{,}055\,(3+\psi)}{\bar{\lambda}_p^2} \le 1{,}0 \quad \text{for } \bar{\lambda}_p > 0{,}673\ \text{, where } (3+\psi) \ge 0 \qquad (4.2)$$ *

Outstand ≡ flange

– outstand compression elements:

$\rho = 1{,}0$ for $\bar{\lambda}_p \le 0{,}748$

$$\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^2} \le 1{,}0 \quad \text{for } \bar{\lambda}_p > 0{,}748 \qquad (4.3)$$

where $\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28{,}4\,\varepsilon\,\sqrt{k_\sigma}}$

Amendment!

ψ is the stress ratio determined in accordance with 4.4(3) and 4.4(4)

$\bar{b}$ is the appropriate width to be taken as follows (for definitions, see Table 5.2 of EN 1993-1-1)

b_w	for webs;
b	for internal flange elements (except RHS);
$b - 3\,t$	for flanges of RHS;
c	for outstand flanges;
h	for equal-leg angles;
h	for unequal-leg angles;

EN 1993-1-5, 4.4

Amendments to EN 1993-1-5: limit between various formulas of r is not equal 0,673,

but

$$0{,}5 + \sqrt{(0{,}085 - 0{,}055\,\Psi)}$$

So, in analysed cases:

For axial force ($\psi = 1{,}0$): $0{,}5 + \sqrt{(0{,}085 - 0{,}055\,\psi)}] = 0{,}673$

For bending moment ($\psi = -1{,}0$): $0{,}5 + \sqrt{(0{,}085 - 0{,}055\,\psi)}] = 0{,}874$

Slendernesses determined for axial force (2,450) and for bending moment (1,002) are greater than limit values for these loads (0,673 and 0,874, respectively), therefore web will be reduced in both cases.

For axial force:

$\rho = [\lambda_p – 0{,}055\ (3 + \psi)] / \lambda_p^2 = [2{,}450 - 0{,}055(3 + 1)] / 2{,}450^2 = 0{,}372$

Height of compressed part after reduction: $1\ 260 \cdot 0{,}372 = 468$ mm

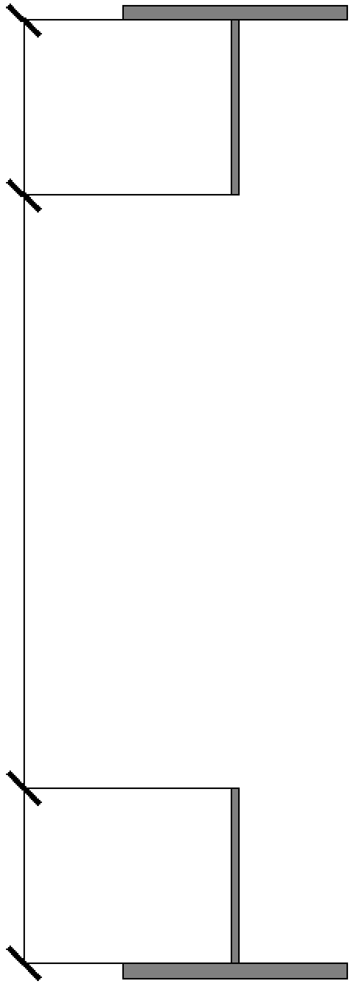
For bending moment:

$\rho = [\lambda_p – 0{,}055\ (3 + \psi)] / \lambda_p^2 = [1{,}002 - 0{,}055(3 - 1)] / 1{,}002^2 = 0{,}888$

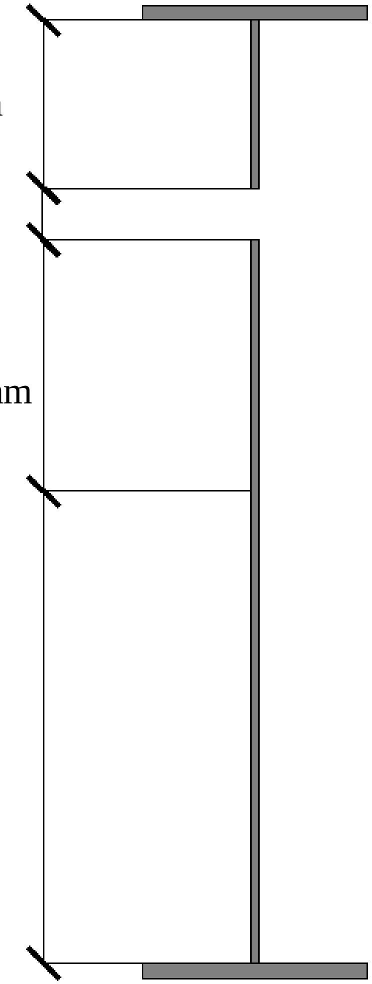
Height of compressed part after reduction: $630 \cdot 0{,}888 = 560$ mm

Effective geometry after III step:

Photo: Author



For axial force



For bending moment

Recalculation of geometry

For axial force

$A_{eff,\ 3,\ N}$ = 171,480 cm^2

For bending moment

$A_{eff,\ 3,\ M}$ = 250,900 cm^2

S_y (about initial centre of gravity) = -285,670 cm^3

$\Delta_y = S_y / A_{eff,\ 3,\ M}$ = -1,1 cm (new center of gravity below the initial one)

z_{top} = 64,1 cm

z_{bottom} = 61,9 cm

$J_{eff,\ 3,\ M}$ = 623 222,349 cm^4

$W_{y,\ top,\ eff,\ 3,\ M} = J_{eff,\ 3,\ M} / z_{top}$ = 9 722,658 cm^3

$W_{y,\ bottom,\ eff,\ 3,\ M} = J_{eff,\ 3,\ M} / z_{bottom}$ = 10 068,212 cm^3

On border web-flange:

$W_{y,\ top,\ eff,\ 3,\ M,\ w\text{-}f}$ = 10 035,787 cm^3

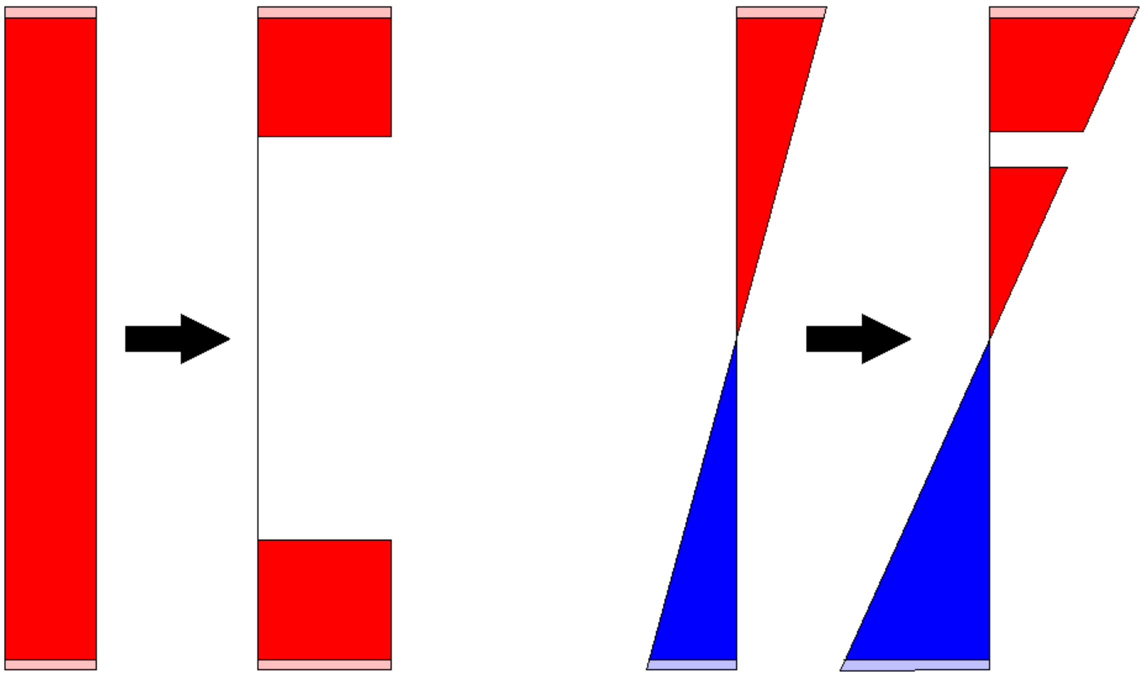
$W_{y,\ bottom,\ eff,\ 3,\ M,\ w\text{-}f}$ = 10 404,380 cm^3

IV step

Web under compression - verification

Stress distribution is calculated for effective geometry after III step: $\mathbf{A_{eff,\,3}}$ **,** $\mathbf{W_{y,\,eff,\,3}}$ **.**

Stress distribution is totally changed.



Rys: Autor

Axial force

New value of stress in web:

$\sigma_{max} = N_{Ed} / A_{eff,\,3,\,N}$ = 64,722 kN / 171,480 cm^2 = 3,774 MPa (previous case 2,503 MPa).

So it is still constant distriburion. Ψ = 1,0 the same as previous. The sane, total height of web (1 260 mm) is under compression. Rest steps of calculation will be made for the same data as previous (#t / 36-46). Geometry after IV step will be completelu the same as after III step.

Bending moment

New value of stress in web :

$\sigma_{max,\ top} = M_{y,\ Ed} / W_{y,\ top,\ eff,\ 3,\ M,\ wf}$ = 3 361,320 kNm / 10 035,707 cm^3 =
= 334,936 MPa (compression)

$\sigma_{max,\ bottom} = -M_{y,\ Ed} / W_{y,\ bottom,\ eff,\ 3,\ M,\ wf}$ = -3 361,320 kNm / 10 404,380 cm^3 =
= -323,068 MPa (tension)

Additionally, due to noew centre of gravity, the height of the compressed part of the web changes to 64.1 cm (previously 63 cm).

$\psi = \sigma_2 / \sigma_1$ **= tension / compression** or **smaller compression / compression**

ψ = -323,068 / 334,936 = -0,965

Ψ = -0,965 → table → $k_\sigma = 7,81 - 6,29\ \psi + 9,78\ \psi^2 = 22,987$

Steel S 355 → f_y = 355 MPa

$\varepsilon = \sqrt{(235 / f_y)} \rightarrow \varepsilon = \sqrt{(235 / 355)} = 0,814$

Slenderness for bending moment:

b – total height of web; a – thickness of weld; t_w – thickness of web;

$c = b - 2\,a\,\sqrt{2} = 1\,260 - 14 = 1\,246$ mm

$\lambda_p = (c / t_w) / (28{,}4\,\varepsilon\,\sqrt{k_\sigma}) = (1\,246 / 11) / (28{,}4 \cdot 0{,}814 \cdot \sqrt{22{,}987}) = 1{,}022$

Limit for bending moment according to amendments ($\psi = -0{,}965$):

$0{,}5 + \sqrt{(0{,}085 - 0{,}055\,\psi)}] = 0{,}872$

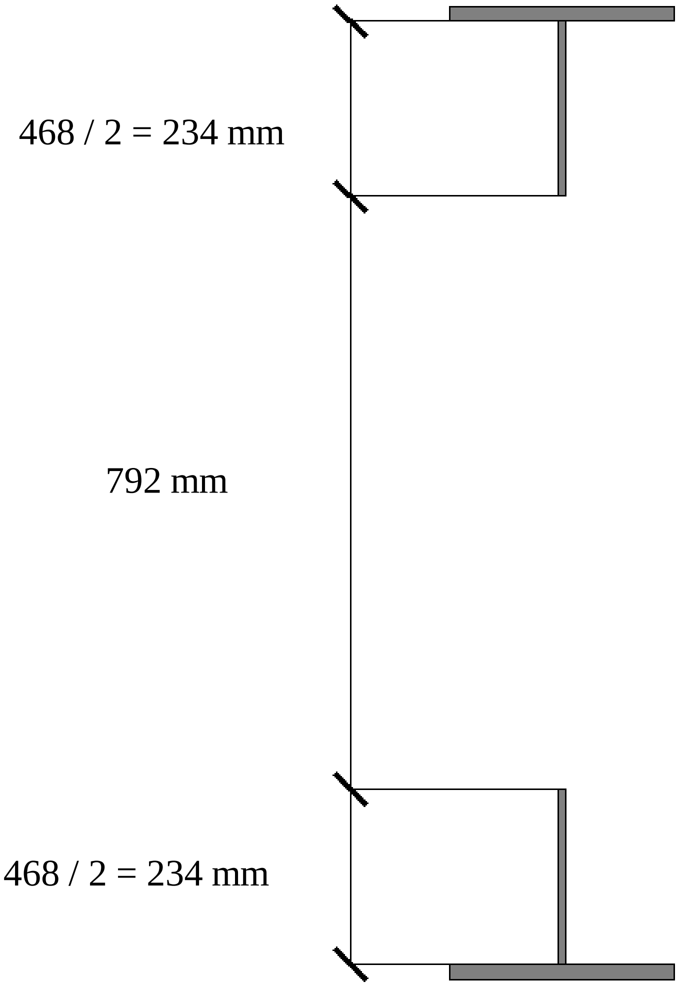
Slenderness is bigger than limit, so there will be reduction of web.

$\rho = [\lambda_p - 0{,}055\,(3 + \psi)] / \lambda_p^2 = [1{,}022 - 0{,}055(3 - 0{,}965)] / 1{,}022^2 = 0{,}871$

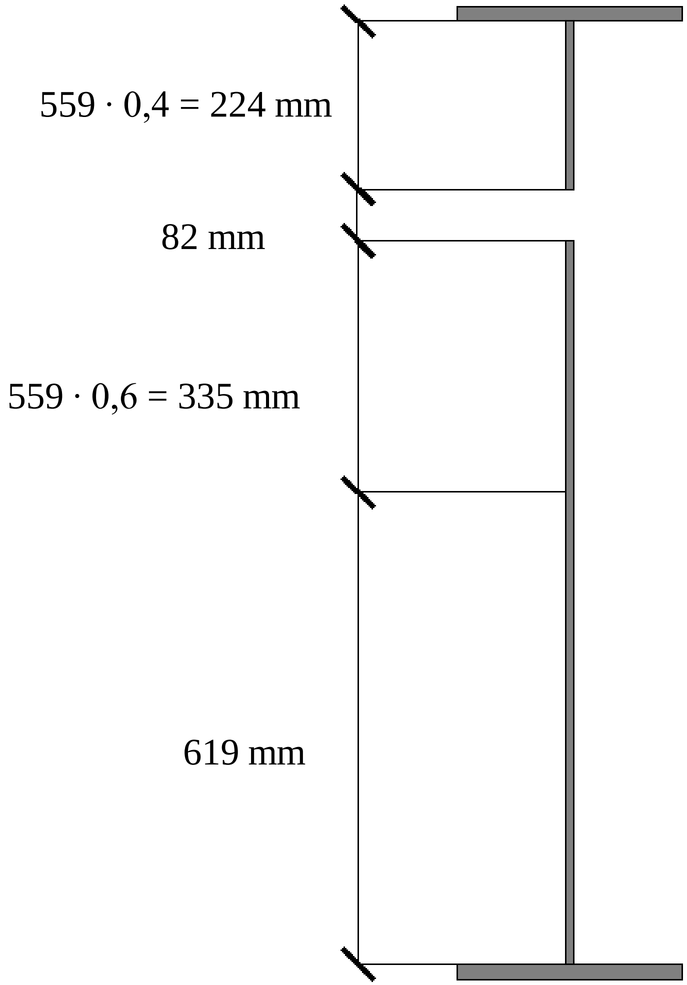
Height of compressed part after reduction: $641 \cdot 0{,}871 = 559$ mm

Effective geometry after IV step:

Rys: Autor



For axial force



For bending moment

Recalculation of geometry

For axial force (no changes):

$A_{eff,\ 4,\ N} = 171{,}480\ cm^2$

For bending moment:

$A_{eff,\ 4,\ M} = 249{,}580\ cm^2$

S_y (about initial centre of gravity) = $-329{,}230\ cm^3$

$\Delta_y = S_y / A_{eff,\ 4,\ M} = -1{,}3$ cm (new center of gravity below the initial one)

$z_{top} = 64{,}3$ cm

$z_{bottom} = 61{,}7$ cm

$J_{eff,\ 4,\ M} = 621\ 804{,}844\ cm^4$

$W_{y,\ top,\ eff,\ 4,\ M} = J_{eff,\ 4,\ M} / z_{top} = 9\ 670{,}371\ cm^3$

$W_{y,\ bottom,\ eff,\ 4,\ M} = J_{eff,\ 4,\ M} / z_{bottom} = 10\ 077{,}874\ cm^3$

Na granicy półki i środnika:

$W_{y,\ top,\ eff,\ 4,\ M,\ wf} = 9\ 980{,}816\ cm^3$

$W_{y,\ bottom,\ eff,\ 4,\ M,\ wf} = 10\ 415{,}492\ cm^3$

		A [cm^2]	$W_{y,\ top}$ [cm^2]	$W_{y,\ bottom}$ [cm^2]
0	Start	258,600	10 382,889	10 382,889
I	Shear lag effect	258,600	10 382,889	10 382,889
II	Flange under compression	258,600	10 382,889	10 382,889
III	Web under compression	171,480	9 722,658	10 068,212
IV	Web under compression - verification	171,480	9 670,371	10 077,874
...	...	...	...	...

Difference betwen two last calculation for web	0,000 %	-0,538 %	0,096 %

Completion of iteration procedure is designer's decision. Usually it ends when it is converged < 2%.

$$\mathbf{N_{Rd} = A_{eff,\,4,\,N}\, f_y / \gamma_{M0} = 6\,087{,}540\ kN}$$

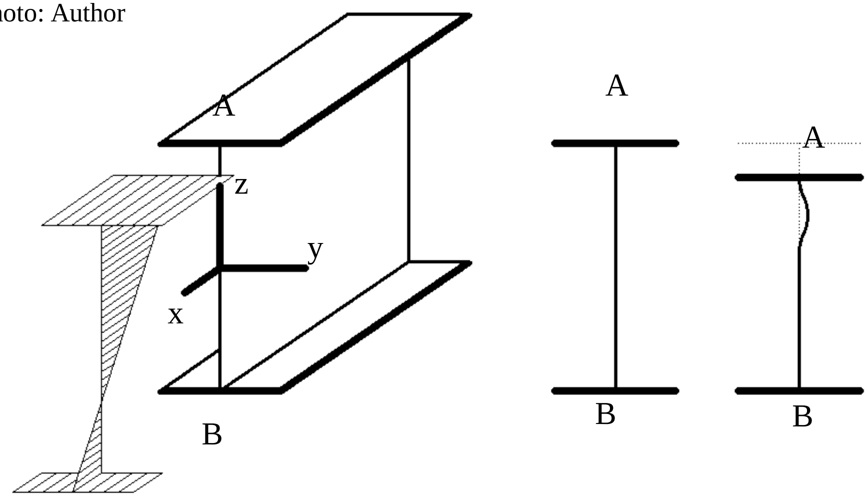
$$\mathbf{M_{Rd} = W_{y,\,top,\,eff,\,4,\,M}\, f_y / \gamma_{M0} = 3\,432{,}982\ kNm}$$

Flange induced buckling

(compression flange buckling in plane of web)

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Photo: Author



Reason: axial stresses σ_x (from bending moment and / or compressive axial force). Second type interaction flage-web: lost of stability for both sub-patrs is dependent each other; behaviors of flange and web are common.

Position of point A changes after lost of stability.

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

The same reason: axial stresses σ_x (from bending moment and / or compressive axial force).
Various forms of instability (Point A not changes / changes its position

Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020

Acceptable situation: effective geometry (→ #t / 22 – 52).

Prohibit situation: only specific proportions of geometry flange : web are accpeted (→ #t / 55).

Example:

steel I-beam

Geometrical condition:

$$h_w / t_w \leq k\,(E / f_{yf})\,[\sqrt{(A_w / A_{fc})}]$$

Class of cross-section	k
1	0,30
2	0,40
3, 4	0,55

EN 1993-1-5 (8.1)

h_w = (total depth of web) = 1 260 mm ; t_w = (thickness of web) = 11 mm

f_{yf} = (yield strength of flange) = f_{yf} **=** 355 MPa ; A_w = (full area of web) = 1 260 · 11 = 138,6 cm^2

A_{fc} = (full area of compressed flange) = 300 · 20 = 60,0 cm^2 ; E = (Young modulus) = 210 GPa

$$h_w / t_w = 1\,260 / 11 = 114{,}546$$

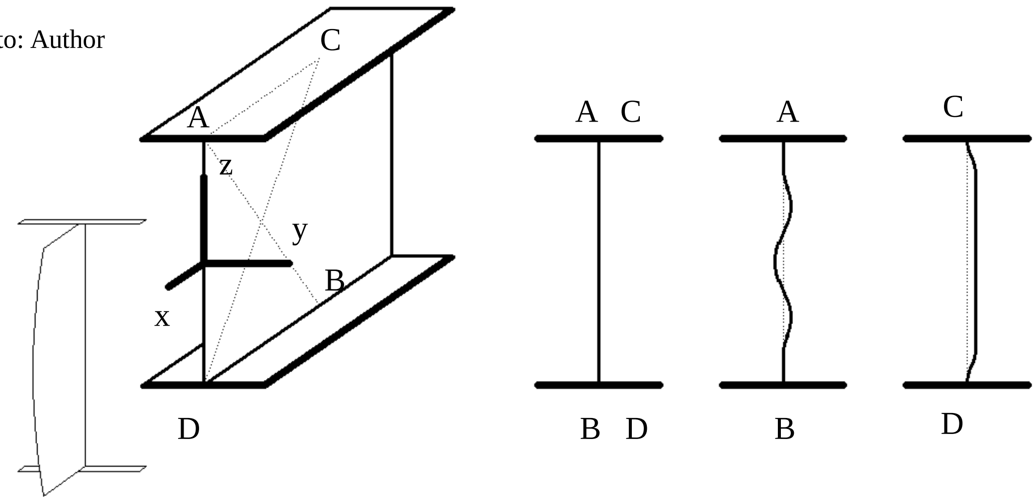
$$k\,(E / f_{yf})\,[\sqrt{(A_w / A_{fc})}] = 0{,}55\,(210\,000 / 355)\,\sqrt{(138{,}6 / 60{,}0)} = 449{,}577$$

OK

Instability of web under shear force

Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013

Photo: Author

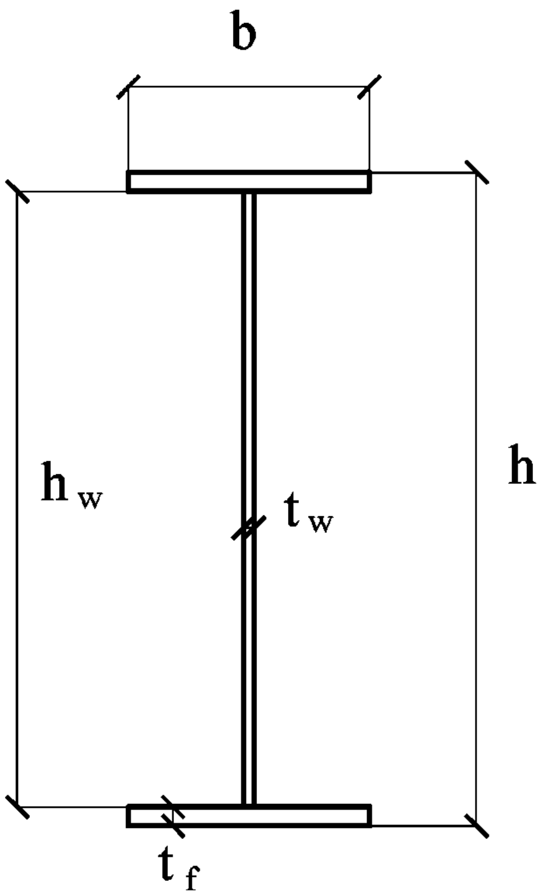


Reason: shear stresses τ_{xz} (from shear force V_z). Along line A-B local compression, along line C-D local tension.

Effective geometry depends on values of external action (proportion between stresses), so N_{Ed} , M_{Ed} must be konwn for calculation.

Paradoxically, value of $V_{z,\,Ed}$ is not important for analysis of this form of instability, but important are N_{Ed} , M_{Ed} .

Geometry:



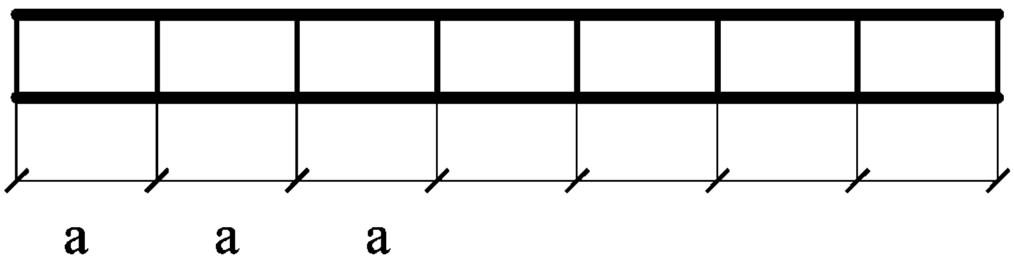


Photo: Author

General formula for resistance:

EN 1993-1-5 (5.1), (5.2)

$$V_{b,Rb} = \min \left[\chi_w f_{yw} h_w t_w / (\gamma_{M1} \sqrt{3}) + V_{bf,Rd} \;\; ; \;\; \eta f_{yw} h_w t_w / (\gamma_{M1} \sqrt{3}) \right]$$

Resistance of web

Resistance of web

Secondary impact of steel grade

Impact of lost of stability

Support from flange

Resistance of web is calculated the same for Ist, IInd, IIIth and IVth class of cross-section

Resistance of web depends on steel grade and geometry of cross-section of web

Impact of lost of stability depends on numer and position of horizontal and vertical stiffeners, geometry of cross-section of web and grade of steel

Support from flange depends on loads (N_{Ed} , M_{Ed}) and resistance of flange

Secondary impact of steel grade depends on steel grande

Resistance of web:

$$f_{yw}\, h_w\, t_w / (\gamma_{M1} \sqrt{3}) = 355 \text{ MPa} \cdot 1\,260 \text{ mm} \cdot 11 \text{ mm} / (1{,}0 \cdot \sqrt{3}) =$$
$$= 2\,842{,}461 \text{ kN}$$

Secondary impact of steel grade:

$f_y \leq 460$ MPa	$f_y > 460$ MPa
$\eta = 1{,}2$	$\eta = 1{,}0$

EN 1993-1-5 5.1.(2)

Impact of lost of stability:

If condition is satisfied, $\chi_w = 1{,}0$

Unstiffened web	Stiffened web
$h_w / t_w \leq 72\, \varepsilon / \eta$	$h_w / t_w \leq 31\, \varepsilon \sqrt{(k_\tau)} / \eta$

EN 1993-1-5 (5.5), (5.6)

$h_w / t_w = 1\,260 / 11 = 114{,}545$

$\varepsilon = \sqrt{(235 / 355)} = 0{,}814$

$$h_w / t_w = 114{,}545 > \leq 72\, \varepsilon / \eta = 72 \cdot 0{,}814 / 1{,}2 = 48{,}72$$

So, $\chi_w < 1{,}0$ and its value must be calculated

Unfortunately, in Eurocode can be found many inconsequences. The most often case is:

General situation is divided into sub-cases

A – full information about way of calculation

B – no information about way of calculation

The most part of such situation concern various phenomenon in rigid bolted joints (for example: resistance for netto area around hols for bolts, stiffness of shear joints, impact of axial force for bending moment resistance).

Rare case is contradictions between different points in Eurocode. An examples are calculation of concrete base resistance under hinged support of columns or calculation of built-up columns.

→ #3 / 95

All these problems will be mentioned in future lectures.

Inconsequence in analysed case:

- EN 1993-1-5 5.1.(2) presesented distinction: unstiffened web ↔ stiffened web;
- EN 1993-1-8 fig. 5.1 accepted welded I-beam without stiffeners over supports (unstiffened web);
- EN 1993-1-8 5.3.(1) presented information only for I-beams with stiffners at least over supports;
- EN 1993-1-8 no presented information about calculation of I-beams completely without stiffeners;
- Therefore, it is not known how to understand the condition at point EN 1993-1-5 5.1.(2): where web is unstiffened and stiffened;
- According to literature, beam with stiffeners over supports only could be treated as beam with unstiffened web;
- No information about calculation for case „beam completely without stiffneners” means, that such solution is not recommended;
- Welded I-beam must have at least stiffenenr over supports.

Impact of lost of stability:

Slenderness of web under shear force (stiffeners over supports only):

$$\lambda_w = h_w / (86{,}4\ t_w\ \varepsilon) =$$

$$=1\ 260 / (86{,}4 \cdot 11 \cdot 0{,}814) = 1{,}629$$

λ_w	$\chi_w =$	
	Rigid end post	Non-rigid end post
$< 0{,}83 / \eta$	min (1 ; η)	
$0{,}83 / \eta \div 1{,}08$	$0{,}83 / \lambda_w$	
$\geq 1{,}08$	$1{,}37 / (0{,}7 + \lambda_w)$	$0{,}83 / \lambda_w$

EN 1993-1-5 tab. 5.1

Rigid and non-rigid end post → Lec #21

In case of the most often applicated stiffeners over supports, calculation is as for non-rigid end post

$$\chi_w = 0{,}510$$

Support from flange:

EN 1993-1-5 (5.8)

M_{Ed}	$V_{bf,Rd}$
$< \rho\, W_f\, f_{yf} / \gamma_{M0}$	$b_f\, t_f^2\, f_{yb}\, [1 - (M_{Ed} / M_{f,Rd})^2] / (c\, \gamma_{M1})$
$\geq \rho\, W_f\, f_{yf} / \gamma_{M0}$	0

$$\rho = 1 - N_{Ed} / [\, (A_{f,\,top} + A_{f,\,bottom})\, f_{yf} / \gamma_{M0}]$$ EN 1993-1-5 (5.9)

$$b_f = \min\,(30\, \varepsilon\, t_f \quad ; \quad b_{f,\,eff})$$

$$c = a\, [\, 0{,}25 + 1{,}6\, b_f\, t_f^2\, f_{yf} / (t_w\, h_w^2\, f_{yw})]$$

EN 1993-1-5 (5.8)

Two cases of impact N_{Ed} , M_{Ed} :

- small value M_{Ed} : small effort of flange; in flange is „enought space" for stresses from N_{Ed} , M_{Ed} and for suport of web;
- big value M_{Ed} : big effort of flange; in flange is „enought space" only for stresses from N_{Ed} , M_{Ed} and flange can't suport web

No vertical stiffeners, interaction between M_{Ed}, N_{Ed}, V_{Ed}

Resistance against $V_{Ed} = 5\,F_{Ed} / 16$ is calculated in interaction with N_{Ed} and $M_{Ed} = 5\,F_{Ed}\,L / 32$

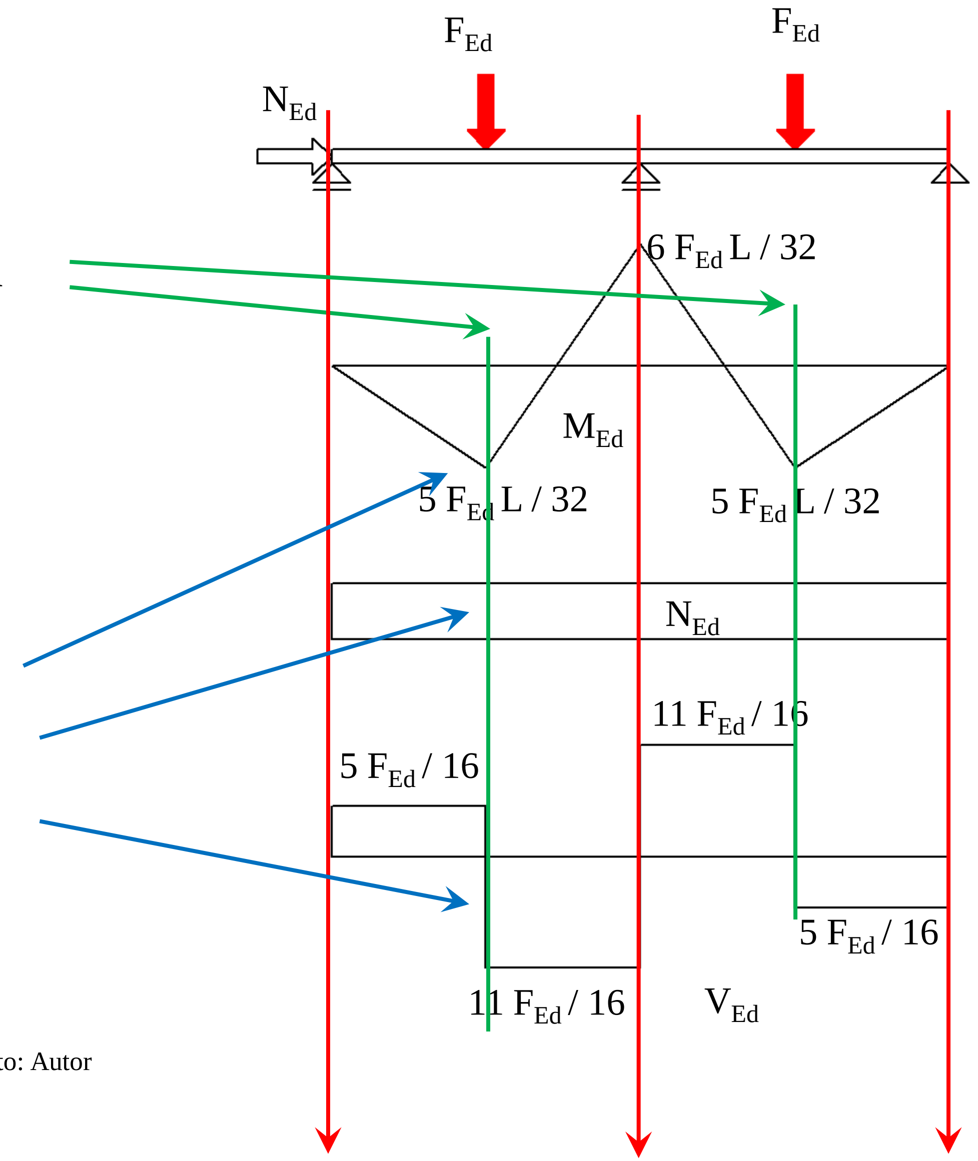


Photo: Autor

Supports, vertical stiffeners, protection against local instability

$A_{f,\,top}$ = top flange area = $300 \cdot 20 = 60{,}0\ cm^2$

$A_{f,\,bottom}$ = bottom flange area = $300 \cdot 20 = 60{,}0\ cm^2$

f_{yf} = (yield strength of flange) = f_y = 355 MPa = f_{yw} = (yield strength of web)

$b_{f,\,eff}$ = (effective width of compressed flange) = 30 cm

t_f = (thickness of flange) = 2,0 cm

t_w = (thickness of web) = 1,1 cm

h_w = (depth of web) = 126 cm

ε = (dimensioless strength of flange) = 0,814

a = (horizontal distance between vertical stiffeners) = 25,0 m (stiffeners over support only)

N_{Ed} = 64,722 kN

$$\rho = 1 - N_{Ed} / [\,(A_{f,\,top} + A_{f,\,bottom})\, f_{yf} / \gamma_{M0}] = 0{,}977$$

$$b_f = \min\,(30\,\varepsilon\, t_f \;\; ; \;\; b_{f,\,eff}) = \min\,(48{,}84 \;\; ; \;\; 30{,}00) = 30\ cm$$

$$c = a\,[\,0{,}25 + 1{,}6\, b_f\, t_f^2\, f_{yf} / (t_w\, h_w^2\, f_{yw})] = 6{,}525\ m$$

$$M_{Ed,\,y} = 3\ 361{,}320\ \text{kNm}$$

Inconsequence in Eurocode: „bending resistance of the cross section consisting of the effective area of the flanges only”, but this resistnace depends on position of effective centre of gravity. There are two possibilities: effective for axial force and effective for bending moment. For simplification, calculation is provide for global center of gravity.

$$W_f = 2\,[(t_f^3\, b_f /12) + t_f\, b_f\, (h / 2 - t_f / 2)^2] / (h / 2) = 4\ 915{,}6\ \text{cm}^3$$

$$\rho\, W_f\, f_{yf} / \gamma_{M0} = 1\ 155{,}166\ \text{kNm}$$

M_{Ed}	$V_{bf,Rd}$
$< \rho\, W_f\, f_{yf} / \gamma_{M0}$	$b_f\, t_f^2\, f_{yb}\, [1 - (M_{Ed} / M_{f,Rd})^2] / (c\, \gamma_{M1})$
$\geq \rho\, W_f\, f_{yf} / \gamma_{M0}$	0

$$V_{b,Rb} = \min\,[\ \chi_w\, f_{yw}\, h_w\, t_w / (\gamma_{M1}\, \sqrt{3}) + V_{bf,Rd}\quad ;\quad \eta\, f_{yw}\, h_w\, t_w / (\gamma_{M1}\, \sqrt{3})] =$$

$$= \min\,[0{,}510 \cdot 2\ 842{,}461 + 0{,}0\quad ;\ 1{,}2 \cdot 2\ 842{,}461\ ]$$

$$\mathbf{V_{b,Rb} = 1\ 449{,}655\ kN}$$

Instability of web under transversal force

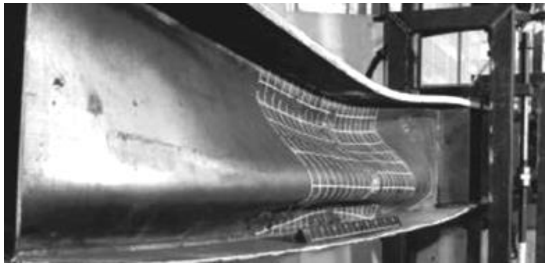
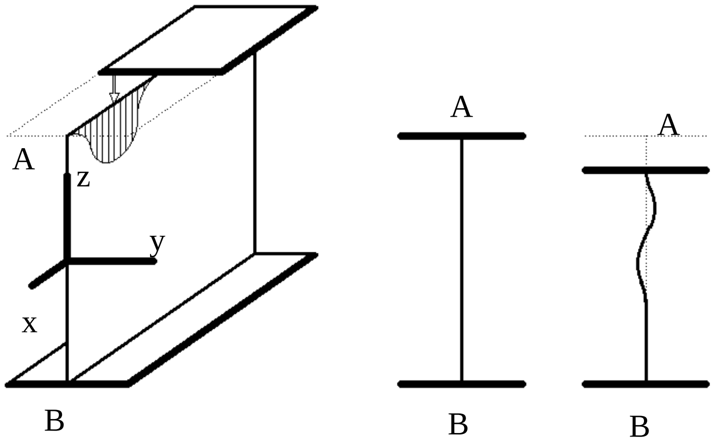


Photo : Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

Photo: Author



Reason: axial stresses σ_z (from transversal force V_z applicated in point). Axial stresses occur in web at contact with flange.

Prevention: transversal stiffeners → Lec #21

Calculation: checking stability → #t / 68 - 73

Transverse force F_s is force applied in point to beam. Of course, transverse force (type of loads) produces shear force V_{Ed} (type of cross-sectional force).

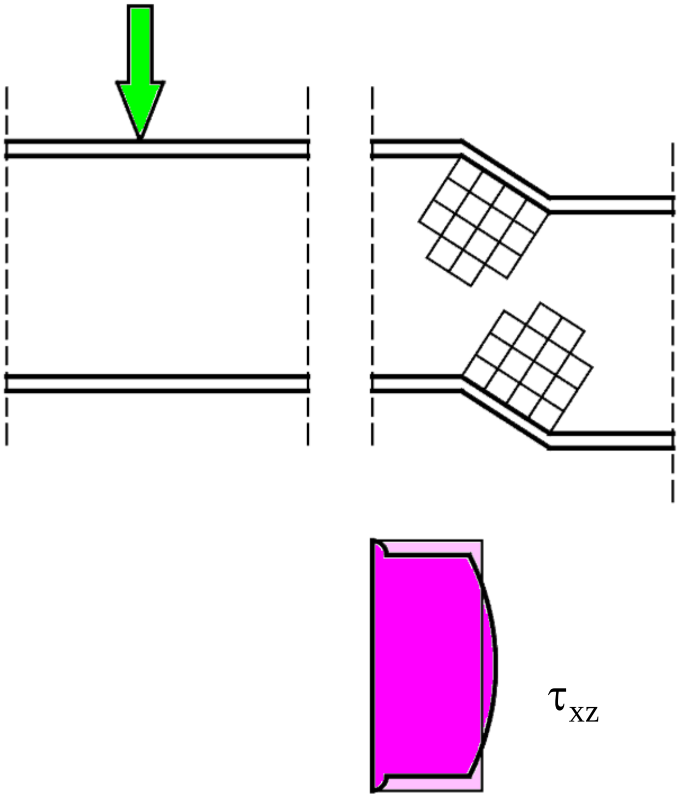
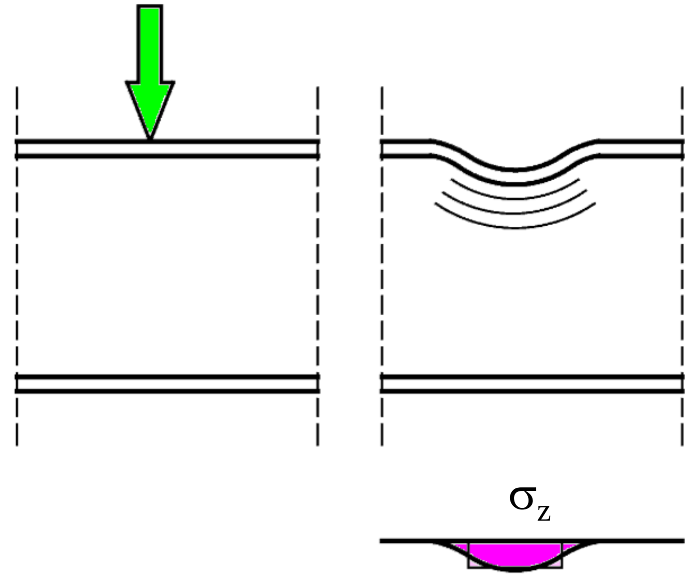


Photo: Author

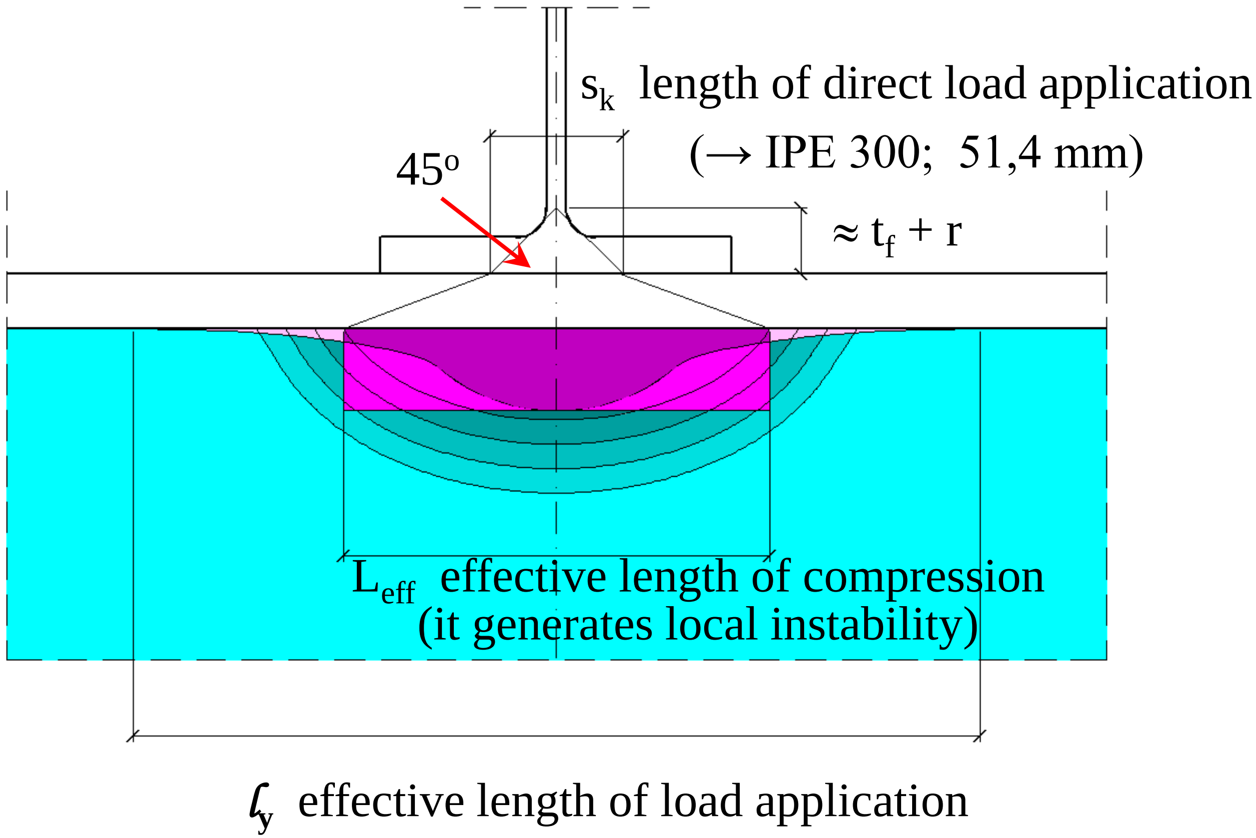


Transverse force: local effect deformation, instability, axial stresses and its idealisation

Shear force: global effect, deformation, instability, shear stresses and its idealisation

Important lengths:

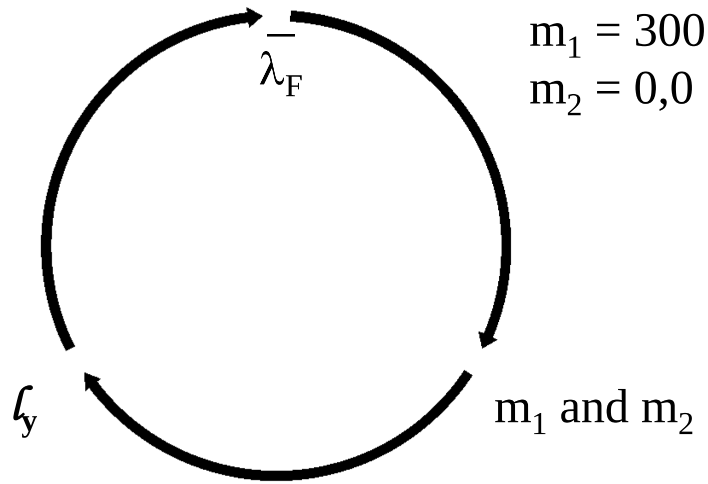
Photo: Author



	$\bar{\lambda}_F \leq 0{,}5$	$\bar{\lambda}_F > 0{,}5$
m_1	$(f_{yf}\, b_f \,/\, f_{yw}\, t_w)$	
m_2	0	$0{,}02\,(h_w \,/\, t_f)^2$

EN 1993-1-5 (6.8), (6.9)

Variables in calculations are implicit and an iterative procedure is necessary. Initial assumptions about values of variables should be made and next verified.



m_1 = 300 [mm] / 11 [mm] = 27,273
m_2 = 0,0

Photo: Author

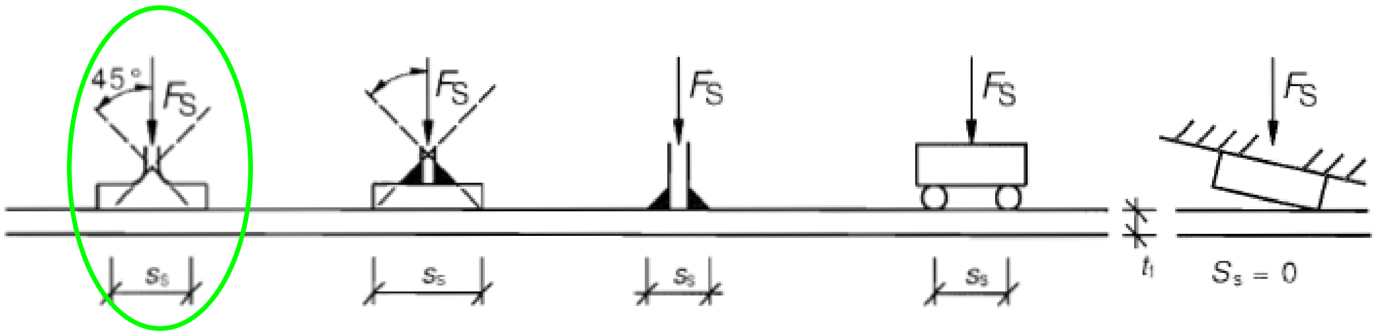
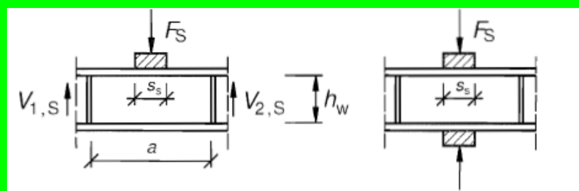
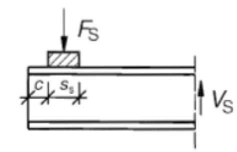


Photo: EN 1993-1-5 fig. 6.2

Photo: EN 1993-1-5 fig. 6.1

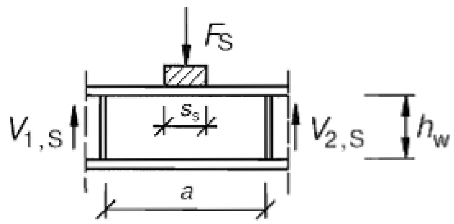
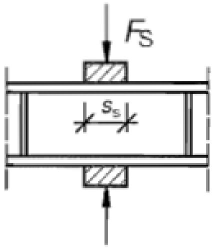
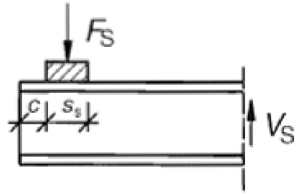
EN 1993-1-5 fig. 6.1		
ℓ_y	$\min\{a\ ;\ s_s + 2\,t_f\,[\,1 + \sqrt{(m_1 + m_2)}\,]\}$	$\min\{\ell_e + t_f\sqrt{[m_1/2 + (\ell_e/t_f)^2 + m_2]}\ ;\ \ell_e + t_f\sqrt{[m_1 + m_2]}\}$

EN 1993-1-5 (6.10) - (6.13)

$$\ell_y = \min\{25{,}00\ ;\ 0{,}051 + 2 \cdot 0{,}02 \cdot [1 + \sqrt{(27{,}273 + 0)}]\} = 0{,}300\ [\mathrm{m}]$$

EN 1993-1-5 fig. 6.1, no longitudinal stiffeners:

Photo: EN 1993-1-5 fig. 6.1

			
k_F =	$6 + 2\ (h_w / a)^2$	$3{,}5 + 2\ (h_w / a)^2$	$\min \{ 2 + 6\ [(s_s + c) / h_w] \ ; \ 6{,}0 \}$

EN 1993-1-5 (6.2) - (6.5)

$$k_F = 6 + 2\ (11 / 25\ 000)^2 = 6{,}000$$

$$F_{cr} = 0{,}9\ k_F\ E\ t_w^{\ 3} / h_w = 1\ 197{,}900\ [\mathrm{kN}]$$

$$\bar{\lambda}_F = \sqrt{(\ell_y\ t_w\ f_{yw} / F_{cr})} = 0{,}805$$

Contradiction with the original assumption $\bar{\lambda}_F \leq 0{,}5$

Recalculation for $\lambda_F > 0,5$:

$m_1 = 27,273$

EN 1993-1-5 (6.2) - (6.5)

$m_2 = 7\,938,0$ (previously 0,0)

$\ell_y = 3,661$ [m] (prewioulsy 0,399 [m])

$k_F = 6,0$

$F_{cr} = 1\,197,900$ [kN]

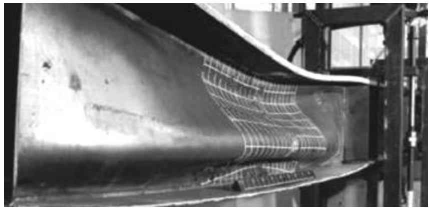
$\overline{\lambda}_F = 2,811$ (ok, it's > 0,5)

$\chi_F = \min(1,0 \;\; ; \;\; 0,5 / \overline{\lambda}_F) = 0,178$

$L_{eff} = \ell_y \chi_F = 0,651$ [m]

$F_{Rd} = f_{yw} L_{eff} t_w / \gamma_{M1}$ EN 1993-1-5 (6.1)

$\mathbf{F_{Rd} = 1\,682,835\ kN}$

Loads E	Critical resistance R	Phenomenon
Axial force N_{Ed} = 64,722 kN Bending moment M_{Ed} = 3 361,320 kNm	N_{Rd} = 6 087,540 kN M_{Rd} = 3 432,982 kNm	Photo: 1 
	Geometrical relation flange-web OK	Photo: 2 
Shear force V_{Ed} = 492,994 kN	$V_{b,Rb}$ = 1 449,655 kN	Photo: 3 
Force applied in point F_{Ed} = 717,082 kN	F_{Rd} = 1 682,835 kN	Photo: 4 

1. Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013
2. 2. Photo: Web Buckling of High Strength Steel Plate Girders Induced by Bending Curvature, S. Nascimento, J. Pedro, A. Biscaya, Wiley Online Library, 9 III 2020
3. . Photo: Saliba, N. Gardner, L. Experimental study of the shear response of lean duplex stainless steel plate girders. Engineering Structures. 1 / 2013
4. Rys: Local Web Buckling in Tapered Composite Beams - A Parametric Study, R. Hobbs, P. Vellasco, Journal of the Brazilian Society of Mechanical Sciences 23-4/2001

Notices:

- fundamental E / R relationship must be checked each time;
- for very slender sections, there are numerous interactions between loads E, which require additional checking few nonlinear relationships; details will be shown in lecture #12;
- critical load capacities R for V and P shown in #t / 74 refer to „naked" web, not reinforced with stiffeners;
- in analysed case, stifeners have been placed above supports only, locally reinforcing resistance of web for P and V;
- information about stiffeners will be shown in lecture #21

Example:

Task only for comparison with steel

aluminum I-beam

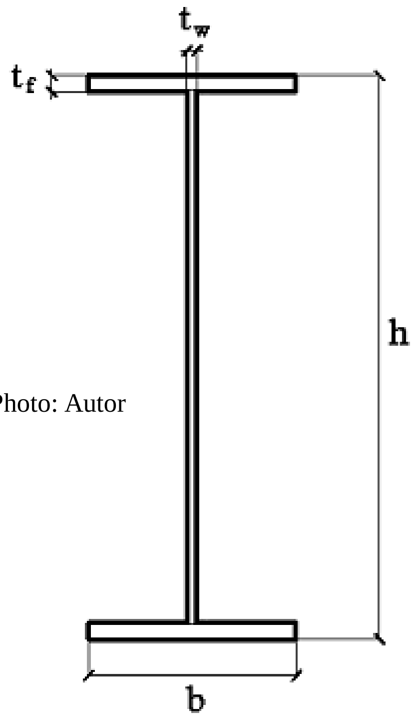


Photo: Autor

Alloy EN-AV 5083 H14

L = 10,000 m

A_0 = 81,600 cm^2

W_0 = 899,730 cm^3

a - thickness of welds = 5 mm

h [mm]	t_w [mm]	b [mm]	t_f [mm]	$M_{Ed,\ y}$ [kNm]	$N_{Ed,\ comp}$ [kN]
300	8	200	15	75,577	228,480

To explain difference in calculation methods, full stress distribution ($M_{Ed,\ y}$ + $F_{Ed,\ comp}$) analysis method is used here.

NOTE

In 2023, work on the new edition of the "aluminum" Eurocode EN 1999-1-1 was completed. Many significant changes were introduced, compared to the previous edition, from 2007 (for example: the classification of aluminum alloys - with respect to local instability - has been changed; 3 groups instead of the previous 2). Much important information was added, and some content from the annexes was moved to the main body. Most of the information from the previous version has been retained, but is located in differently numbered chapters.

EN 1991-1-1:2023 is now in used, instead of EN 1999-1-1:2007.

The transition period expires in 2028.

<table>
<tr><th rowspan="2">Alloy EN AW</th><th rowspan="2">Temper[a]</th><th rowspan="2">Thick-ness mm[a]</th><th>f_o[a]</th><th>f_u</th><th>A_{50} (A) [a, c]</th><th>$f_{o,haz}$[b]</th><th>$f_{u,haz}$[b]</th><th colspan="2">HAZ-factor[b]</th><th rowspan="2">BC[d]</th><th rowspan="2">n_p[a, e]</th></tr>
<tr><th colspan="2">N/mm²</th><th>%</th><th colspan="2">N/mm²</th><th>$\rho_{o,haz}$[a]</th><th>$\rho_{u,haz}$</th></tr>
<tr><td rowspan="5">5083</td><td rowspan="3">O/H111</td><td>≤ 6,3</td><td>125</td><td>275</td><td>15</td><td>125</td><td>275</td><td rowspan="3">1</td><td rowspan="3">1</td><td>C</td><td rowspan="3">6</td></tr>
<tr><td>6,3 < t ≤ 80</td><td>115</td><td>270</td><td>14</td><td>115</td><td>270</td><td>C</td></tr>
<tr><td>80 < t ≤ 120</td><td>110</td><td>260</td><td>(12)</td><td>110</td><td>260</td><td>C</td></tr>
<tr><td>H12|H22/H32</td><td>≤ 40</td><td>250 | 215</td><td>305[f]</td><td>7 | 10</td><td rowspan="2">155</td><td rowspan="2">275</td><td>0,62 | 0,72</td><td>0,90</td><td>A|B</td><td>22 | 13</td></tr>
<tr><td>H14|H24/H34</td><td>≤ 25</td><td>280 | 250</td><td>340</td><td>4 | 8</td><td>0,55 | 0,62</td><td>0,81</td><td>A|B</td><td>22 | 14</td></tr>
</table>

Photo: EN 1999-1-1 tab. 5.3

Alloy EN-AV 5083 H14

EN 1999-1-1, tab 5.3:

$f_0 = 280$ MPa

$f_u = 340$ MPa

$\rho_{0\ haz} = 0{,}55$

Class A

Ist step

Welded zones

EN 1999-1-1 8.1.6.3.(3) - effective cross-section for welds

(3) For a MIG weld with preheating and/or interpass cooling to 80 °C or less for 7xxx alloys and to 100 °C or less for 6xxx alloys and work-hardened 3xxx and 5xxx series alloys when multi-pass welds are laid, values of b_{haz} should be taken as follows:

$0 < t \le 6$ mm: $b_{haz} = 20$ mm

$6 < t \le 12$ mm: $b_{haz} = 30$ mm

$12 < t \le 25$ mm: $b_{haz} = 35$ mm

$t > 25$ mm: $b_{haz} = 40$ mm

(4) For TIG welds for in-line butt or fillet welds in 6xxx, 7xxx or work-hardened 3xxx and 5xxx series alloys, b_{haz} should be taken as:

$0 < t \le 6$ mm: $b_{haz} = 30$ mm

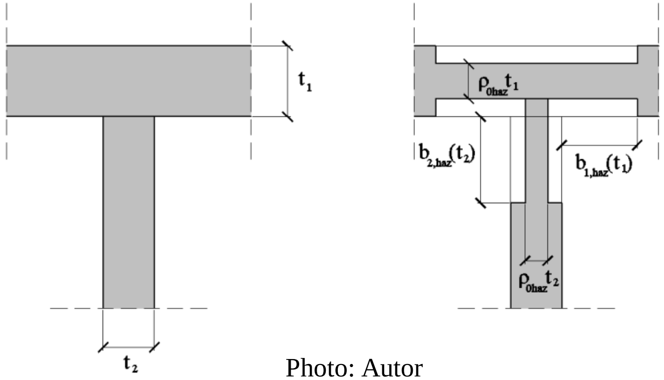
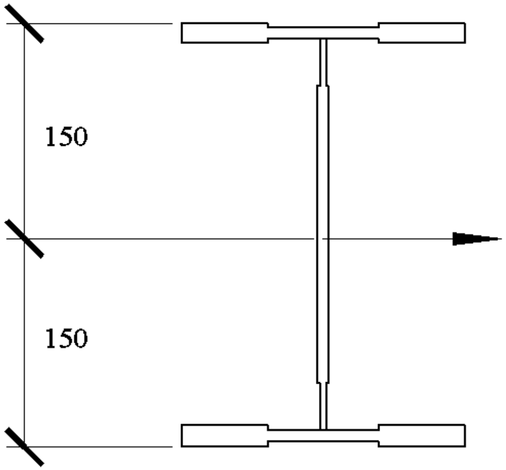


Photo: Autor

$t_f = 15$ mm $\rightarrow b_{haz,\,f} = 35$ mm; $t_{f,\,haz} = t_f\ \rho_{0\ haz} = 15 \cdot 0{,}55 = 8$ mm

$t_w = 8$ mm $\rightarrow b_{haz,\,w} = 30$ mm; $t_{w,\,haz} = t_w\ \rho_{0\ haz} = 8 \cdot 0{,}55 = 4$ mm

Symmetrical cross-section



$A_{eff,\,1} = 68{,}560\ cm^2$

$W_{y,\ eff,\,1} = 732{,}733\ cm^3$

Photo: Autor

IInd step

Flange – too wide or not?

L_e = length of the beam = 10 m; b = width of the flange = 200 mm
$\rightarrow b_0 = b / 2 = 100$ mm

$b_0 < L_e / 50 = 200$ mm

Flange not too wide → shear lag in flanges can be neglected → after IInd step geometry is the same as at start

$b_{eff\ 2} = b_{initial}$

*Otherwise (if flange **is** too wide):*

$b_{eff1} = b_0 \cdot \beta$

β – reduction factor for wide flanges; different value for different part of beam:

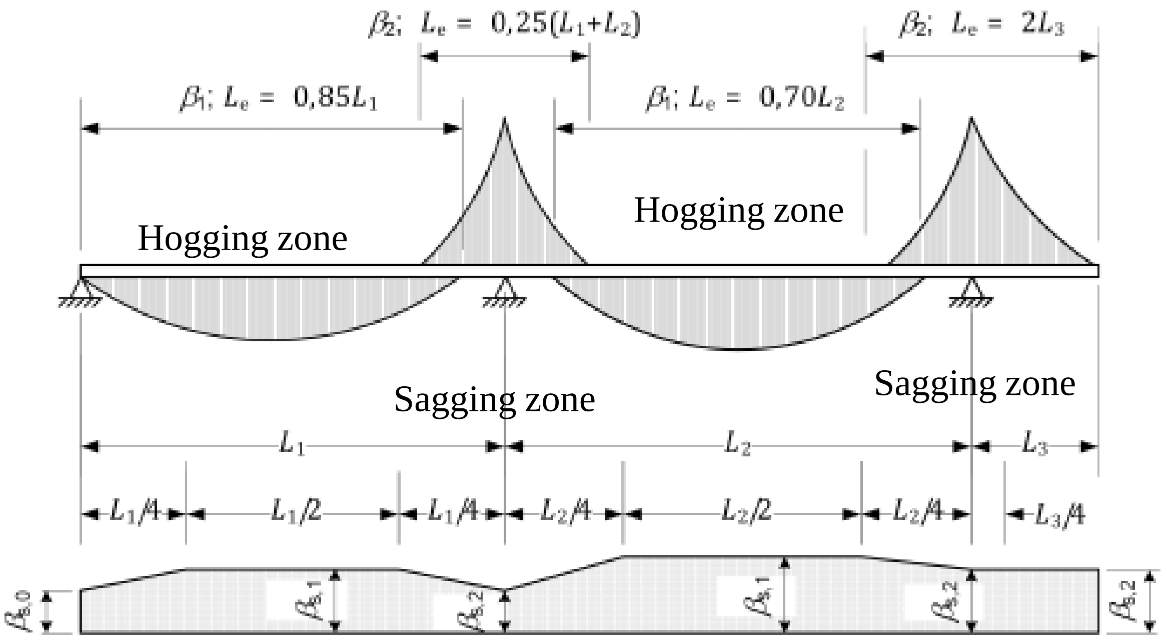


Figure J.2 — Effective length L_e for continuous beam and distribution of the effective width

For one-span-beam: $L_e = L$

Photo: EN 1999-1-1, fig J.1

Table J.1 — Effective width factor β_s

κ	Location for verification	β_s
$\kappa \le 0{,}02$		$\beta_s = 1{,}0$
$0{,}02 < \kappa \le 0{,}70$	sagging bending	$\beta_s = \beta_{s,1} = \dfrac{1}{1+6{,}4\kappa^2}$
	hogging bending	$\beta_s = \beta_{s,2} = \dfrac{1}{1+6{,}0\left(\kappa - 0{,}0004/\kappa\right)+1{,}6\kappa^2}$
$\kappa > 0{,}70$	sagging bending	$\beta_s = \beta_{s,1} = \dfrac{1}{5{,}9\kappa}$
	hogging bending	$\beta_s = \beta_{s,2} = \dfrac{1}{8{,}6\kappa}$
All κ	end support	$\beta_{s,0} = (0{,}55 + 0{,}025/\kappa)\beta_{s,1}$ but $\beta_{s,0} \le \beta_{s,1}$
All κ	cantilever	$\beta_s = \beta_{s,2}$ at support and at the end

Key:

$\kappa = a_0 b_0 / L_e$ with $\alpha_0 = \sqrt{1 + A_{st}/(b_0 t)}$

in which A_{st} is the area of all longitudinal stiffeners within the width b_0 and other symbols are as defined in Figure J.1 and Figure J.2.

EN 1999-1-1, tab. J.1

$L_e = 10\ m$

No longitudinal stiffeners $\rightarrow A_{sl} = 0 \rightarrow \alpha_0 = 1$

$b_0 = 100\ mm$

$\kappa = 1 \cdot 100 / 10\ 000 = 0{,}010 \rightarrow \kappa < 0{,}02 \rightarrow \beta = 1{,}0$

$b_{eff1} = b_{initial} \cdot \beta = b_{initial}$

Generally: after this step we must recalculate area of cross-section $A_{eff\ 1}$ *and sectional modulus* $W_{y\ eff\ 1}$ *; cross-section is still symmetrical.*

IIIrd step

Compressed flange

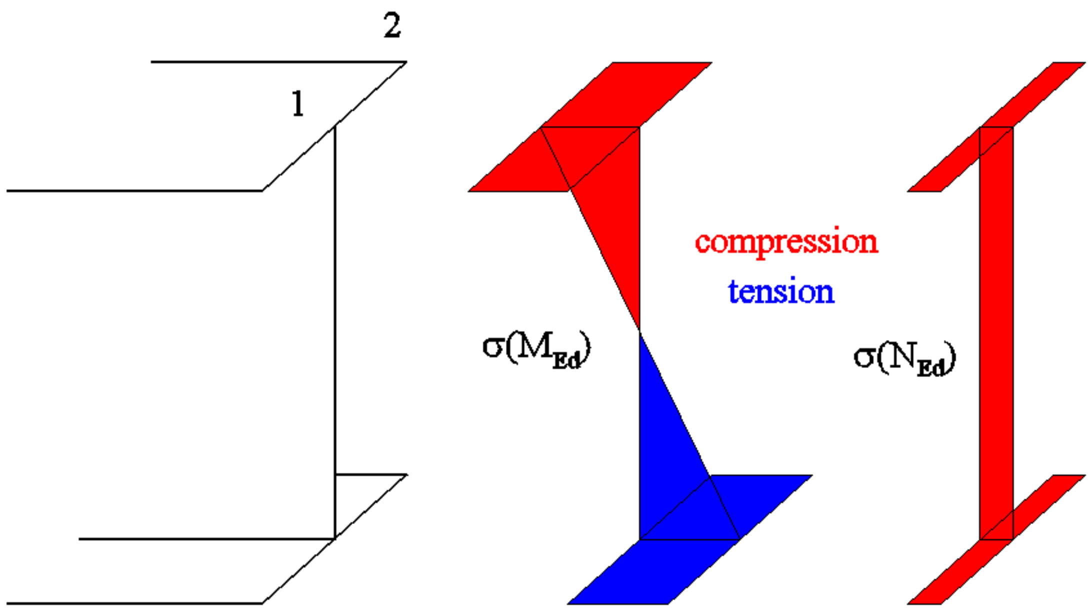


Photo: Autor

σ_1 – compression, σ_2 - tension

Flange $\rightarrow \sigma = \text{const} \rightarrow \sigma_1 = \sigma_2 \rightarrow \psi = \sigma_2 / \sigma_1 = 1{,}0$

$\eta = 0{,}7 + 0{,}3\ \psi$

$\beta = \eta\ b / t$

$\varepsilon = \sqrt{(250 / f_0)}$

$\rho_c = \min \{ [C_1 / (\beta / \varepsilon)] - [C_2 / (\beta / \varepsilon)^2] ; 1{,}0\}$

Table 8.3 — Constants C_1 and C_2 in formulae for ρ_c

	Buckling Class according to Table 5.3 and Table 5.4	Curve (see Fig. 8.5)	Internal part		Outstand part	
			C_1	C_2	C_1	C_2
Without welds	A	a	32	220	10	24
	B	b	30,5	209	9,5	22
	C	c	29	198	9	20
With welds	A	aw	29	198	9	20
	B	bw	28,2	193	8,5	18
	C	cw	27,6	189	8	16

EN 1999-1-1, tab 8.3

Internal ≡ web

Outstand ≡ flange

Alloy EN-AV 5083 H14, welded → Class A with welds → web: C_1 = 29; C_2 = 198

flange: C_1 = 9; C_2 = 20

$\eta = 0{,}7 + 0{,}3\ \psi = 1$

$\beta = \eta\ b / t_f = \eta\ (b - 2\ a \sqrt{2} - t_w) / 2\ t_f = 1 \cdot (200 - 14 - 8)\ (2 \cdot 15) = 5{,}93$

$\varepsilon = \sqrt{(250 / f_0)} \rightarrow \varepsilon = \sqrt{(250 / 280)} = 0{,}945$

$\rho_c = \min \{ [C_1 / (\beta / \varepsilon)] - [C_2 / (\beta / \varepsilon)^2] ; 1{,}0\} = 0{,}926$

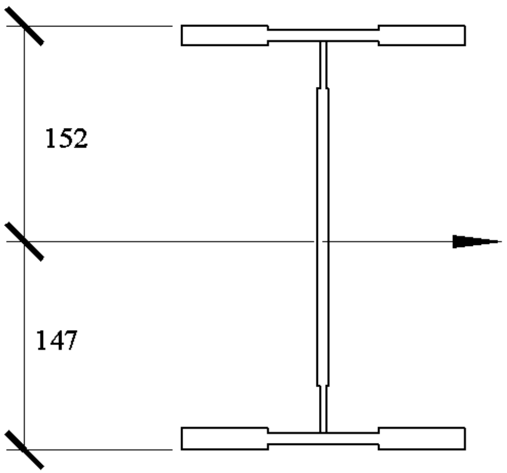
$t_{f,\ eff,\ 3} = t_f\ \rho_c = 15 \cdot 0{,}926 = 14$ mm

$t_{f,\ haz,\ eff,\ 3} = \min (t_{w,\ eff,\ 3}\ ;\ t_{f,\ haz}) = 8$ mm

Unsymmetrical cross-section:

top flange 14 mm / 8 mm;

bottom flange 15 mm / 8 mm



$A_{eff,\ 3} = 67{,}34\ cm^2$

$W_{y,\ top,\ eff,\ 3} = 705{,}992\ cm^3$

$W_{y,\ bottom,\ eff,\ 3} = 728{,}503\ cm^3$

Photo: Autor

IVth step

Compressed web

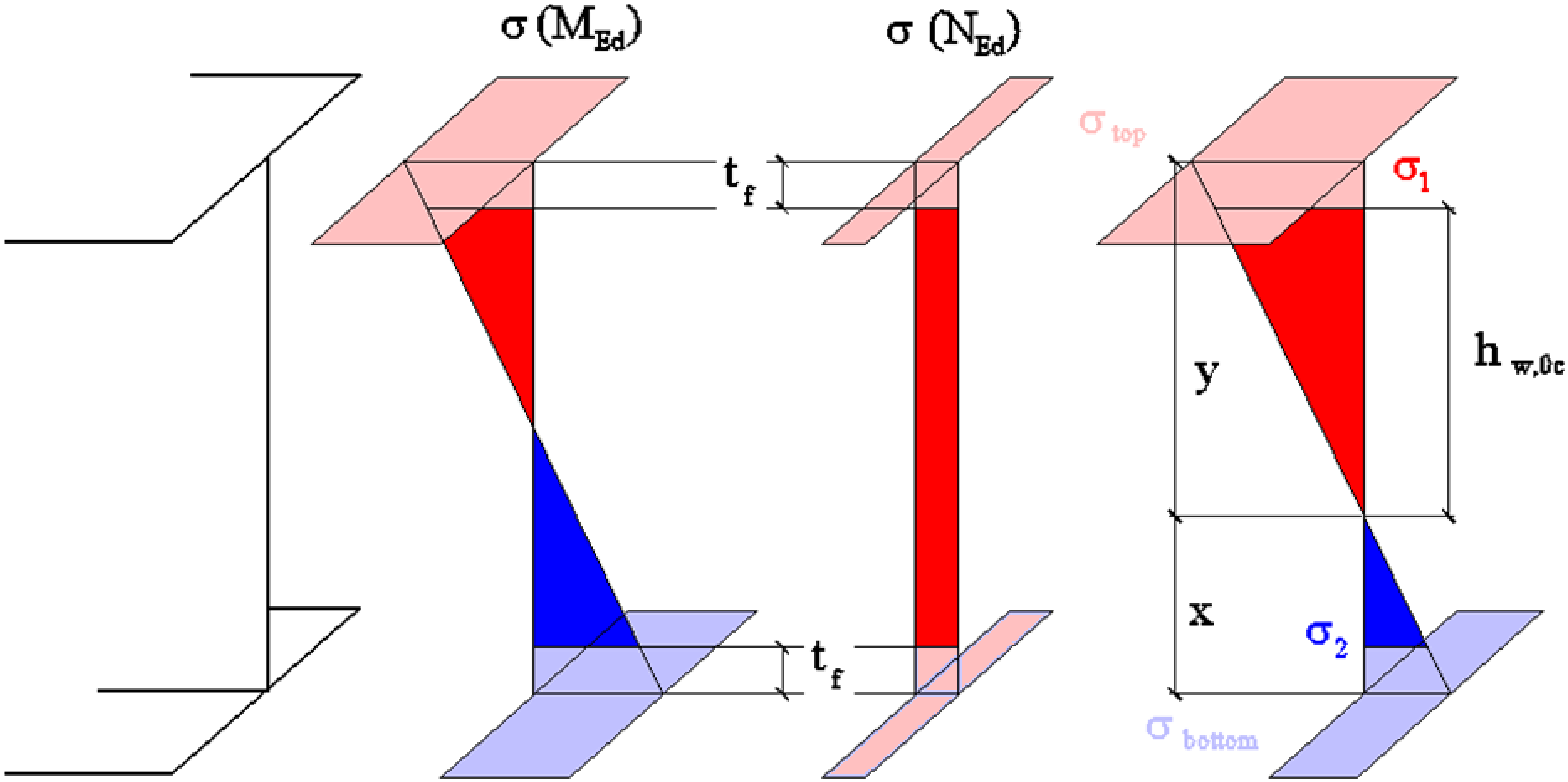


Photo: Autor

σ_1 - compression

σ_2 - tension

σ_{top} = 141,039 MPa

σ_1 = 134,025 MPa

σ_2 = -62,856 MPa

σ_{bottom} = -69,900 MPa

$$\psi = \sigma_2 / \sigma_1 = -0{,}469$$

$$\eta = 0{,}7 + 0{,}3\ \psi = 0{,}559$$

$$\beta = \eta\ b / t_w = 0{,}559 \cdot 270 / 8 = 18{,}866$$

$$\varepsilon = \sqrt{(250 / f_0)} \rightarrow \varepsilon = \sqrt{(250 / 280)} = 0{,}945$$

$$\rho_c = \min \{ [C_1 / (\beta / \varepsilon)] - [C_2 / (\beta / \varepsilon)^2] ; 1{,}0\} = 0{,}956$$

$t_{w,\ eff,\ 4} = t_4\ \rho_c = 8 \cdot 0{,}956 = 7$ mm

$t_{w,\ haz,\ eff,\ 4} = \min\ (t_{w,\ eff,\ 4}\ \ ;\ \ t_{w,\ haz}) = 4$ mm

Web: 4 mm / 8 mm / 7 mm / 4 mm

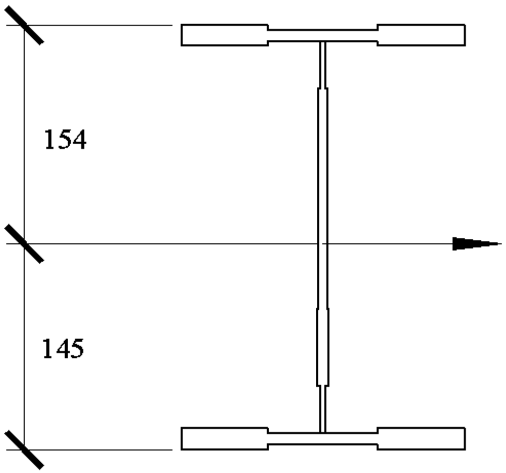


Photo: Autor

$A_{eff,\ 4} = 65{,}78\ cm^2$

$W_{y,\ top,\ eff,\ 4} = 692{,}972\ cm^3$

$W_{y,\ bottom,\ eff,\ 4} = 736{,}910\ cm^3$

Vth step

Compressed web - recalculation

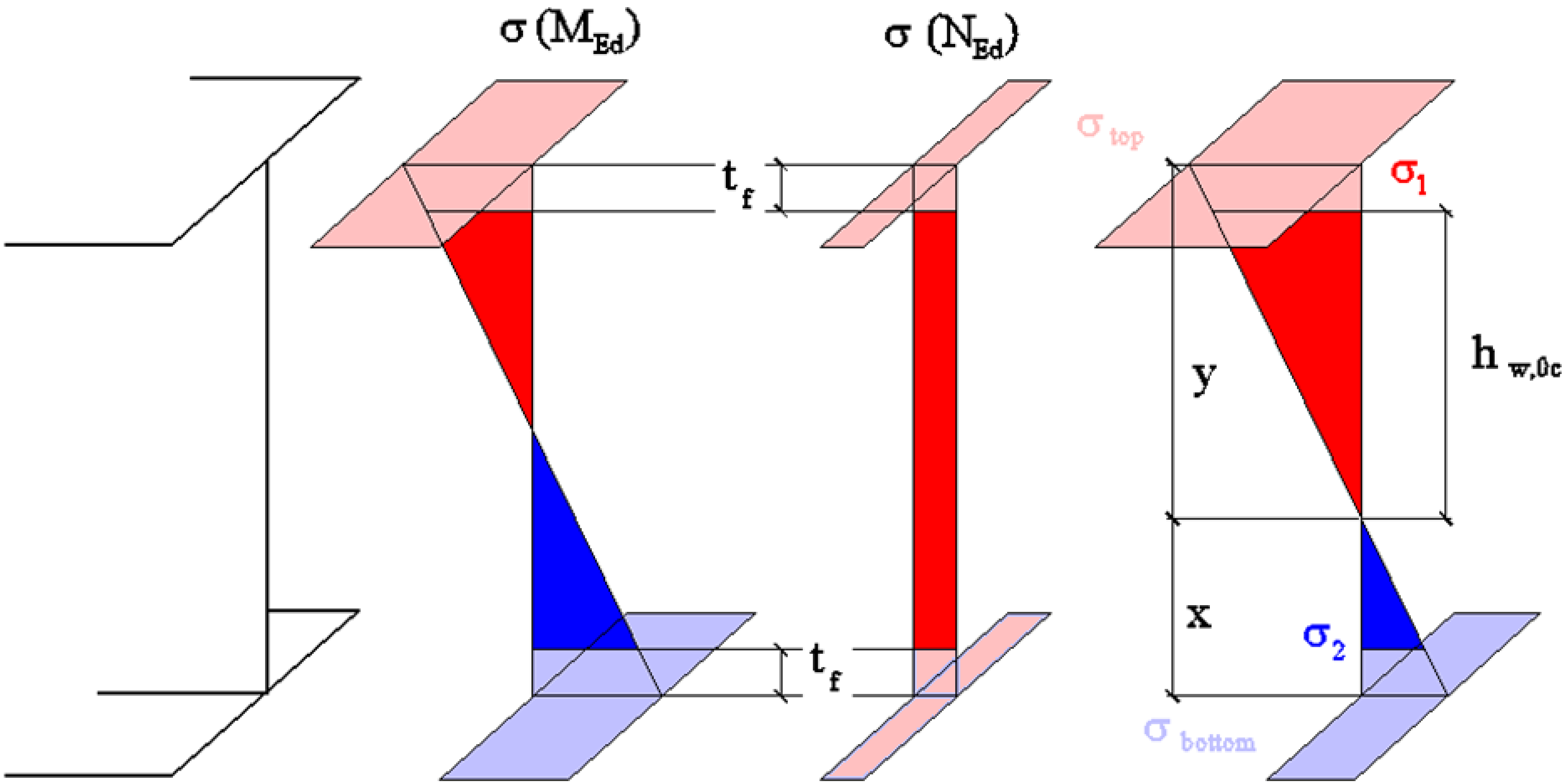


Photo: Autor

σ_1 - compression

σ_2 - tension

σ_{top} = 143,887 MPa

σ_1 = 136,814 MPa

σ_2 = -61,621 MPa

σ_{bottom} = -68,753 MPa

$$\psi = \sigma_2 / \sigma_1 = -0{,}450$$

$$\eta = 0{,}7 + 0{,}3\ \psi = 0{,}565$$

$$\beta = \eta\ b / t_w = 0{,}559 \cdot 270 / 8 = 19{,}069$$

$$\varepsilon = \sqrt{}\ (250 / f_0) \rightarrow \varepsilon = \sqrt{}\ (250 / 280) = 0{,}945$$

$$\rho_c = \min \{\ [C_1 / (\beta / \varepsilon)] - [C_2 / (\beta / \varepsilon)^2]\ ;\ 1{,}0\} = 0{,}950$$

$t_{w,\,eff,\,5} = t_5\,\rho_c = 8 \cdot 0{,}950 = 7\ \mathrm{mm}$

$t_{w,\,haz,\,eff,\,5} = \min\,(t_{w,\,eff,\,5}\ \ ;\ \ t_{w,\,haz}) = 4\ \mathrm{mm}$

The same cross-section as after IVth step

$A_{eff,\,5} = 65{,}78\ \mathrm{cm}^2$

$W_{y,\ top,\,eff,\,5} = 692{,}972\ \mathrm{cm}^3$

$W_{y,\ bottom,\,eff,\,5} = 736{,}910\ \mathrm{cm}^3$

		A [cm^2]	$W_{y,\ top}$ [cm^2]	$W_{y,\ bottom}$ [cm^2]
0	Start	81,600	900,480	900,480
eff 1	Welded zone	68,560	732,733	732,733
eff 2	Shear lag in flange	68,560	732,733	732,733
eff 3	Reduction of compressed flange	67,34	705,992	728,503
eff 4	1^{st} reduction of comperssed part of web	65,78	692,972	736,910
eff 5	2^{nd} reduction of comperssed part of web	65,78	692,972	736,910

Difference between step IV^{th} and V^{th}:	0,000 %	0,000 %	0,000 %

All differences < 2,0%, enough.

For aluminum structures, as for steel structures, the same phenomena must be checked:

- proportion between flange and web;
- shear resistance with possibility of loss of web stability;
- resistance under force in point with possibility of loss of web stability;

Formulas for aluminum differ in some details from those for steel.

Local instability, local buckling - niestateczność lokalna

Shear lag in flange - efekt szerokiego pasa

Hogging - strefa przęsłowa

Sagging - strefa podporowa

Plate buckling effect - niestateczność ścianek

Thank you for attention

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