Metal Structures

Laboratory I

Geometrical characteristics, steel products

LABORATORY OBJECTIVE’S

Presentation various types of steel cross-sections;

Reminder about geometrical characteristics from Strength of Materials;

Calculation of geometrical characteristisc for massvie cross-sections;

LABORATORY EXERCISES # 1

Student..

Topic:

1. Using a ruler, thickness gauge and ultrasonic thickness gauge, identify the profile of plate girder with corrugated web.
2. Using a ruler, thickness gauge and ultrasonic thickness gauge, identify the profile used for supporting node of truss.
3. Calculate geometrical characteristics for cross-section as follow:

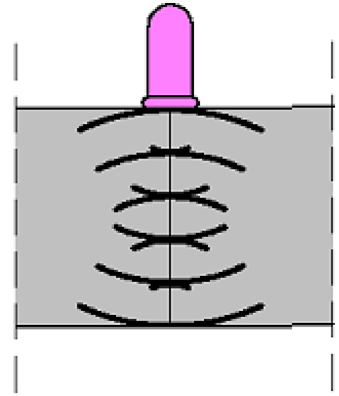
   I ....

   L ....

   [.....................................
   Shape ....

Two separated grades:

- For points 1+2 → $L_{1,\,1}$
- For point 3 → $L_{1,\,2}$



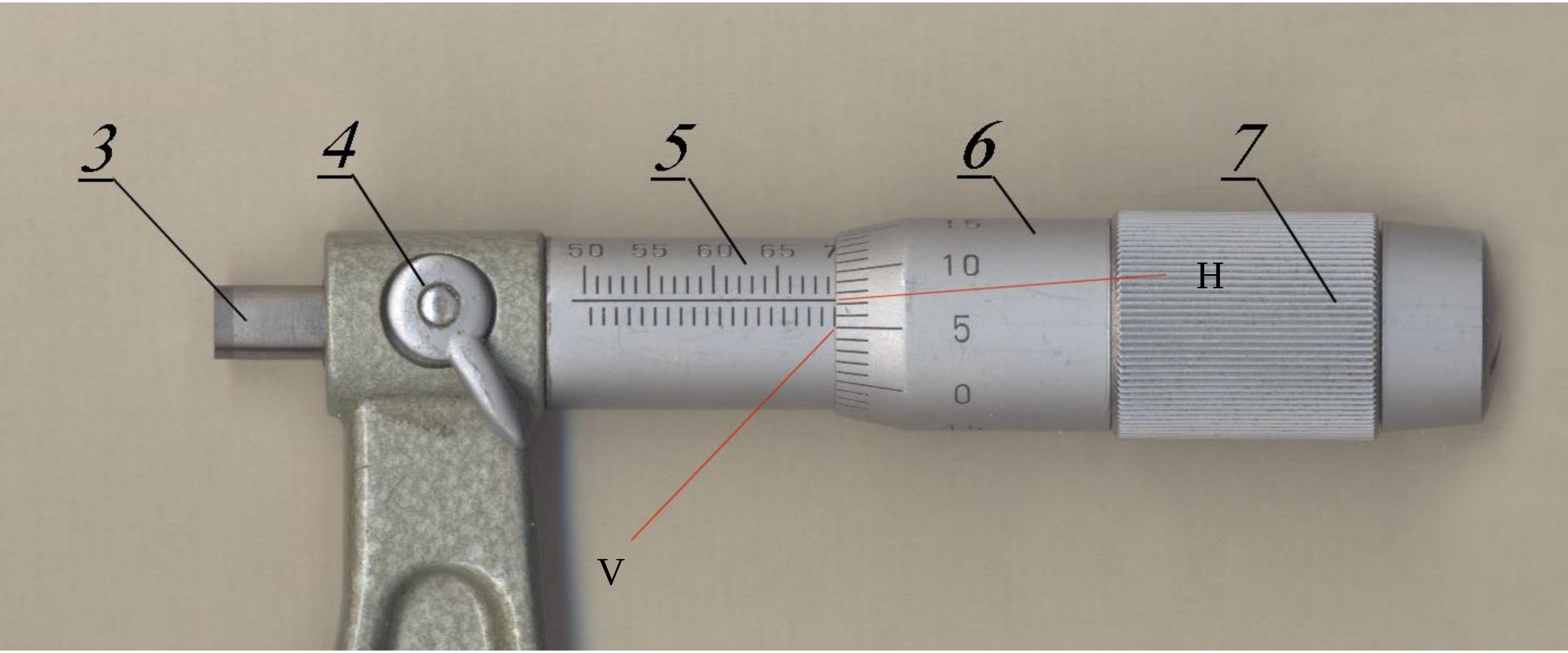
Ultrasonic gauge:

2 x Thickness = Velocity / Time

Ruler,

micrometric gauge

Photo.: Autor



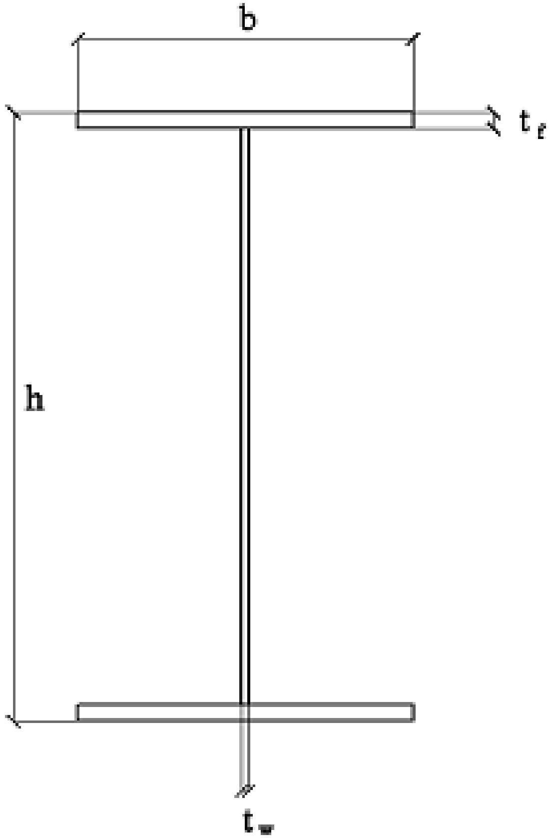
Micrometric gauge: how to use

Photo: wikipedia

V: vertical line – more than 71 mm

H: horizontal line – between 7 and 8 (about 7,5); 1/100 scale

Result: 71 + 7,5/100 = 71,075 mm



Describe	Symbol	Value [mm]	Average value [mm]
Ruler	h		
Ruled	b		
Thickness gauge	t_f		
Thickness gauge	t_w		
Ultrasonic thickness gauge			

Photo: Autor

Steel products

I - beam:

- Hot rolled
- Welded

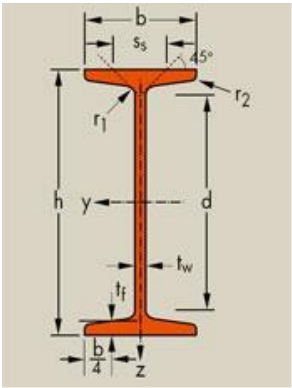
Hot rolled:

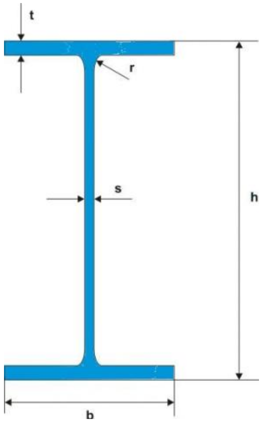
I H

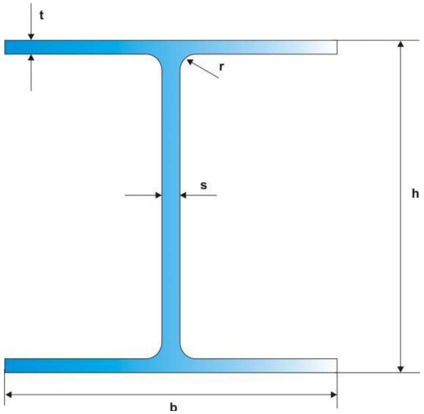
IP

HE

Photo: optimax.pl







IPN

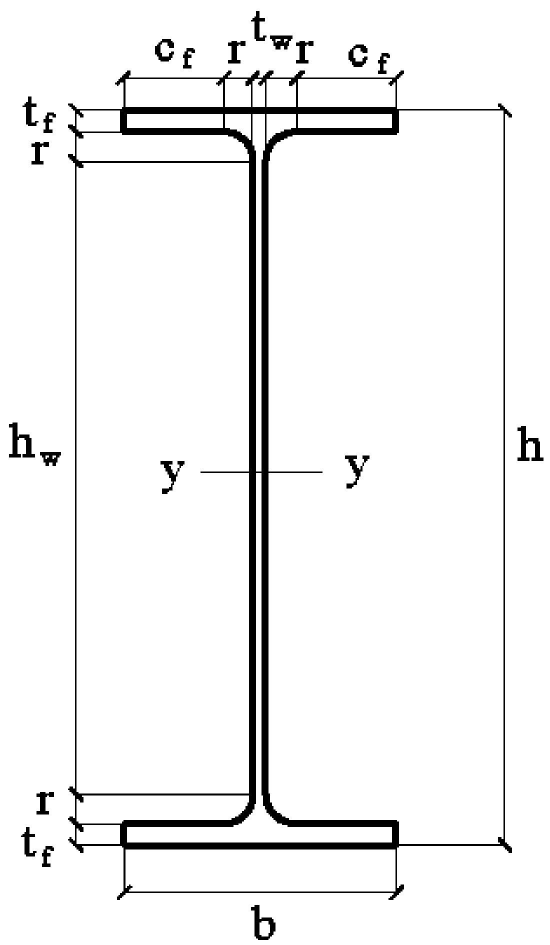
IPE, IPE-A, IPE-AA IPE-O

HEB, HEA, HEAA, HEM

Photo: hmsteel.pl

Photo: hmsteel.pl

Photo: Autor



IPN h → h [mm]

IPE h → h [mm]

HEB h → h [mm]

h (IPE-O) > h (IPE) > h (IPE-A) > h (IPE-AA)

h (HEM) > h (HEB) > h (HEA) > h (HEAA)

h (IPN 200) = h (IPE 200) = h (HEB 200) = 200 mm

h (IPE-O 200) = 202 mm

h (IPE-A 200) = 197 mm

H (IPE-AA 200) = 196,4 mm

h (HEM 200) = 220 mm

h (HEA 200) = 190 mm

h (HEAA 200) = 186 mm

Welded:

- Plane web
    - IKS
    - HKS

Photo: weldingweb.com

- Corrugated web

Photo: hxssvic.en.ec21.com

Other types of products (the most important):

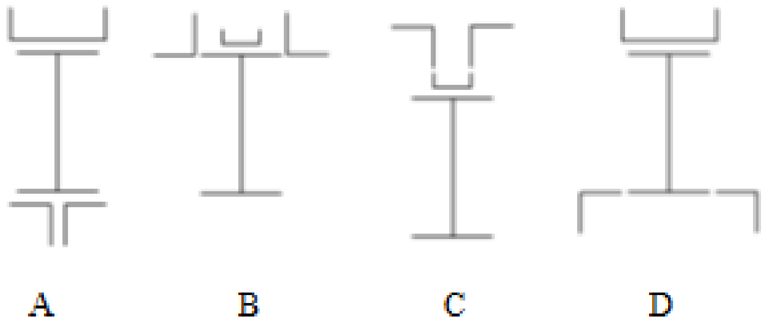
- Plates (sheets steel)
- Channel sections
- Angle sections
- Ropes
- Sigma-beams
- Hollow sections
- Z-bars
- Cold-formed sections

Photo: Autor

More informtion about steel products was presented on lectures (→ Lec #1)

Third point

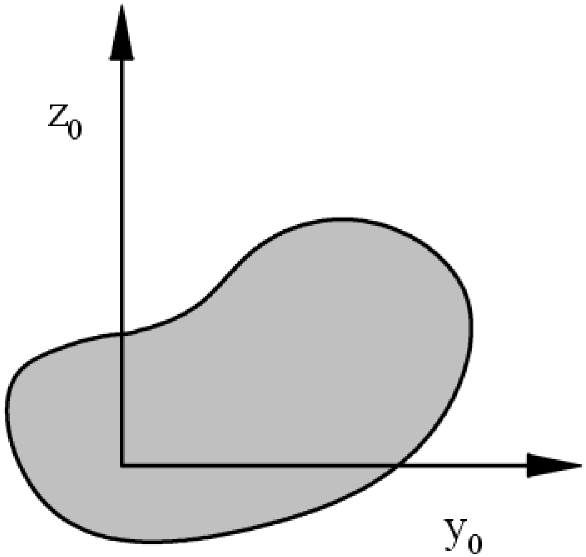
Nr	Name	I	L	C	Shape
1		IPE 300	60x60x6	120	A
2		IPEA 300	50x50x5	160	B
3		IPEO 300	65x65x9	180	C
4		HEB 300	45x45x5	120	D



Calculation of geometrical characteristics (A, $A_{V,\,y}$, $A_{V,\,z}$, J_y, J_z, i_y, i_z, $W_{y,\,el,\,top}$, $W_{y,\,el,\,bottom}$, $W_{z,\,el}$, $W_{y,\,pl}$, $W_{z,\,pl}$)

Photo.: Autor

Geometrical characteristics



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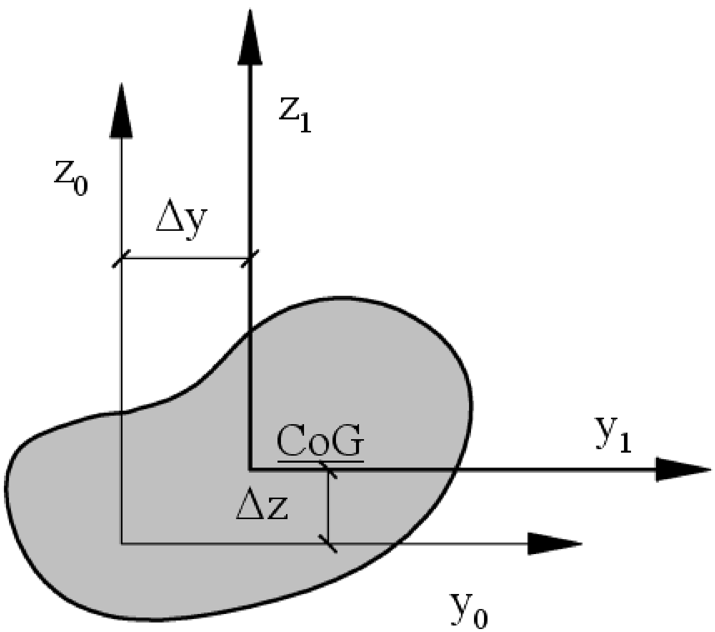
Area of cross-section [m²]:

$$A = \int \int dz\,dy$$

Static moment [m^3]:

$$S_y = \int\int z \, dz \, dy$$

$$S_z = \int\int y \, dz \, dy$$



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$S_y = S_z = 0$ when axis goes through Centre of Gravity:

$$\Delta y = S_z / A$$

$$\Delta z = S_y / A$$

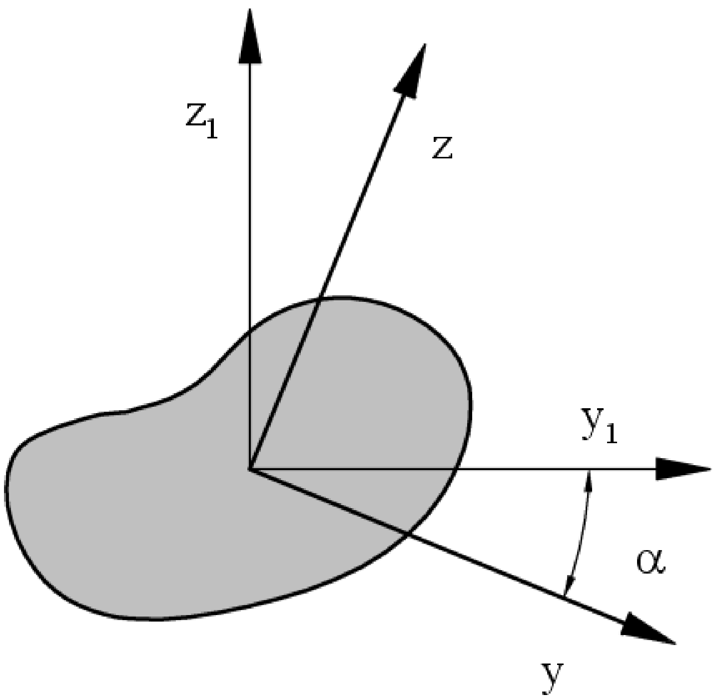
Moment of inertia [m^4]:

$$J_y = \int\int z^2 \, dz \, dy$$

$$J_z = \int\int y^2 \, dz \, dy$$

Centrifugal moment (moment of deviation) [m^4]:

$$D_{yz} = \int\int y \, z \, dz \, dy$$



$D_{yz} = 0$ for principal axes of inertia:

$$J_y = \{(J_{y1} + J_{z1}) + \sqrt{[(J_{y1} - J_{z1})^2 + 4\, D_{y1z1}{}^2\,]}\} / 2$$

$$J_z = \{(J_{y1} + J_{z1}) - \sqrt{[(J_{y1} - J_{z1})^2 + 4\, D_{y1z1}{}^2\,]}\} / 2$$

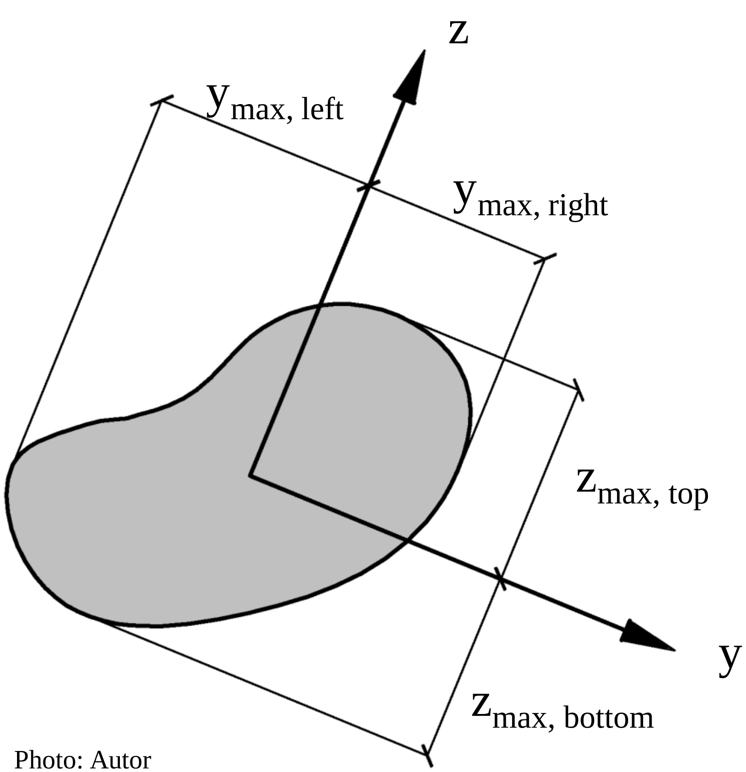
$$\text{tg}\, \alpha = D_{y1z1} / (J_{y1} - J_y)$$

Photo: Autor

Radius of gyration [m]:

$$i_y = \sqrt{(J_y / A)}$$

$$i_z = \sqrt{(J_z / A)}$$



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Sectional modulus [m^3]:

$$W_{y, top} = J_y / | z_{max, top}|$$

$$W_{y, bottom} = J_y / | z_{max, bottom}|$$

$$W_{z, left} = J_z / | y_{max, left}|$$

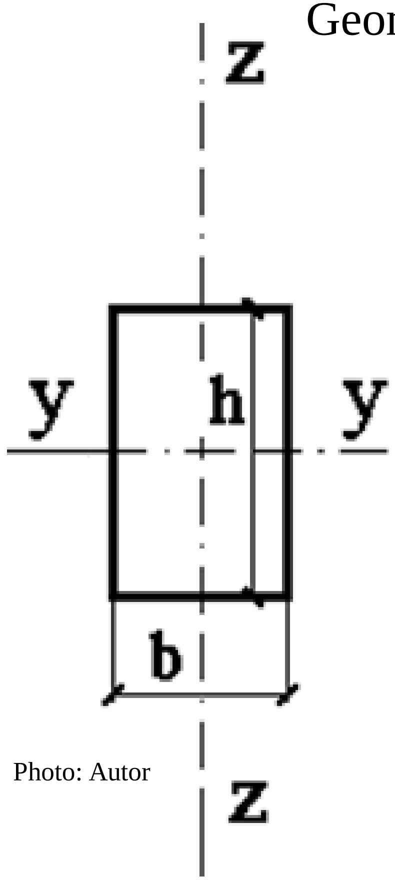
$$W_{z, right} = J_z / | y_{max, right}|$$

For bi-symmetrical cross-section:

$$W_{y, top} = W_{y, bottom} = W_y$$

$$W_{z, left} = W_{z, right} = W_z$$

Geometrical characteristics - rectangular cross-section:



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$$A = b\,h$$

$$J_y = b\,h^3 / 12$$

$$W_y = J_y / z_{max} = J_y / (0{,}5h) = b\,h^2 / 6$$

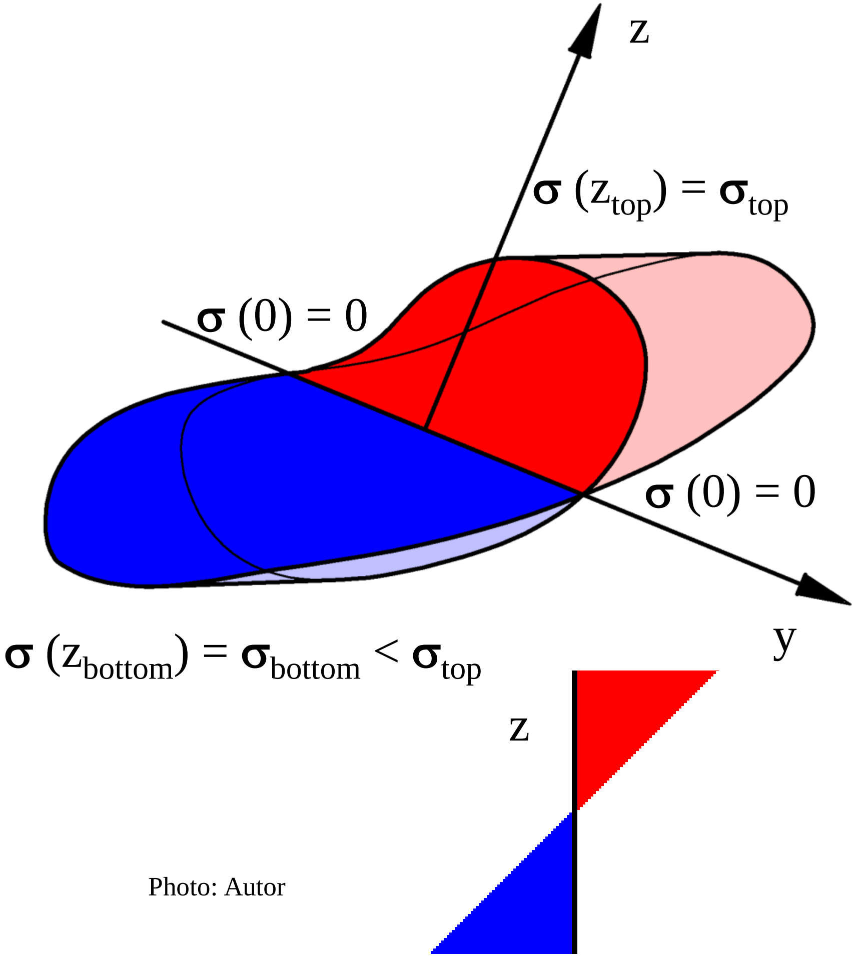
$$i_y = \sqrt{(J_y / A)} = h / (2 \sqrt{3})$$

More information about sectional modulus:

$$W_{y,\,top} = J_y \,/\, |\, z_{max,\,top}|$$
$$W_{y,\,bottom} = J_y \,/\, |\, z_{max,\,bottom}|$$

Why sectional modulus is important?

Let's analyse stress distribution as follow:



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- value of stress depends only on z:

  $\boldsymbol{\sigma}(y, z) = \boldsymbol{\sigma}(z);$

- $z = 0 \rightarrow \boldsymbol{\sigma} = 0;$
- $\boldsymbol{\sigma}(z_{max}) = \boldsymbol{\sigma}(z_{top}) = \boldsymbol{\sigma}_{top}$
- <u>there is linear function $\boldsymbol{\sigma}(z)$</u>:

  $\boldsymbol{\sigma}(z) = \boldsymbol{\sigma}_{top}\,(z / z_{top})$

What is value of axial force, makes this one stresses:

$$\boldsymbol{\sigma}(z) = \boldsymbol{\sigma}_{top} \, (z \,/\, z_{top})$$

$$N = \int\int [\boldsymbol{\sigma}(z)] \, dz \, dy = \int\int [\boldsymbol{\sigma}_{top} \, (z \,/\, z_{top})] \, dz \, dy = (\boldsymbol{\sigma}_{top} \,/\, z_{top}) \int\int z \, dz \, dy =$$

$$= \| \; \#t \,/\, 14\!: \; S_y = \int\int z \, dz \, dy \quad ; \quad \underline{CoG}\!: S_y = 0 \; \| = (\boldsymbol{\sigma}_{top} \,/\, z_{top}) \, 0 = 0$$

What is value of bending moment, makes this one stresses:

$$\boldsymbol{\sigma}(z) = \boldsymbol{\sigma}_{top}\,(z / z_{top})$$

$$M_y = \int\int \{z\,[\boldsymbol{\sigma}(z)]\}\;dz\,dy = \int\int \{z\,[\boldsymbol{\sigma}_{top}\,(z / z_{top})]\}\;dz\,dy =$$

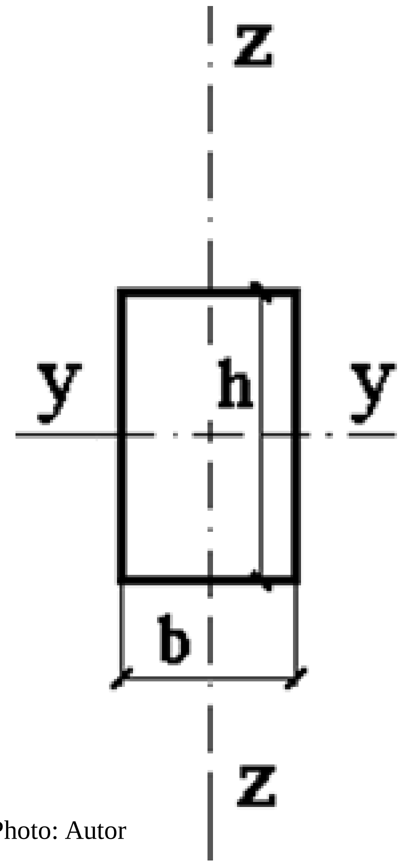
$$= (\boldsymbol{\sigma}_{top} / z_{top}) \int\int (z^2)\;dz\,dy =$$

$$= \| \;\#t / 15: J_y = \int\int z^2\,dz\,dy\; \| = (\boldsymbol{\sigma}_{top} / z_{top})\,J_y =$$

$$= \| \;\#t / 16: W_{y,\,top} = J_y / |\,z_{\,top}|\; \| = \boldsymbol{\sigma}_{top}\,W_{y,\,top}$$

$$M_y = \boldsymbol{\sigma}_{max}\,W_{y,\,max}$$

$$M_{y,\,max} = f_y\,W_{y,\,max}$$



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What is the best type of cross-section, if we have limitations as follow:

$A \longrightarrow$ small (small dead weight)

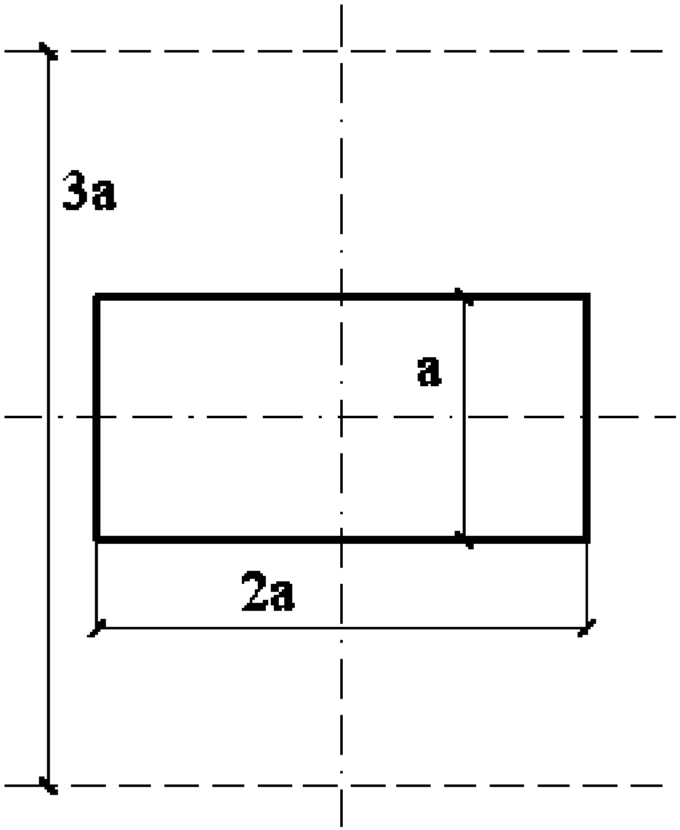
$W_y \longrightarrow$ large (large resistance of cross-section)

$h \longrightarrow$ limited (design limitation)

For example:

$A = 2\ a^2$

$h \leq 3a$



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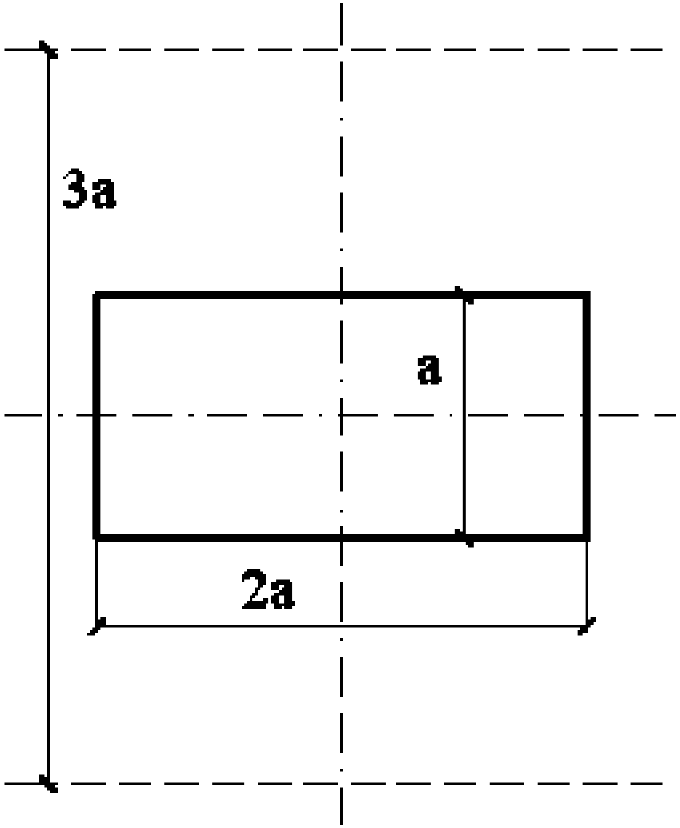
Shape of cross-section:

$J_y = ?$

$W_y = ?$

$i_y = ?$

$M_{y,\,max} = ?$



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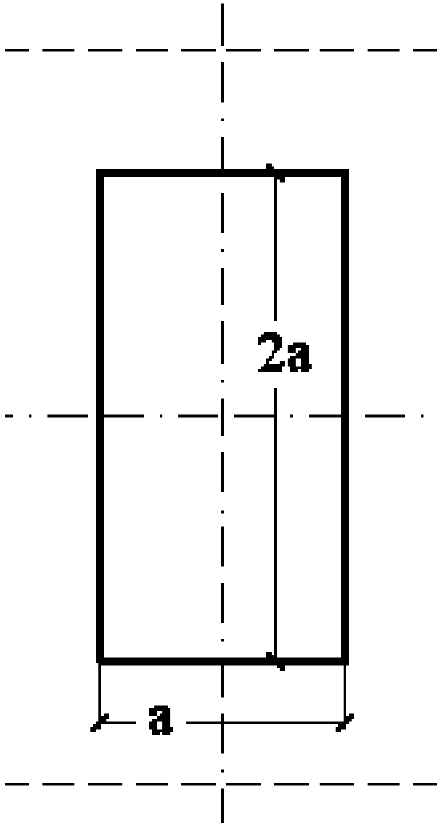
$$b = 2a \quad h = a$$

$$J_y = b\,h^3 / 12 = 0{,}167\ a^4$$

$$W_y = b\,h^2 / 6 = 0{,}333\ a^3$$

$$i_y = \sqrt{(J_y / A)} = 0{,}289\ a$$

$$M_{y,\,max} = 0{,}333\ a^3\,f_y$$



$$b = a \quad h = 2a$$

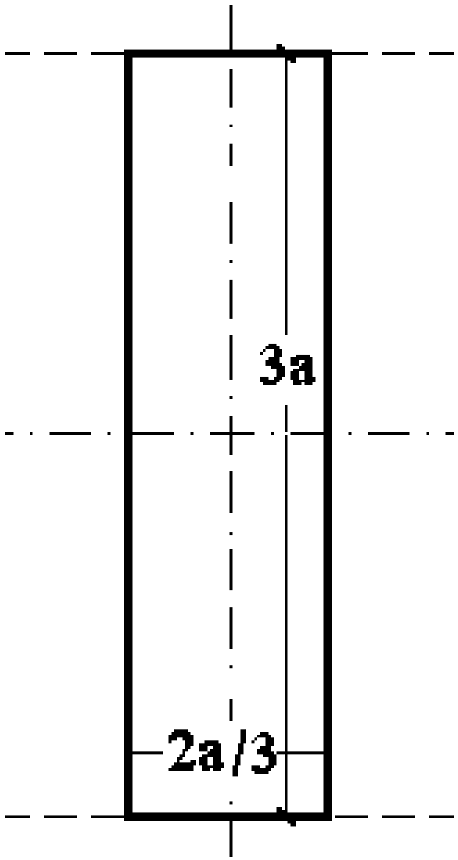
$$J_y = b\,h^3 / 12 = 0{,}667\ a^4$$

$$W_y = b\,h^2 / 6 = 0{,}667\ a^3$$

$$i_y = \sqrt{}\,(J_y / A) = 0{,}577\ a$$

$$M_{y,\,max} = 0{,}667\ a^3\,f_y$$

Photo: Autor



$$b = 2\,a / 3 \quad h = 3\,a$$

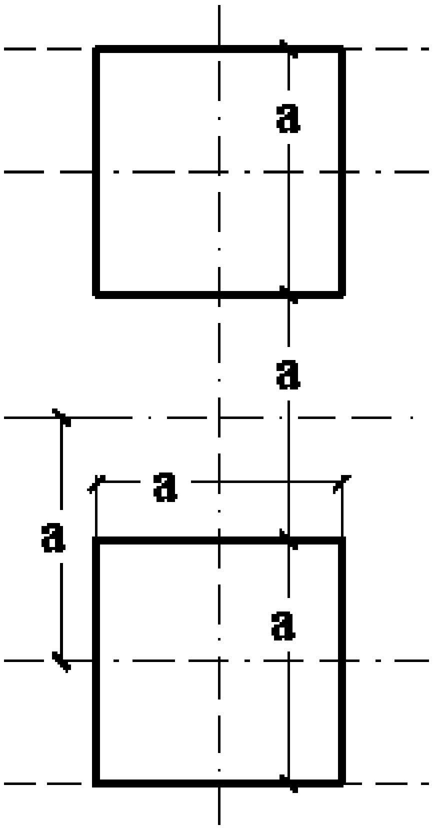
$$J_y = b\,h^3 / 12 = 1{,}500\ a^4$$

$$W_y = b\,h^2 / 6 = 1{,}000\ a^3$$

$$i_y = \sqrt{(J_y / A)} = 0{,}866\ a$$

$$M_{y,\,max} = 1{,}000\ a^3\, f_y$$

Photo: Autor



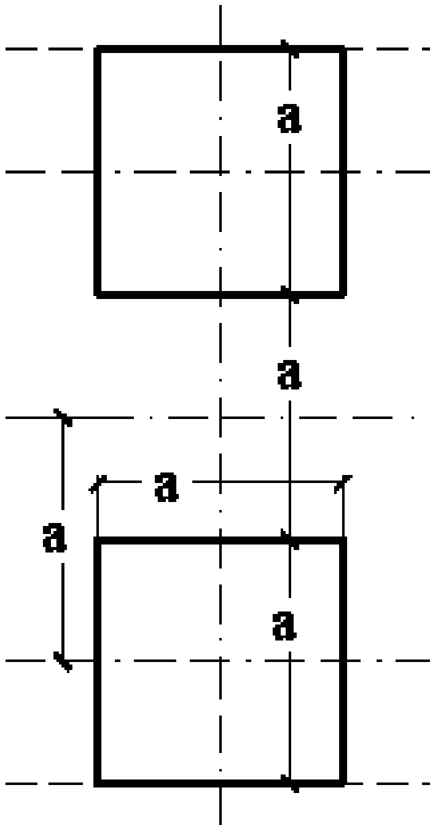
$J_y = ?$

$W_y = ?$

$i_y = ?$

$M_{y,\,max} = ?$

Photo: Autor



$$J_y = 2\,[\,b\,h^3 / 12\,] + 2\,[\,A_1\,d^2\,]$$

Own stiffness Steiner's theorem

$b = a \quad h = a \quad d = a \quad z = 1{,}5\,a \quad A_1 = a^2$

$$J_y = 2\,[\,b\,h^3 / 12\,] + 2\,[A_1\,d^2\,] =$$

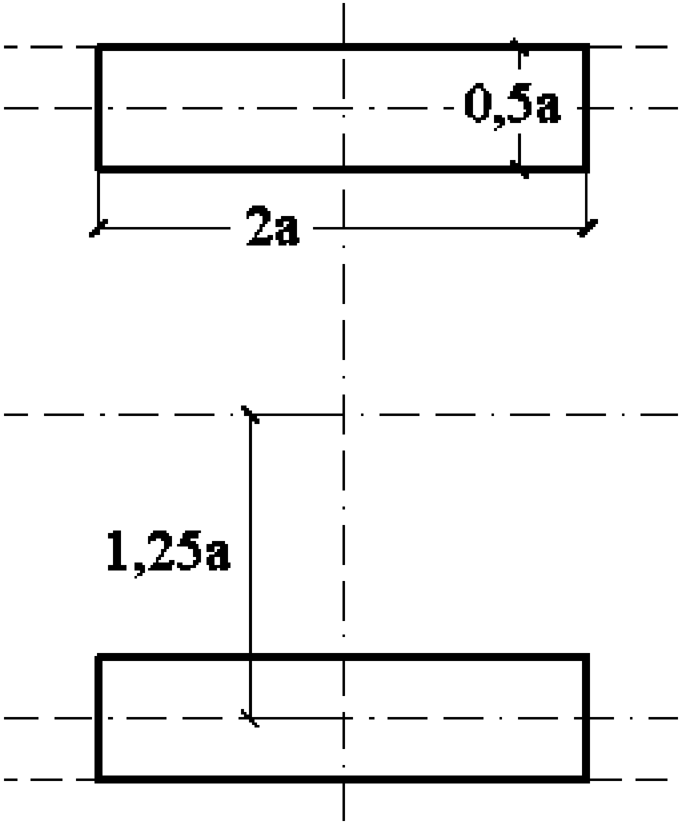
$$= 0{,}167\,a^4 + 2{,}000\,a^4 = 2{,}167\,a^4$$

$$W_y = J_y / z = 1{,}444\,a^3$$

$$i_y = \sqrt{(J_y / A)} = 1{,}041\,a$$

$$M_{y,\,max} = 1{,}444\,a^3\,f_y$$

Photo: Autor



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$b = 2a \quad h = 0{,}5a \quad d = 1{,}25\ a \quad z = 1{,}5\ a \quad A_1 = a^2$

$J_y = 2\ [\ b\ h^3 / 12\ ] + 2\ [A_1\ d^2\ ] =$

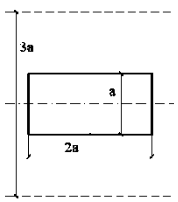
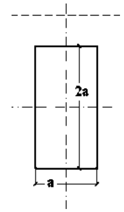
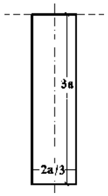
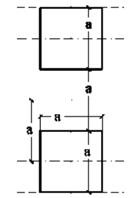
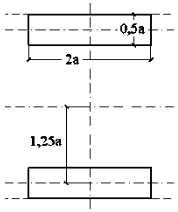
$= 0{,}042\ a^4 + 3{,}125\ a^4 = 3{,}167\ a^4$

$W_y = J_y / z = 2{,}111\ a^3$

$i_y = \sqrt{(J_y / A)} = 1{,}258\ a$

$M_{y,\ max} = 2{,}111\ a^3\ f_y$

Photo: Autor

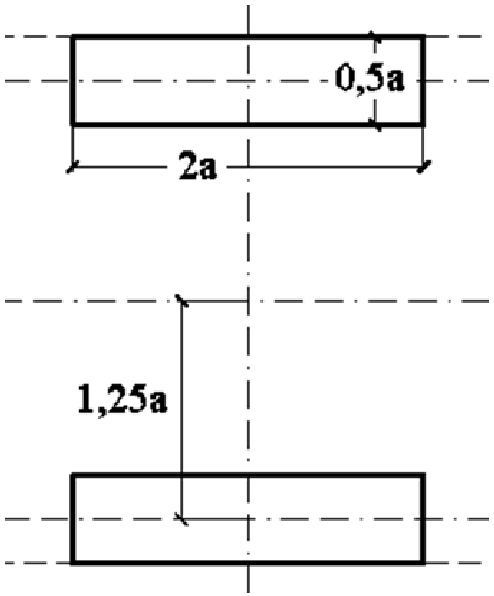
					
A	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$
J_y	$0{,}167\ a^4$	$0{,}667\ a^4$	$1{,}500\ a^4$	$2{,}167\ a^4$	$3{,}167\ a^4$
W_y	$0{,}333\ a^3$	$0{,}667\ a^3$	$1{,}000\ a^3$	$1{,}444\ a^3$	$2{,}111\ a^3$
i_y	0,289 a	0,577 a	0,866 a	1,041 a	1,258 a
M_y	$0{,}333\ a^3 f_y$	$0{,}667\ a^3 f_y$	$1{,}000\ a^3 f_y$	$1{,}444\ a^3 f_y$	$2{,}111\ a^3 f_y$
J_y	1,000	4,000	9,000	13,000	19,000
W_y	1,000	2,000	3,000	4,333	6,333
i_y	1,000	2,000	3,000	3,602	4,221
$M_{y,\ max}$	1,000	2,000	3,000	4,333	6,333

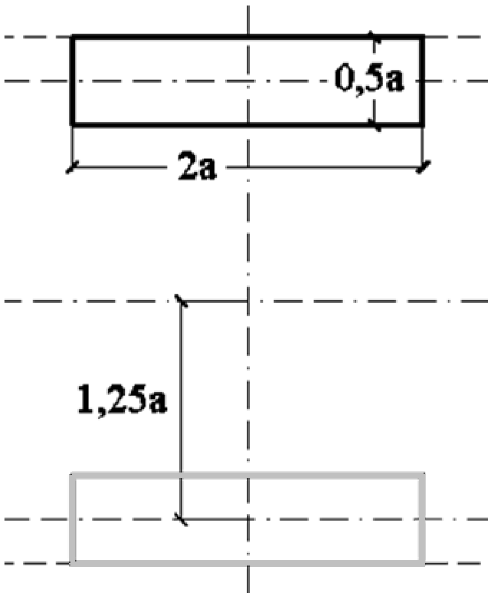
There is an error in the calculation.

What kind of error?

Steiner's theorem can be used only when exist rigid connection between separated part of cross-section.

Upper and lower branch of cross-section do not cooperate each other, when there is no connecting element. In this case, load acts only on one branch of cross-section.





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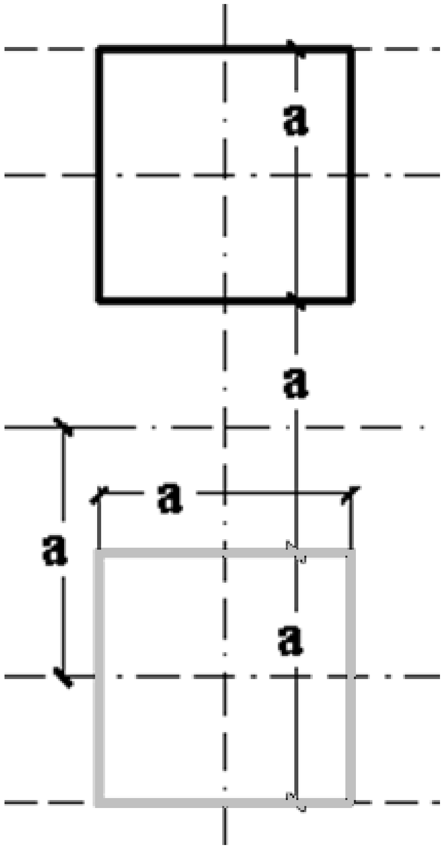


No connection - no Steiner's theorem, one branch only

$J_y = 2\ [\ b\ h^3 / 12\ ]$

$b = a \quad h = a \quad z = 1{,}5a$

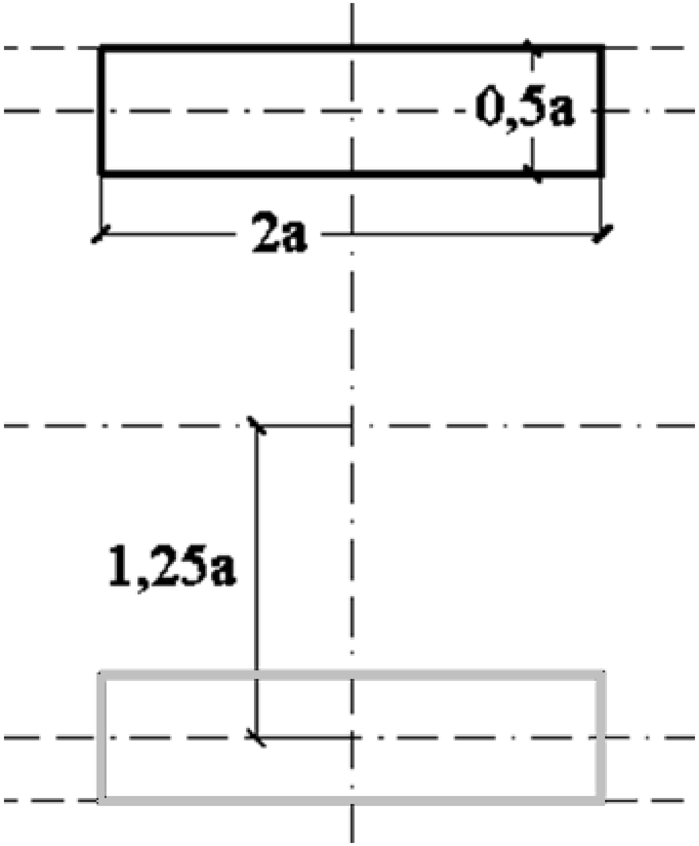
$J_y = a^4 / 12 = 0{,}083\ a^4$

$W_y = J_y / z = 0{,}056\ a^3$

$i_y = \sqrt{(J_y / A)} = 0{,}204\ a$

$M_{y,\ max} = 0{,}056\ a^3\ f_y$

Photo: Autor



No connection - no Steiner's theorem, one branch only

$$J_y = 2\ [\ b\ h^3 / 12\ ]$$

$$b = 2a \quad h = 0{,}5a \quad x = 1{,}5\ a$$

$$J_y = 0{,}021\ a^4$$

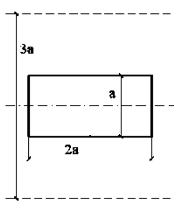
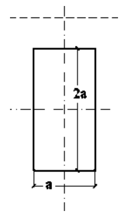
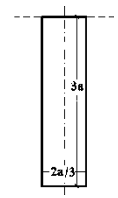
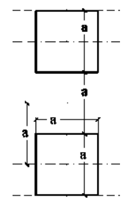
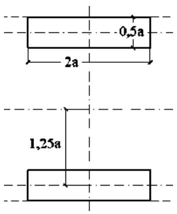
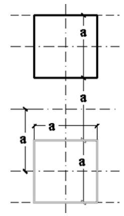
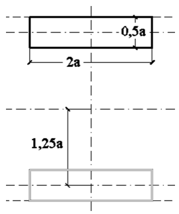
$$W_y = J_y / z = 0{,}014\ a^3$$

$$i_y = \sqrt{(J_y / A)} = 0{,}102\ a$$

$$M_{y,\ max} = 0{,}014\ a^3 f_y$$

Photo: Autor

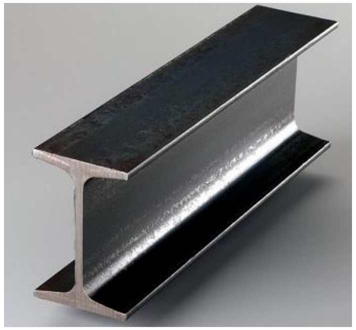
Photo: Autor

				Steiner's theorem		No steiner's theorem	
							
A	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$	$2a^2$
J_y	0,167 a^4	0,667 a^4	1,500 a^4	2,167 a^4	3,167 a^4	0,167 a^4	0,042 a^4
W_y	0,333 a^3	0,667 a^3	1,000 a^3	1,444 a^3	2,111 a^3	0,111 a^3	0,028 a^3
i_y	0,289 a	0,577 a	0,866 a	1,041 a	1,258 a	0,289 a	0,145 a
M_y	0,333 $a^3 f_y$	0,667 $a^3 f_y$	1,000 $a^3 f_y$	1,444 $a^3 f_y$	2,111 $a^3 f_y$	0,111 $a^3 f_y$	0,028 $a^3 f_y$
J_y	1,000	4,000	9,000	13,000	19,000	0,083	0,021
W_y	1,000	2,000	3,000	4,333	6,333	0,056	0,014
i_y	1,000	2,000	3,000	3,602	4,221	0,204	0,102
$M_{y,\ max}$	1,000	2,000	3,000	4,333	6,333	0,056	0,014

The best shape of cross-section for bending member:

I

Photo: discountsteel.com



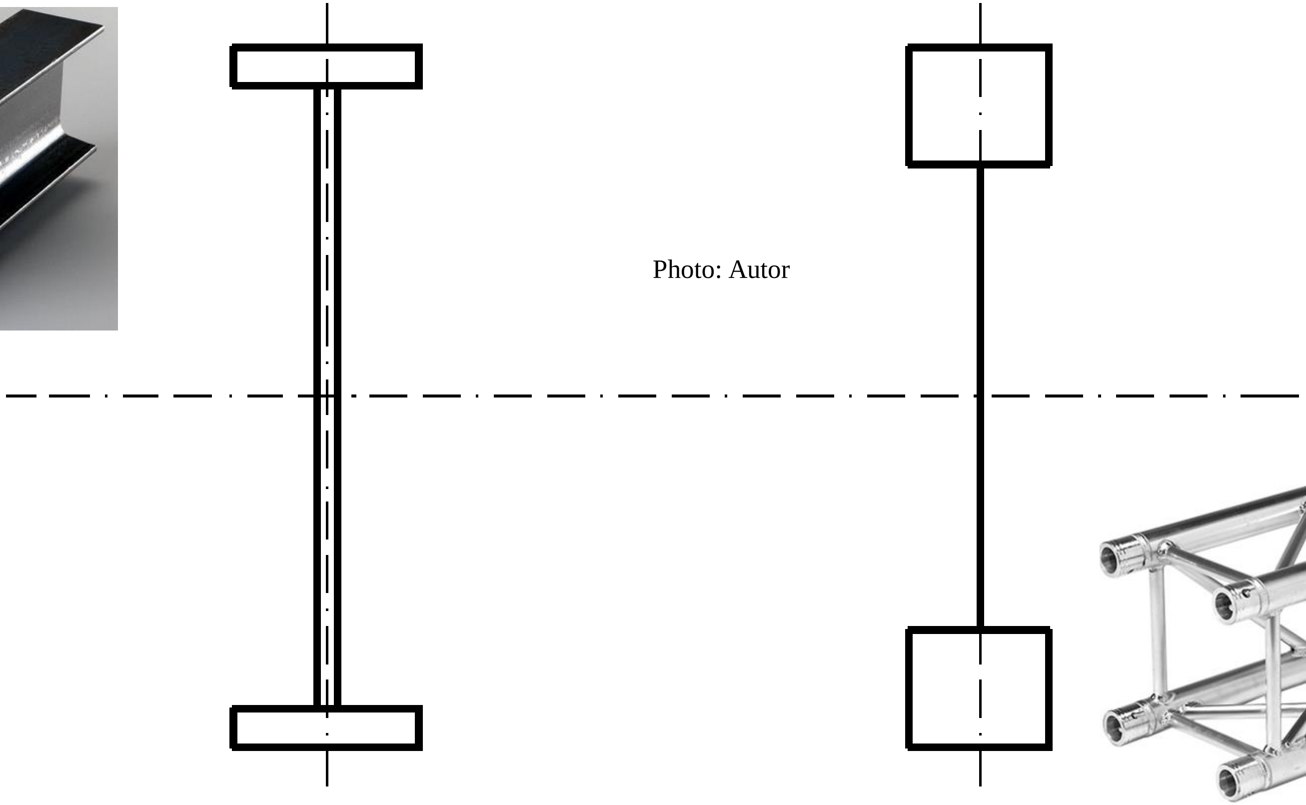


Photo: Autor

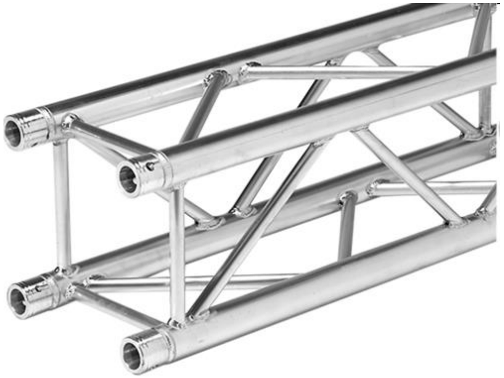
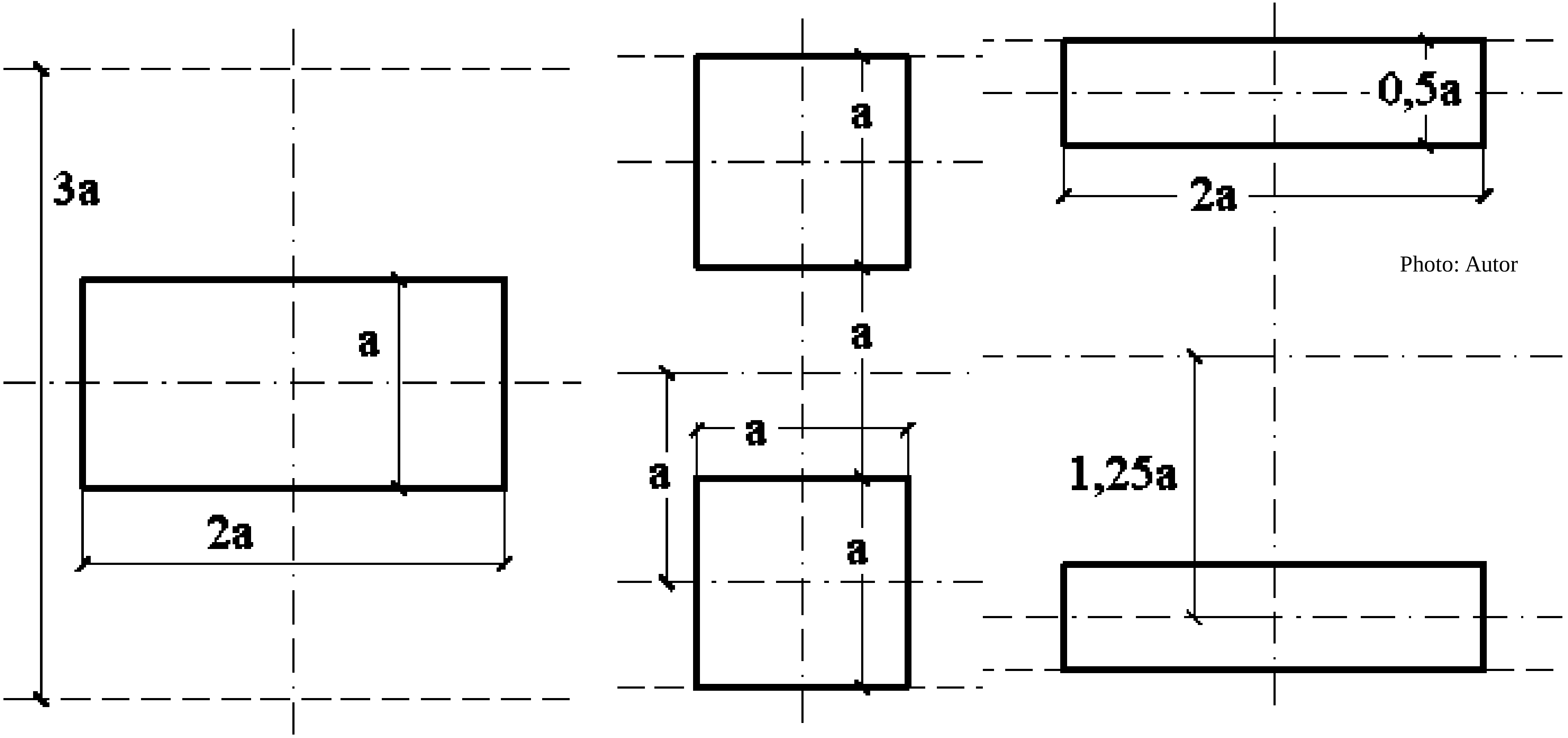


Photo: conference-truss-hire.co.uk

Cross-section of I-beam

Cross-section of truss



Own stiffness = 0,167 a^4 | Own stiffness = 0,167 a^4 | Own stiffness = 0,042 a^4

Steiner's theorem = 0,000 a^4 | Steiner's theorem = 2,000 a^4 | Steiner's theorem = 3,125 a^4

Total = 100,0% + 0,0% | Total = 7,7% + 92,3% | Total = 1,3% + 98,7%

For rectangles in horizontal position and in a distance from centre of gravity, their own stiffness is neglegible.

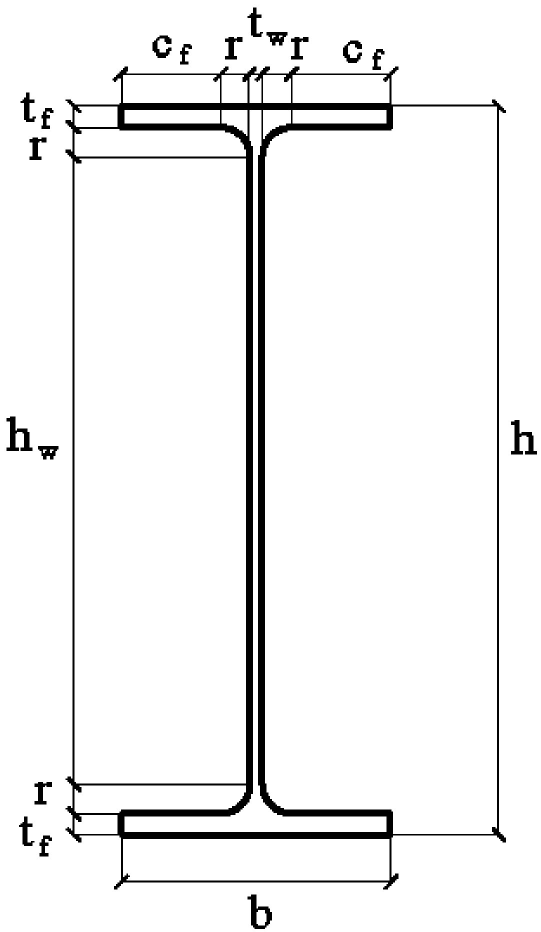


Photo: Author

IPE A 600 [mm]

h	b	t_f	t_w	r
597	220	17,5	9,8	24

J_y = 82 920 cm^4

Three rectangles; for horizontal: Steiner's theorem + own stiffness:

$$J_y = (59{,}7 - 2 \cdot 1{,}75)^3 \cdot 0{,}98 / 12 +$$

$$+ 2 \cdot [\,(1{,}75^3 \cdot 22 / 12) + 1{,}75 \cdot 22 \cdot (59{,}7 / 2 - 1{,}75 / 2)^2\,] =$$
$$= 79\,161 \text{ cm}^4 \quad (95{,}47\%)$$

Three rectangles; for horizontal: Steiner's theorem only:

$$J_y = (59{,}7 - 2 \cdot 1{,}75)^3 \cdot 0{,}98 / 12 +$$

$$+ 2 \cdot [1{,}75 \cdot 22 \cdot (59{,}7 / 2 - 1{,}75 / 2)^2\,] = 79\,142 \text{ cm}^4$$
(95,44%)

Previous calculations are true for elastic analysis (linear distribution of stresses). But there are three possibilities during analysis of steel structures: elastic, elasto-plastic and total plastic behaviour in analysis of steel structures. What about plastic analysis?

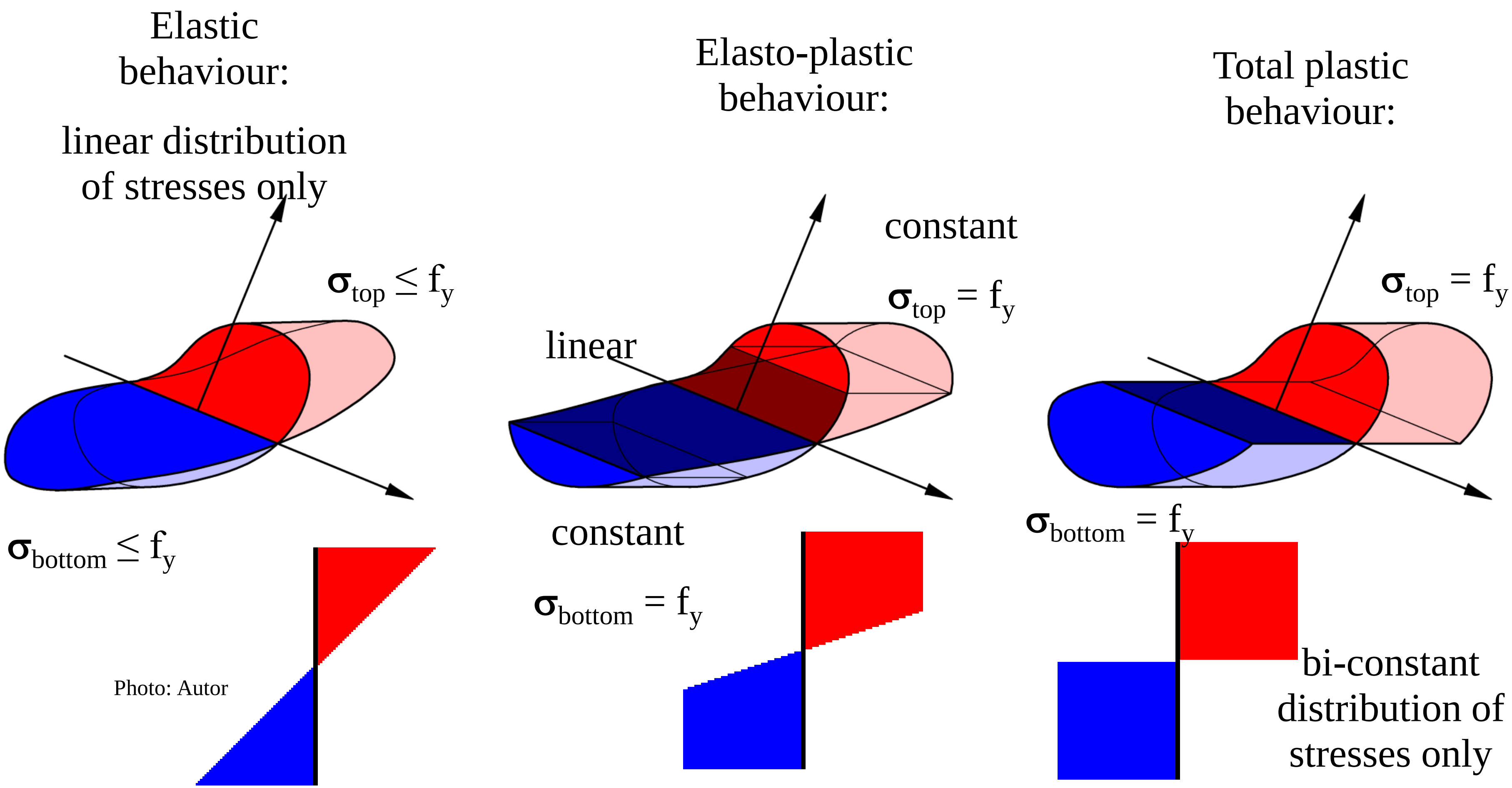
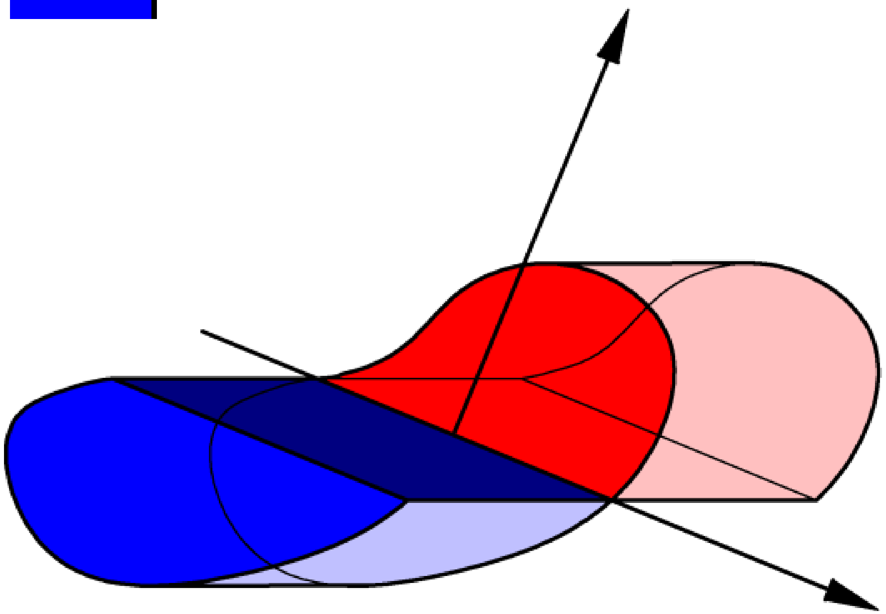


Photo: Autor

Generally:



$$M_y = \iint_A \sigma(y, z)\, z\, dz\, dy$$

and

$$N = \iint_A \sigma(y, z)\, dz\, dy$$

There is taken into consideration cross-sectional forces as for previous situation:

$$N = 0$$

and

$$M_y \neq 0$$

Photo: Autor

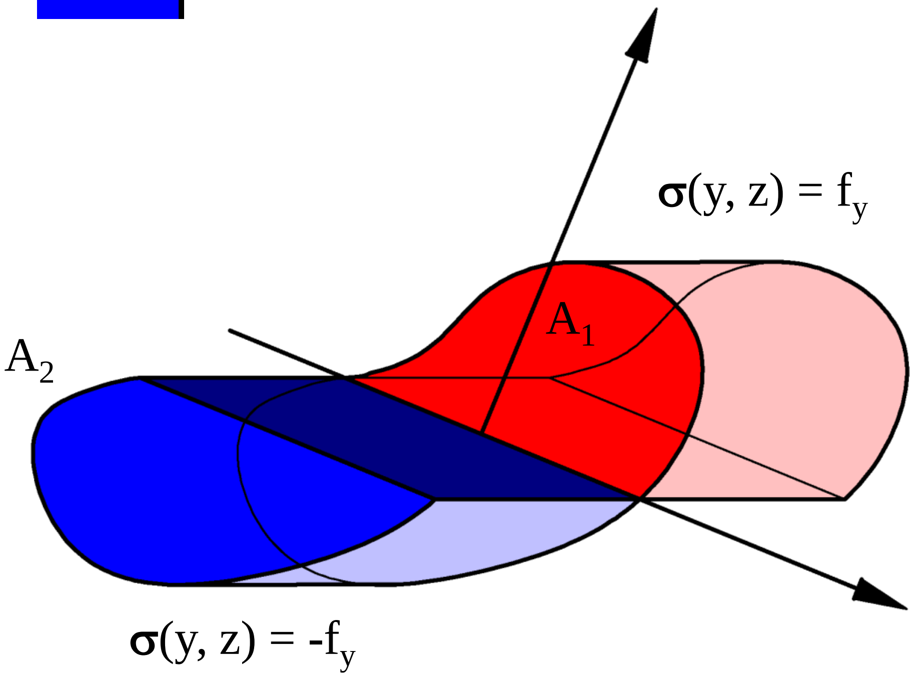


Photo: Autor

$$N = \int_A \int \boldsymbol{\sigma}(y, z)\, dz\, dy =$$

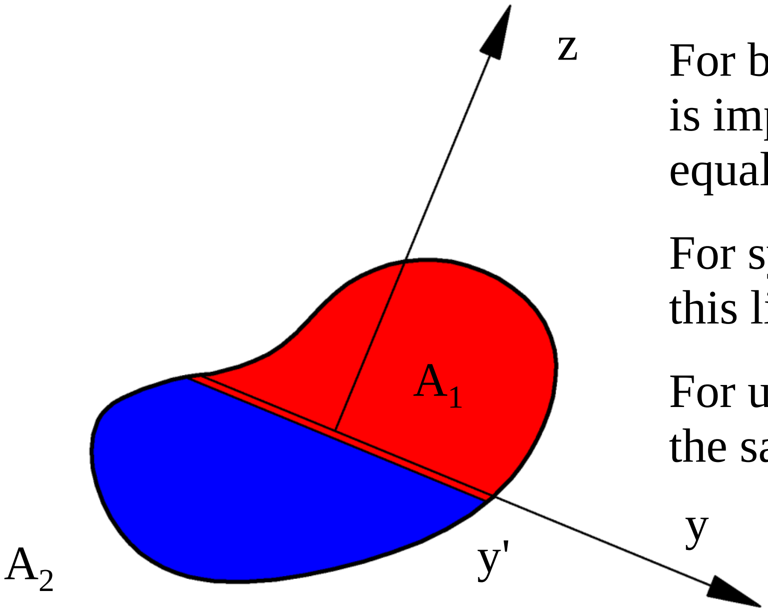
$$= \int_{A1} \int f_y\, dz\, dy + \int_{A2} \int -f_y\, dz\, dy =$$

$$= f_y \int_{A1} \int dz\, dy - f_y \int_{A2} \int dz\, dy =$$

$$= \| \text{ \#t / 13: } A = \int \int dz\, dy \| =$$

$$= A_1 f_y - A_2 f_y$$

$$N = 0 \rightarrow A_1 = A_2$$



For bending moment and total plastic behaviour, there is important line, which divides cross-section into two equal surface area ($A_1 = A_2 = A / 2$)

For symmetrical cross-section (y = axis of symmetry) this line is y axis.

For unsymmetrical cross-section, this is y' axis, not the same as y axis.

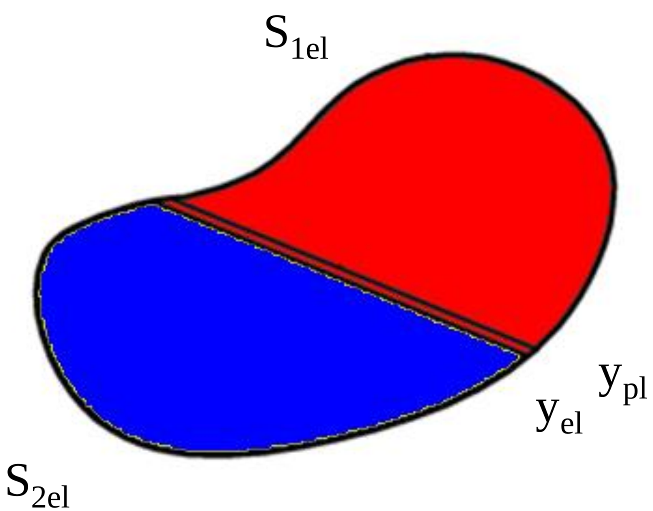
Photo: Autor

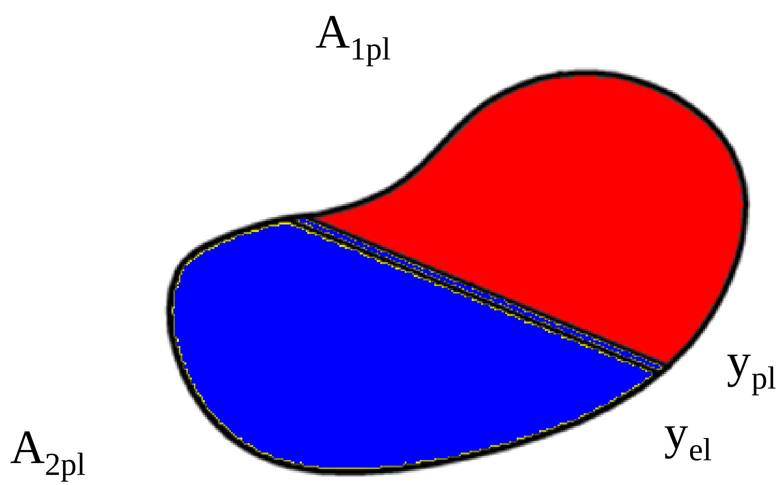
Elastic behaviour: $S_{1el}(y_{el}) = -S_{2el}(y_{el})$;

$A_{1el} \neq A_{2el}$;

Plastic behaviour: $A_{1pl} = A_{2pl}$;

$S_{1pl}(y_{pl}) \neq -S_{2pl}(y_{pl})_{l}$;





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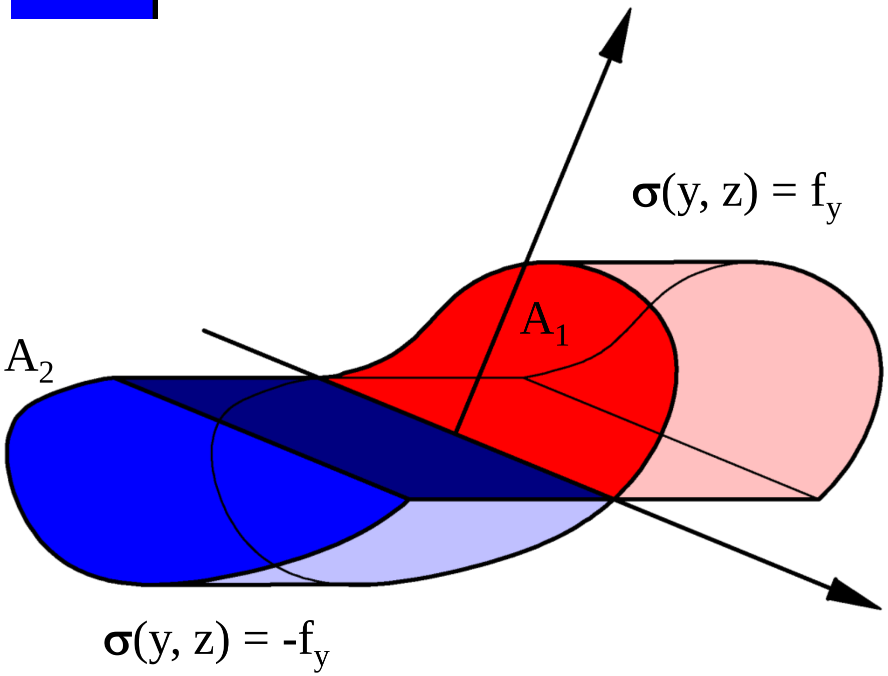


$$A_1 = A_2 = A / 2$$

$$M_y = \int\int \{z\,[\boldsymbol{\sigma}(z)]\}\ dz\ dy =$$

$$= \int_{A1}\int (z\ f_y)\ dz\ dy + \int_{A2}\int (-z\ f_y)\ dz\ dy =$$

$$= f_y\ (\int_{A1}\int z\ dz\ dy - \int_{A2}\int z\ dz\ dy\ ) =$$

$$= \|\ \#t / 14:\ S_y = \int\int\ z\ dz\ dy\ \| =$$

$$= f_y\ [\ |S(A_1)_{y'}| + |S(A_2)_{y'}|]$$

Photo: Autor

$M_y = f_y\,[\,|S(A_1)_{y'}| + |S(A_2)_{y'}|\,]$

It can be presented as:

$M_y = f_y\, W_{pl,\, y'}$

$W_{pl,\, y'}$ (plastic sectional modulus) is specific geometrical characteristic, important for total plastic behaviour of material.

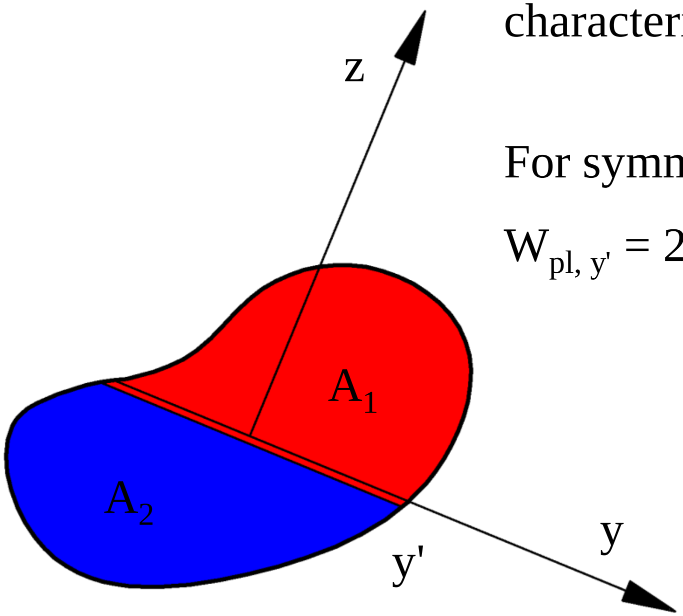


Photo: Autor

For symmetrical cross-section (y = y'):

$W_{pl,\, y'} = 2$ (static moment of half of cross-section about axis y = y')

Generally (i, j = y, z):

$W_{el,\, i} = J_i \,/\, |\, j_{max}|$

$W_{pl,\, i} > W_{el,\, i}$

In addition to the elastic and plastic sectional modulus, it is sometimes necessary to consider the effective sectional modulus. It appears in the situation of very slender cross-sections (thin and high web), for which the local stability of compressed part may be lost.

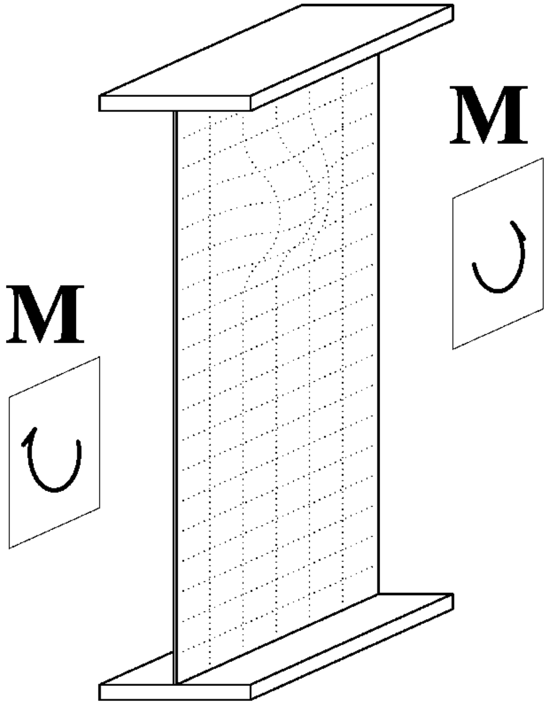


Photo: Autor

Photo: ijird.com

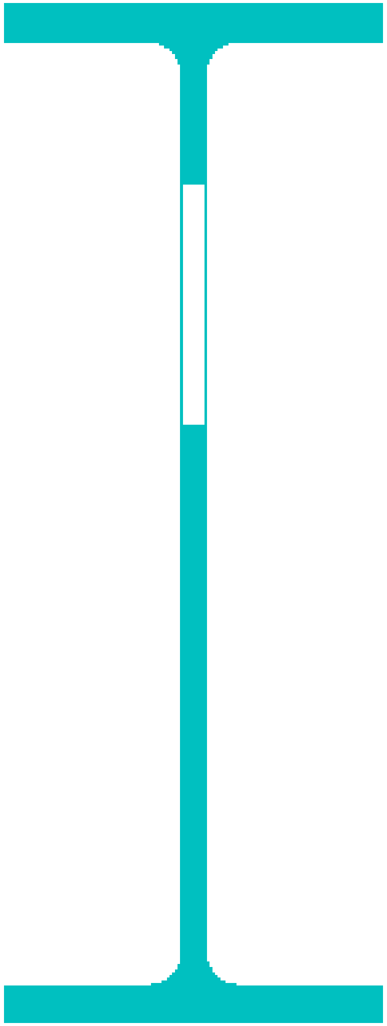


Photo: Autor

The part that is subject to local instability is ignored. Effective cross-section is a fictitious cross-section from which fragments of flanges / web (subject to instability) have been removed.

When considering the effective cross-section, it is necessary to find the effective cross-sectional area, effective center of gravity, effective moment of inertia and effective sectional modulus. More information will be presented on Lab #2.

$$W_{eff} \leq W_{el} < W_{pl}$$

$$A_{eff} \leq A$$

Shear force

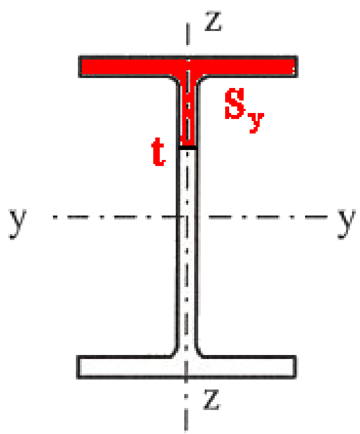


Photo: Autor

According to Mechanics of Materials:

$\tau = V\, S_y(z) / J_y\, t(z)$

It can be presented as:

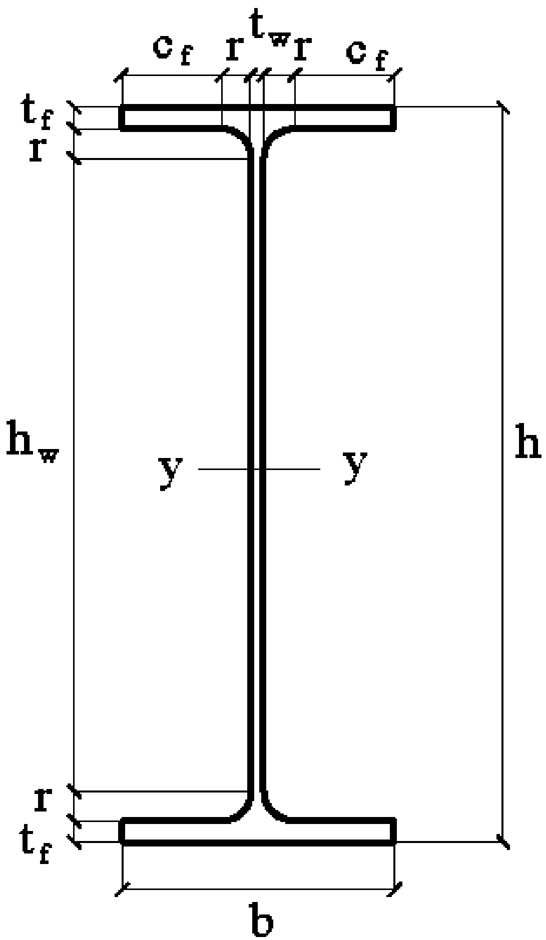
$\tau = V / A_V$

τ = max when A_V = min; A_V = min for t = min and S_y = max.

$A_V = J_y\, t(z = 0) / S_y(z = 0)$

A_V is specific geometrical characteristic ("active area"), important for shear force.

Photo: Autor



Generally:

$t_f << h$

$J_y \approx h^3\, t_w / 12 + 2\, [t_f\, b\, (h / 2)^2] =$

$= h^2\, (h\, t_w / 12 + t_f\, b / 2)$

S_y (top half of cross-section) $\approx$

$\approx t_w\, (h / 2)\, (h / 4) + t_f\, b\, (h / 2) =$

$= h\, (h\, t_w / 8 + t_f\, b / 2)$

$$A_V = J_y\, t_w / S_y =$$

$$= t_w [h^2 (h\, t_w / 12 + t_f\, b / 2)] / [h (h\, t_w / 8 + t_f\, b / 2)] =$$

$$= t_w\, h [(h\, t_w / 12 + t_f\, b / 2) / (h\, t_w / 8 + t_f\, b / 2)] =$$

$$= \| \; h\, t_w \approx t_f\, b = A' \; \| = t_w\, h [(A'/ 12 + A' / 2) / (A' / 8 + A' / 2)] =$$

$$= t_w\, h \{(A' / 2) [(1 / 6) +1]\} / \{(A' / 2) [(1 / 4) + 1]\} =$$

$$= t_w\, h [(1 / 6) +1] / [(1 / 4) + 1] = t_w\, h (1{,}167 / 1{,}250) \approx t_w\, h$$

$$A_V \approx t_w\, h$$

Generalisation:

A_V can be approximated as area of rectangulars, paralell to direction of shear force.

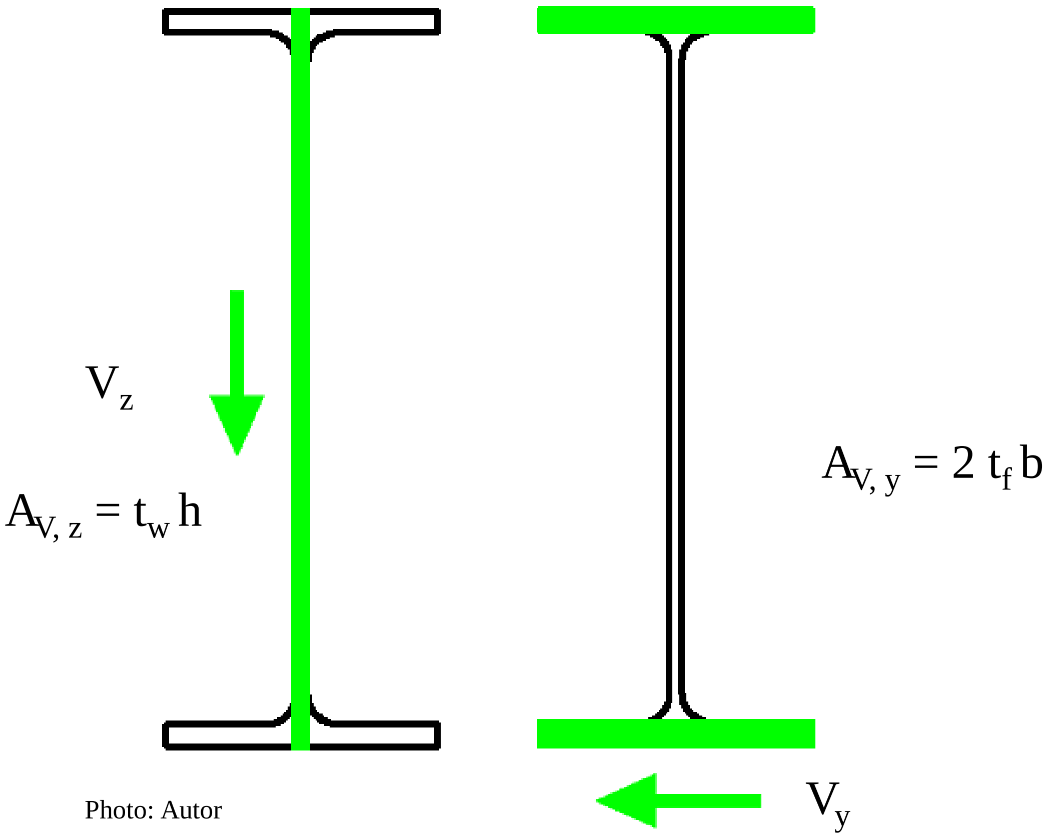


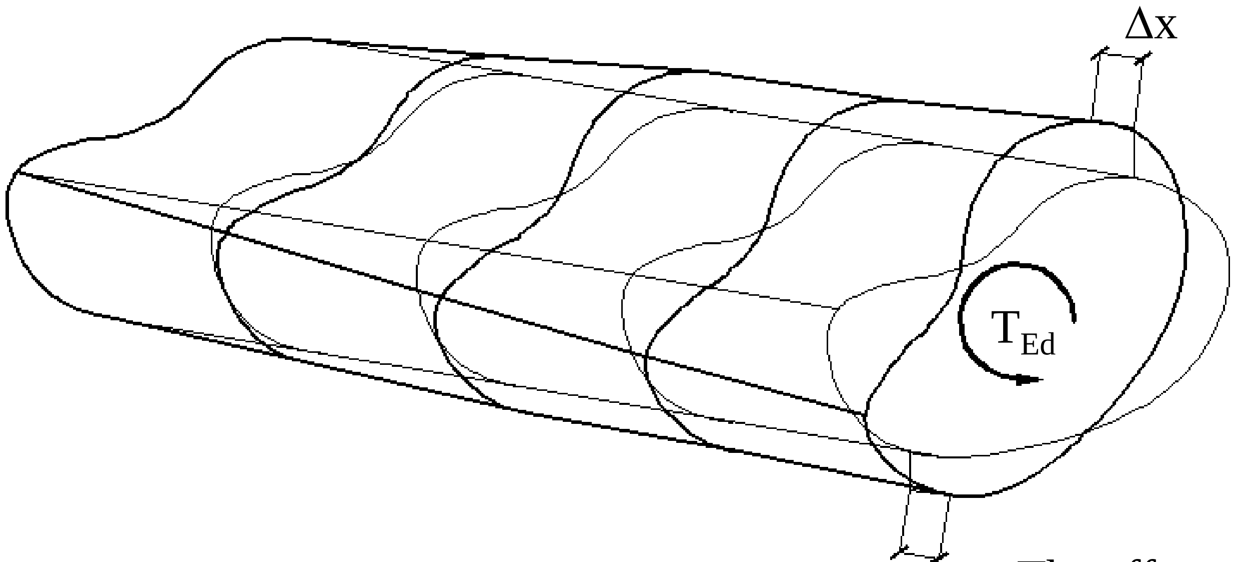
Photo: Autor

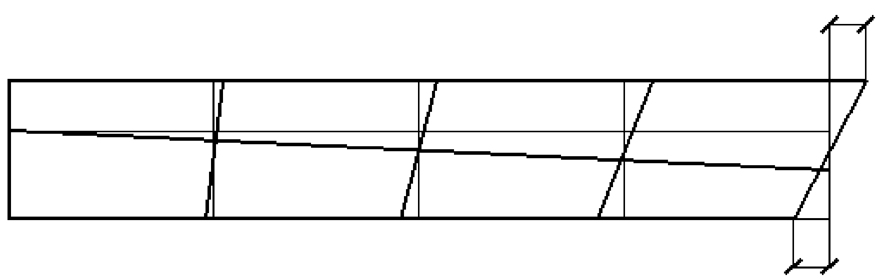
Torsion is the most complex type of load. They should be considered in several different ways:

- round bars;
- circular hollow sections;
- square or rectangular hollow sections;
- simple cross-sections (L T ┼);
- rest open cross-sections.

Differentiation consists in various deformations of twisted bars, various stress distributions or various formulas for geometric characteristics.

General case:





Rys: Autor

The effect of the torsional moment is not only the rotation of the section, but also its deplanation - initially flat and parallel sections become non-flat and non-parallel. Not all cross-sectional shapes are subject to deplanation, and for those parts which are subject to deplanation have negligible values.

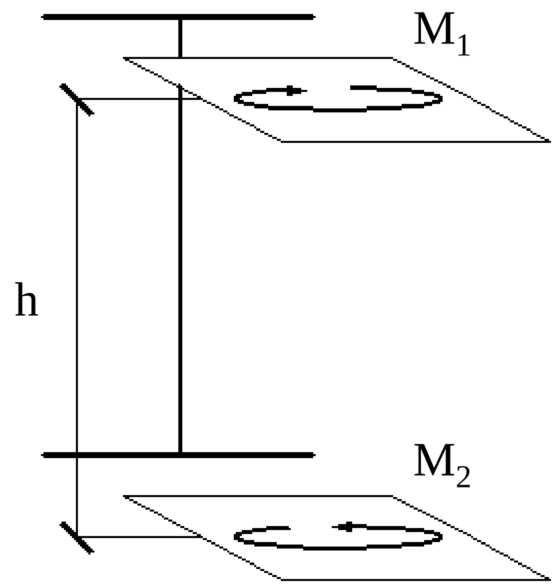
Cross-section	Deplanation		Torsional moment T_{Ed}	Remarks
Round (bar, hollow section)	Does not exists		$T_{Ed} = T_{t, Ed}$	-
L T +	Very small	Free	$T_{Ed} = T_{t, Ed}$	-
		Restricted by supports	$T_{Ed} \approx T_{t, Ed}$	-
Rest	Important	Free	$T_{Ed} = T_{t, Ed}$	-
		Restricted by supports	$T_{Ed} = T_{t, Ed} + T_{w, Ed}$	In addition, B_{Ed} it should be taken into account

$T_{t, Ed}$ – St Venant torsion moment (free deplanation of cross-section);

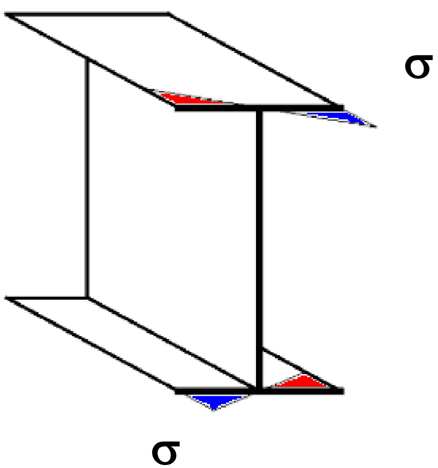
$T_{w, Ed}$ – warping torsional moment (restricted deplanation of cross-section);

B_{Ed} - bimoment

I-beam deplanation: both flanges are deformed in opposite directions.



In case of restricted torsion, specific distribution of stresses (σ not ony τ) in flanges is inducted.



Rys: Autor

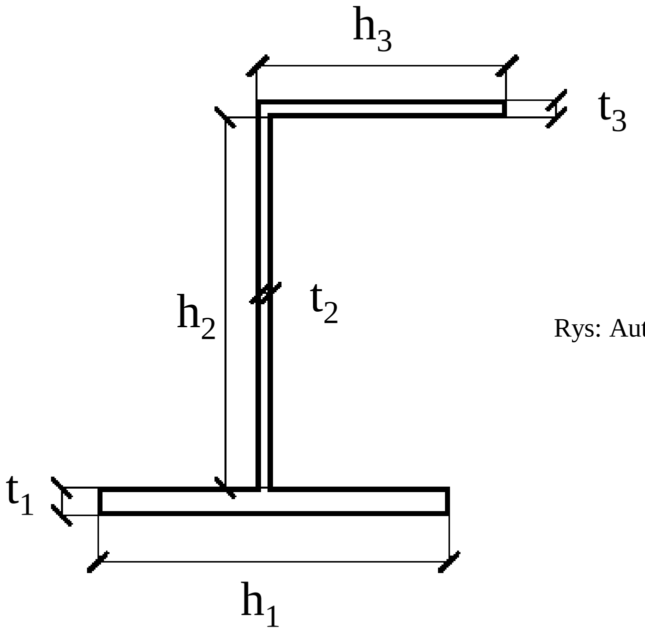
Axial stresses can be presented as the effect of bi-moment [Nm^2]:

$$B = h\,M$$

$$M = |M_1| = |M_2|$$

Bi-moment is taken into consideration during analisis of thin-walled structures also.

Different ways of bar deformation make it necessary to consider two separate groups of geometrical characteristics:
The first group (J_T W_T^*) concerns the torsion of the cross section and resistance due to the torsional moment.
The second one (J_ω W_ω) is related to the section deplanation and resistance due to the bi-moment.



Rys: Autor

In analogy to bending, the moment of inertia at torsion and the sectional modulus at torsion can be presented:

$$W_T^* = J_T / t_{max}$$

$$t_{max} = \max(\, t_1 \;;\; t_2 \;;\; t_3 \ldots)$$

Values for I-beam are presented in special tables.

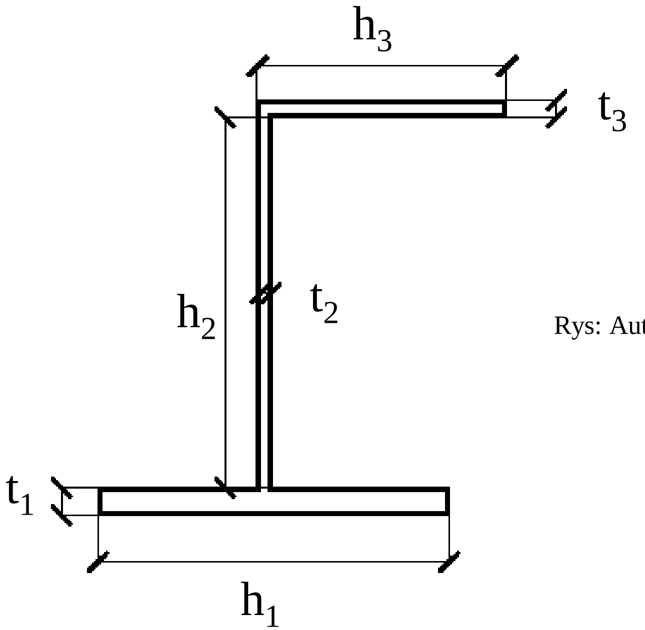
Notations pages 104-108 / Bezeichnungen Seiten 104-108

Désignation / Designation / Bezeichnung		Valeurs statiques / Section properties / Statische Kennwerte												Classification ENV 1993-1-1						HISTAR
		axe fort y-y / strong axis y-y / starke Achse y-y					axe faible z-z / weak axis z-z / schwache Achse z-z							pure bending y-y			pure compression			
	G kg/m	I_y cm^4	$W_{el.y}$ cm^3	$W_{pl.y}$♦ cm^3	i_y cm	A_{vz} cm^2	I_z cm^4	$W_{el.z}$ cm^3	$W_{pl.z}$♦ cm^3	i_z cm	s_s mm	I_t cm^4	$I_w \times 10^{-3}$ cm^6	S 235	S 355	S 460	S 235	S 355	S 460	
IPE A 360	50.2	14520	811.8	906.8	15.06	29.76	944.3	111.1	171.9	3.84	50.69	26.51	282	1	1	-	4	4	-	
IPE 360	57.1	16270	903.6	1019	14.95	35.14	1043	122.8	191.1	3.79	54.49	37.32	313.6	1	1	-	2	4	-	
IPE O 360	66.0	19050	1047	1186	15.05	40.21	1251	145.5	226.9	3.86	59.69	55.76	380.3	1	1	-	1	3	-	
IPE A 400	57.4	20290	1022	1144	16.66	35.78	1171	130.1	202.1	4.00	55.60	34.79	432.2	1	1	-	4	4	-	
IPE 400	66.3	23130	1156	1307	16.55	42.69	1318	146.4	229.0	3.95	60.20	51.08	490	1	1	-	3	4	-	
IPE O 400	75.7	26750	1324	1502	16.66	47.98	1564	171.9	269.1	4.03	65.30	73.10	587.6	1	1	-	2	3	-	

Rys: europrofil.lu

Approximated formulas could be used also:

$$J_T \approx \alpha\,(h_1\,t_1^{\,3} + h_2\,t_2^{\,3} + h_3\,t_3^{\,3} + \dots)\,/\,3$$



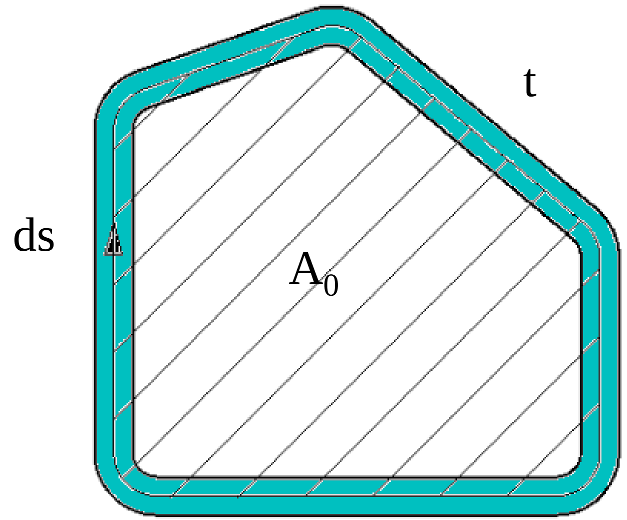
Rys: Autor

Cross-section	α
Hot-rolled I-beam	1,20
Welded I-beams with vertical stiffeners	1,50
L	1,00
C	1,12
T	1,40

For hollow cross-sections (circular, rectangular, square), the moment of inertia at torsion and the torsional sectional modulus are calculated differently:

$$J_T = 4\,A_0^2 \,/\, \int (ds \,/\, t)$$

$$W_T^* = 2\,t\,A_0$$



Rys: Autor

For round bars:

$$J_T = \pi\, r^4 / 2$$

$$W_T^* = \pi\, r^3 / 2$$

The most complicated topic in field of calculating cross-sectional characteristics is to count those associated with deplanation. This issue is beyond scope of course of metal structures. It should be presented during lectures from Strength of Materials.

$$W_w = J_w / \omega$$

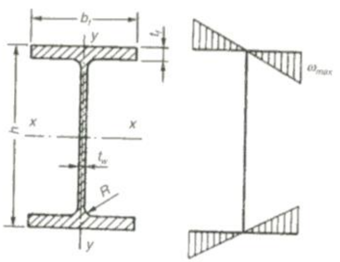
J_w – warping moment of inertia / warping constant [m^6]

W_w – warping sectional modulus [m^4]

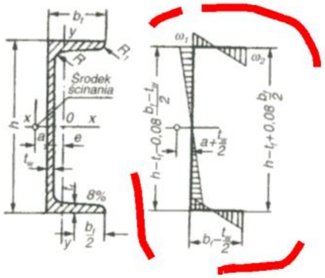
ω – sectional coordinate [m^2]

Warping sectional modulus is not such important for metal structures as sectional coordinate. Warping constant is the most important.

Geometrical characteristics associated with torsion and deplanation are important not only in the case of torsion, but also in the case of thin-walled sections (sometimes aluminum or cold-formed steel sections are calculated using this method), and for matters related to the loss of stability of the element under compression or bending (→ Lecture # 5). For "basic" cross-sections (I-sections, C-sections), the warping of inertia and the sectional coordinate are given in the tables.



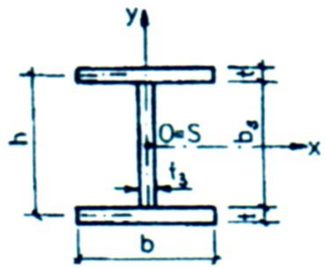
Rys.: Tablice do projektowania konstrukcji metalowych, W. Bogucki, M. Żyburtowicz, Arkady Wa-wa 1996



Wskaźnik wytrzymałości		Promień bezwładności		Pole wycinkowe	Wycinkowy moment bezwładności	Wycinkowy wskaźnik wytrzymałości	Moment bezwładności przy skręcaniu	Giętnokrętna charakterystyka 1/cm
W_x	W_y	i_x	i_y	ω_{max}	I_ω	W_ω	I_τ	$k=\sqrt{\frac{GI_\tau}{EI_\omega}}$
cm³		cm		cm²	cm⁶	cm⁴	cm⁴	
20,0	3,69	3,24	1,06	8,6	118	13,7	0,70	0,0477
34,2	5,79	4,07	1,24	13,0	351	27,1	1,20	0,0363
53,0	8,65	4,90	1,45	18,1	889	49,1	1,74	0,0275
77,3	12,3	5,74	1,65	24,3	1980	81,5	2,45	0,0218
109	16,7	6,58	1,84	31,3	3958	126	3,61	0,0188

Wskaźnik wytrzymałości		Promień bezwładności		Pole wycinkowe		Wycinkowy moment bezwładności	Wycinkowy wskaźnik wytrzymałości	Moment bezwładności przy skręcaniu	Giętnokrętna charakterystyka 1/cm
W_x	W_y	i_x	i_y	ω_2	ω_2	I_ω	W_ω	I_τ	$k=\sqrt{\frac{GI_\tau}{EI_\omega}}$
cm³		cm		cm²		cm⁶	cm⁴	cm⁴	
5,33	2,63	1,32	1,04	1,77	2,75	8,46	3,08	0,66	0,172
3,63	0,78	1,47	0,55	2,04	3,18	13,2	4,15	1,02	0,173
8,31	3,18	1,74	1,13	2,56	4,13	19,9	4,81	0,76	0,121
10,6	3,75	1,92	1,13	2,86	4,60	30,6	6,65	1,14	0,120
17,7	5,07	2,52	1,25	4,10	6,97	80,9	11,6	1,88	0,0946

If necessary, you can use approximate formulas:

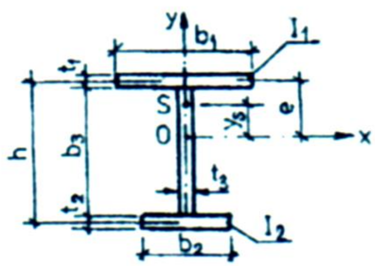
<table>
<tr><th>Przekrój</th><th>Cechy geometryczne</th></tr>
<tr><td>
</td><td>

$y_s = 0$

$I_\omega = \dfrac{I_y h^2}{4}$

$I_T = \dfrac{1}{3}(2\, b t^3 + b_3\, t_3^3)$

$r_x = 0$

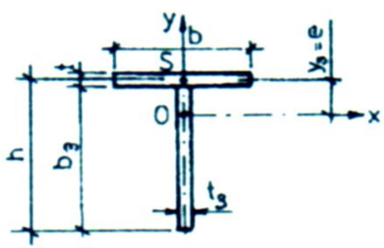
</td></tr>
<tr><td>
</td><td>

$y_s = \dfrac{1}{I_y}[e\, I_1 - (h - e)\, I_2] = e - \dfrac{I_2}{I_y}\, h$

$I_\omega = \dfrac{I_1\, I_2\, h^2}{I_1 + I_2}$

$I_T = \dfrac{1}{3}(b_1\, t_1^3 + b_2\, t_2^3 + b_3\, t_3^3)$

$r_x = \dfrac{1}{I_x}\left\{y_s\, I_y + b_1\, t_1\, e^3 - b_2\, t_2\, (h - e)^3 + \dfrac{t_3}{4}\,[e^4 - (h - e)^4]\right\}$

</td></tr>
<tr><td>
</td><td>

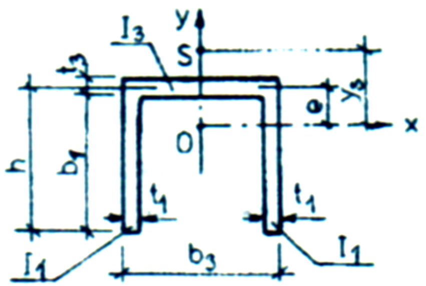
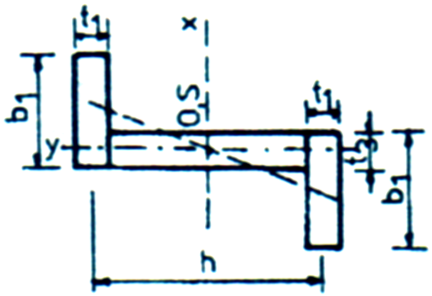
$y_s = e$

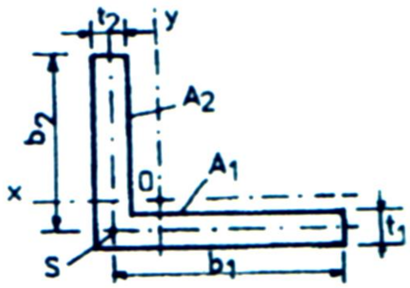
$I_\omega \cong 0$

$I_T = \dfrac{1}{3}(b\, t^3 + b_3\, t_3^3)$

$r_x = \dfrac{1}{I_x}\left\{e\, I_y + b\, t\, e^3 + \dfrac{t_3}{4}\,[e^4 - (h - e)^4]\right\}$

</td></tr>
</table>

	$y_s = e + \frac{I_1\, h}{I_y}$ $I_\omega = \frac{h^2}{3} \cdot \frac{I_1^2 + 2\, I_1\, I_3}{I_y}$ $I_T = \frac{1}{3}(2\, b_1\, t_1^3 + b_3\, t_3^3)$
	$y_s = 0$ $I_\omega = \frac{h^2}{4}\, I_y$ $I_r = \frac{1}{3}(2\, b_1\, t_1 saup2 + h\, t_3^2)$

Przekrój	Cechy geometryczne
	$y_s = e_y - \frac{t}{2}$ $I_\omega = \frac{A_1^3 + A_2^3}{36}$ $I_r = \frac{1}{3}(b_1 t_1^3 + b_2 t_2^3)$

Oznaczenia:

0 — środek ciężkości
S — środek ścinania
I_1 I_2 (I_3) — momenty bezwładności półek (środnika) względem osi symetrii
I_y — moment bezwładności figury względem osi symetrii

J. Żmuda, „Podstawy projektowania konstrukcji metalowych”, TiT Opole 1992

Example of calculation

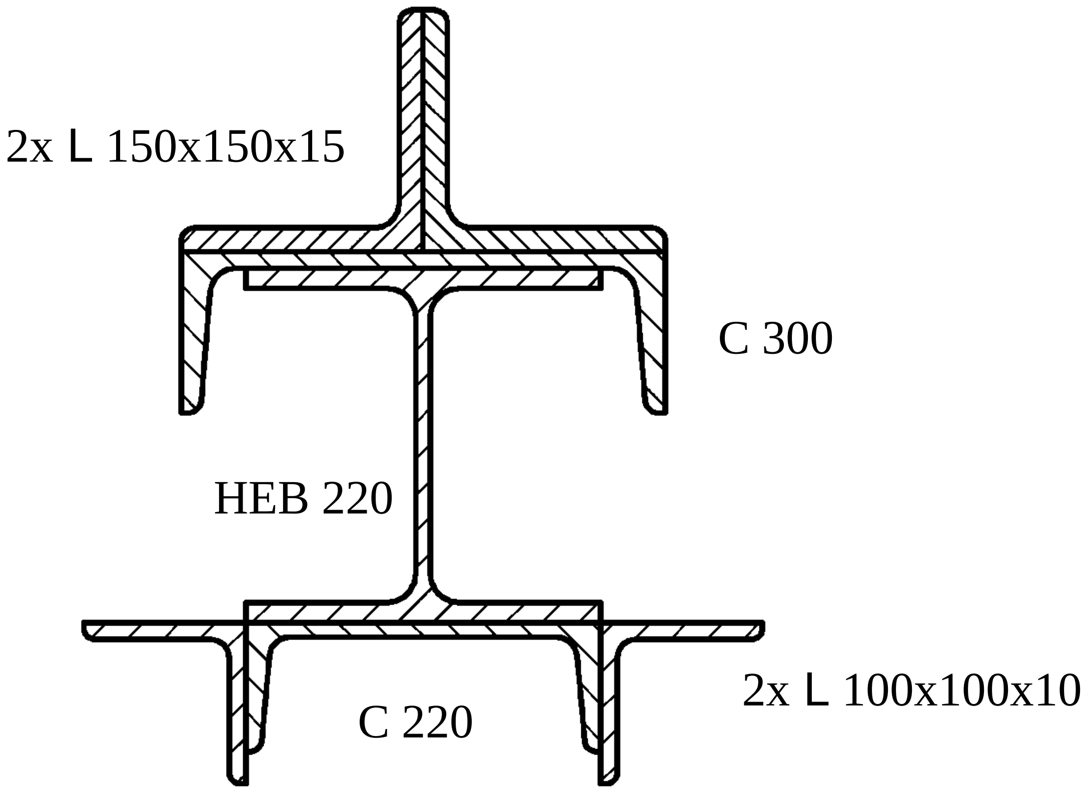
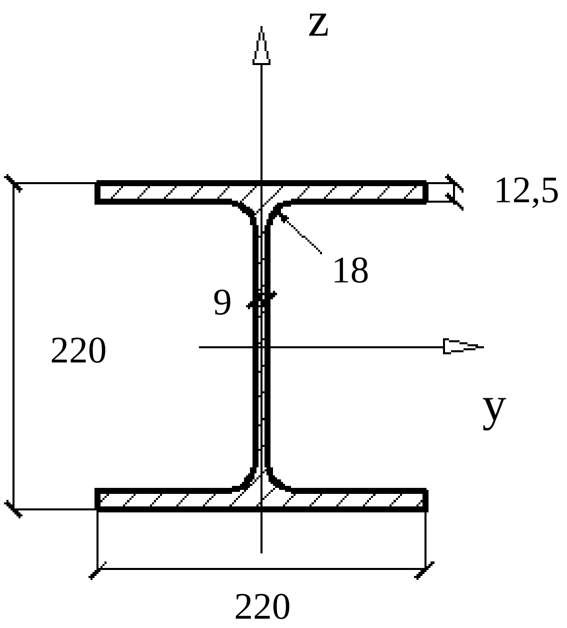
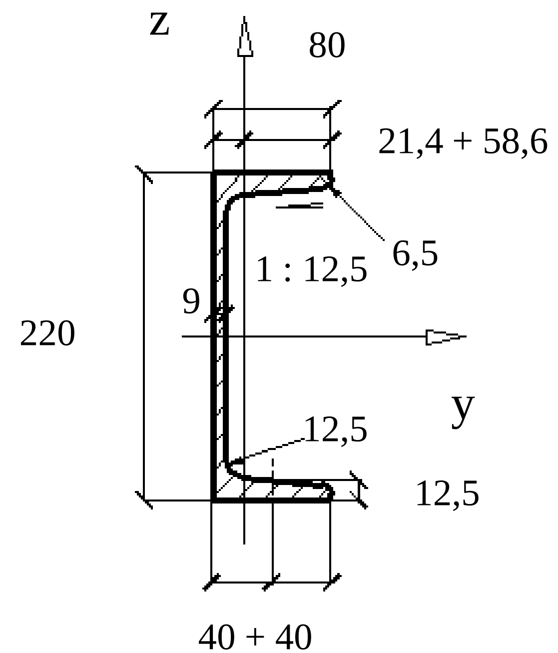


Photo.: Autor

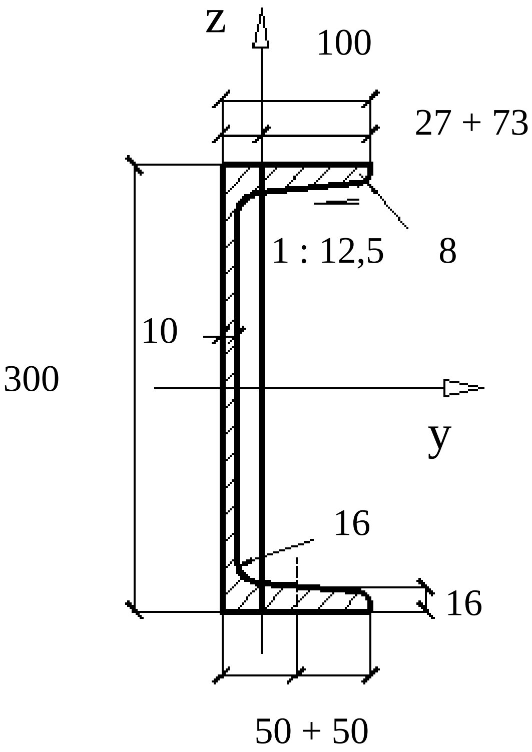


HEB 220

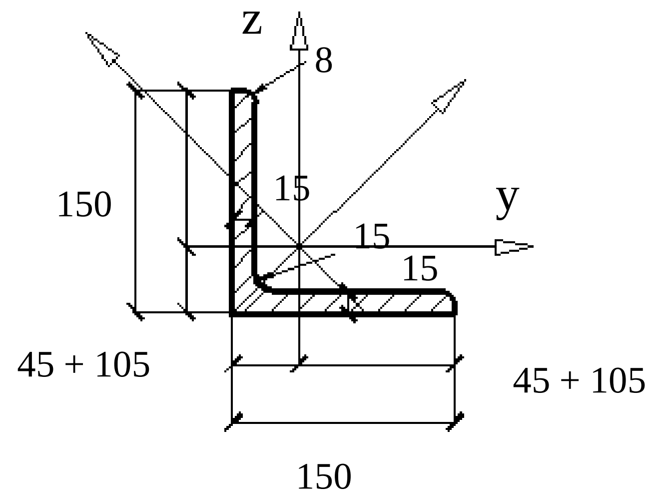


C 220

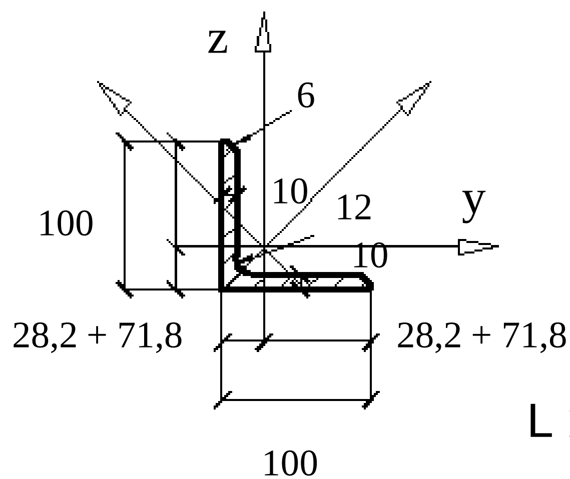
Photo.: Autor



C 300



L 150x150x15



L 100x100x10

Photo.: Autor

	A [cm^2]	J_y [cm^4]	J_z [cm^4]	e + f [cm]
HEB 220	91,0	8 090	2 840	-
C 220	37,4	2 690	197	2,14 + 5,86
C 300	58,8	8 030	495	2,70 + 7,30
L 100x100x10	19,2	177	177	2,82 + 7,18
L 150x150x15	46,0	1 100	1 100	4,50 + 10,50

Both C-sections are rotated to horizontal position. This means, their **local axis y** is parallel to **global axis z** and their local axis z is parallel to global axis y. It's important for calculation of moments of inertia (in global axes of course).

$$A = \Sigma A_i = 2\,A\,(\text{L } 150x150x15) + A\,(C\,300) + A\,(HEB\,220) + A\,(C\,220) +$$

$$+\,2\,A\,(\text{L } 100x100x10) =$$

$$= 2 \cdot 46{,}0 + 58{,}8 + 91{,}0 + 37{,}4 + 2 \cdot 19{,}2 =$$

$$= 317{,}6\ cm^2$$

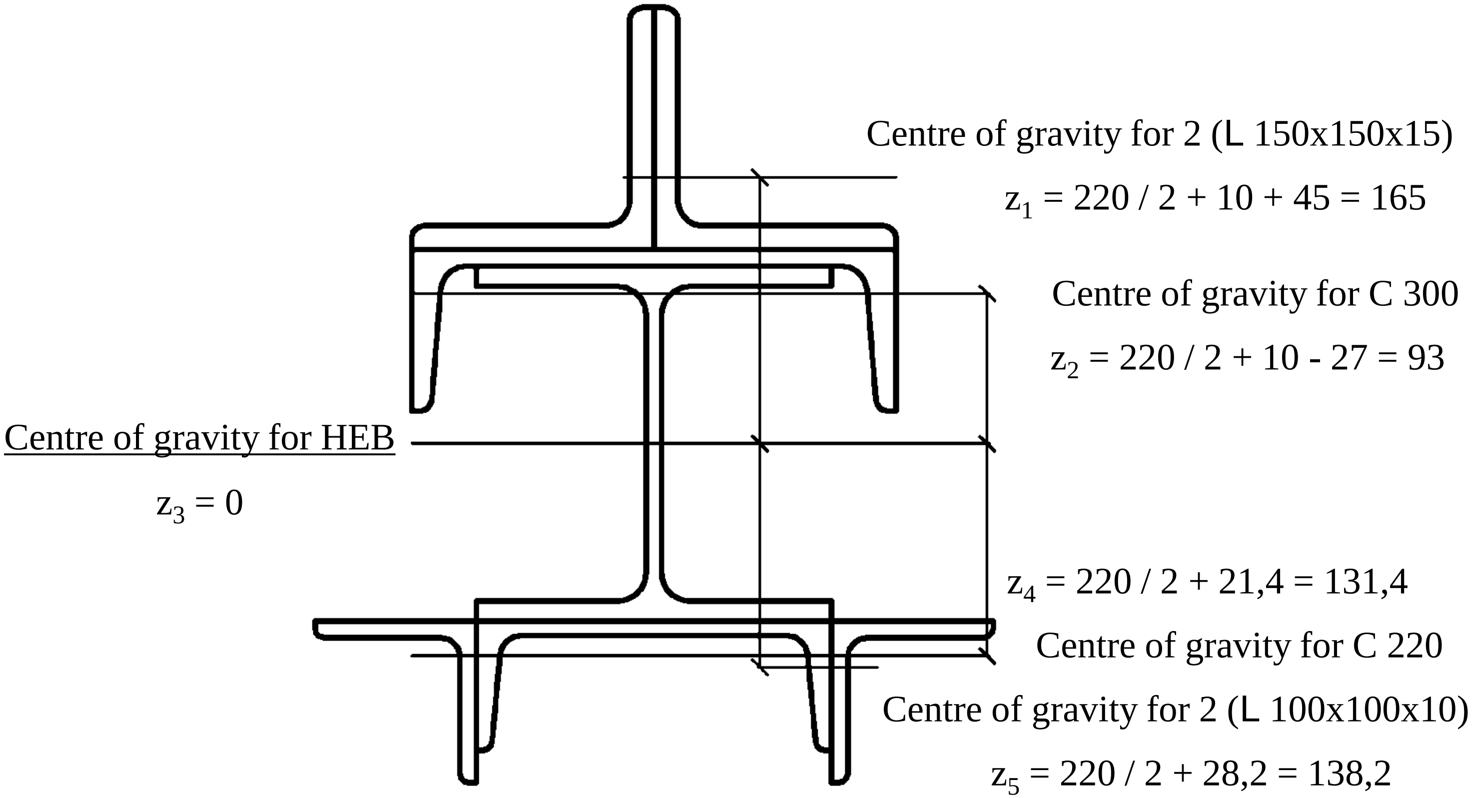


Photo.: Autor

$$S_y = \Sigma S_{i,y} = 2A(\text{L } 150x150x15)\, z_1 +$$

$$+ A(\text{C } 300)\, z_2 + A(\text{HEB } 220)\, z_3 +$$

$$+ A(\text{C } 220)\, z_4 + 2A(\text{L } 100x100x10)\, z_5 =$$

$$= 2 \cdot 46{,}0 \cdot 16{,}5 + 58{,}8 \cdot 9{,}3 +$$

$$+ 91{,}0 \cdot 0 - 37{,}4 \cdot 13{,}14 - 2 \cdot 19{,}2 \cdot 13{,}82 =$$

$$= 1042{,}716 \text{ cm}^3$$

$$z_{3\text{-}1} = S_y \;/\; A = 1042{,}716 \;/\; 317{,}6 = 3{,}28 \text{ cm}$$

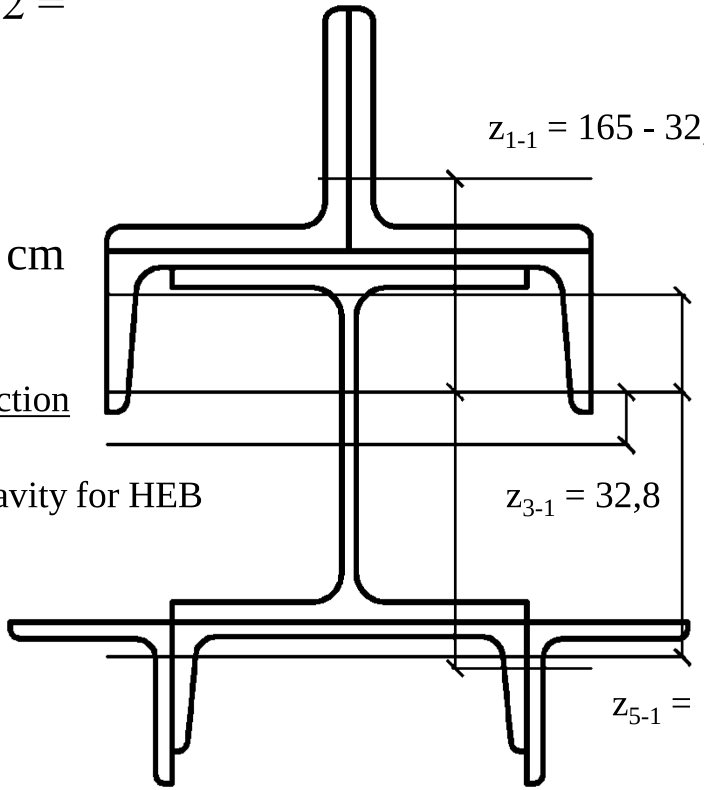


Photo.: Autor

$$J_y = [2\,J_y\,(L\,150x150x15) + 2\,A\,(L\,150x150x15)\,z^2_{1\text{-}1}] +$$

$$+ [J_z\,(C\,300) + A\,(C\,300)\,z^2_{2\text{-}1}] + [J_y\,(HEB\,220) + A\,(HEB\,220)\,z^2_{3\text{-}1}] +$$

$$+ [J_z\,(C\,220) + A\,(C\,220)\,z^2_{4\text{-}1}] + [2\,J_y\,(L\,100x100x10) + 2\,A\,(L\,100x100x10)\,z^2_{5\text{-}1}] =$$

$$= [2 \cdot 1\,100 + 2 \cdot 46{,}0 \cdot 13{,}22^2] + [495 + 58{,}8 \cdot 6{,}02^2] + [8\,090 + 91{,}0 \cdot 3{,}28^2] +$$

$$+ [197 + 37{,}4 \cdot 16{,}42^2] + [2 \cdot 177 + 2 \cdot 19{,}2 \cdot 17{,}1^2] =$$

$$= 51\,836{,}840\ \text{cm}^4$$

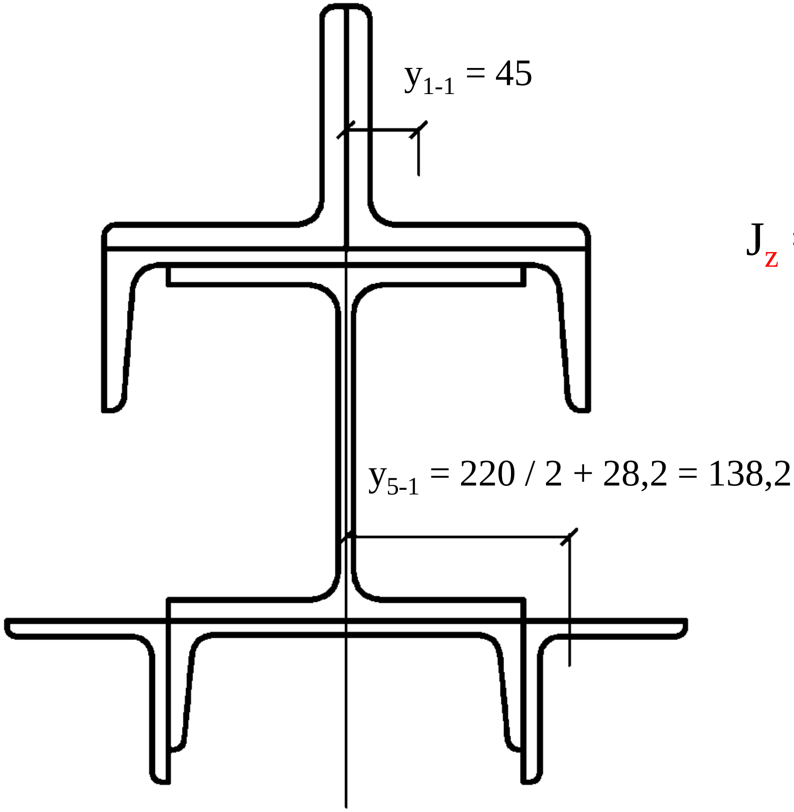


Photo.: Autor

$$J_z = [2\, J_z\, (L\ 150x150x15) + 2\, A\, (L\ 150x150x15)\, y^2_{1\text{-}1}] +$$
$$+ [J_y\, (C\ 300)] + [J_z\, (HEB\ 220)] +$$
$$+ [J_y\, (C\ 220)] + [2\, J_y\, (L\ 100x100x10) +$$
$$+ 2\, A\, (L\ 100x100x10)\, y^2_{5\text{-}1}] =$$
$$= [2 \cdot 1\ 100 + 2 \cdot 46{,}0 \cdot 4{,}5^2] + [8\ 030] + [2\ 840] +$$
$$+ [2\ 690] + [2 \cdot 177 + 2 \cdot 19{,}2 \cdot 13{,}82^2] =$$
$$= 25\ 311{,}108\ cm^4$$

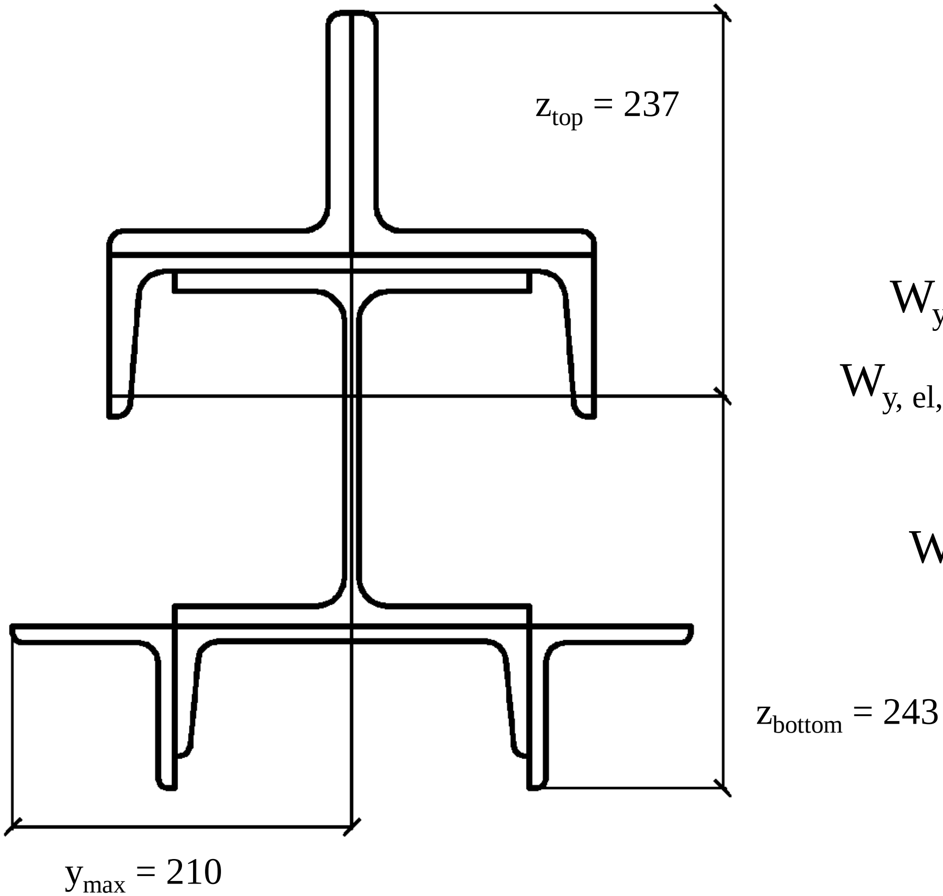


Photo.: Autor

$$i_y = \sqrt{(J_y / A)} = 12,776 \text{ cm}$$

$$i_z = \sqrt{(J_z / A)} = 8,927 \text{ cm}$$

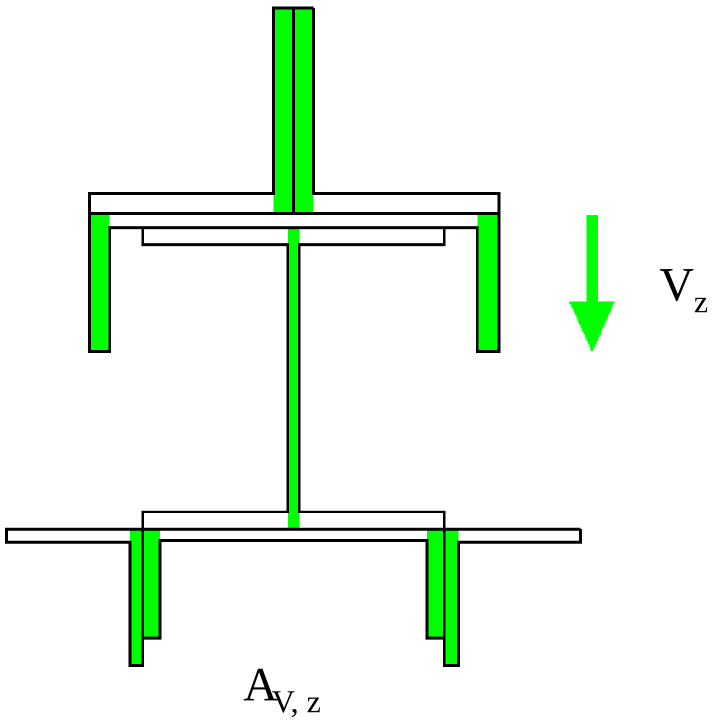
$$W_{y,\ el,\ top} = J_y / z_{top} = 2\ 185,364 \text{ cm}^3$$

$$W_{y,\ el,\ bottom} = J_y / z_{bottom} = 2\ 134,960 \text{ cm}^3$$

$$W_{z,\ el} = J_z / y_{max} = 1\ 205,291 \text{ cm}^3$$

"Active area" for shear force.

For calculations, cross-section is approximated by rectangulars.



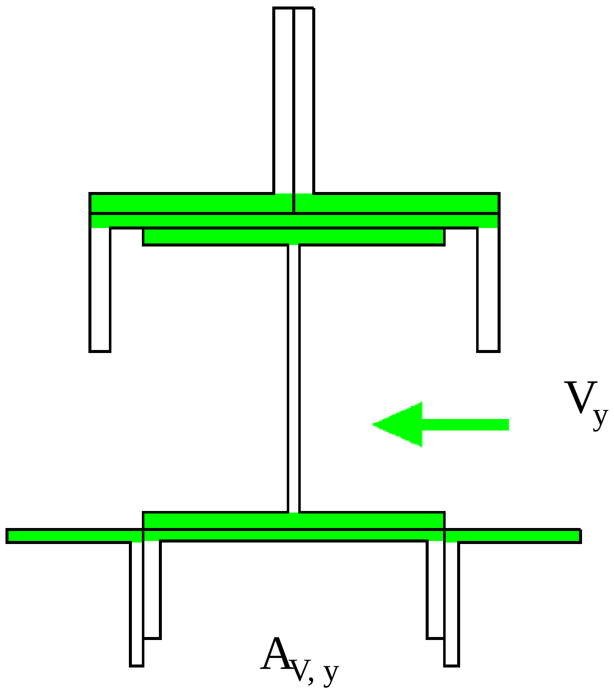


Photo.: Autor

$A_{V,z}$ = 2x (amr L 150x150x15) + 2x (flange C 300) + (web HEB 220) +

+ 2x (flange C 220) + 2x (arm L 100x100x10) =

$= 2 \cdot (15 \cdot 1{,}5) + 2 \cdot (10 \cdot 1{,}6) + 22 \cdot 0{,}9 + 2\text{x}\,(8 \cdot 1{,}25) + 2 \cdot (10 \cdot 1) = 136{,}8$

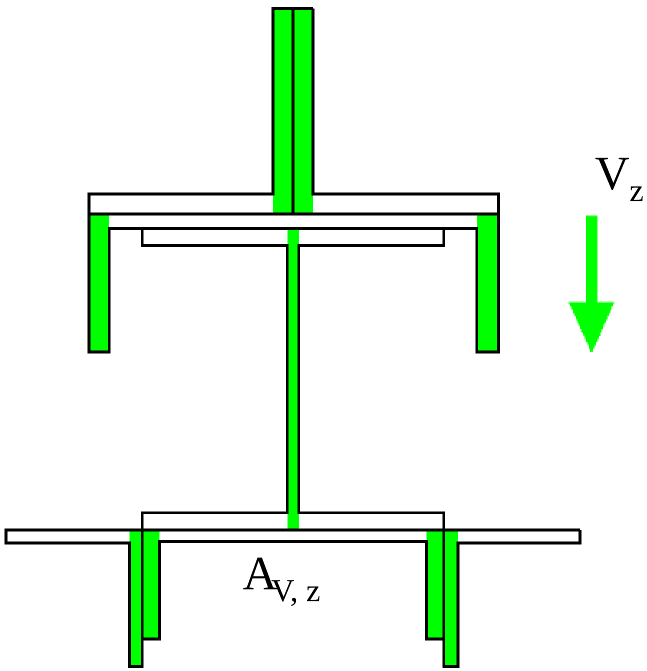
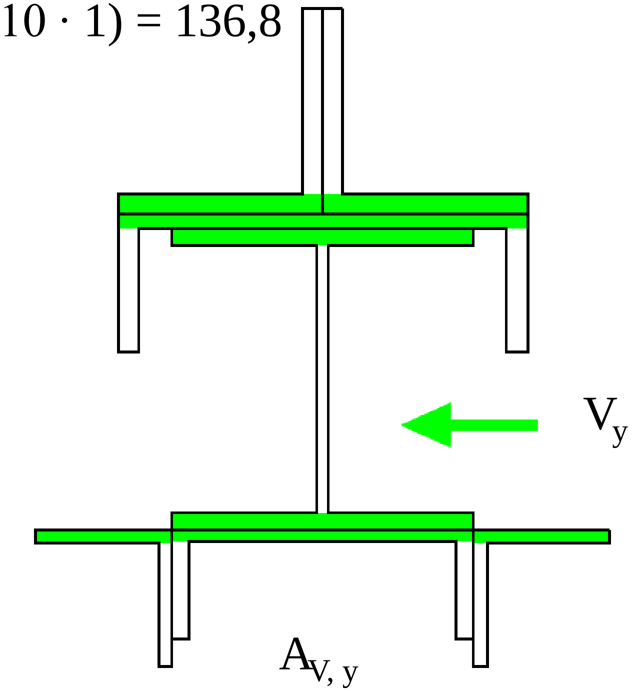


Photo.: Autor



$A_{V,y}$ = 2x (amr L 150x150x15) + (web C 300) + 2x (flange HEB 220) +

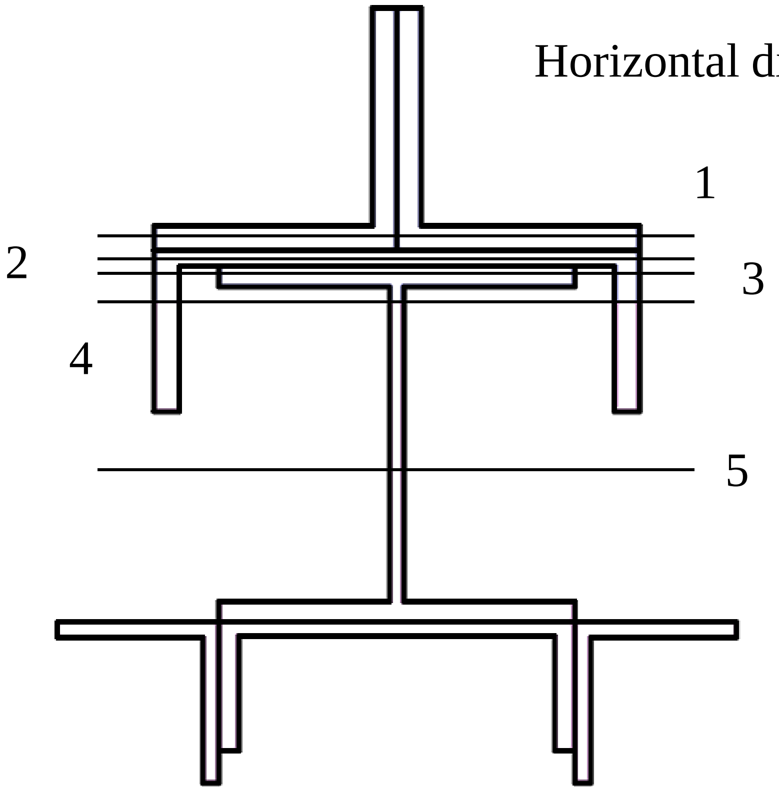
+ (web C 220) + 2x (arm L 100x100x10) =

$= 2\text{x}\,(15 \cdot 1{,}5) + 30\text{x}1 + 2 \cdot (22 \cdot 1{,}25) + 22 \cdot 0{,}9 + 2 \cdot (10 \cdot 1) = 169{,}8$

Plastic analysis: division of cross-section on two part of the same area.

For calculations, cross-section is approximated by rectangulars.

Vertical axis z (= axis of symmetry) divides area into two identical parts.



Horizontal direction - many possibilities:

1) Line goes through amrs L 150x150x15;

2) Line goes through web C 300;

3) Line goes through flange HEB 220;

4) Line goes through web 220 and flanges C 300;

5) Line goes through web HEB 220 out of flanges C300;

Photo.: Autor

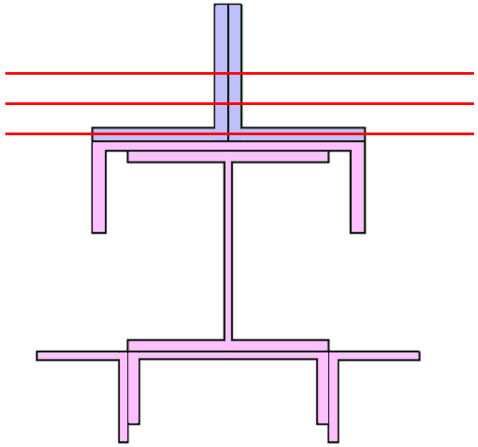
Possibility 1

Line goes through amrs L 150x150x15

The lowest position of line (max top part) divides total cross-section into two parts:

Top part: 2x L 150x150x15;

Bottom part: C 300 + HEB 220 + 2x L 100x100x10 + C 220



Area (top) = 2x 46,0 = 92,0;

Area (bottom) = 58,8 + 91,9 + 2x 19,2 + 37,4 = 225,6;

Area (top) is too small. On the other hand, it is max area for possibility 1.

Photo.: Autor

Possibility 1 is impossible.

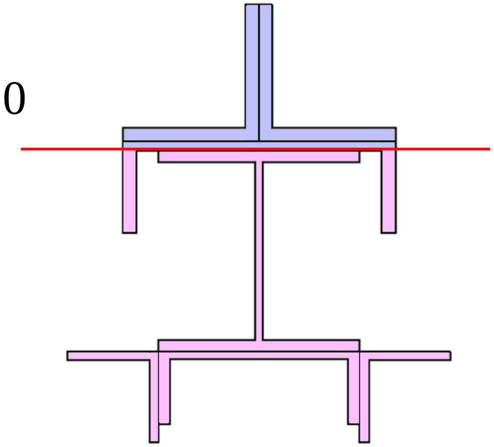
Possibility 2

Line goes through web C 300

The lowest position of line (max top part) divides total cross-section into two parts:

Top part: 2x L 150x150x15 + web C 300

Bottom part: flanges C 300 + HEB 220 + 2x L 100x100x10 + C 220



Area (top) = 2x 46,0 + 1,0 · 30,0 = 122,0;

Area (bottom) = 28,8 + 91,9 + 2x 19,2 + 37,4 = 196,6;

Area (top) is too small. On the other hand, it is max area for possibility 2.

Possibility 2 is impossible.

Photo.: Autor

Possibility 5

Line goes through web HEB 220 out of flanges C300;

Top part: 2x L 150x150x15 + C 300 + top flange HEB 220 + top part of web HEB 220;

Bottom part: 2x L 100x100x10 + C 220 + bottom flange HEB 220 + bottom part of web HEB 220

Area (top) = Area (bottom)

Area (top flange HEB 220) = Area (bottom flange HEB 220); flanges can be neglected

Area (top) = 2x 46,0 + 58,8 + top web = 150,8 + top web;

Area (bottom) = 2x 19,2 + 37,5 + bottom web = 75,9 + bottom web;

But total aera of web (top + bottom) ≈ 17. It is too small to make equivalent top and bottom par.

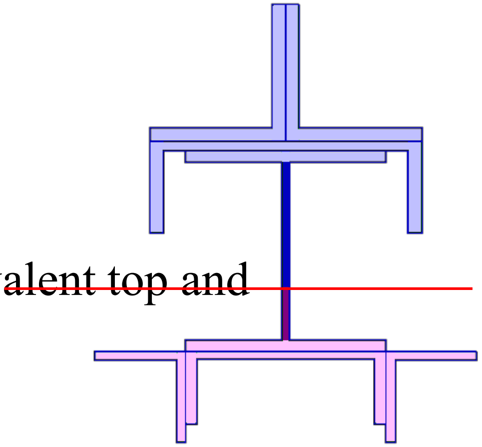


Photo.: Autor

Possibility 5 is impossible.

Initial analysis of areas of sub-patrs shows, that only two possibilities should be taken into consideration:

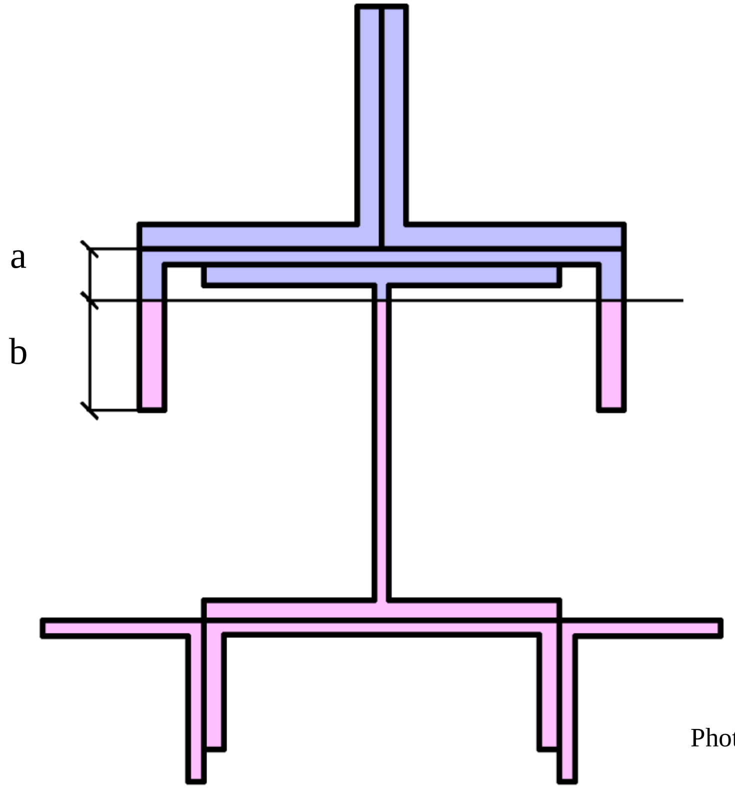
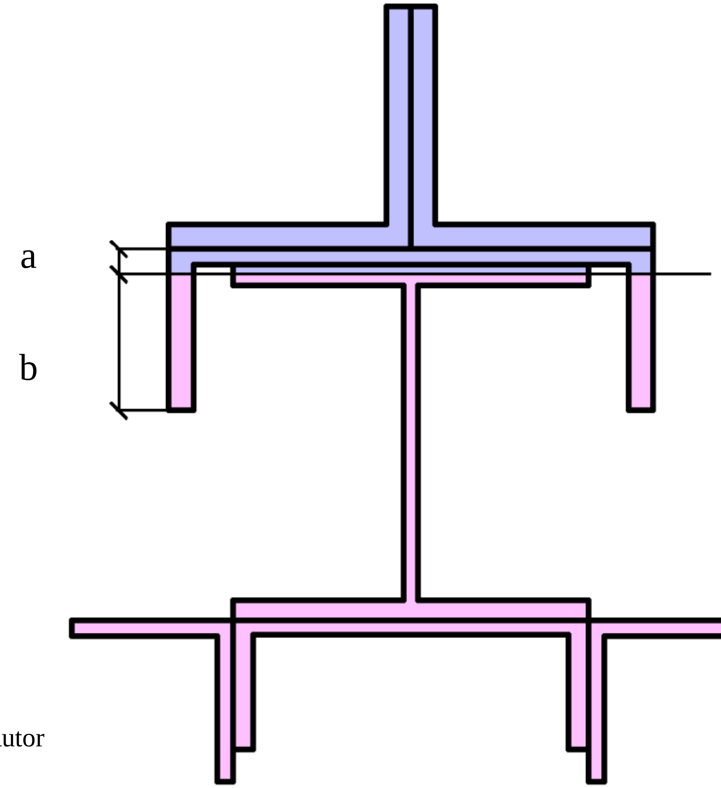


Photo.: Autor



4) Line goes through web HEB 220 out of flanges C300

(22,5 mm ≤ a ≤ 100,0 mm,   a + b = 100,0 mm)

3) Line goes through flange of HEB 220 and flanges C300

(10,0 mm ≤ a < 22,5 mm,   a + b = 100 mm)

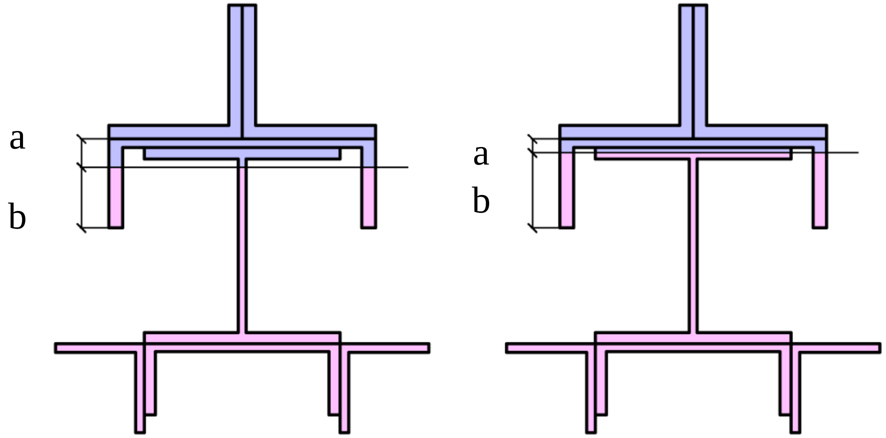


Photo.: Autor

	4	3
$A_1 =$ $= A / 2 = A_2$	2x (L 150x150x15) web C 300 part flange C 300 whole top flange HEB 220 part web 220	2x (L 150x150x15) web C 300 part flange C300 part top flange HEB 220 no web 220
$A_2 =$ $= A / 2 = A_1$	no top flange HEB 220 part web HEB 220 part flange C 300 bottom flange HEB 220 C 220 2x (L 100x100x10)	part top flange HEB 220 whole web HEB 220 part flange C 300 bottom flange HEB 220 C 220 2x (L 100x100x10)

	4		3
	2 · (46,0) 30 · 1 (a - 1) · 1,6 22 · 1,25 (a - 1 - 1,25) · 0,9		2 · (46,0) 30 · 1 (a - 1) · 1,6 (a - 1) · 22 0
	0 [22 - 1,25 - (a - 1)] · 0,9 (10 - a) · 1,6 22 · 1,25 37,4 2 · (19,2)		[1,25 - (a - 1)] · 22 0,9 · (22 - 2 · 1,25) (10 - a) · 1,6 22 · 1,25 37,4 2 · (19,2)

	4		3
	92		92
	30		30
	(a - 1) · 1,6		(a - 1) · 1,6
	27,5		(a - 1) · 22
	(a - 2,25) · 0,9		0
	0		(2,25 - a) · 22
	(21,75 - a) · 0,9		17,55
	(10 - a) · 1,6		(10 - a) · 1,6
	27,5		27,5
	37,4		37,4
	38,4		38,4

	4		3
	149,5 1,6a - 1,6 0,9a - 2,025		122 1,6a - 1,6 22a - 22
	19,575 - 0,9a 16 - 1,6a 103,3		49,5 - 22a 16 - 1,6a 120,85

	4		3
	2,5a + 145,875		23,6a + 98,400
	138,875 - 2,5a		186,450 - 23,6a

4		3
2,5a + 145,875 = = 138,875 - 2,5a		23,6a + 98,400 = = 186,450 - 23,6a

4		3
5,0a = -7,000		47,2a = 88,050

4		3
a = -1,4 cm		a = 1,9 cm

Conclusion:

Possibility 4, initial assumption 22,5 mm ≤ a ≤ 100,0 mm, result a = -14 mm, contradiction, possibility rejected

Possibility 3, initial assumption 10,0 mm ≤ a < 22,5 mm, result a = 19, mm, OK., possibility accepted

$$W_{y,pl} = |S_y\ (A_1)| + |S_y\ (A_2)|$$

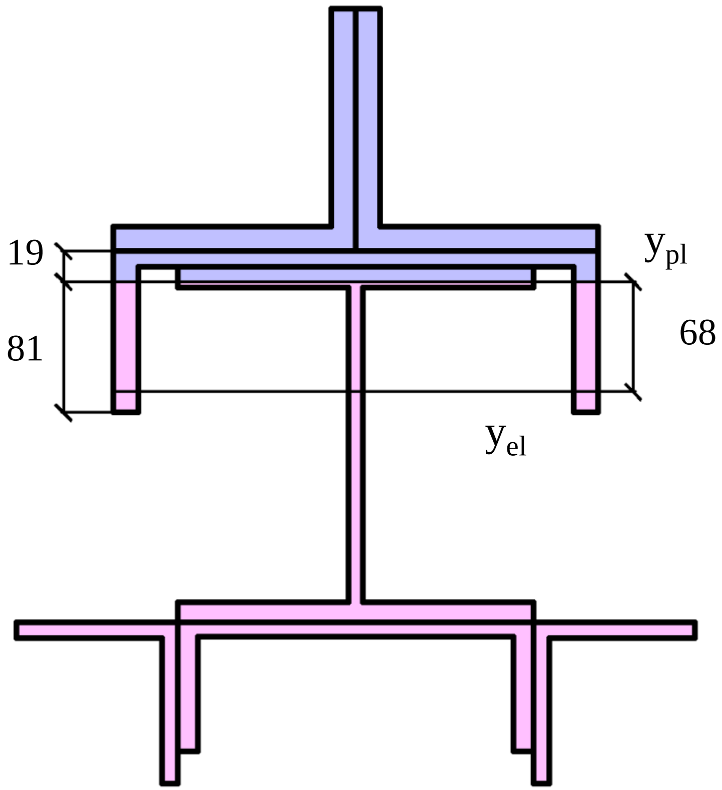


Photo.: Autor

$S_y\ (A_1) = S_y$ [2x (L 150x150x15)] +

+ S_y (web C 300) + S_y [2x (top part flange C 300)] +

+ S_y (top part top flange HEB 220)

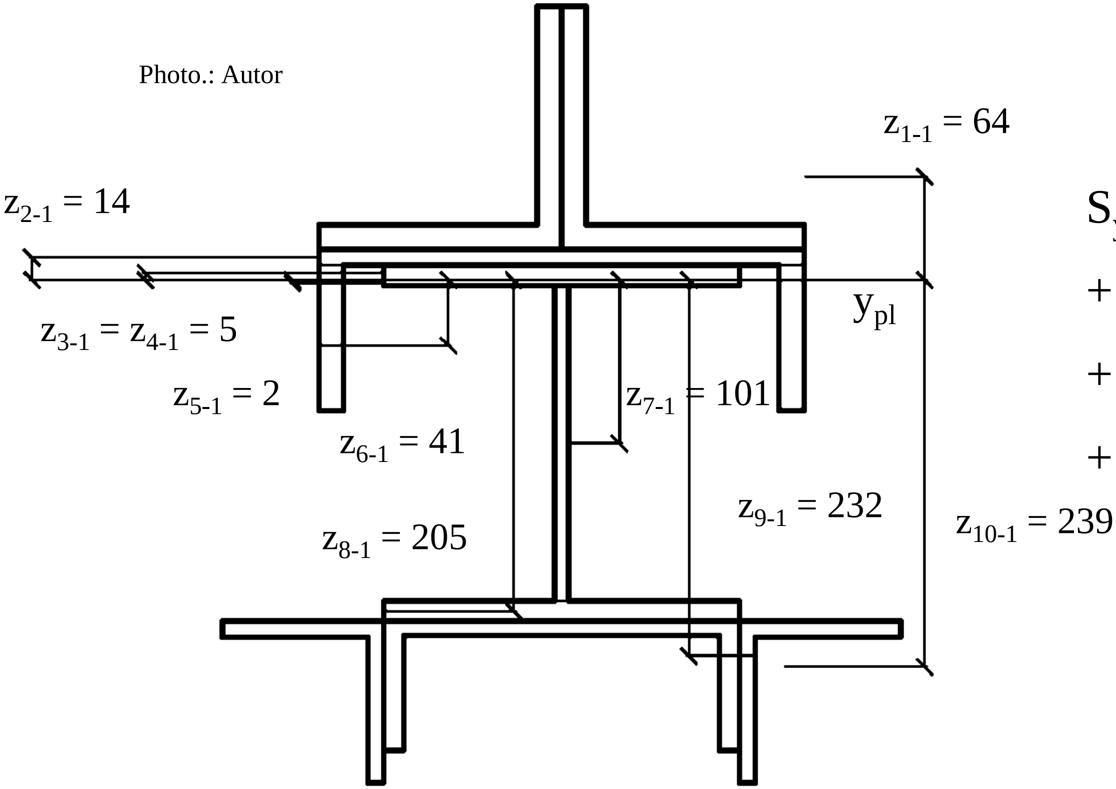
$S_y\ (A_2) = S_y$ (bottom part top flange HEB 220) +

+ S_y [2x (bottom part flange C 300)] +

+ S_y (web HEB 220) + S_y (bottom flange HEB 220) +

+ S_y (C 220) + S_y [2x (L 100x100x10)]

Photo.: Autor



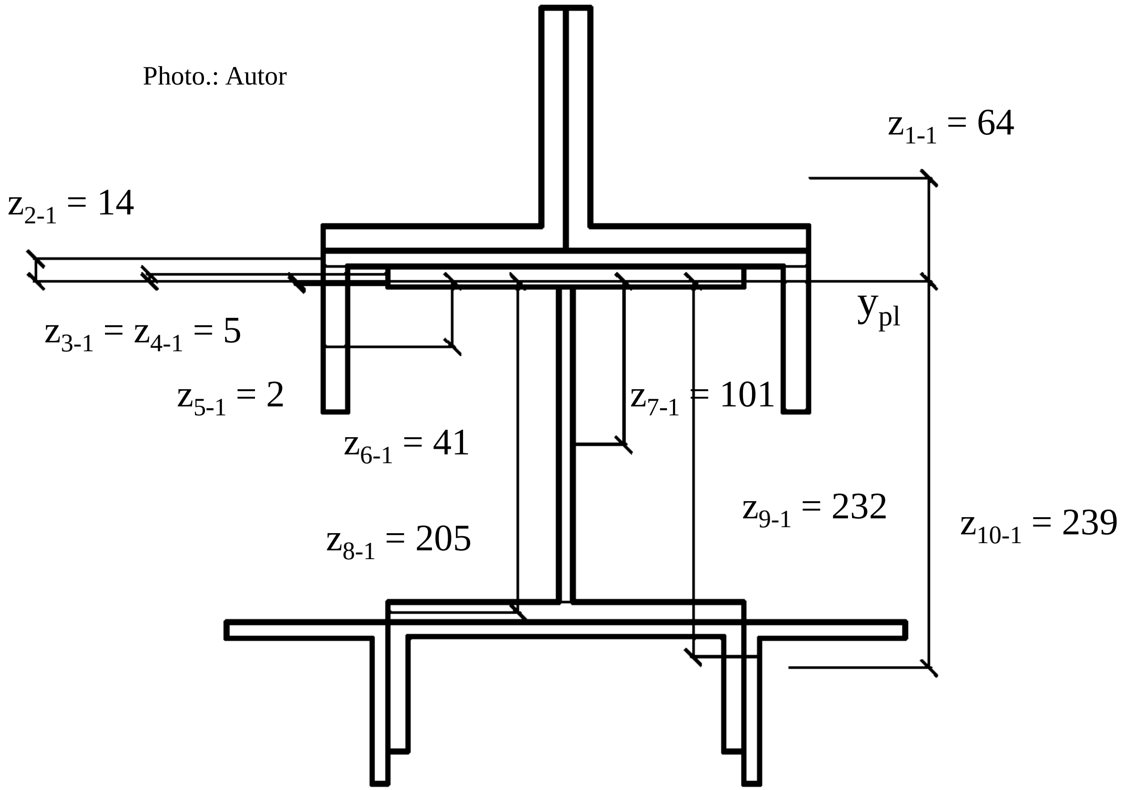
S_y (A_1) = $z_{1\text{-}1}$ [2x A(L 150x150x15)] +

+ $z_{2\text{-}1}$ A(web C 300) +

+ $z_{3\text{-}1}$ [2x A(top part flange C 300)] +

+ $z_{4\text{-}1}$ A(top part top flange HEB 220)

$S_y (A_1) = 6{,}4 \cdot 2 \cdot 46{,}0 + 1{,}4 \cdot 30 \cdot 1 + 0{,}5 \cdot 2 \cdot 1{,}6 \cdot 0{,}9 + 0{,}5 \cdot 22 \cdot 0{,}9 = 642{,}14$ [cm^3]



S_y (A_2) = $z_{5\text{-}1}$ A(bottom part top flange HEB 220) + $z_{6\text{-}1}$ [2x A(bottom part flange C 300)] +

+ $z_{7\text{-}1}$ A(web HEB 220) + $z_{8\text{-}1}$ A(bottom flange HEB 220) + $z_{9\text{-}1}$ A(C 220) +

+ $z_{10\text{-}1}$ [2x A(L 100x100x10)]

$$|S_y\,(A_2)| = 0{,}2 \cdot 0{,}4 \cdot 22 + 2 \cdot 4{,}1 \cdot 8{,}1 \cdot 1{,}6 + 10{,}1 \cdot 19{,}5 \cdot 0{,}9 + 20{,}5 \cdot 1{,}25 \cdot 22 +$$

$$+ 23{,}2 \cdot 37{,}4 + 23{,}9 \cdot 2 \cdot 19{,}2 = 2\,640{,}66\ [\mathrm{cm}^3]$$

$$W_{z,\,pl} = |S_z\,(A_1)| + |S_z\,(A_2)| = 2x\,|S_z\,(A_1)| = 2x\,|S_z\,(A_2)|$$

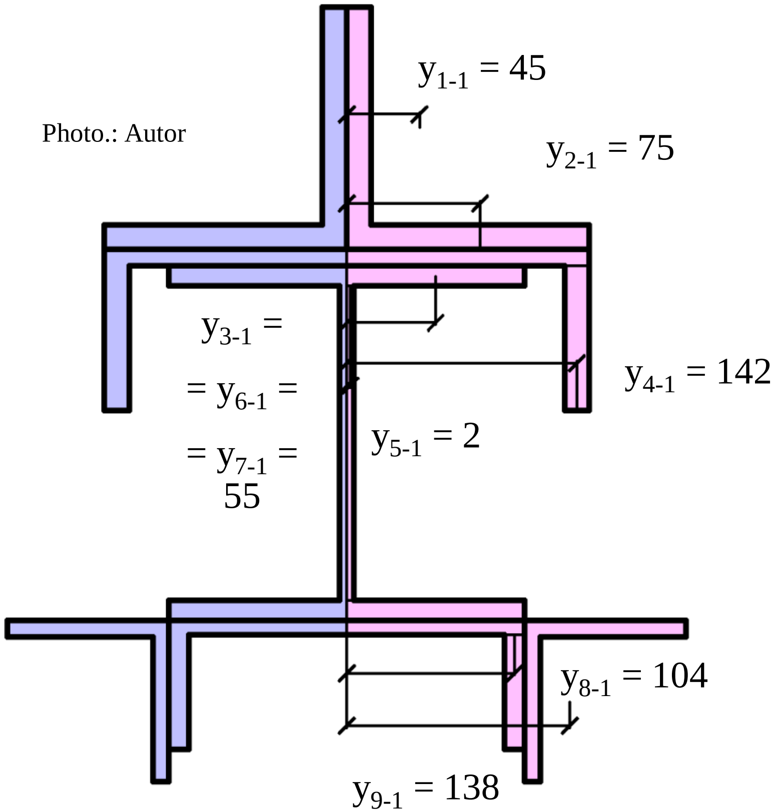


Photo.: Autor

$|S_z\,(A_2)| = S_z$ (L 150x150x15) +

+ S_z (left part web C 300) +

+ S_z (left part top flange HEB 220)] +

+ S_z (left flange C 300) +

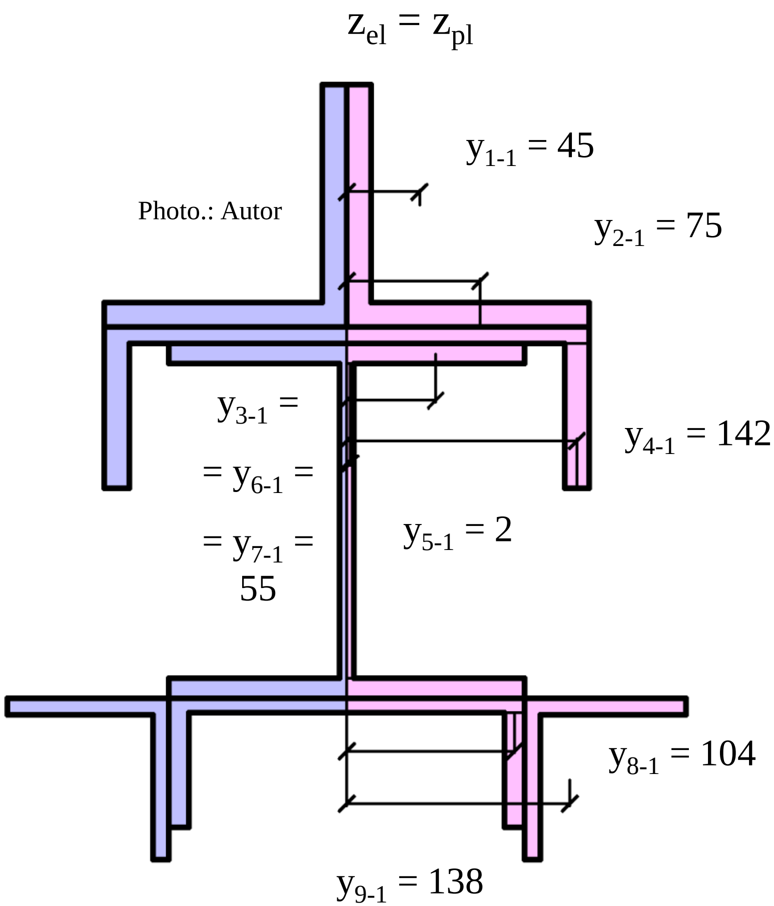
+ S_z (left part web HEB 220) +

+ S_z (left part bottom flange HEB 220)] +

+ S_z (left part web C 220) +

+ S_z (left flange C 220) +

+ S_z (L 100x100x10)



$$
\begin{aligned}
|S_z\,(A_2)| &= y_{1-1}\,A(\text{L } 150\text{x}150\text{x}15) + \\
&+ y_{2-1}\,A(\text{left part web C 300}) + \\
&+ y_{3-1}\,A(\text{left part top flange HEB 220})] + \\
&+ y_{4-1}\,A(\text{left flange C 300}) + \\
&+ y_{5-1}\,A(\text{left part web HEB 220}) + \\
&+ y_{6-1}\,A(\text{left part bottom flange HEB 220})] + \\
&+ y_{7-1}\,A(\text{left part web C 220}) + \\
&+ y_{8-1}\,A(\text{left flange C 220}) + \\
&+ y_{9-1}\,A(\text{L } 100\text{x}100\text{x}10) = \\
&= 4{,}5 \cdot 46 + 7{,}5 \cdot 15 \cdot 1 + 5{,}5 \cdot 11 \cdot 1{,}25 + \\
&+ 14{,}2 \cdot 9 \cdot 1{,}6 + 0{,}2 \cdot 19{,}5 \cdot 0{,}5 + 5{,}5 \cdot 11 \cdot 1{,}25 + \\
&+ 5{,}5 \cdot 11 \cdot 0{,}9 + 10{,}4 \cdot 1{,}25 \cdot 7 + 13{,}8 \cdot 19{,}2 = \\
&= 1087{,}59\ [\text{cm}^3]
\end{aligned}
$$

$$W_{y,\,pl} = |S_y\,(A_1)| + |S_y\,(A_2)| = 642{,}14\ [cm^3] + 2\,640{,}66\ [cm^3] = 3\,282{,}80\ \ [cm^3]$$

$$W_{z,\,pl} = |S_z\,(A_1)| + |S_z\,(A_2)| = 2x\ |S_z\,(A_1)| = 2x\ |S_z\,(A_2)| = 2\,175{,}18\ \ [cm^3]$$

Characteristic	Value
A [cm^2]	317,6
$A_{V,\, y}$ [cm^2]	169,8
$A_{V,\, z}$ [cm^2]	136,8
J_y [cm^4]	51 836,840
J_z [cm^4]	25 311,108
i_y [cm]	12,78
i_z [cm]	8,93
$W_{y,\, el,\, top}$ [cm^3]	2 185,36
$W_{y,\, el,\, bottom}$ [cm^3]	2 134,96
$W_{z,\, el}$ [cm^3]	1 205,29
$W_{y,\, pl}$ [cm^3]	3 282,80
$W_{z,\, pl}$ [cm^3]	2 175,18

Verification of results:

$$A_{V,\,y} + A_{V,\,z} \approx A \quad \text{OK}$$

$$\min\,(W_{y,\,el,\,top}\ ;\ W_{y,\,el,\,bottom}) < W_{y,\,pl} \quad \text{OK}$$

$$W_{z,\,el} < W_{z,\,pl} \quad \text{OK}$$

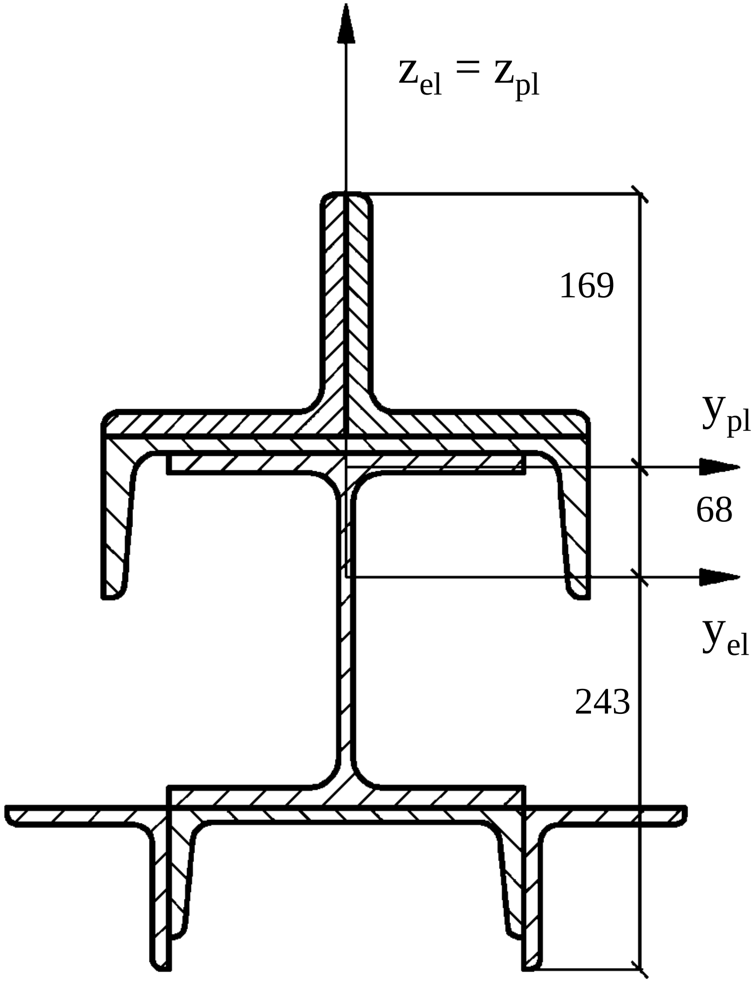


Photo.: Autor

I-beam dwuteownik

web   środnik (dwuteownika)

flange półka, pas (dwuteownika)

chord pas (kratownicy)

braces   skratowanie (kratownicy, czyli słupki + krzyżulce)

corrugated web falisty środnik

hot rolled goracowalcowany

welded spawany

cold formed zimnogięty

plate / sheet steel blacha (w zależności od kontekstu)

channel section ceownik

angle steel kątownik

rope lina

sigma-beam przekrój sigma

hollow section rura

z-bar zetownik

Thank you for attention

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