Metal Structures

Design Project III

Steel hall – examples of calculation (part I)

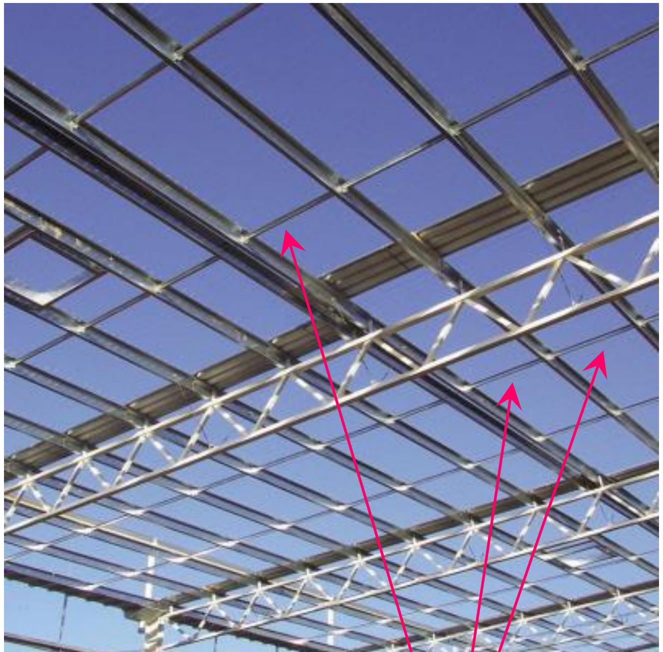
Ist example of calculations – continous suspended cold-formed purlin

Photo: kingspan.com

Photo: stratco.com.au

Purlins on a pitched roof are always bent in two directions. In case of hot-rolled purlins, bending resistance about weak axis is about 20% of resistance about strong one (IPE 300). For cold-formed purlins, it is only about 10% (Z-400-3). Even very small load about weak axis can lead to exceeding resistance of cold-formed purlin. Presented method of preventing this problem is to use suspensions.

Photo: stratco.com.au



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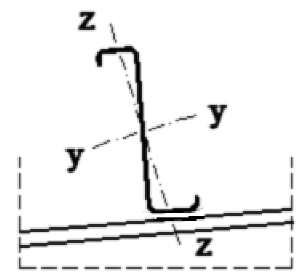
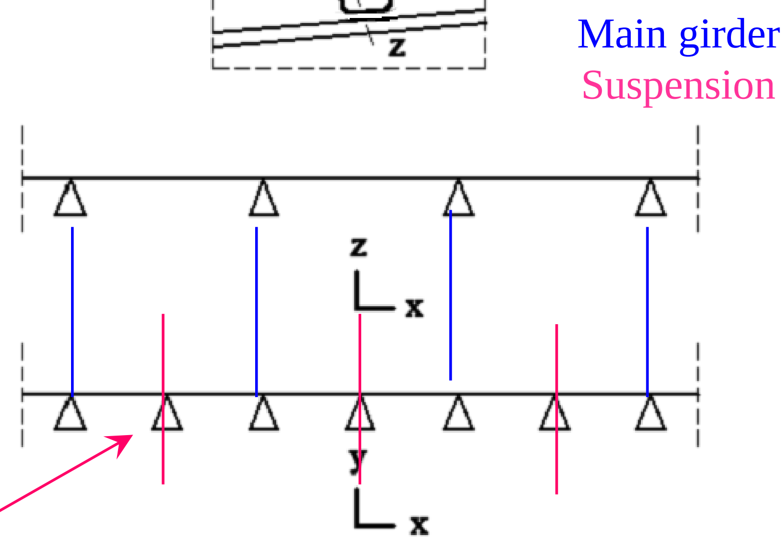
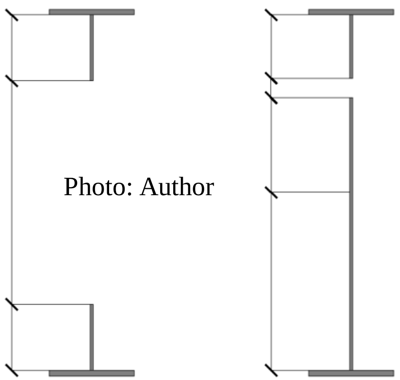




Suspensions are additional bars (for example: round ones) that connect purlins in plane of roof. This creates additional supports about weak axis. Span of purlin about strong axis is, for example, 6.0 m, and about weak axis, for example, 6.0 / 2 = 3.0 m. Thanks to this, bending moments about weak axis are significantly smaller than without suspension; about 25% „initial” value.

Correct calculation of cold-formed purlins should be based on EN 1993-1-3; discussed in separate subject (Steel Thin-Walled Structure) within the Metal Structure Diploma Profile. Method presented in EN 1993-1-3 is based on a similar principle to calculation of welded I-beams presented within Lab #2: effective cross-section geometry (we ignore parts exposed to loss of local stability).



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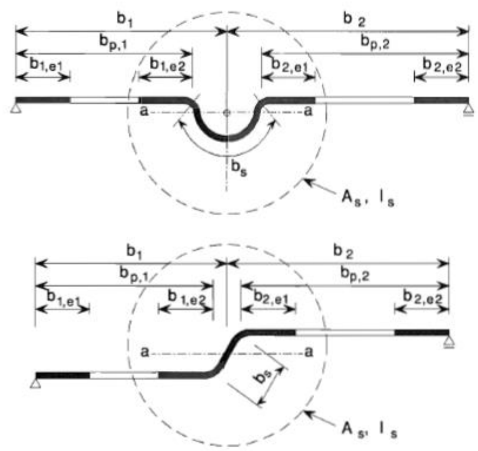


Photo: EN 1993-1-3 fig. 5.9

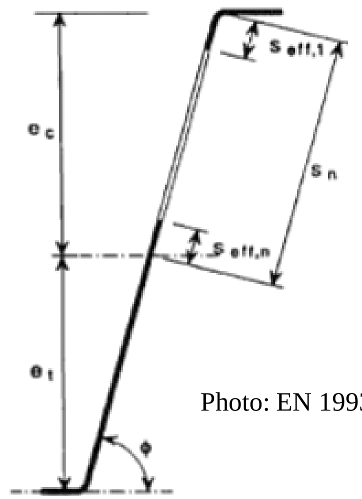


Photo: EN 1993-1-3 fig. 5.12

Method presented in this project is only **<u>very rough simplification of a complex problem</u>**. Aim of project is to familiarize You with influence of suspensions on work of purlins, and not with calculation of cold-bent purlins as such.

Resistance of purlin could be analised in axes y z paralell to web and flange or in main central axes η ζ .

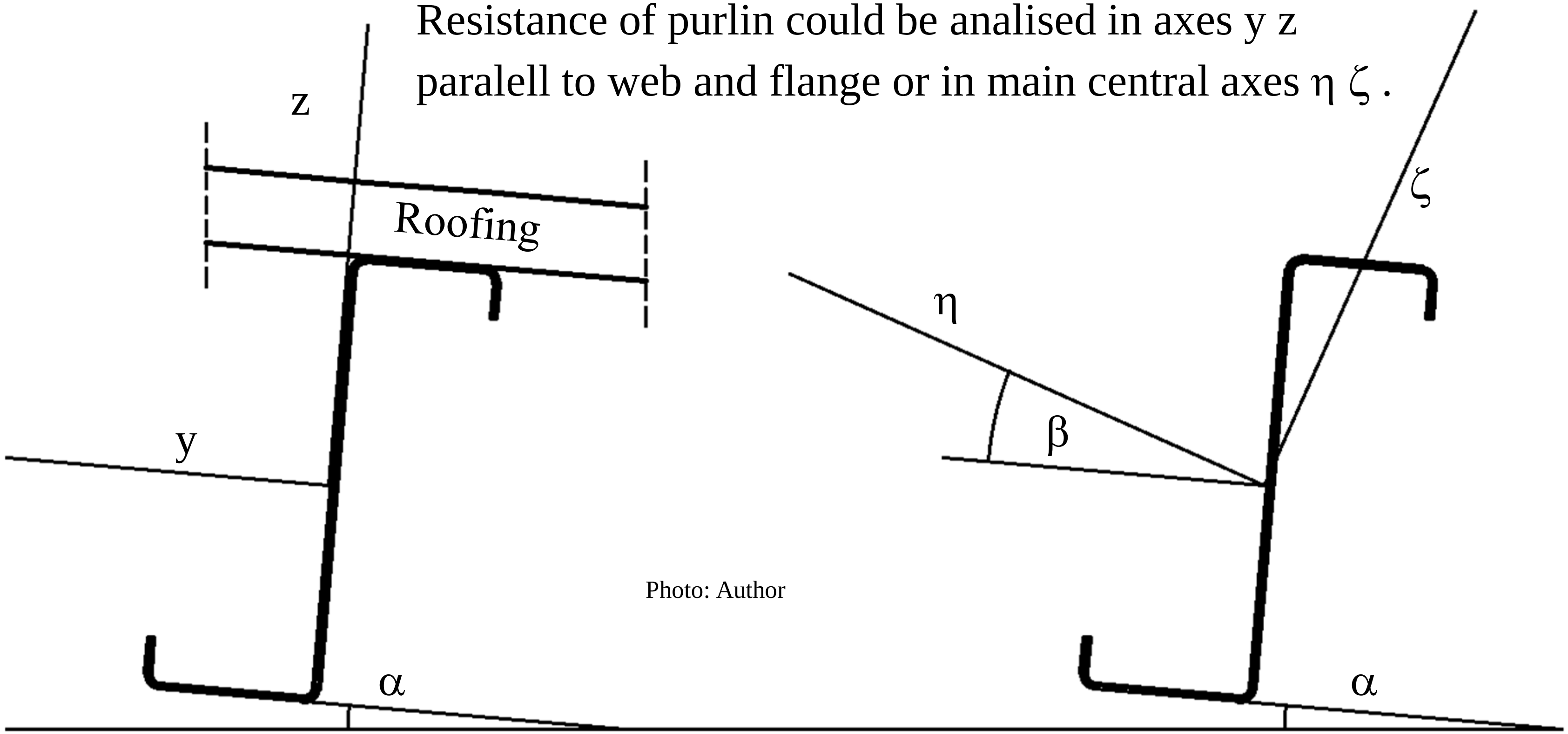


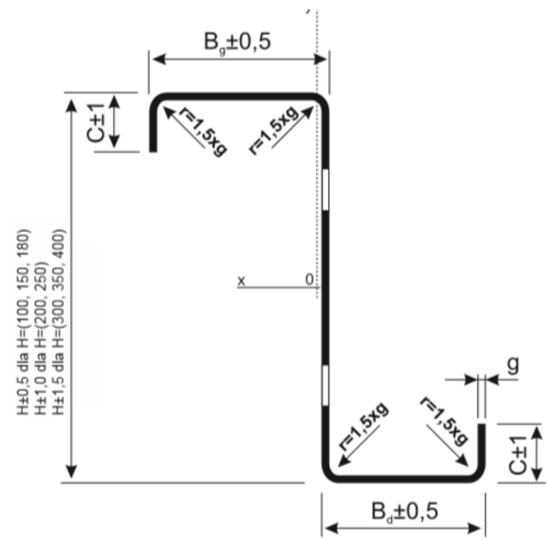
Photo: Author

I[st] stage: cooperation between purlin and roofing (through ~ 10 years since erecton of structure); maybe in next years the same.

II[nd] stage: no cooperation between purlin and roofing (for example: after many years after erection of structure - complex of small destructions in bolts, purlins and roofing eliminates cooperation).

H	Bg	Bd	r	Cg	Cd	g	masa	FA	x	y	Jx	Jy	Wx	Wy	ix	iy
[mm]							kg/mb	[cm2]	[cm]		[cm4]		[cm3]		[cm]	
280	75	65	2,25	20	20	1,50	5,25	6,68	0,267	14,233	735,26	56,17	51,66	7,82	10,49	2,90
			3,00	20	20	2,00	6,94	8,85	0,291	14,234	965,09	72,62	67,80	10,14	10,44	2,87
			3,75	20	20	2,50	8,62	10,98	0,315	14,235	1187,47	87,98	83,42	12,33	10,40	2,83
			4,50	22	22	3,00	12,47	15,89	0,329	17,746	2631,82	154,33	148,30	19,00	12,87	3,12
400	85	75	2,25	22	22	1,50	6,94	8,84	0,241	20,259	1893,91	83,80	93,48	10,21	14,63	3,08
			3,00	22	22	2,00	9,21	11,73	0,265	20,260	2495,26	108,83	123,16	13,30	14,59	3,05
			3,75	22	22	2,50	11,44	14,58	0,289	20,261	3081,93	132,47	152,11	16,23	14,54	3,01
			4,50	22	22	3,00	13,65	17,39	0,313	20,262	3653,93	154,77	180,34	19,02	14,49	2,98

Photo: pruszynski.com.pl



I^st stage: calculations in axes y-x (horizontal-verical):

Z 400-3:

$W_y = 180{,}34\ cm^3$

$W_z = 19{,}02\ cm^3$

Vertical part of web = $(40 - 2 \cdot 1{,}5 \cdot 0{,}3) \cdot 0{,}3 = 11{,}73\ cm^2$

Horizontal part of flages = $2 \cdot (8{,}5 - 2 \cdot 1{,}5 \cdot 0{,}3) \cdot 0{,}3 = 4{,}56\ cm^2$

II^nd^ stage: AutoCAD 2021, functions REGION and MASSPROP (main central axes):

J_η = 3 767,86 cm^4

J_ζ = 84,46 cm^4

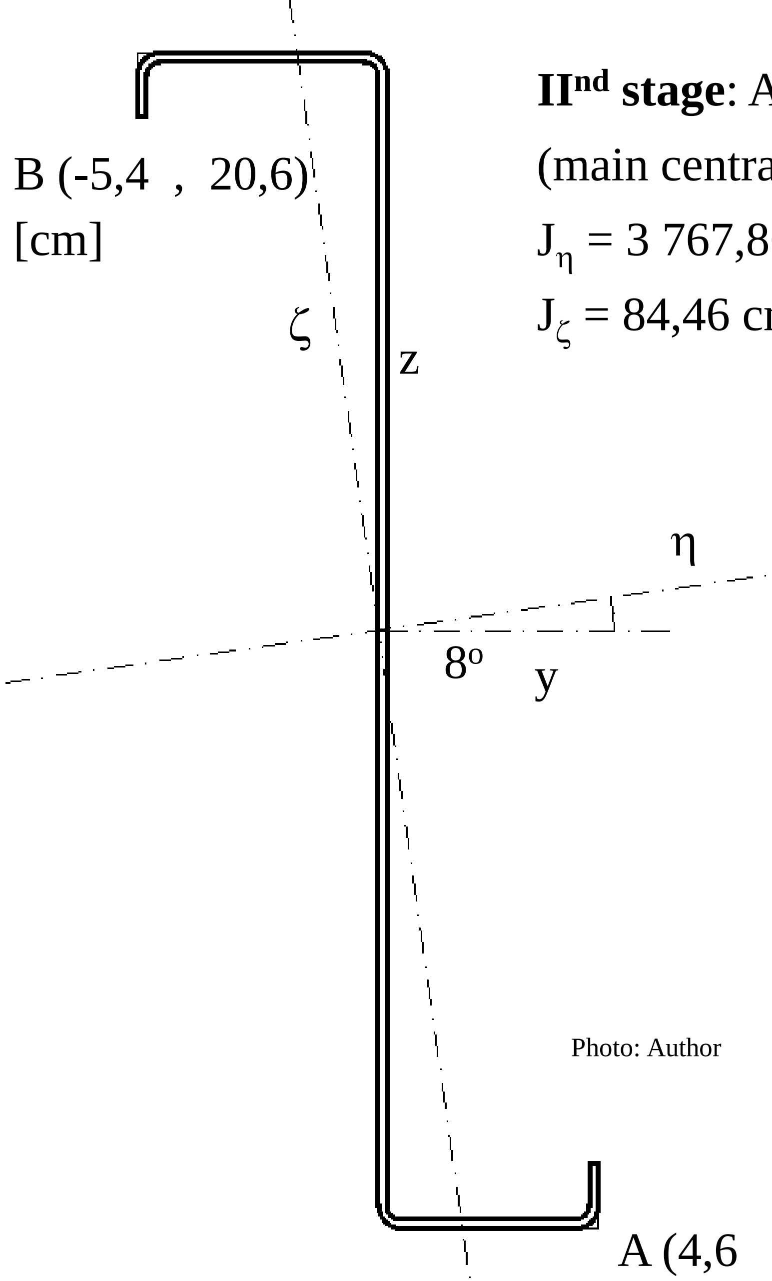


Photo: Author

η_A = 4,6 cm

η_B = 5,6 cm

ζ_A = 21,2 cm

ζ_B = 20,6 cm

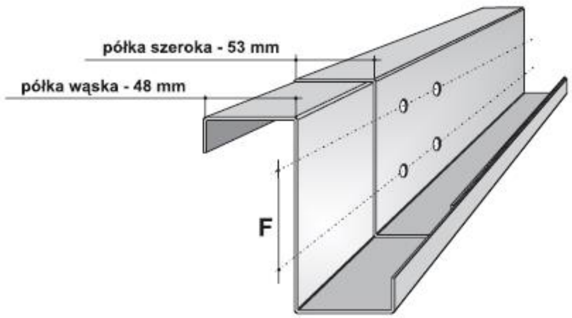
$W_{A,\eta} = J_\eta / \zeta_A$ = 177,73 cm^3

$W_{A,\zeta} = J_\zeta / \eta_A$ = 18,36 cm^3

$W_{B,\eta} = J_\eta / \zeta_B$ = 182,91 cm^3

$W_{B,\zeta} = J_\zeta / \eta_B$ = 15,08 cm^3

In case of cold-formed purlins, a single cross-section is used in spans and a double one above supports (overlap joint). Without precise geometrical calculations, we will assume that resistance above support is 1,80 times resistance in span.



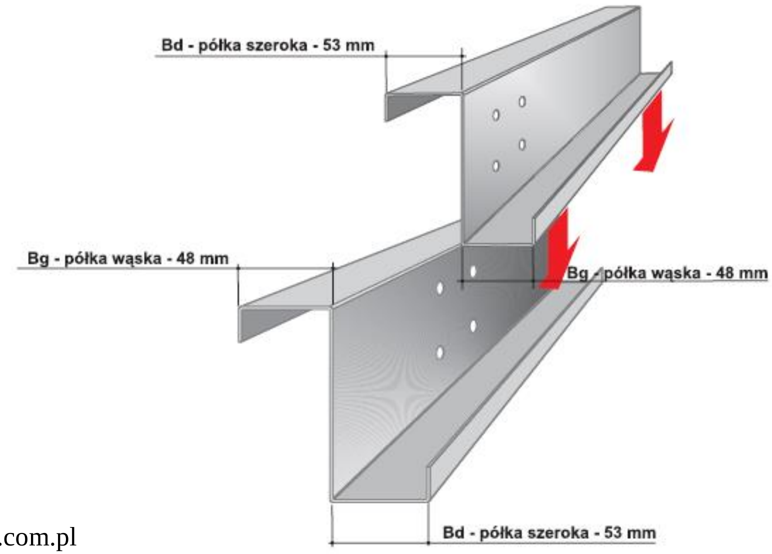


Photo: pruszynski.com.pl

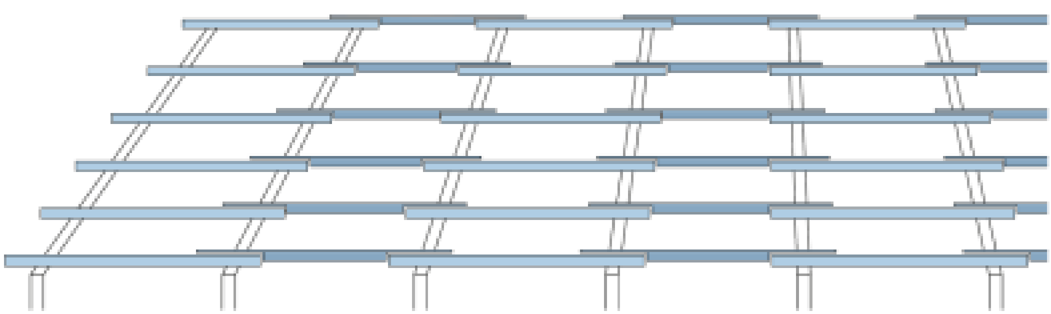


Photo: schrag.pl

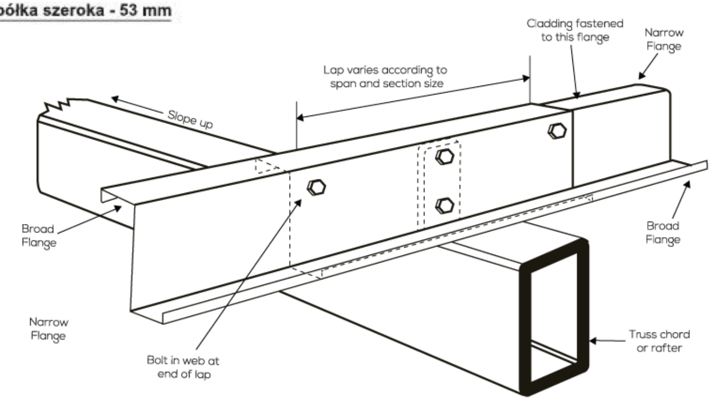


Photo: forminometal.com

Dead-weight of roofing, snow load and wind action on roof are the same as for Ist design project. Dead-weigth of purlin for cold-formed is smaller as for hot-rolled one. **<u>For simplification</u>**, dead-weight of purlin could be not recalculated. So, for Ist stage, we can take the same values of loads, as for Ist design project (→ Des #1 examp / 24):

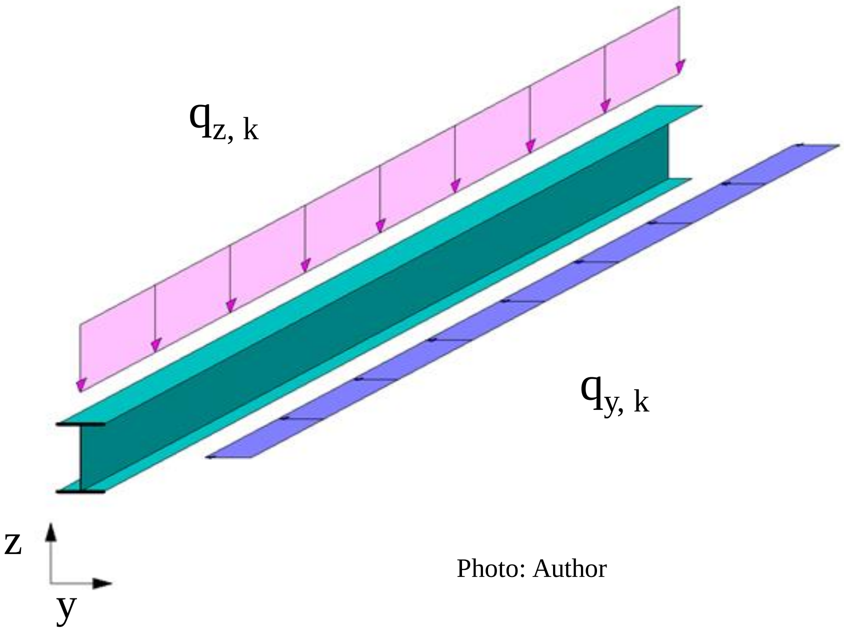


Photo: Author

q_z = 10,331 kN / m

q_y = 0,863 kN / m

II[nd] stage: for analysis in main central axes, loads must be additionally recalculated by 8°. This means, in calculation must be taken into consideration values of $\sin(\alpha + \beta) = \sin(13^o)$ and $\cos(\alpha + \beta) = \cos(13^o)$, instead $\sin(\alpha) = \sin(5^o)$ and $\cos(\alpha) = \cos(5^o)$.

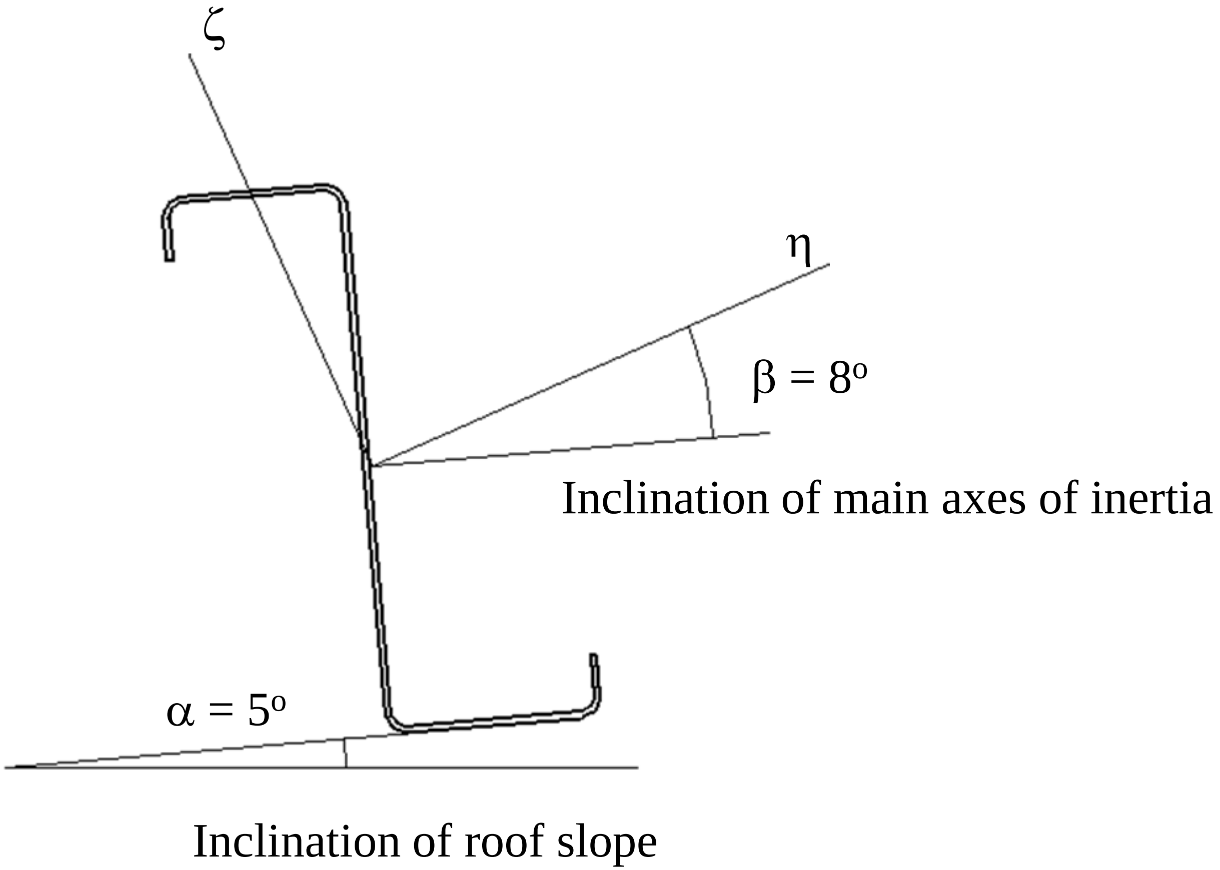


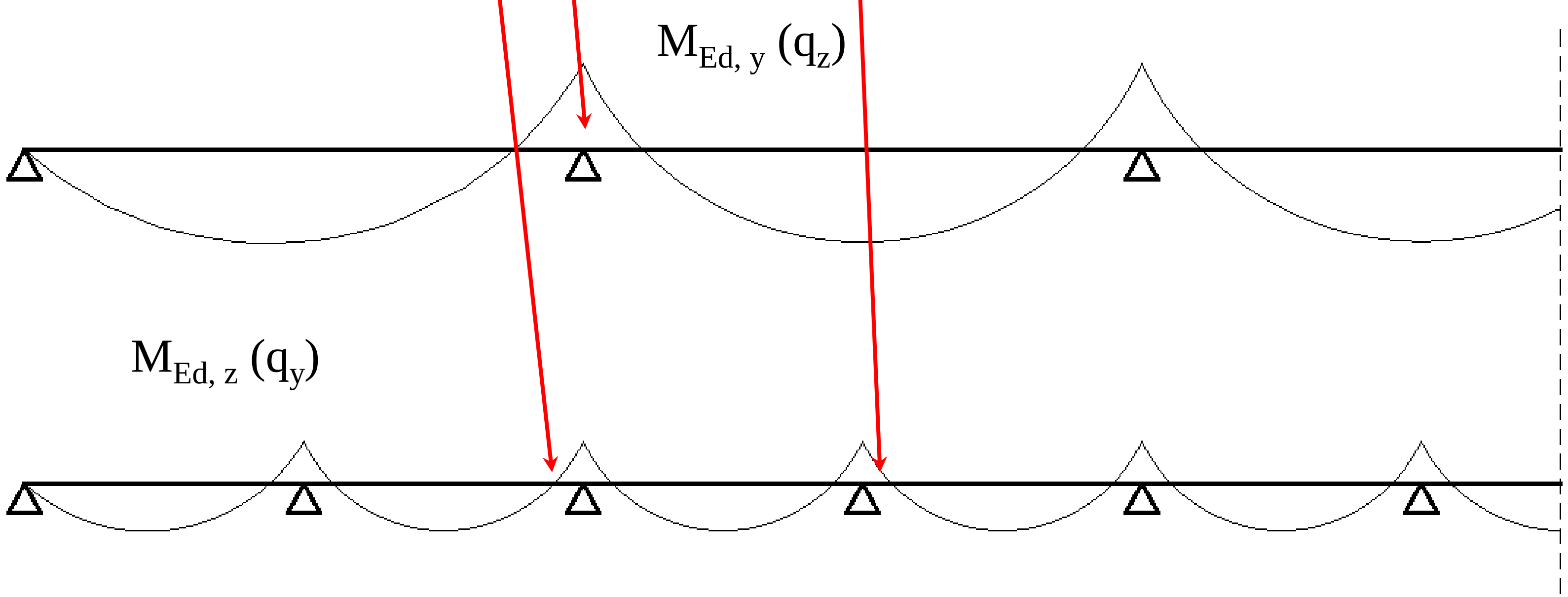
Photo: Author

Loads	I^{st} stage (5^o, according to Des #1 examp / 24)	II^{nd} stage (13^o, in analogy to Des #1 examp / 24)
s_k = 2,50 [kN / m^2] g_k = 0,16 [kN / m^2] w_k = 0,20 / -0,25 [kN / m^2] i_k = 0,60 [kN / m^2]	q_z = 10,331 kN / m q_y = 0,863 kN / m	q_z = 10,116 kN / m q_y = 2,109 kN / m

Purlin is supported:

- in direction z by main roof girder (bending about stron axis);
- in direction y by main roof girder and additional suspension for purlins in half between girders (bending about weak axis).

This types of supports make two warious static schemes in both directions.



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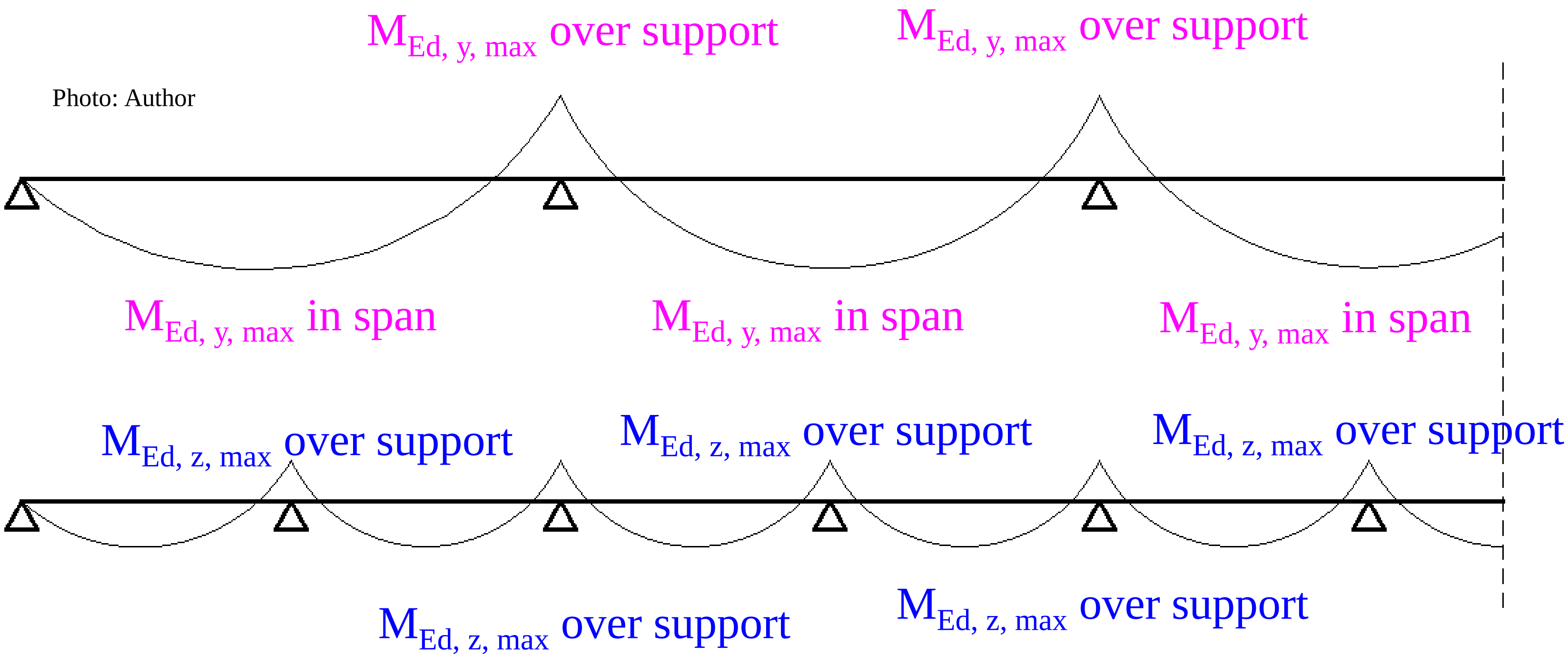


Photo: Author

Such type of static scheme makes, that max values $M_{Ed, y, max}$ – over supports or in spans – correspond max values $M_{Ed, z, max}$ – only over supports. About strong axis importanr are bending moments over supports and in spans. About weak asix – only owver supports.

For long, continous multispan purlin, values of bending moments in spans for central part are the same. Over supports – the same rule. So, for calculation, important are only values for first, second and third span and for supports B and C (second and third).

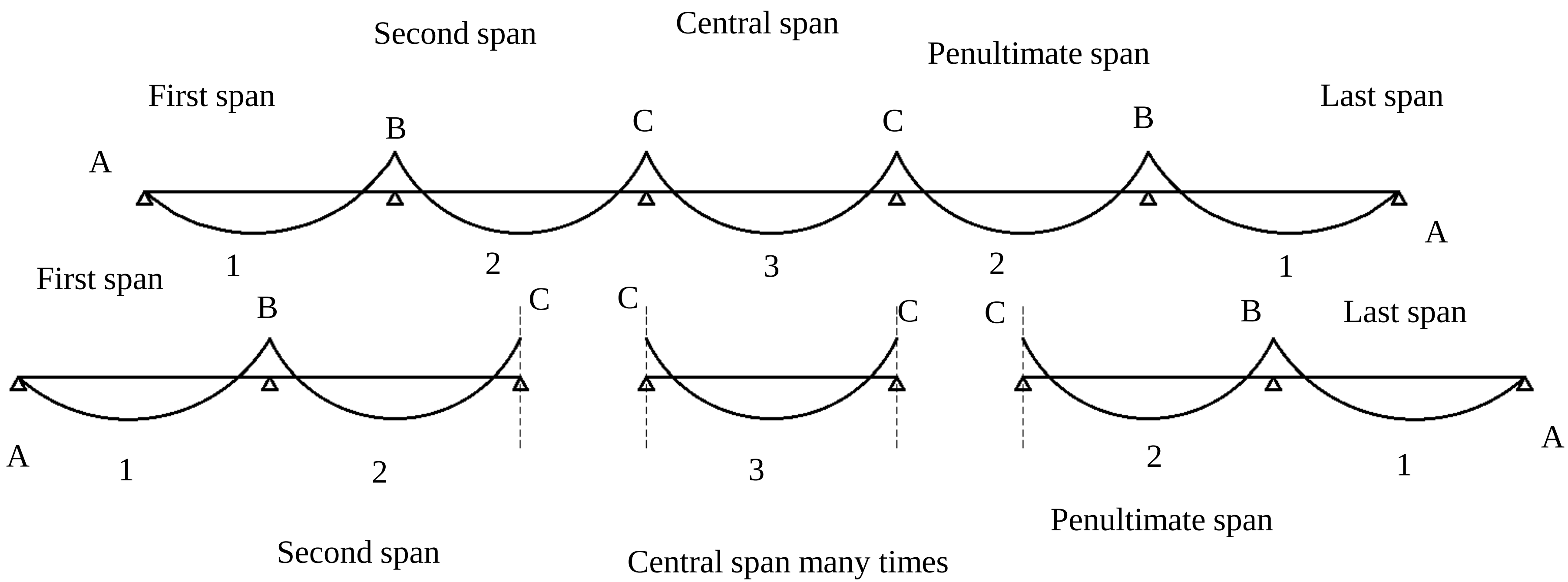


Photo: Author

Multi-span beam could be calculated in computer programme or with using of tables.

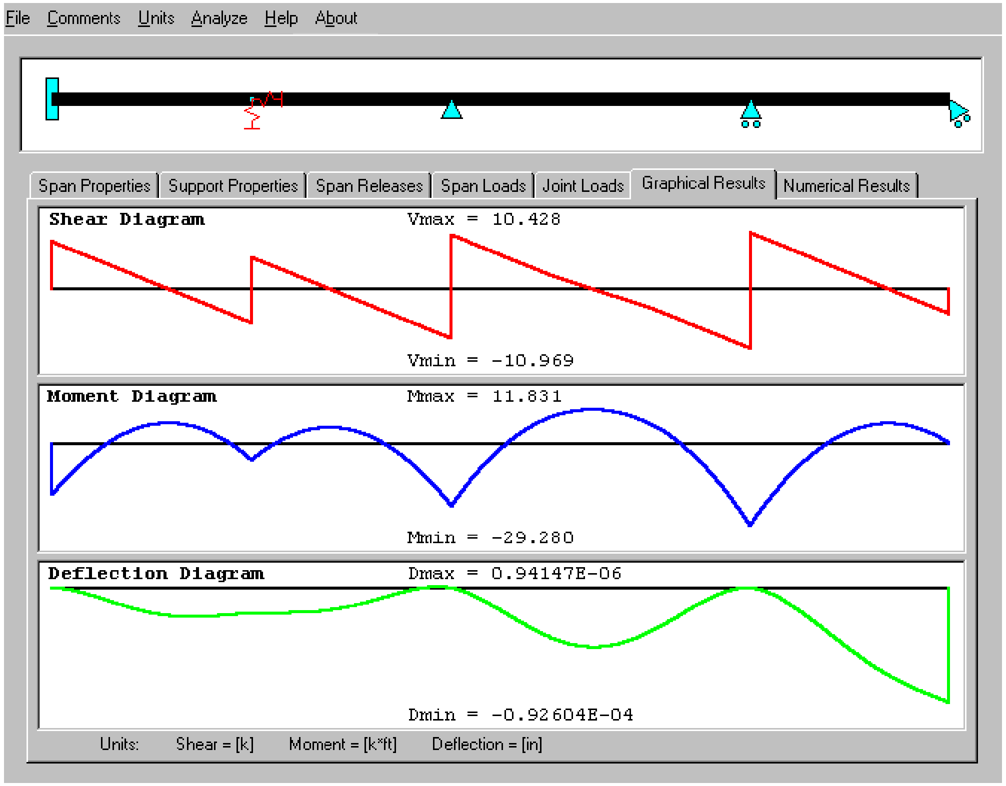
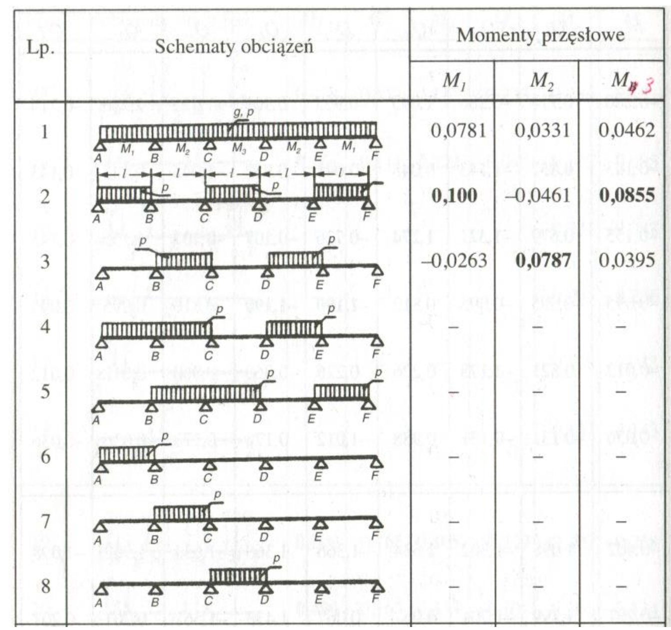


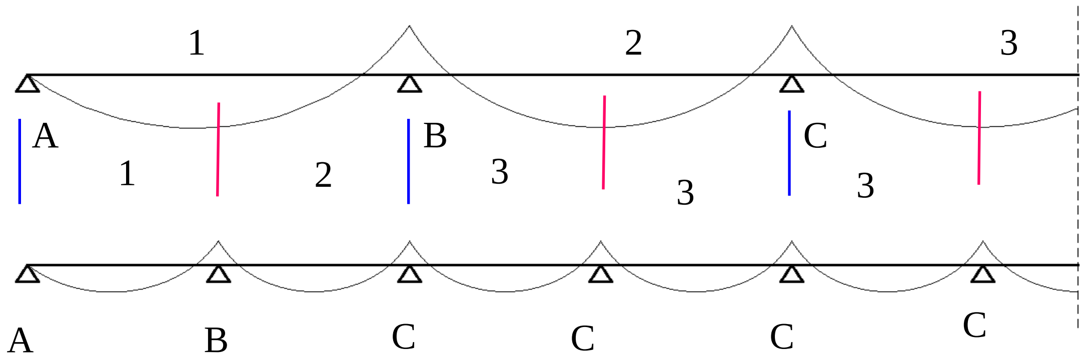
Photo: civilhelp.wordpress.com



Lp.	Schematy obciążeń	Momenty przęsłowe		
		M_1	M_2	M_3
1		0,0781	0,0331	0,0462
2		**0,100**	–0,0461	**0,0855**
3		–0,0263	**0,0787**	0,0395
4		–	–	–
5		–	–	–
6		–	–	–
7		–	–	–
8		–	–	–

Photo: Tablice do projektowania konstrukcji metalowych, W. Bogucki, M. Żyburtowicz, Arkady 1996 Warszawa

Points	Value	Effect of q_z (6m)	Effect of q_y (3m)	Effect of q_y , no suspension (6 m, for comparison)
A-A	R_{A-A} [kN]	24,484	1,022	2,045
1-B	R_{1-B} [kN]	-	2,931	5,861
	M_{1-B} [kNm]	29,047	0,815	3,262
B-C	R_{B-C} [kN]	70,168	2,522	5,043
	M_{B-C} [kNm]	39,051	0,614	2,454
2-C	R_{2-C} [kN]	-	2,522	5,043
	M_{2-C} [kNm]	12,310	0,614	2,454
C-C	R_{C-C} [kN]	60,374	2,522	5,043
	M_{C-C} [kNm]	29,381	0,614	2,454
3-C	R_{3-C} [kN]	-	2,522	5,043
	M_{3-C} [kNm]	17,183	0,614	2,454



Direction z (about strong axis)

Main girder
Suspension

I^st stage

Direction y (about weak axis)

Photo: Author

I^{st} stage

Point	Cross-section	$\sigma_{x,y}$ = $= M_{Ed,\,y} / W_y$ [MPa]	$\sigma_{x,z}$ = $= M_{Ed,\,z} / W_z$ [MPa]	$\tau_z = V_z / A_{vy}$ [MPa]	$\tau_y = V_y / A_{vz}$ [MPa]	σ_{HMH} [MPa]	σ_{HMH} / f_y
1-B	Single	161,070	42,850	0,000	3,300	204,000	0,868
B-C	Twice (1,80)	120,393	17,934	17,791	1,577	141,744	0,603

According to correct calculation, effort will be bigger than 0,868 (effective gemoetry is smaller than total, so effort increases). Additionally, impact of lateral buckling must be taken into consideration: top flange could be treated as protected by roofing, but bottom one (compressed next to supports) is not protected.

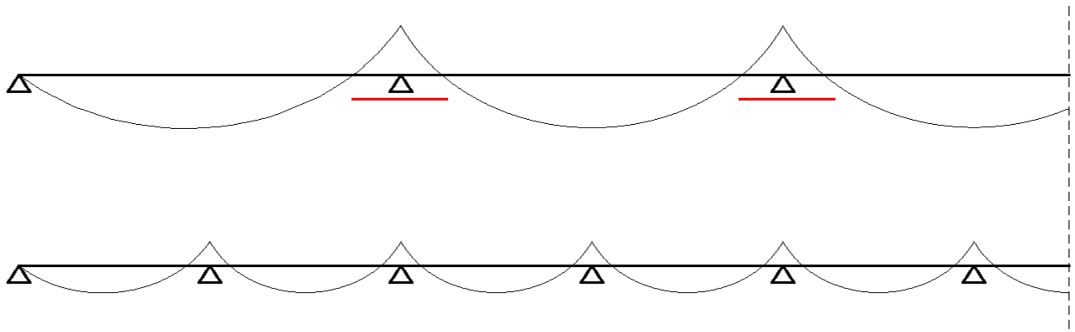
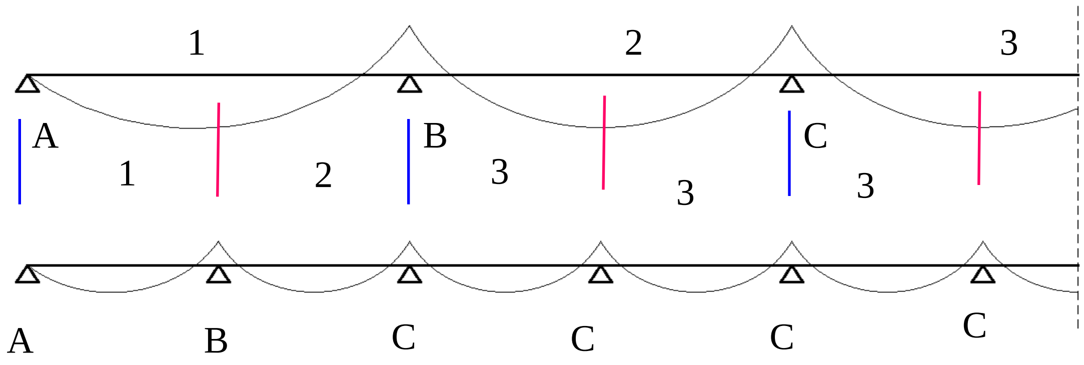


Photo: Author

Points	Value	Effect of q_z (6m)	Effect of q_y (3m)	Effect of q_y , no suspension (6 m, for comparison)
A-A	R_{A-A} [kN]	23,974	2,498	4,995
1-B	R_{1-B} [kN]	-	7,163	14,326
	M_{1-B} [kNm]	28,442	1,992	7,967
B-C	R_{B-C} [kN]	68,708	6,163	12,327
	M_{B-C} [kNm]	38,238	1,500	6,002
2-C	R_{2-C} [kN]	-	6,163	12,327
	M_{2-C} [kNm]	12,054	1,500	6,002
C-C	R_{C-C} [kN]	59,118	6,163	12,327
	M_{C-C} [kNm]	28,770	1,500	6,002
3-C	R_{3-C} [kN]	-	6,163	12,327
	M_{3-C} [kNm]	16,825	1,500	6,002



Direction z (about strong axis)

Main girder
Suspension

IInd stage

Direction y (about weak axis)

Photo: Author

II^{nd} stage

Point	Cross-section	$\sigma_{x,y} = M_{Ed,y} / W_y$ [MPa]	$\sigma_{x,z} = M_{Ed,z} / W_z$ [MPa]	$\tau_z = V_z / A_{vy}$ [MPa]	$\tau_y = V_y / A_{vz}$ [MPa]	σ_{HMH} [MPa]	σ_{HMH} / f_y
1-B	Single	157,718	104,717	0,000	8,064	262,806	1,118
B-C	Twice (1,80)	117,887	43,827	17,421	3,854	164,640	0,701

This stage is much more dangerous than previous one. Effective geometry is smaller than total, so effort will be even bigger than 1,118. addidtionally, in case of no cooperation with roofing, critical length for lateral buckling is much more longer than previous; this effect additionally increases effort.

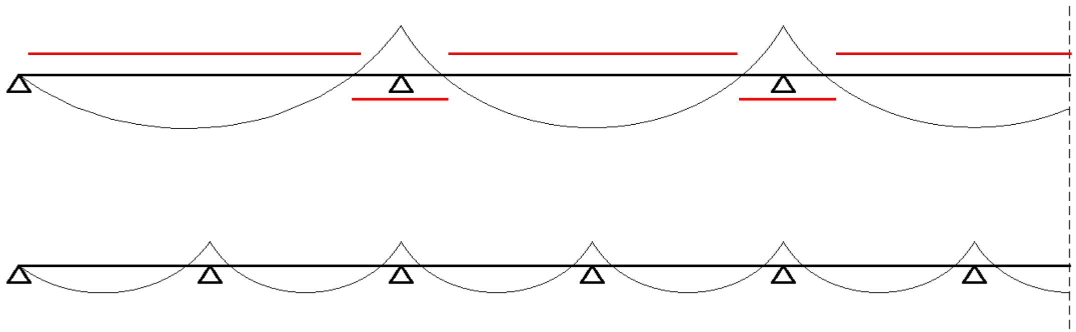


Photo: Author

Deflections:

Value of deflections can be approximated by one of two ways:

- Values from „normal" static calculations;
- Values from formula:

$$\Delta = 0{,}50\ [5\ g\ L^4 / (384\ E\ J)] + 0{,}75\ [5\ q\ L^4 / (384\ E\ J)]$$

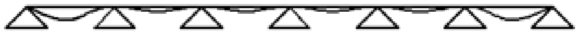
where

Δ - deflection; g – dead weight; q – live load; L – length of one span

→ Des #2 / 31

Δ_z

Photo: Author



Δ_y

Suspension makes, that max Δ_z corresponds with $\Delta_y = 0$. Δ_y is neglegible (values without adding new purlins).

According to Des #2 examp1 / 22, 23:

$$g_z + q_z = 0{,}540 \text{ kN / m} + 9{,}791 \text{ kN / m}$$

According to #t / 6:

$$J_y = 3653{,}93 \text{ cm}^4$$

$$\Delta_z = 0{,}50 \, [5 \, g_z \, L^4 / (384 \, E \, J_y)] + 0{,}75 \, [5 \, q_z \, L^4 / (384 \, E \, J_y)] = 0{,}017 \text{ m}$$

$$\Delta_{acc} \text{ (\#11 / 95)} = L / 200 = 0{,}030 \text{ m}$$

$$\Delta_z / \Delta_{acc} = 0{,}567 < 1 \text{ OK}$$

Effort (II[nd] stage) >> Effort (I[st] stage)

It is important to notice in time that purlin is no longer cooperate with roofing and to start renovation in time. It's good explanation, why **periodic inspections** are such important.

Photo: flyability.com

Photo: structures.com.sg

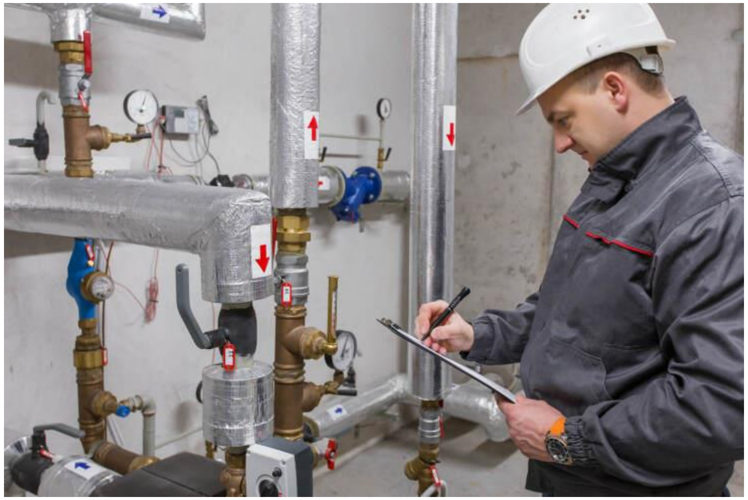


Photo: extradom.pl

Resistance condition fo purlin must be met in stage I. If a problem occurs in stage II, note: "*Durnig technical inspection, special attenton should be paid to cooperation of purlin and roofing, especially when time of exploitation had exceeded 10 years*" should be included in construction card.

Hangers

Reactions in additional supports in direction y are loads applied to suspension bars (#t / 16).

According to #t / 16, this force = 2,931 kN (values without adding new purlins).

Round bar Φ 8mm; $A = 0{,}503\ cm^2$

$N_{t,Rd} = A\, f_y / \gamma_{M0} = 11{,}812\ kN$

$F / N_{t,Rd} = 0{,}248 <$ OK

Compression in suspended bars is not taken into consideration. Such slenderness members losts stability under even small compressive force.

There is possible compression in part of hangers. There will be permanently deformations as the effect of buckling.

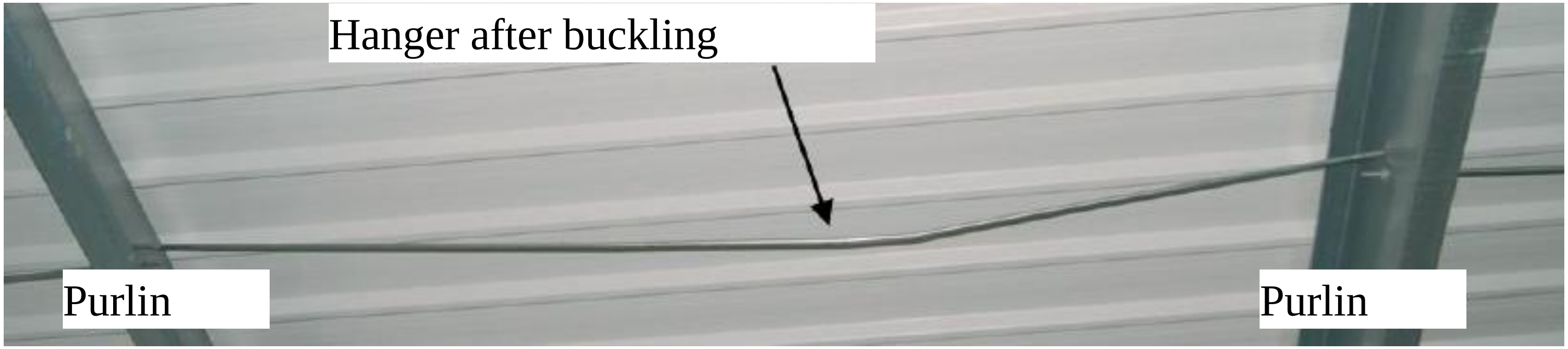
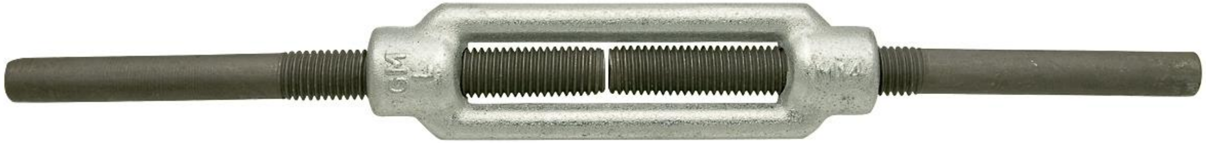


Photo: A. Biegus, Przyczyny przedawaryjnego stanu technicznego płatwi hali stalowej, Budownictwo i Architektura 12 / 2013, 173-180

→ #8 / 34

Photo: dromet.pl



We need rigging screws to repair hangers.

IInd example of calculations – main column in frame

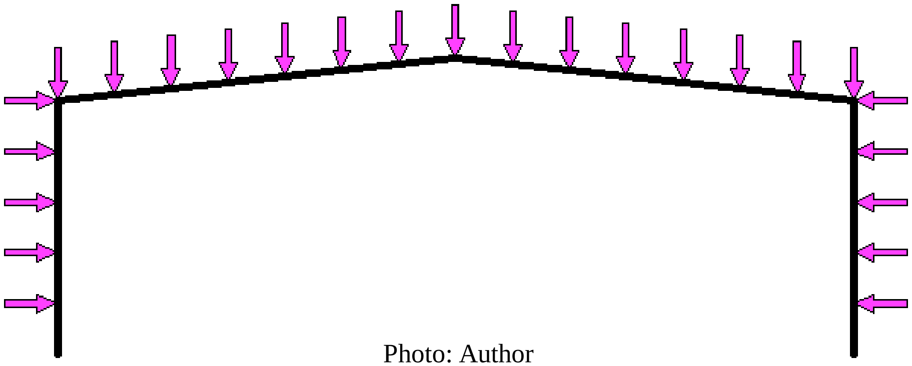
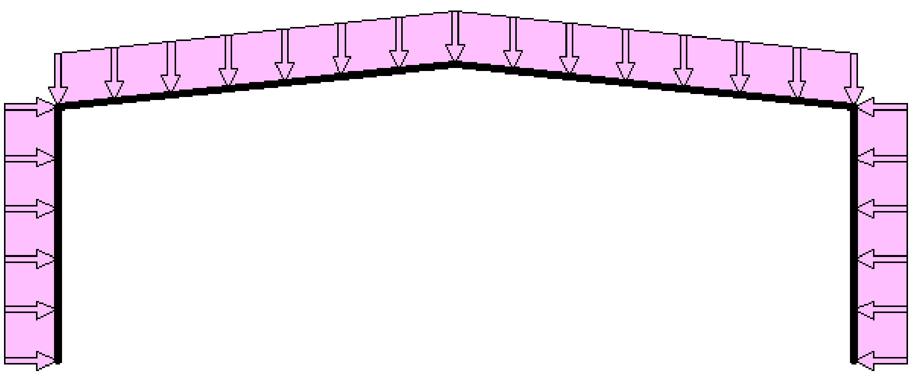


Photo: Author

Forces shoul be applied in point – as effect of contact with purlins and wall girts.



For big number of purlins and girts, loads could be applied as continous.

External actions as forces in points

Vertical force, girder: dead-weight, snow action, vertical compound from wind action

Horizontal force, girder: horizontal compound from wind action

Common point of column and girder, common horizontal and vertical forces

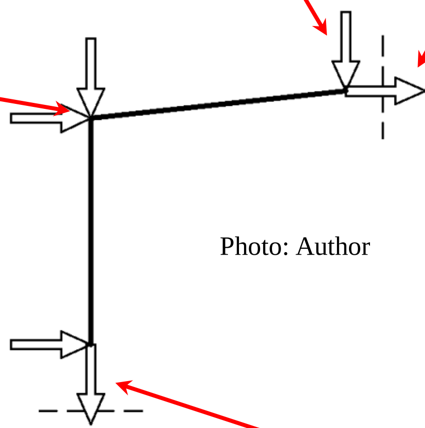


Photo: Author

Vertical force, cloumn: dead-weight

Horizontal force, column: wind action

General notice

Main roof girder is calculated according to the same algorithm, as main column. The same:

- resistance,
- stability,
- deformation

must be taken into consideration.

Resistance is checked completely the same for girder and column.

Stability for girder is calculated for specific critical lengths.

Deformation for girder is vertical deflection of ridge.

I-beam roof girder

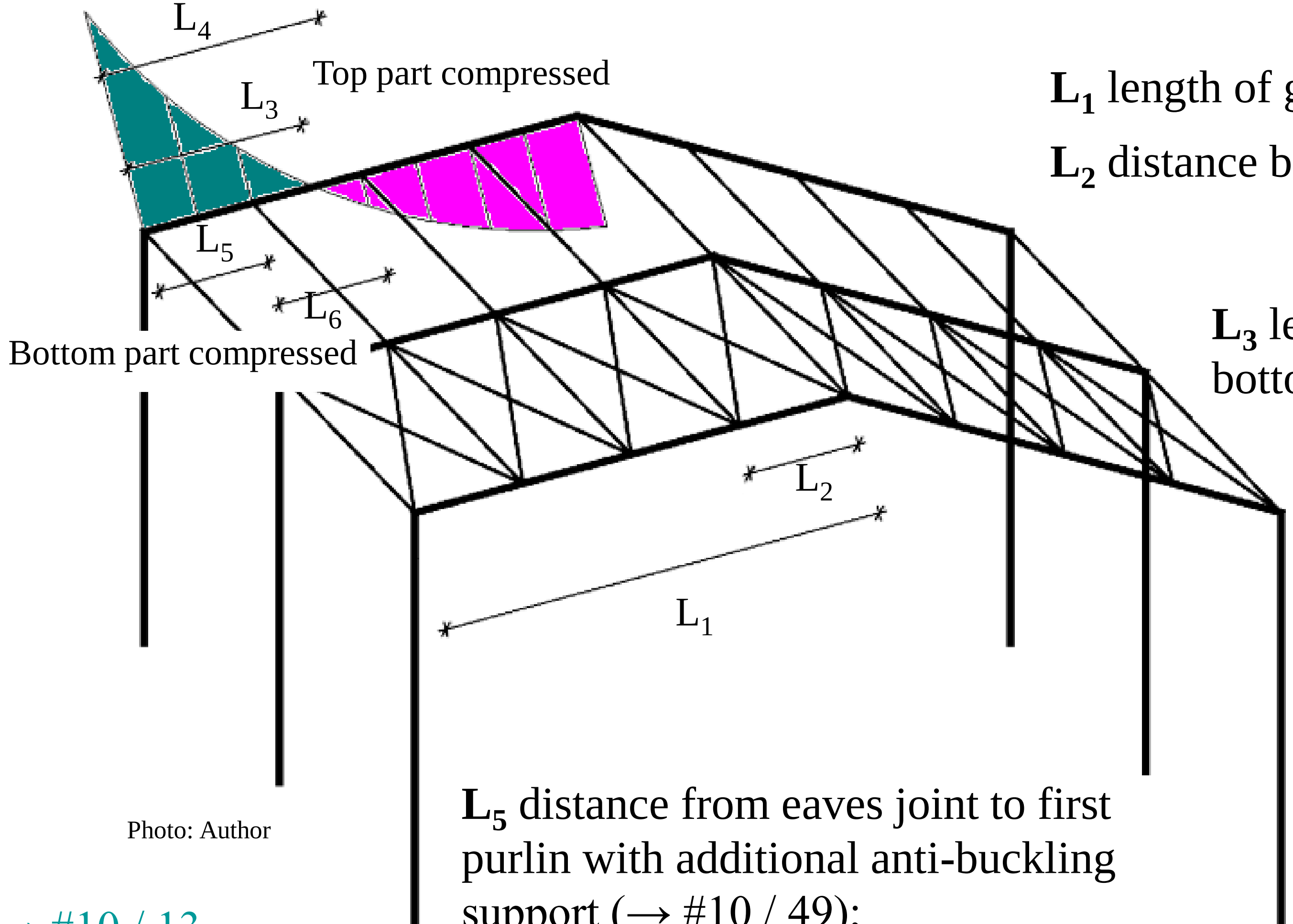


Photo: Author

L_1 length of girder;

L_2 distance between bracings

L_3 length of girder where bottom part is compressed;

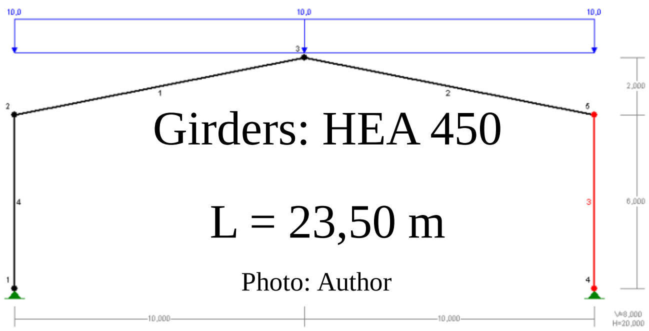
L_4 distance from eaves joint to first purlin in top part compressed;

L_5 distance from eaves joint to first purlin with additional anti-buckling support (→ #10 / 49);

→ #10 / 13

L_6 distance between the closest purlins with additional anti-buckling supports (→ #10 / 49);

Resistance and stability of main columnn



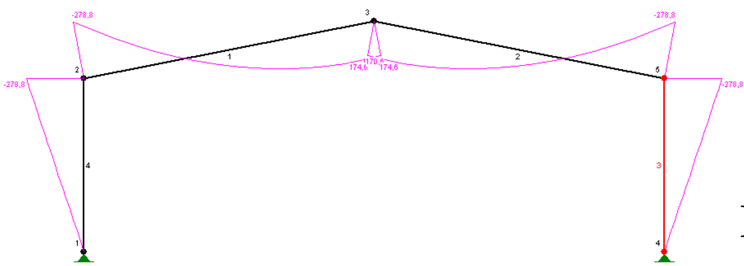
Girders: HEA 450

L = 23,50 m

Photo: Author

Columns: HEA 450

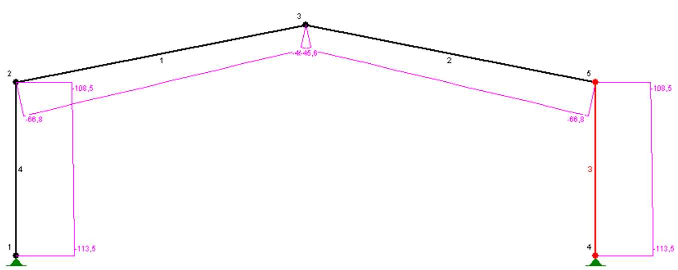
H = 5,45 m



$M_{Ed, top}$ = 311,6 kNm

$M_{Ed, bottom}$ = 0,0 kNm

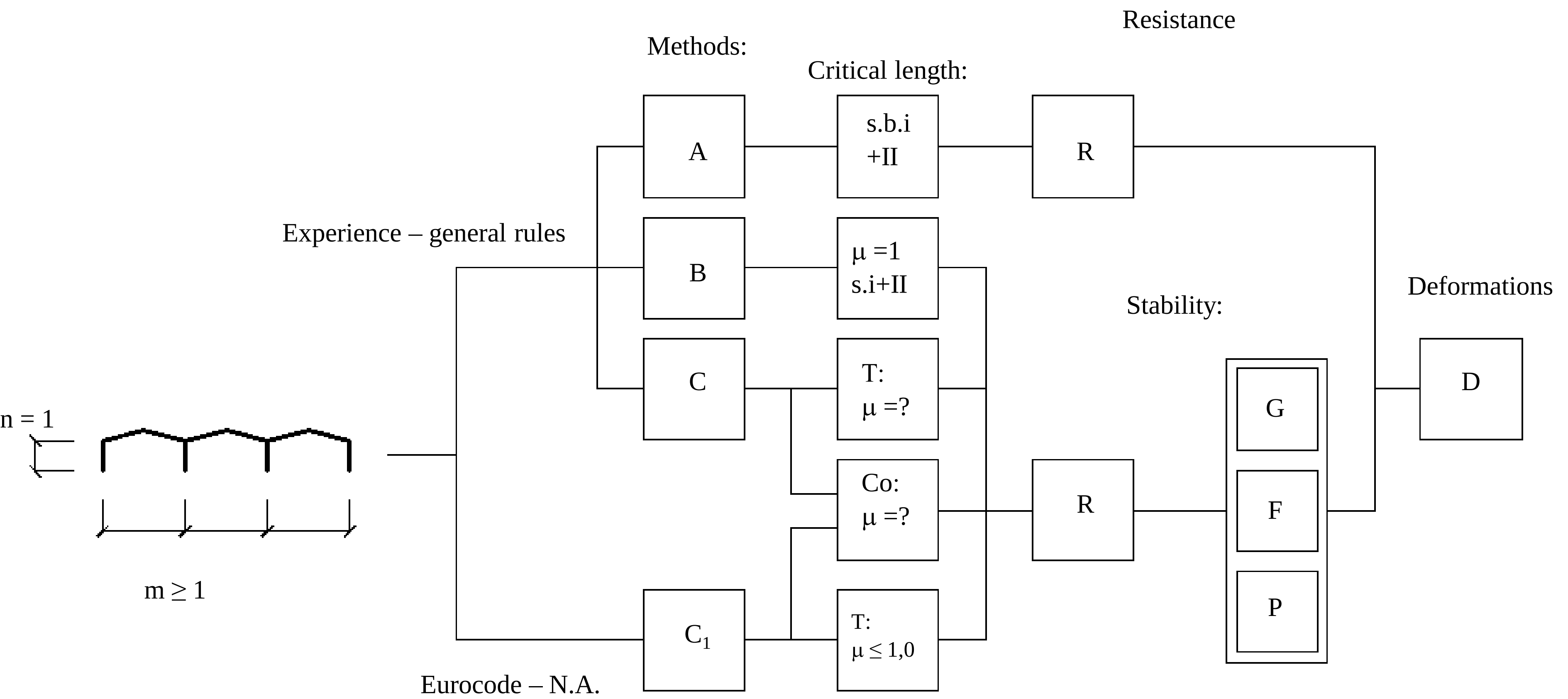
$V_{Ed, max}$ = 57,2 kNm



$N_{Ed, top}$ = 93,4 kN

$N_{Ed, bottom}$ = 97,6 kN

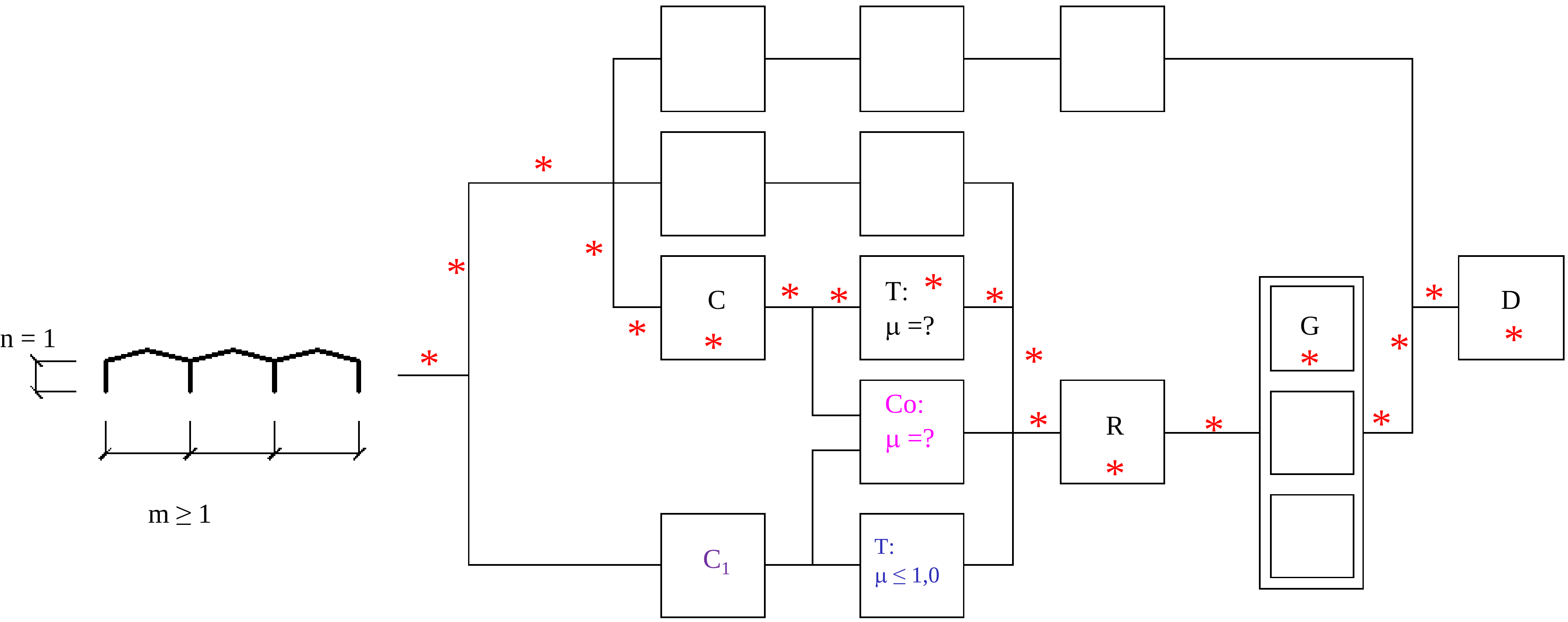
Single-storey frame



→ #13 / 96

Photo: Author

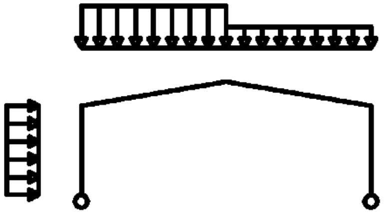
Recommended algorithm for Your design project:



Few additional information (→ #t / 34)

Photo: Author

Procedure "C"



„Normal" loads are taken into consideration only.

→ #13 / 57

Photo: Author

„Normal” loads in procedure C means loads from dead-weight, snow, wind, imposed, thermal, etc. In opposite to method A or B, effects from imperfection and second order effects are not taken into consideration. Cross-sectional forces for method C are presented in #t / 38.

In method A or B, more combinations of actions are taken into consideration in comparison to C. But analisis of instability in these two methods is simpler, than in C.

In method C, critical length of column in plane of frame must be calculated.

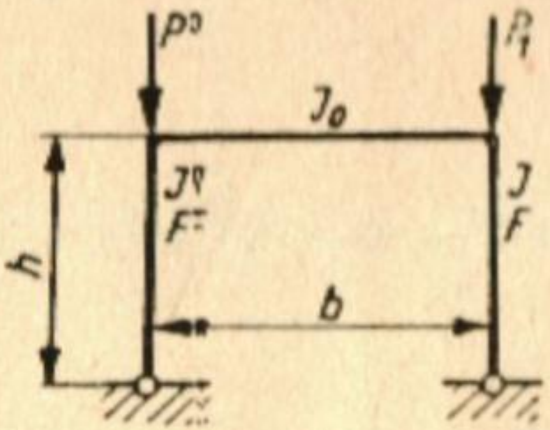
According to EN 1993-1-1 N.A. 9 (C_1 on #t / 32), one-storey steel frame can be calculated without second order effects and incluences of imperfection and, additionaly, as non-sway frame. Critical length of column in non-sway frame is not greater than height of column. If we do not make accurate calculations of μ (of course $\mu \leq 1,0$), we can taken into consideration $\mu = 1,0$, the same as in method B.

Critical length can be calculated by computer ($\mu = ?$). More information is presented on #13 / 70.

Single-storey frame

"Tablice do projektowania konstrukcji metalowych", W. Bogucki, M. Żyburtowicz, Arkady, Warszawa 1984

$(\nu = \mu)$

Rodzaj ramy	Oznaczenia	Współczynnik
	$n = \frac{P_1}{P}$ $c = \frac{Jb}{J_0 h}$ $s = \frac{4J}{b^2 F}$	$\sqrt{0{,}5(1+n)} \times$ $\times \sqrt{4 + 1{,}4(c+6s) + 0{,}02(c+6s)^2}$

Data for calculation:

$h = 5{,}45$ m

$b = 23{,}50$ m

F = area of cross-section of column = A (HEA 450) = 98,8 cm^2

J_0 , J = moments of inertia for cross section of girder and columns = J (HEA 450) =

= 33 740 cm^4

P_0, P_1 – equivalent vertical loads from roof; for symmetrical frame $P_0 = P_1$

$n = P_1 / P_0 = 1{,}000000$

$c = J\, b / (J_0\, h) = 4{,}311927$

$s = 4\, J / (b^2\, F) = 0{,}000247$

$\mu = \{\sqrt{[0{,}5(1 + n)]}\}\ \{\sqrt{[4+1{,}4(c + 6s) + 0{,}02\ (c + 6s)^2]}\} = 3{,}227$

Critical length of column in plane of frame is equal:

$5{,}45 \text{ m} \cdot 3{,}227 = 17{,}587 \text{ m}$

Critical length of column in out-of-plane of frame, because of vertical bracings between columns, is equal:
5,45 m

The same for critical length foe lateral buckling.

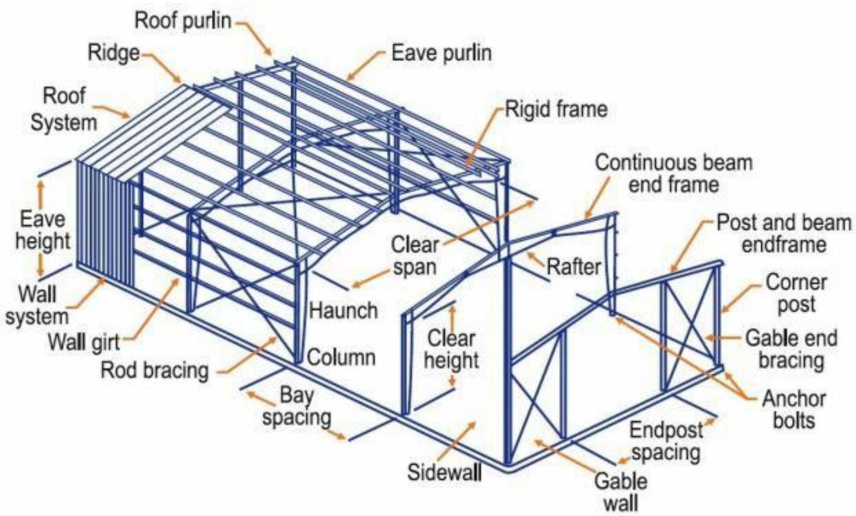


Photo: greenterrahomes.com

Photo: steelwarehousechina.com

Resistance must be analised in each characteristic point separately. For frame as follow: In top point of column and in bottom point of column.

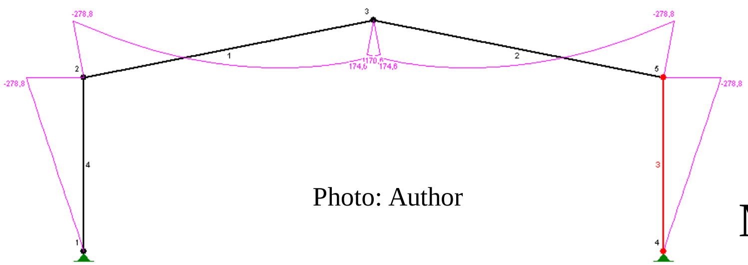
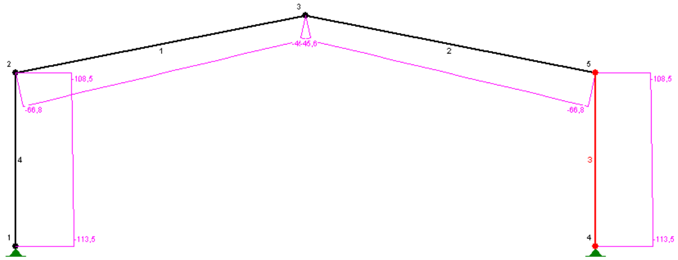


Photo: Author

$M_{Ed,\ top} = 311{,}6\ kNm$

$M_{Ed,\ bottom} = 0{,}0\ kNm$

$V_{Ed,\ max} = 57{,}2\ kNm$



$N_{Ed,\ top} = 93{,}4\ kN$

$N_{Ed,\ bottom} = 97{,}6\ kN$

Shear force the same should be analised separately for top and bottom poin, but for such small value can be analised max value.

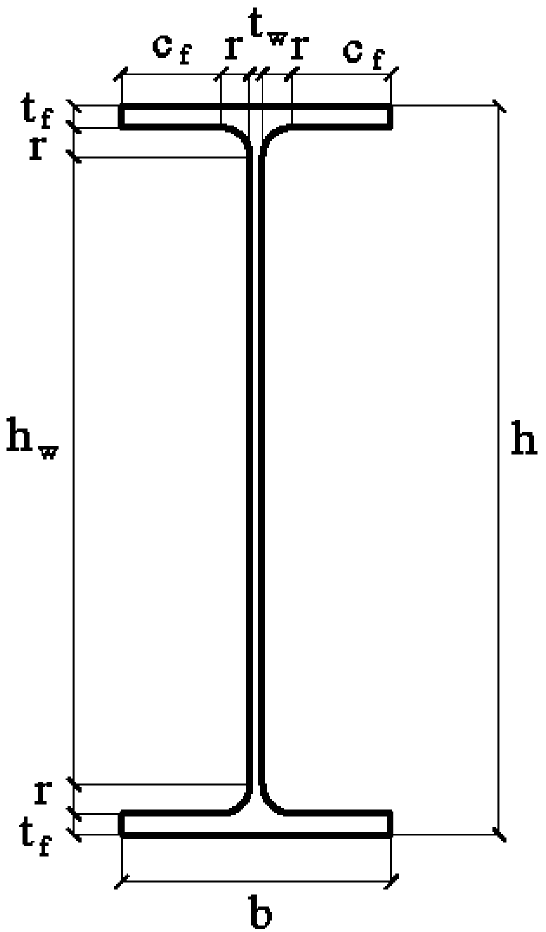


Photo: Author

HEA 450 [mm]

h	b	t_f	t_w	r
440	300	21	11,5	27

$A = 178{,}0\ cm^2$

$J_y = 63\ 720\ cm^4$ $\quad$ $i_y = 18{,}9\ cm$

$J_z = 9\ 465\ cm^4$ $\quad$ $i_z = 7{,}3\ cm$

$W_{y,\ pl} = 3\ 216\ cm^3$

$W_{z,\ pl} = 965{,}5\ cm^3$

$A_{V,\ z} \approx 92{,}4\ cm^2$

$M_{Rd,\ y} = 755{,}760\ kNm$

$N_{Rd} = 4183{,}0\ kN$

$V_{Rd} = 1253{,}658\ kN$

Resistance: top point

$M_{Ed,\ top} = 311{,}6$ kNm

$N_{Ed,\ top} = 93{,}4$ kN

$V_{Ed,\ max} = 57{,}2$ kNm

$V_{Ed,\ max} / V_{Rd} = 0{,}046 < 1{,}0$ OK

$V_{Ed,\ max} / V_{Rd} = 0{,}046 < 0{,}5 \rightarrow$ no interaction between shear force and bending moment

$M_{Ed,\ top} / M_{Rd,\ y} = 0{,}412 < 1{,}0$ OK

$N_{Ed,\ top} / N_{Rd} = 0{,}022 < 1{,}0$ OK

min ($0{,}25\ N_{pl,\ Rd}$; $0{,}5\ h_w\ t_w\ f_y / \gamma_{M0}$) = min (1045,750 ; 464,830) = 464,830 kN

$N_{Ed,\ top} < 464{,}830$ kN $\rightarrow$ no interaction between axial force and bending moment

Resistance: bottom point

$M_{Ed,\ bottom} = 0,0$ kNm

$N_{Ed,\ bottom} = 97,6$ kN

$V_{Ed,\ max} = 57,2$ kNm

$V_{Ed,\ max} / V_{Rd} = 0,046 < 1,0$ OK

$V_{Ed,\ max} / V_{Rd} = 0,046 < 0,5 \rightarrow$ no interaction between shear force and bending moment

$M_{Ed,\ top} / M_{Rd,\ y} = 0,0 < 1,0$ OK

$N_{Ed,\ top} / N_{Rd} = 0,023 < 1,0$ OK

No bending moment → no interaction with shear force and axial force

Stability is analised for total member. This means global calculation for maximum values of N_{Ed}, $M_{y, Ed}$, $M_{z, Ed}$ even if they are in three different cross-sections.

$$M_{Ed} = M_{Ed, top} = 311{,}6 \text{ kNm}$$

$$N_{Ed} = N_{Ed, bottom} = 97{,}6 \text{ kN}$$

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"French" and "German" methods:

$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} (M_{y,\,Ed} + \Delta M_{y,\,Ed}) / (\chi_{LT} M_{y,\,Rk} / \gamma_{M1}) +$$
$$+ k_{yz} (M_{z,\,Ed} + \Delta M_{z,\,Ed}) / (M_{z,\,Rk} / \gamma_{M1}) \leq 1{,}0$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + k_{zy} (M_{y,\,Ed} + \Delta M_{y,\,Ed}) / (\chi_{LT} M_{y,\,Rk} / \gamma_{M1}) +$$
$$+ k_{zz} (M_{z,\,Ed} + \Delta M_{z,\,Ed}) / (M_{z,\,Rk} / \gamma_{M1}) \leq 1{,}0$$

EN 1993-1-1 (6.61), (6.62)

	k_{yy} , k_{yz} , k_{zy} , k_{zz}
"French" method	EN 1993-1-1 App. A, table A1, A2
"German" method	EN 1993-1-1 App. B, table B1, B2, B3

Interaction between flexural buckling in plane of frame (L_{cr} = 17,587 m, buckling about stron axis of HEA 450) and lateral buckling (L_{cr} = 5,450 m):

$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) +$$
$$+ k_{yz} (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk} / \gamma_{M1}) \leq 1,0$$

Interaction between flexural buckling out of plane of frame (L_{cr} = 5,450 m, buckling about weak axis of HEA 450) and lateral buckling (L_{cr} = 5,450 m):

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + k_{zy} (M_{y, Ed} + \Delta M_{y, Ed}) / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) +$$
$$+ k_{zz} (M_{z, Ed} + \Delta M_{z, Ed}) / (M_{z, Rk} / \gamma_{M1}) \leq 1,0$$

No bending moment out of plane of truss $\rightarrow M_{z,\,Ed} = 0{,}0$ kNm

HEA 450: I[st] class of cross section, no diference between global and effective center of gravity, no eccentricities $\rightarrow e_z = 0{,}0$ m ; $e_y = 0{,}0$ m

$$\Delta M_{y,\,Ed} = N_{Ed}\, e_z = 0{,}0 \text{ kNm}$$

$$\Delta M_{z,\,Ed} = N_{Ed}\, e_y = 0{,}0 \text{ kNm}$$

$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} M_{y,\,Ed} / (\chi_{LT} M_{y,\,Rk} / \gamma_{M1}) \leq 1{,}0$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + k_{zy} M_{y,\,Ed} / (\chi_{LT} M_{y,\,Rk} / \gamma_{M1}) \leq 1{,}0$$

Generally: both formulas

$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} M_{y, Ed} / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) \leq 1{,}0$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) + k_{zy} M_{y, Ed} / (\chi_{LT} M_{y, Rk} / \gamma_{M1}) \leq 1{,}0$$

must be satisfied for roof girder and for column. Calculation of lateral buckling will be presented in detail for column and briefly for roof girder.

3.3. Momenty krytyczne przy zwichrzeniu można obliczać wg poniższych wzorów, przyjmując znak (-), gdy środek ścinania znajduje się w strefie rozciąganej przekroju lub znak (+), w pozostałych przypadkach, przy czym w przypadku przekrojów bisymetrycznych zwrot osi *Y* należy przyjmować przeciwnie do kierunku obciążenia poprzecznego, a przy jego braku - w stronę pasa ściskanego.

a) belka jednoprzęsłowa podparta widełkowo ($\mu_x = \mu_y = \mu_\omega = 1$) i zginania stałym momentem

$$M_{cr} = \pm b_y N_y + \sqrt{\left(b_y N_y\right)^2 + i_s^2 \, N_y N_z} \qquad \text{(Z1-7)}$$

One-span beam, M = const, hinge on both ends

b) belka jak w pozycji a) o przekroju bisymetrycznym ($b_y = 0$)

$$M_{cr} = i_s \sqrt{N_y N_z} \qquad \text{(Z1-8)}$$

As previous, and, additionally, bi-symmetrical cross-section

c) belka jednoprzęsłowa - rozwiązanie ogólne

$$M_{cr} = \pm A_0 N_y + \sqrt{\left(A_0 N_y\right)^2 + B^2 \, i_s^2 \, N_y N_z} \qquad \text{(Z1-9)}$$

General formula for one-span beam

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gdzie: $A_0 = A_1 b_y + A_2 a_s$; A_1, A_2, B - wg tabl. Z1-2;

Photo: PN B-3200

Symbols: $N_y \rightarrow N_{cr,\,z}$; $N_z \rightarrow N_{cr,\,T}$

P - hinge

U - rigid support

$x \rightarrow z$; $y \rightarrow x$; $z \rightarrow x$

Tablica Z1-2

	Obciążenie belki (w płaszczyźnie symetrii przekroju *YZ*)	Warunki podparcia[1]				Współczynniki				
		w płaszczyźnie		μ_y	μ_ω	A_1	A_2	B	C_1	C_2
		YZ	*XZ*							
M linear or constant	Moment stały (β = 1) lub zmienny liniowo[2]	P P P	P P U	1 1 0,5	1 0,5 0,5	1/β 1,33/β 1/β	0 0 0	1/β 1,15/β 1/β	2 - 2	0 - 0
q = const	Obciążenie równomiernie rozłożone	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,61 1,23 0,68 0,27	0,53 0,52 0,29 1,61	1,14 1,31 0,97 1,88	0,93 - 1,43 0,15	0,81 - 0,61 0,91
Force applied on the half of span	Siła skupiona w środku rozpiętości	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,55 1,07 0,62 0	0,76 0,87 0,50 1,23	1,37 1,46 1,12 1,23	0,60 - 1 0	0,81 - 0,81 1,62

[1] P - podparcie obustronnie przegubowe (swobodne); U - obustronne utwierdzenie;
μ_y, μ_ω - współczynniki długości wyboczeniowej w płaszczyźnie *XY* i przy skręcaniu.
[2] Współczynnik β należy przyjmować wg tabl. 12 - poz. a).

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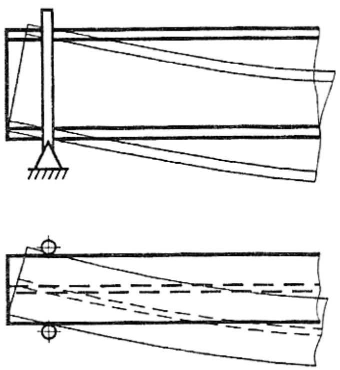
Photo: PN B-3200

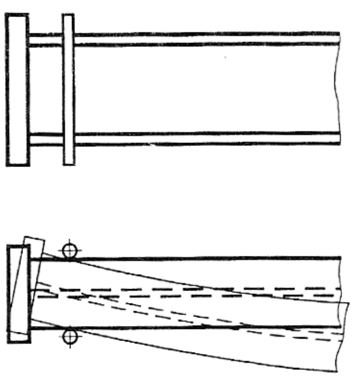
Support		Critical length factor		Situation	Comments
about y	about z	about z	rotation		
P = Hinge	P = Hinge	1,0	1,0	A	Often used
P = Hinge	P = Hinge	1,0	0,5	B	Rarely used
P = Hinge	U = Rigid	0,5	0,5	C	
U = Rigid	U = Rigid	0,5	0,5	D	Often used

Photo: K. Rykaluk, *Konstrukcje stalowe, podstawy i elementy*, DWE Wrocław 2001



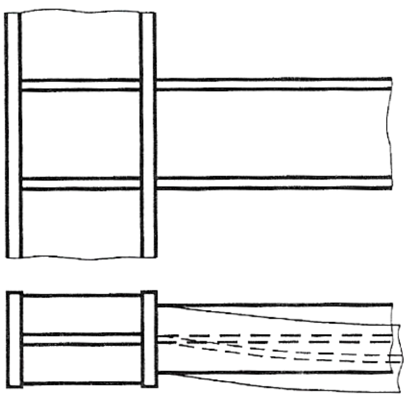
A

B



C – technically difficult to perform

D



→ #5 / 70

	Roof girder	Column
Initian assumption – one-span beam	Two-span member (two beams joined in ridge); simplification; according to #t / 29: L_1 for calculation χ_y L_2 for calculation χ_z min (L_3 ; L_4 ; L_6) for χ_{LT}	Yes; according to #t / 36: 5,45 m · 3,227 = 17,587 m for calculation χ_y 5,45 m for calculation χ_z 5,45 m for χ_{LT}
Supports	2 times rigid (on ridge and in eave); case D according to #t / 49	1 rigid (on eave) and 1 hinge (suport); no such a case in table on #t / 49; in simplification cade D will be taken into consideration
Loads	Gropup of forces in points; so dense that they can be roughly treated as q = conts	Gropup of forces in points; so dense that they can be roughly treated as q = conts

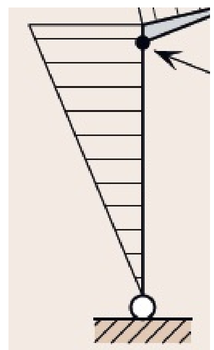
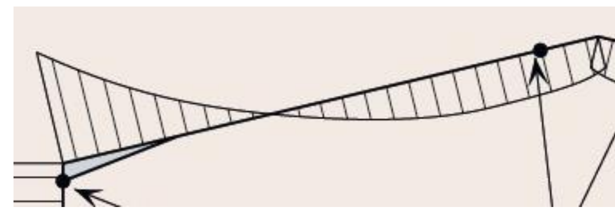
Parameters for one-span beam, rigid supports, q = const (according to #t / 48)	$A_1 = 0,27$; $A_2 = 1,61$; $B = 1,88$; $C_1 = 0,15$; $C_2 = 0,91$	
Distribution of bending moments; final recalculation of χ_{LT} (factor k_c)		

Photo: steelconstruction.info

HEA 450 :

$z_g = h / 2 = 300$ mm

$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z L)^2 = 4\,269{,}837$ kN

$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z l_{0z})^2 = 6\,604{,}602$ kN

$i_s = \sqrt{}\ ([_y{}^2 + i_z{}^2 + (h/2)^2] = 29{,}91$ cm

$N_{cr,\,T} = [\pi^2 EJ_w / (\mu_t L)^2 + GJ_t] / i_s{}^2 = 5\,446{,}494$ kN

$M_{cr} = - A_0 N_{cr,\,z} + \sqrt{}\ [(A_0 N_{cr,\,z})^2 + B^2 i_s{}^2 N_{cr,\,z} N_{cr,\,T}]$

$A_0 = A_1 b_y + A_2 a_s$

$b_y = y_s – r_y / 2$

r_y - arm of cross-section’s assymetry; for bi-symmetrical I-beam = 0

y_s - coordinate of shear center; for bi-symmetrical I-beam = 0

a_s - distance: shear center – load point; for bi-symmetrical I-beam it’s of beam’s cross-section; for HEA 450 = h / 2 = 220 mm

$A_0 = 0 + A_2 a_s = 0{,}354$ m

$M_{cr} = 1\,377{,}877$ kNm

$\lambda_y = \sqrt{(A\, f_y / N_{cr,\, y})} = 0{,}990$ $\lambda_z = \sqrt{(A\, f_y / N_{cr,\, z})} = 0{,}633$

$\Phi_y = 1{,}128$ $\Phi_z = 1{,}306$

$\chi_y = 0{,}599$ $\chi_z = 0{,}408$

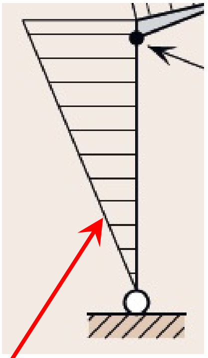
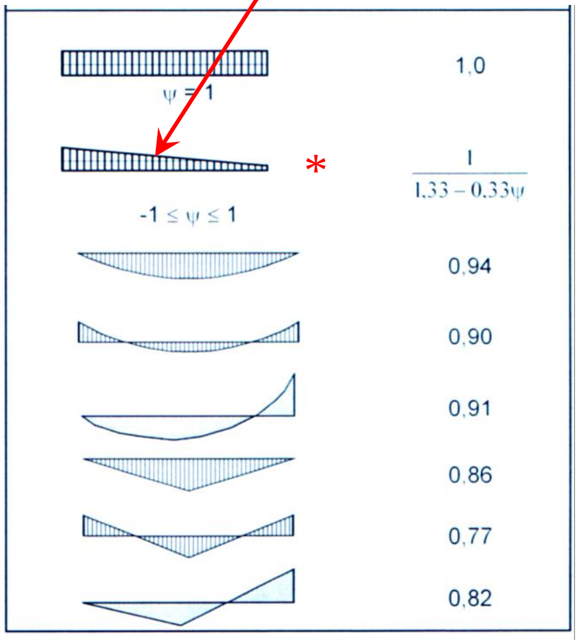


Photo: steelconstruction.info

HEA 450 → EN 1993-1-1 tab. 6.4 → buckling curve c → EN 1993-1-1 tab. 6.3 → $\alpha_{Lt} = 0{,}49$

k_c:

Photo: EN 1993-1-1 tab 6.6



$\lambda_{LT} = \sqrt{(W_{y,\, pl}\, f_y / M_{cr})} = 0{,}741$

$\Phi_{LT} = 0{,}907$

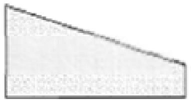
$\chi_{LT} = \min\{1/[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] \;;\; 1/\lambda_{LT}^2 \;;\; 1{,}0\} = 0{,}699$

$k_c = 1 / 1{,}33 = 0{,}752$

$f = 0{,}877$

$\chi_{LT,\, mod} = 0{,}797$

"German" method EN 1993-1-1 tab. B.3

Moment diagram	Range	C_{my} and C_{mz} and C_{mLT}	
		Uniform loading	Concentrated loading
	$-1 \leq \Psi \leq 1$	max $(0,6 + 0,4\,\Psi \;\; ; \;\; 0,4)$	
For members with sway buckling mode, the equivalnet uniform moment factor should be taken $C_{my} = 0,9$ or $C_{mz} = 0,9$ respectively.			
C_{my}, C_{mz} and C_{mLT} should be obtained according to the bending moment between the relevant braced points as follows:			
Moment factor:	bending axis:	points braced in direction:	
C_{my}	y-y	z-z	
C_{mz}	z-z	y-y	
C_{mLT}	y-y	y-y	

$\Psi = 0$

$C_{my} = C_{mz} = C_{mLT} = \max(0,6 + 0,4\,\Psi \;\; ; \;\; 0,4) = 0,6$

But „For members with sway buckling mode…" → This is sway mode of instability for such frame.

$C_{my} = C_{mz} = 0,9$

$C_{mLT} = 0,6$

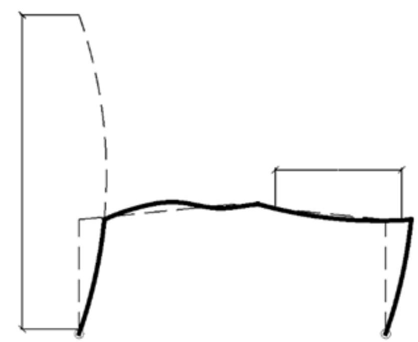
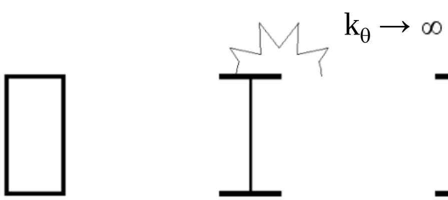


Photo: Author

For "German" method, members are divided into susceptible to torsional deformations and not susceptible to torsional deformations.

Cross-section	Comments
	No lateral buckling, no need for buckling interaction analysis → #13 / 83
 Short member: $1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] \geq 1{,}0$	Member not susceptible to torsional deformations, EN 1993-1-1 tab. B.1
 $1 / [\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}] < 1{,}0$	Member susceptible to torsional deformations, EN 1993-1-1 tab. B.2

EN 1993-1-1 tab. B.1

"German" method,
interaction factors

→ #13 / 84

$$n_y = N_{Ed}\,\gamma_{M1} / (\chi_y\, N_{Rd})$$

$$n_z = N_{Ed}\,\gamma_{M1} / (\chi_z\, N_{Rd})$$

Interaction factors	Cross-section	I[st], II[nd] class	III[rd], IV[th] class
k_{yy}	I, H, RHS	$C_{my} \cdot \min\{1 + 0{,}6\,\lambda_y\, n_y\ ;\ 1 + 0{,}6\, n_y\}$	$C_{my} \cdot \min\{1 + (\lambda_y - 0{,}2)\, n_y\ ;\ 1 + 0{,}8\, n_y\}$
k_{yz}	I, H, RHS	$0{,}6\, k_{zz}$	k_{zz}
k_{zy}	I, H, RHS	$0{,}6\, k_{yy}$	$0{,}8\, k_{yy}$
k_{zz}	I, H	$C_{mz} \cdot \min\{1 + (2\,\lambda_z - 0{,}6)\, n_z\ ;\ 1 + 1{,}4\, n_z\}$	$C_{mz} \cdot \min\{1 + 0{,}6\,\lambda_z\, n_z\ ;\ 1 + 0{,}6\, n_z\}$
	RHS	$C_{mz} \cdot \min\{1 + (\lambda_z - 0{,}2)\, n_z\ ;\ 1 + 0{,}8\, n_z\}$	
For I- and H-sections and RHS, under axial compression and unaxial bending $M_{y,\,ED}$, the coefficient k_{zy} may be = 0			

$n_y = N_{Ed}\, \gamma_{M1} / (\chi_y\, N_{Rd}) = 0{,}038$

$n_z = N_{Ed}\, \gamma_{M1} / (\chi_z\, N_{Rd}) = 0{,}056$

Interaction factors	Cross-section	I^{st}, II^{nd} class
k_{yy}	I, H, RHS	$C_{my} \cdot \min\{1 + 0{,}6\, \lambda_y\, n_y \;;\; 1 + 0{,}6\, n_y\} = 0{,}919$
For I- and H-sections and RHS, under axial compression **and unaxial bending $M_{y,\,ED}$, the coefficient k_{zy} may be = 0**		

$$N_{Ed} / (\chi_y N_{Rk} / \gamma_{M1}) + k_{yy} M_{y,Ed} / (\chi_{LT} M_{y,Rk} / \gamma_{M1}) \leq 1,0$$

$$N_{Ed} / (\chi_z N_{Rk} / \gamma_{M1}) \leq 1,0$$

$97,6 / (0,599 \cdot 4183,0 / 1,0) + 0,919 \cdot 311,6 / (0,797 \cdot 755,76 / 1,0) = 0,514 \leq 1,0$ OK

$97,6 / (0,408 \cdot 4183,0 / 1,0) = 0,058 \leq 1,0$ OK

Deformation

Horizontal displacement of top point of column:
0,002 m

Accepted value:
H / 150 = 0,036 m

$\Delta / \Delta_{acc} = 0{,}056 < 1{,}0$ OK

Thank you for attention

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