Metal Structures

Design Project II

Floor girders – examples of calculation (part III)

IVth example - supports of primary beam

Photo: homeownercosts.co.uk

Photo: Author

Primary beam is supported in three points:

- left end – masnory wall;
- half of span – steel column;
- right end – masonry wall;

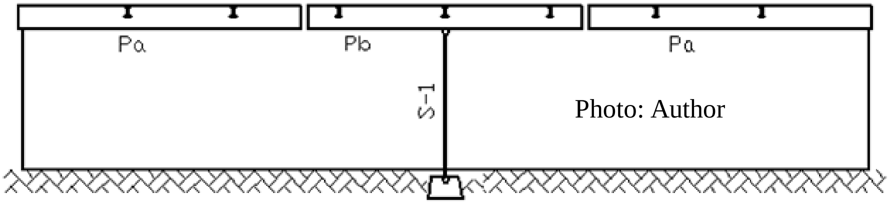


Photo: Author

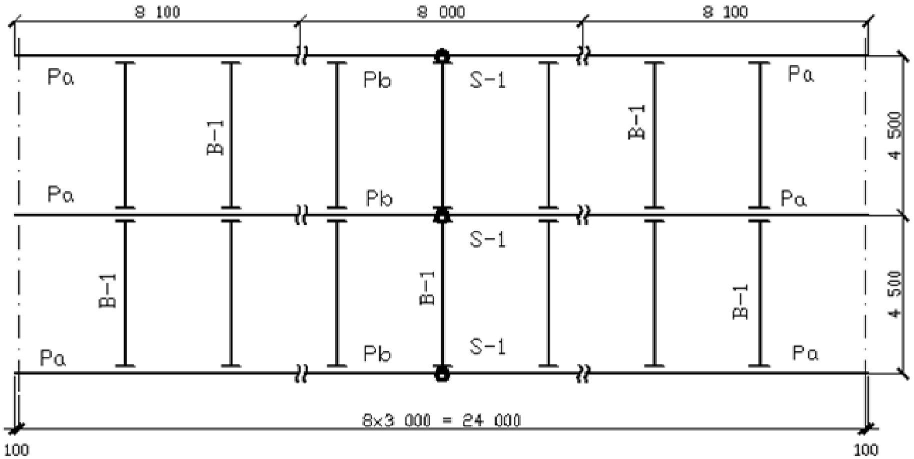


Photo: Author

New complex of data:

S235

Primary beam HEA 700

Span 2 x 12,00 m

Column HEA 240

Height 4,0 m

Reactions:

1 184,235 kN central support

391,112 kN side supports

Masonry wall:

total thickness 40 cm

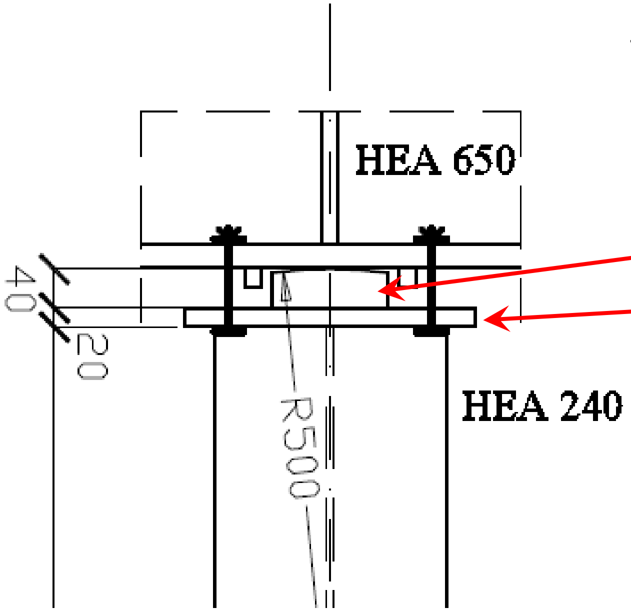
f_d = 4,0 MPa

E_d = 3,6 GPa

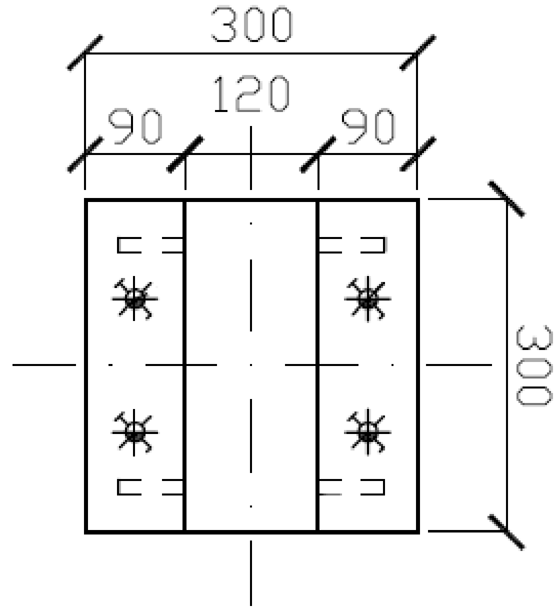
Analysis of supporting joint concerns few important questions:

- Initial analysis (category of joint, class of bolt, dimension, length, geometry) → #t / 5 - 6
- Stiffness of joint (according assumption – hinge joint) → #t / 7 - 13
- Resistance of rockers → #t / 14 - 19
- Resistance of masonry wall → #t / 20 - 22
- Resistance of head of column → #t / 23 - 30
- Stiffener → example V^{th}

Initial assumptions



Support consists on
rocker
bottom plate.



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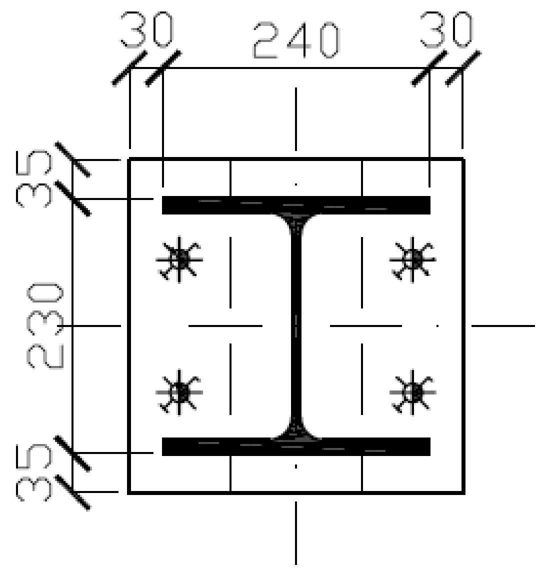


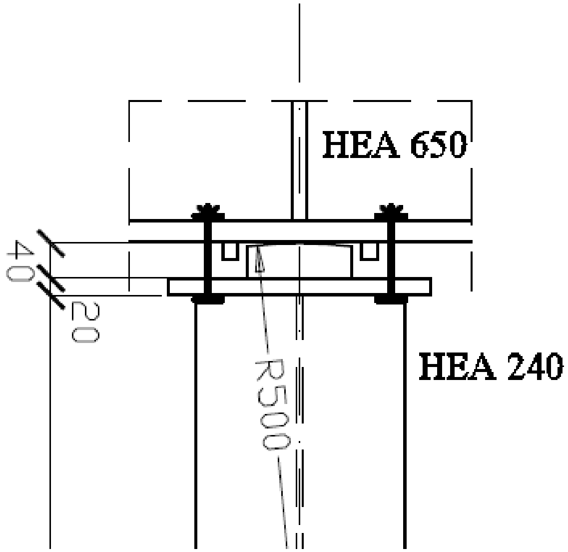
Bottom plate is in contact with:

masonry wall→ #t / 20 - 22

or

steel column→ #t / 23 - 30.

The same geometry of support will be adopted in both cases. Bigger force (reaction on side support / central support will be aplicated in calculatins.



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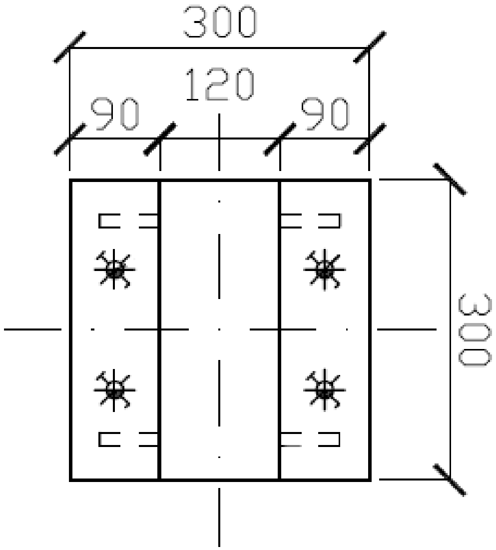


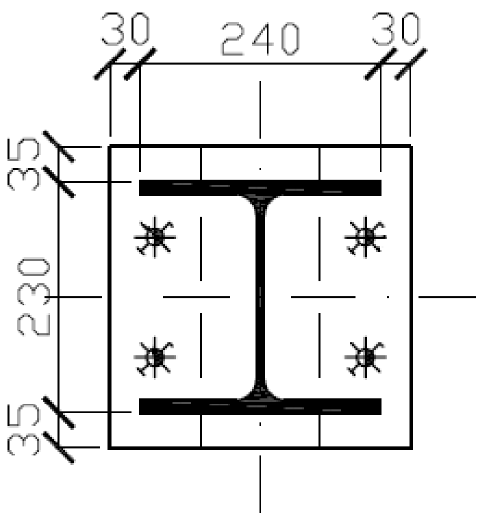
Thickness of bottom plate similar to thickness of bottom flange of primay beam; adopted 20 mm

Thickness of rocker about 2x bigger than thickness of bottom plate; adopted 40 mm

Radius of round part (if is neccessible) about 10x bigger than rocker thickness; adopted 500 mm

Bottom plate dimnesnin: bigger than envelope of column cross-section, but not greated than masonry wall thickness; adopted 300x300 mm

Rocker width about 1/3 bottom plate width; adopted 120 mm



Stiffness

For „normal” truss and beams important factor is proportion between stiffness of Girder (truss or I-beam) and Support (column, wall…):

$$S_{Girder} / S_{Support}$$

$$S_{Girder} = E_G\, J_G / L_G \qquad\qquad S_{Support} = E_S\, J_S / h_S$$

Stiffness of truss could be approximated by:

$$S_{truss} \approx 3{,}5\; q\, L_G^{\,2} \quad ; \quad q = (\Sigma\, F_i) / L_G$$

→ #14 / 29

or (→ #9 / 11):

$$J_{truss} \approx 0{,}35\; h_{1,\,truss}^{\;2}\, A_{chord}$$

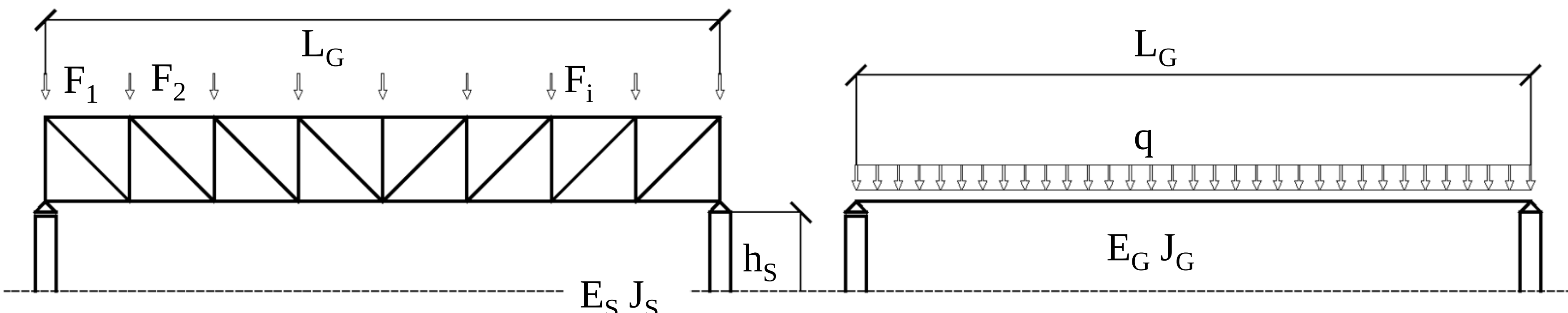


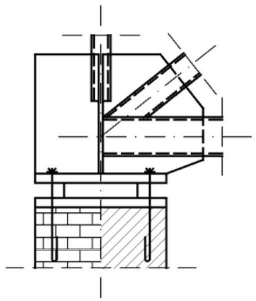
Photo: Author

Recommended technical solutions (based on experience):

(Big)	Various proportion S_G / S_S		(Small)
No rocker	Flat rocker	Round rocker	Elastomeric rocker

Elastomeric rocker is used, first of all, for bridges. For other type of structures is used very rare.

Photo: anthony-johnson-engineering.co.uk



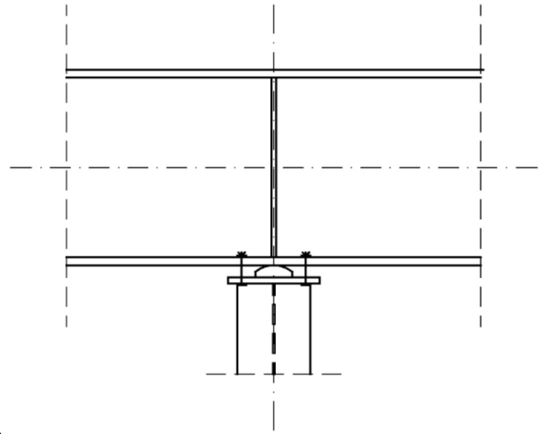


Photo: Author

Photo: steelconstruction.info

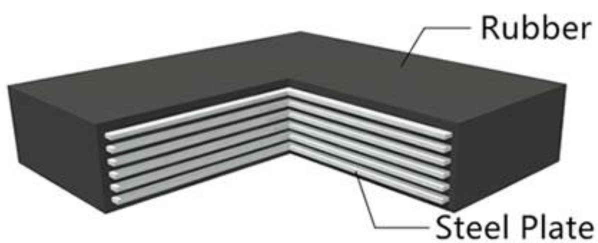


Photo: bridgebearing.org

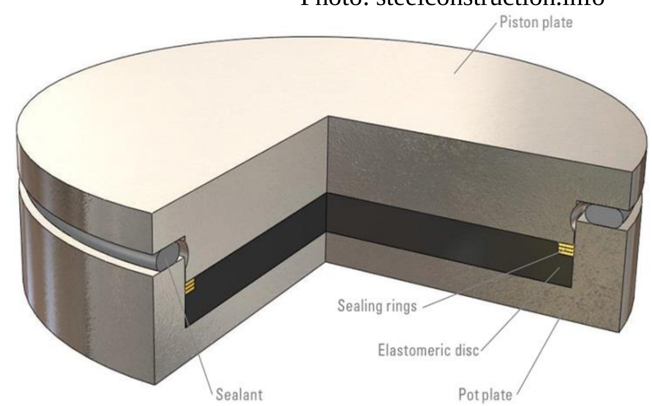


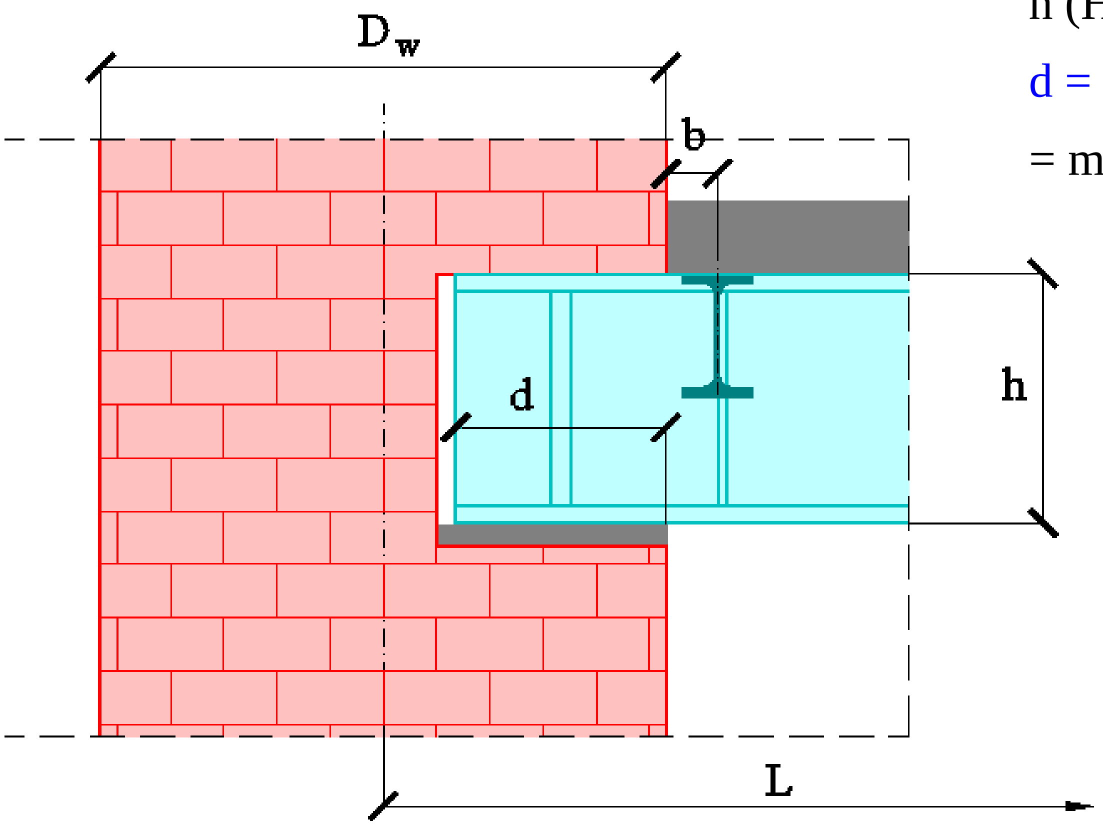
Photo: canambridges.com

→ #14 / 30

Recommendations for CC3 are not necessarily valid for lower Consequence Classes. Structure should be as cheap as possible. Detailed design of supports can be very important for CC3. For lower classes, with a smaller assumed safety margin, it may not be necessary: too expensive in relation to safety of structure. Small differences between ideal and real behavior of structure can be accepted. Carrying out / resigning from thorough analysis should result from experience of designer.

CC	$S_G / S_S > 20$	$20 \geq S_G / S_S > 10$	$10 \geq S_G / S_S > 5$	$5 \geq S_G / S_S$
3	No rocker	Flat rocker	Round rocker	Elastomeric rocker
2	No rocker	No rocker / flat rocker	Flat rocker / round rocker	Round rocker
1	There is usually no detailed analysis			

→ #14 / 33



$L = 2\,L_{span} = 24{,}0$ m

h (HEA 700) = 690 mm

$d = \max(h / 10 \;\; ; \;\; D_w / 2) =$

$= \max(69 \text{ mm} \;\; ; \;\; 200 \text{ mm}) = 200 \text{ mm}$

Photo: Author

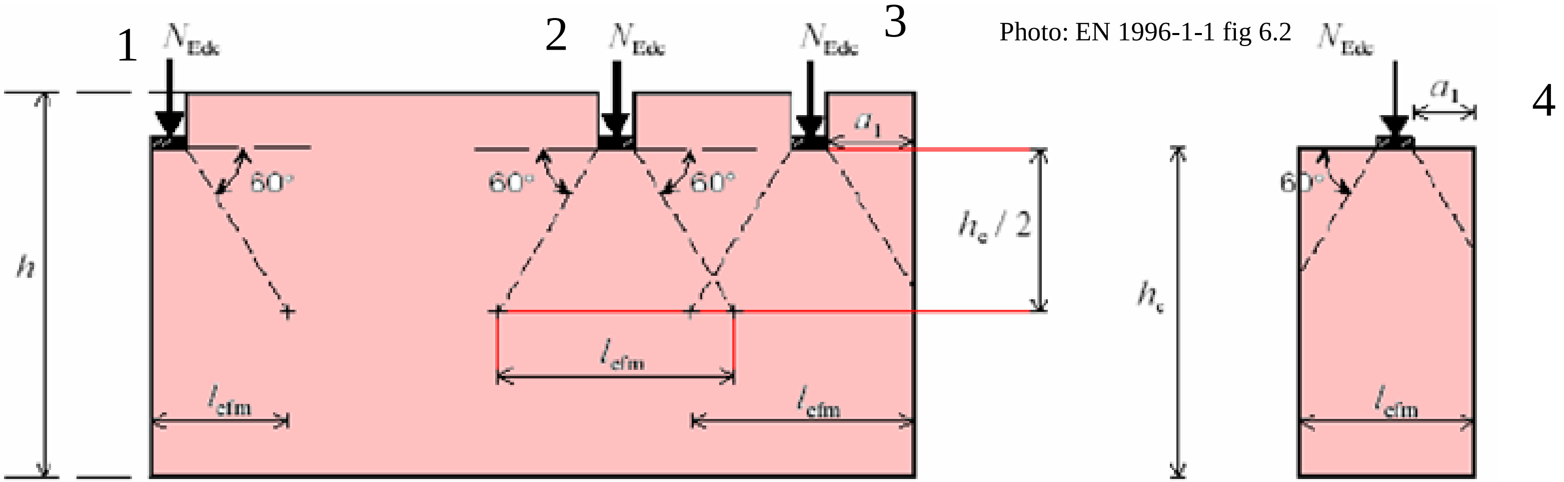


Photo: EN 1996-1-1 fig 6.2

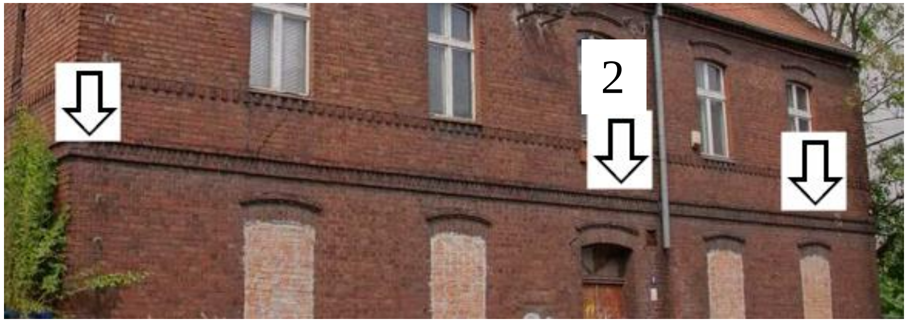
1  3

Photo: projektonwestor.pl

 4

Photo: houzz.co.nz

Case 2 will be taken into consideration in analysis; primary beams in central part of long masonry wall.

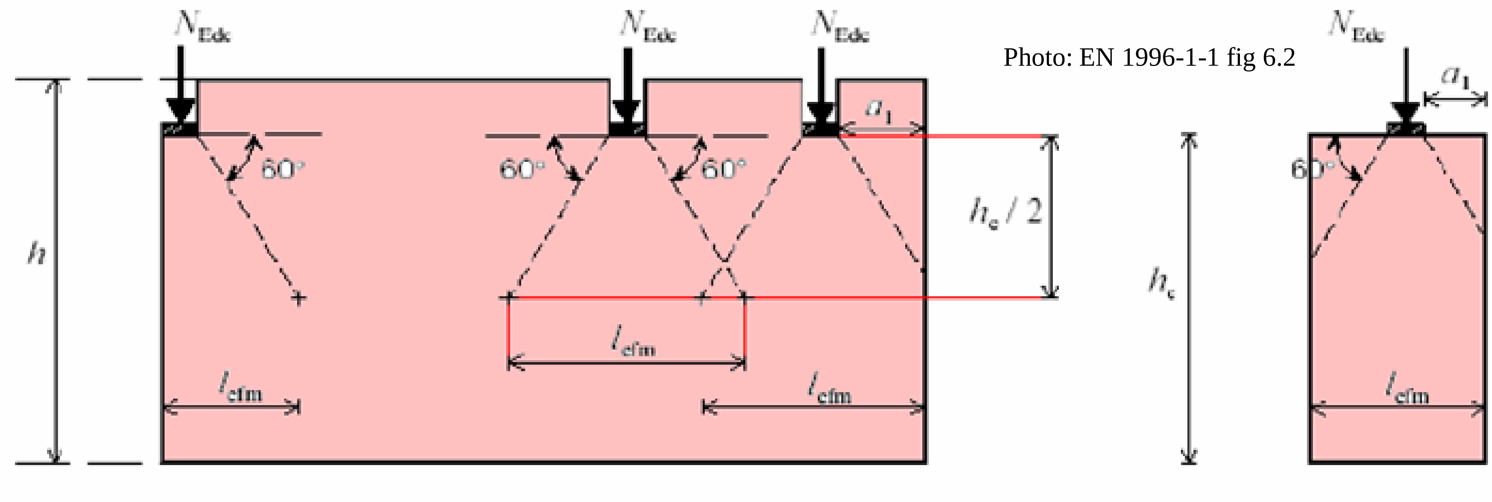


Photo: EN 1996-1-1 fig 6.2

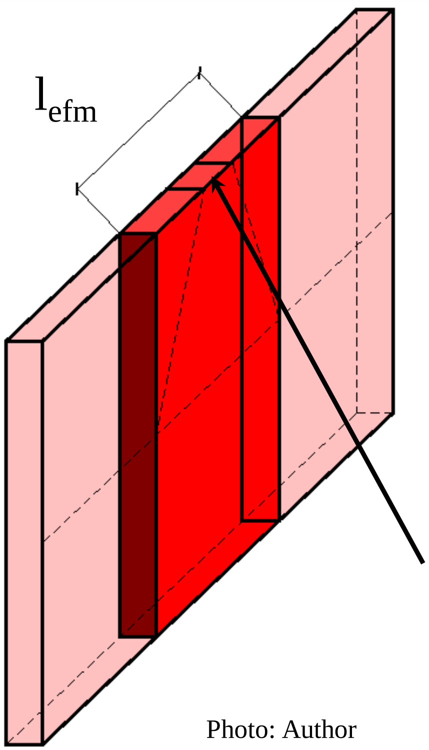


Photo: Author

h_c = 4,5 m

l_{efm} = b(HEA 700) + 2 x (h_c / 2) / tg 60° = 30 cm + 259,8 cm =

= 2,989 m

A_b = 20 cm x 30 cm = 600 cm^2

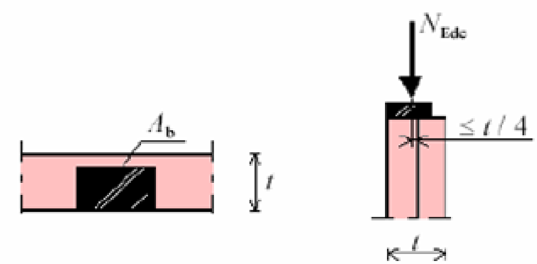


Photo: EN 1996-1-1 fig 6.2

A = 40 cm x 259,8 cm =

= 10 392,3 cm^2

J = [$(40\ cm)^3$ x 259,8 cm] / 12 =

= 1 385 600 cm^4

The most popular, CC2, will be analysed.

Masonry wall:

E_{mw} = 3,6 GPa

H_{mw} = 4,50 m

J_{mw} = 1 385 600 cm^4 = 0,013856 m^4

S_{mw} = 11 084,8 kN m

S_{pb} / S_{mw} = 1,700 → round rocker

Primary beam (HEA 700):

E_{pb} = 210,0 GPa

L_{pb} = 24,0 m

J_{pb} = 215 300 cm^4

S_{pb} = 18 838,75 kN m

Column (HEA 240):

E_c = 210,0 GPa

H_c = 4,0 m

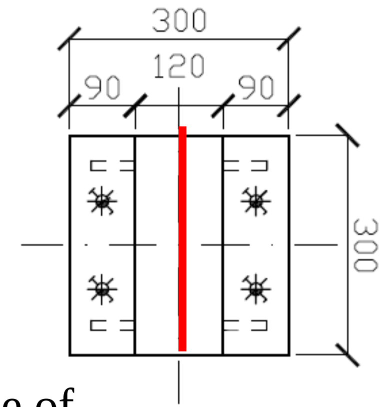
J_c = 7 763 cm^4

S_c = 4 075,58 kN m

S_{pb} / S_c = 4,622 → round rocker

Rocker resistance

Photo: Author

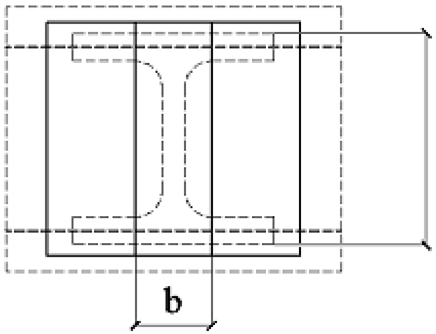


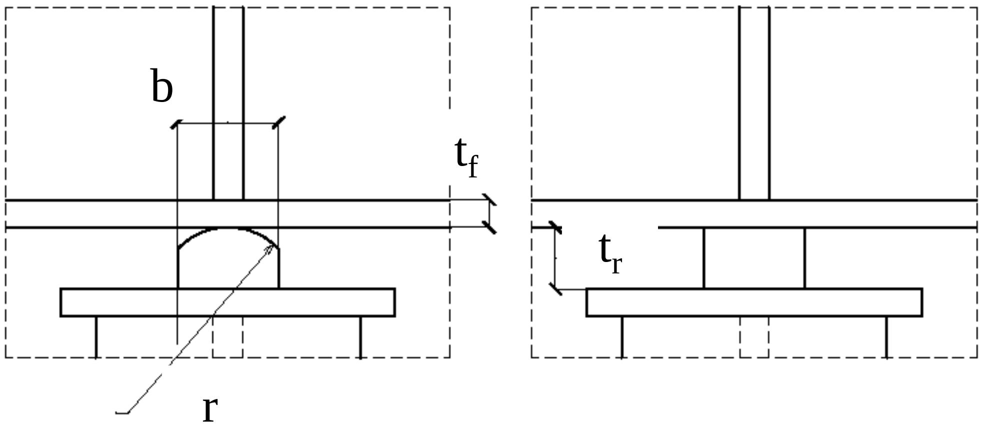
Four points are important for checking resistance:

- behavior of flat surface of beam's flange in contact with round surface of rocker (beam's resistance);
- behavior of round surface of rocker in contact with flat surface of beam's flange (rounded plate's resistance);
- contact two flat surfaces (rocker – botom plate);
- welds between rocker and bottom plate;

In both cases (rocker on masonry wall and rocker on column's top) contact between flange and rounded plate is the same: across flange, on distance 300 mm. This is linear contact; theoretical withd of it is equal 0.
Geometry the same for both cases; more important will be case with bigger force (column's rocker).

$L \approx b_f \approx h_c$





Rocker-beam compression

EN 1337

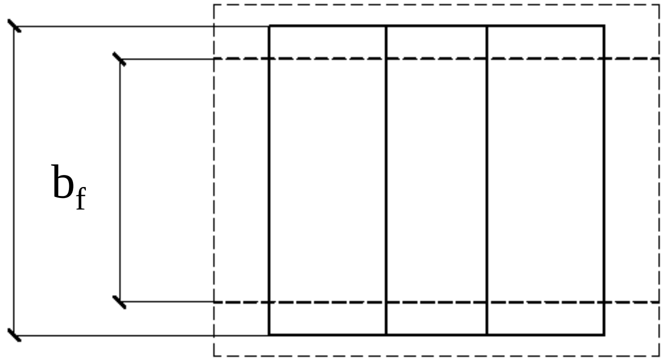


Photo: Author

Condition for beam (flange resistance):

$[N_{Ed} / b_f] / [23\ r\ f_u^2 / (E\ \gamma_M)] \leq 1{,}0 \qquad \gamma_M = 1{,}0$

Condition for rounded plate:

$[N_{Ed} / L] / [f_y\ (2\ t_r + b) / \gamma_M] \leq 1{,}0 \qquad \gamma_M = 1{,}1$

$t_r = 40$ mm

$b = 120$ mm

$L = 300$ mm

$r = 500$ mm

S235

$f_y = 235$ MPa

$f_u = 360$ MPa

Condition for beam:

$[N_{Ed} / b_f] / [23 \text{ r } f_u^2 / (E \gamma_M)] \leq 1,0$

$[1\,184,235 \text{ kN} / 0,3 \text{ m}] / [23 \cdot 0,5 \text{ m} \cdot (360 \text{ MPa})^2 / (210 \text{ GPa} \cdot 1,0)] =$

$= [3\,947,450 \text{ kN/m}] / [7\,079,143 \text{ kN/m}] = 0,558 < 1,0$ OK

Condition for rounded plate:

$[N_{Ed} / L] / [f_y (2 t_r + b) / \gamma_M] \leq 1,0$

$[1\,184,235 \text{ kN} / 0,3 \text{ m}] / [235 \text{ MPa} \cdot (2 \cdot 0,04 \text{ m} + 0,3 \text{ m}) / 1,1] =$

$= [3\,947,450 \text{ kN/m}] / [81\,181,818 \text{ kN/m}] = 0,049 < 1,0$ OK

Rocker-cap plate weldings

Lecture #17, Example 3a
for
$M_{Ed} = 0$; $V_{Ed} = 0$

$$\sqrt{[(\sigma_\perp)^2 + 3(\tau_\parallel^2 + \tau_\perp^2)]} \le f_u / (\beta_w \gamma_{M2})$$

$$\sigma_\perp \le 0{,}9 f_u / \gamma_{M2}$$

$$\gamma_{M2} = 1{,}25$$

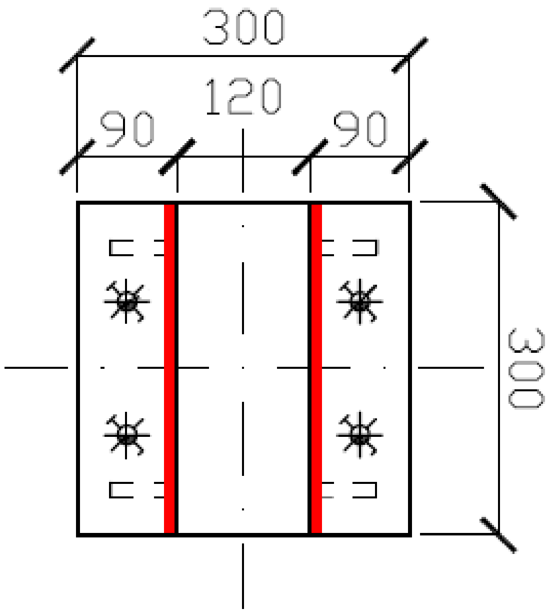


Photo: Author

t_1 = thickness of bottom plate = 20 mm

t_2 = thickness of rounded plate = 40 mm

$0{,}2\ t_2 \le a \le 0{,}7\ t_1 \rightarrow a = 10$ mm

l = length of weld = 300 mm

N_{Ed} = 1 184,235 kN

f_u = S235 = 360 MPa

β_w = S235 = 0,8

$A = 2\ a\ l = 60\ cm^2$

$\sigma\ (N_{Ed}) = N_{Ed}\ /\ A = 197{,}373\ MPa$

$\sigma_\perp = \tau_\perp = \sigma\ /\ \sqrt{2} = 139{,}565\ MPa$

$\tau_\parallel = 0{,}000\ MPa$

$\sqrt{[(\sigma_\perp)^2 + 3(\tau_\parallel^{\ 2} + \tau_\perp^{\ 2})]} = 279{,}126\ MPa$

$f_u\ /\ (\beta_w\ \gamma_{M2}) = 360{,}0\ MPa$

$279{,}126\ /\ 360{,}0 = 0{,}775$

condition satisfied

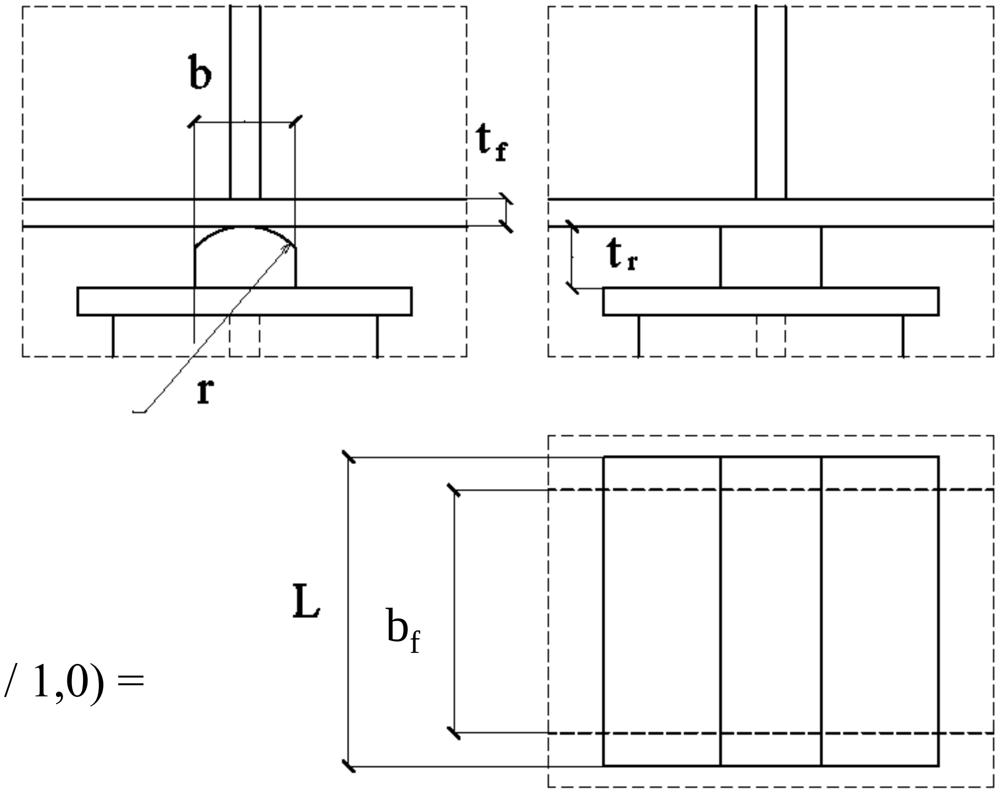
$\sigma_\perp = 139{,}565\ MPa$

$0{,}9\ f_u\ /\ \gamma_{M2} = 259{,}200\ MPa$

$139{,}565\ /\ 259{,}200 = 0{,}538$

condition satisfied

Plate – plate contact



$$N_{Ed} \;/\; (L \; b \; f_y \;/\; \gamma_{M0}) \leq 1{,}0$$

$$1\;184{,}235 \text{ kN} \;/\; (0{,}3 \text{ m} \cdot 0{,}12 \text{ m} \cdot 235 \text{ MPa} \;/\; 1{,}0) =$$

$$= 0{,}140 < 1{,}0 \text{ OK}$$

Photo: Author

Resistance of masonry wall below support

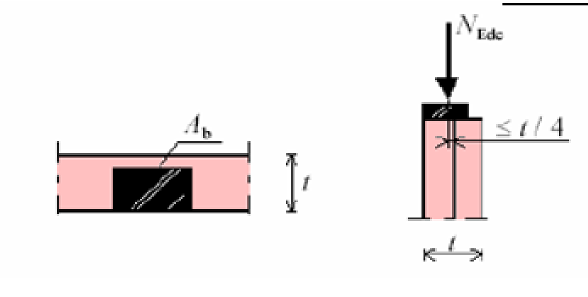


Photo: EN 1996-1-1 fig 6.2

$$N_{Edc} / N_{Rdc} \leq 1{,}0$$

$$N_{Rdc} = b\, A_b\, f_d$$

$$A_{ef} = l_{efm}\, t$$

$$b = \min \{ 1{,}25 + a_1 / (2\, h_c) \;\; ; \;\; 1{,}5 \;\; ; \;\; \max [(1 + 0{,}3\, a_1 / h_c) \cdot (1{,}5 - 1{,}1\, A_b / A_{ef}) \;\; ; \;\; 0] \}$$

In case of support in central part of long masonry wall, distance a_1 from support to end of wall is very big. For sure, first and third proposal for b will be bigger than 1,5.

b = 1,5

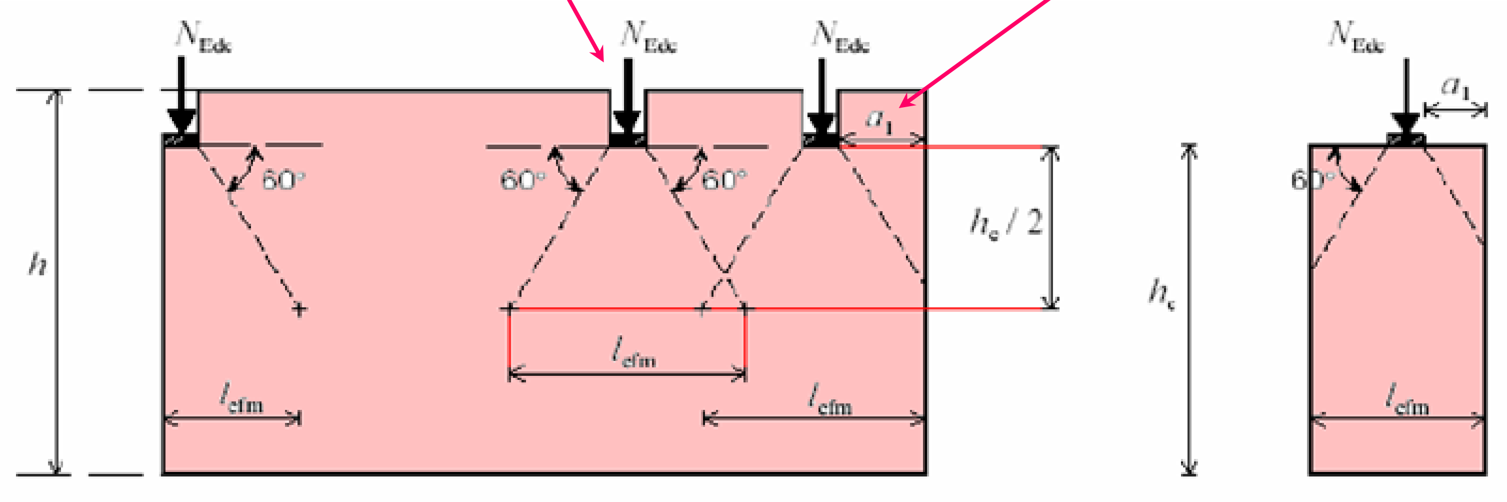


Photo: EN 1996-1-1 fig 6.2

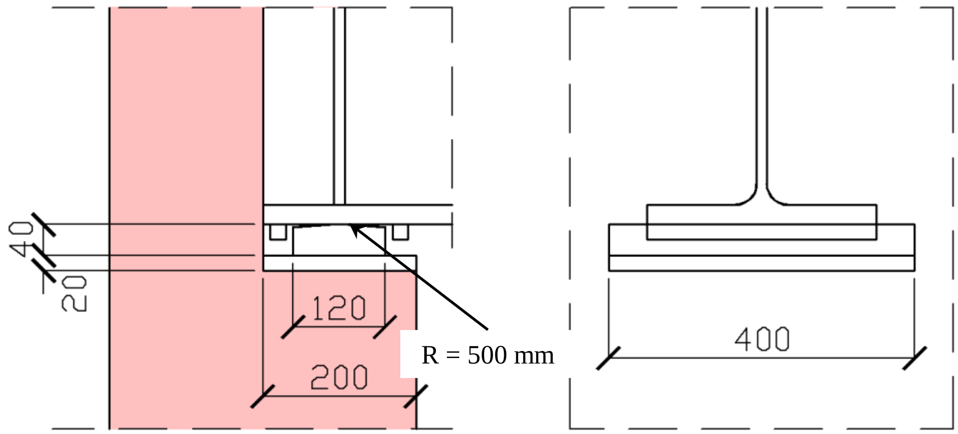


Photo: Author

At now, geometry of support on wall must be analysed once again – with additional member: rocker. Rocker is, in this case, complex of two plates: top with round up surface (common name „rocker” concerns top plate) 120x400, 40 mm thickness, r = 500 mm), and bottom, in contact with masonry wall (200x400, 20 mm thickness).

$$b = 1,5$$

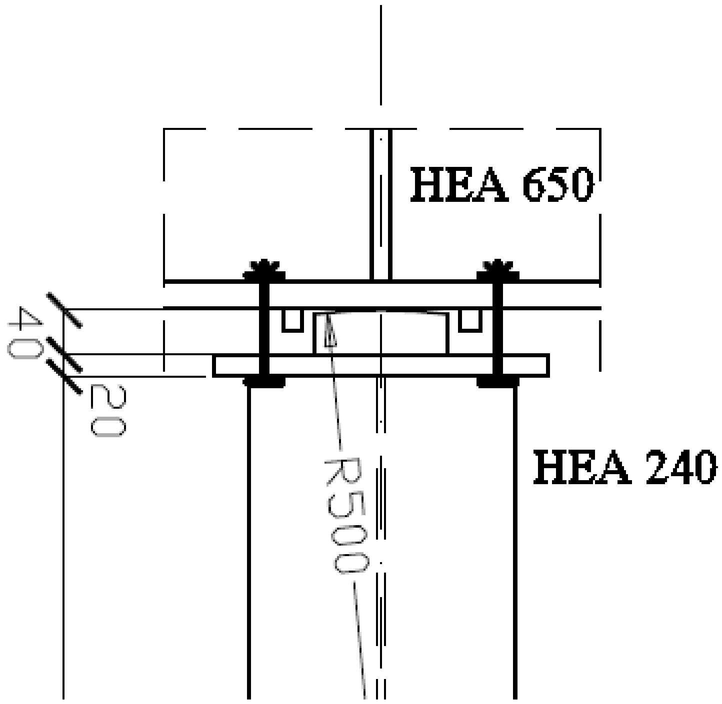
$$A_b = 20 \text{ cm x } 40 \text{ cm} = 800 \text{ cm}^2$$

$$f_d = 4,0 \text{ MPa}$$

$$N_{Rdc} = b\, A_b\, f_d = 480,0 \text{ kN}$$

$$N_{Edc} = 391,112 \text{ kN}$$

$$N_{Edc} / N_{Rdc} = 0,815 < 1,0 \text{ OK}$$



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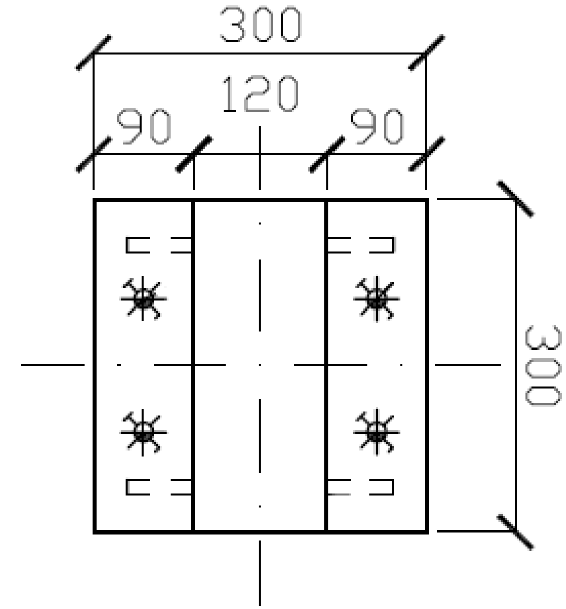
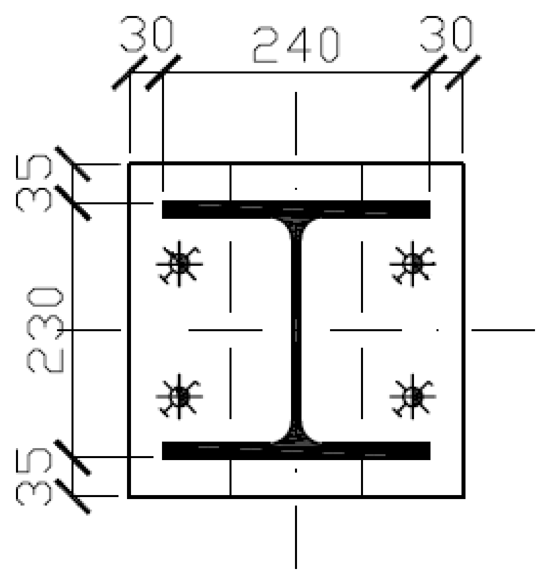
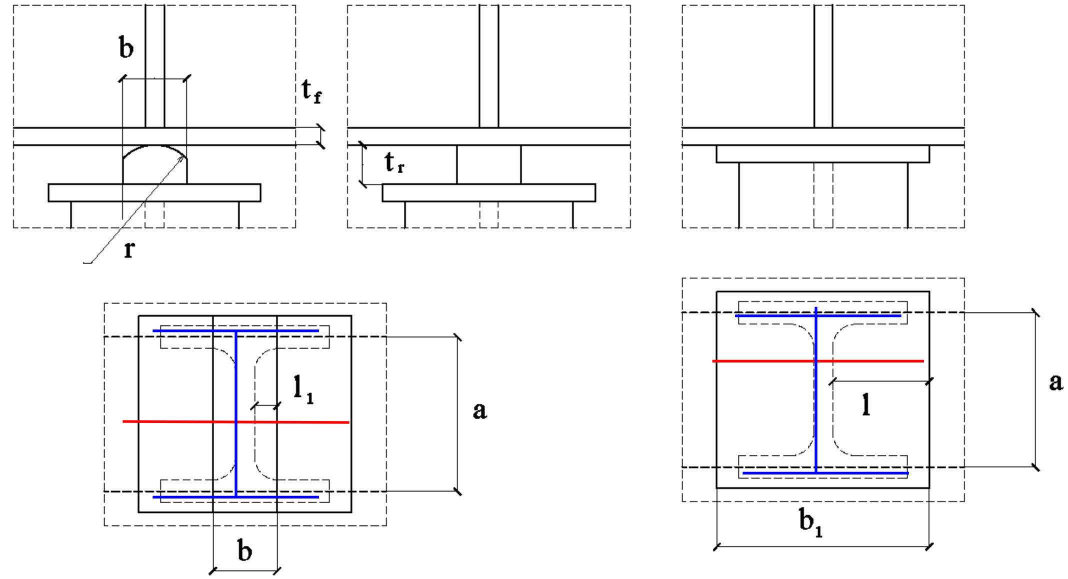




Geometry of rocker on column's heas is a litte another, but general point - round rocker 40 mm thickness, 500 mm radius - is the same in both situations.

Resistance of head of column below support

Photo: Author



Bottom plate of rocker in supported on three parts of column: two flanfes and web.

Along cross-section, marked by red line, bottom plate can be treated as short cantilever (span l_1), loaded by constant load = stresses between rocker and bottom plate

HEA 240:

$h = 230$ mm

$t_f = 12$ mm

$t_w = 7{,}5$ mm

$l_1 = (120 \text{ mm} – 7{,}5 \text{ mm}) / 2 = 56 \text{ mm}$

$a = 230 \text{ mm} – 2\ (12 / 2) = 218 \text{ mm}$

$q = \sigma = N_{Ed}\ /\ (\ L\ b\ ) = 1\ 184{,}235 \text{ kN} / (0{,}3 \text{ m} \cdot 0{,}12 \text{ m}) = 32\ 895{,}417 \text{ kN} / \text{m}^2$

$M_{Ed,\ max}$ (cantilever) $= q\ a\ (l_1)^2 / 2 = 11{,}244$ kNm

Cross-section of short cantilever (along red line);

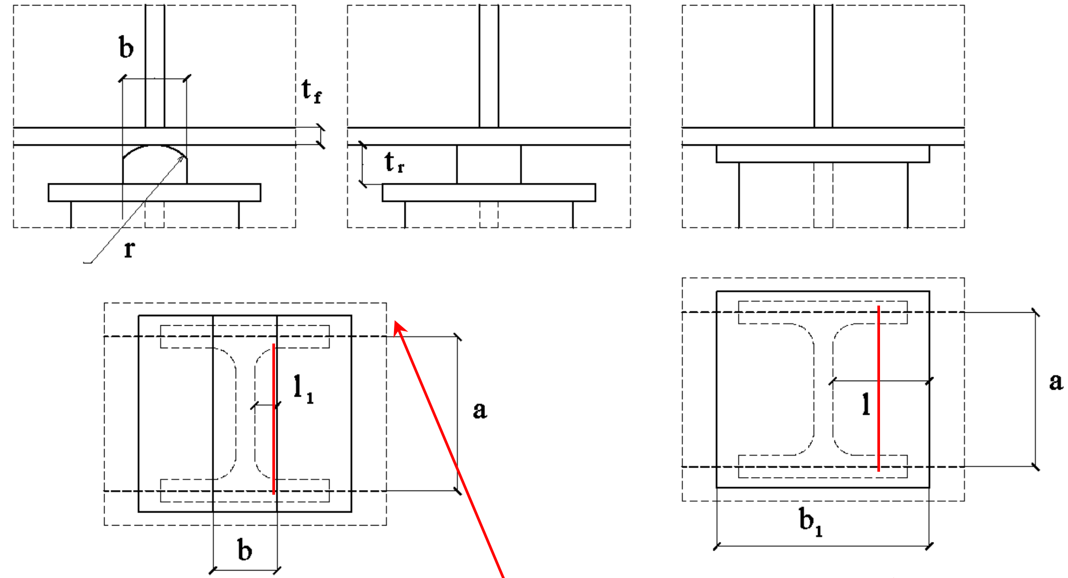
thickness t_p ;

width a = 0,218 m;

$W = a\, t_{p1}^{\,2} / 6$

$M_{Rd} = W\, f_y / \gamma_{M0}$

Photo: Author



$$M_{Ed,\,max} / M_{Rd} \leq 1{,}0 \rightarrow M_{Ed,\,max} / [f_y\, a\, t_{p1}^{\,2} / (6\, \gamma_{M0})] \leq 1{,}0 \rightarrow$$

$$\rightarrow 6\, \gamma_{M0}\, M_{Ed,\,max} / (f_y\, a\, t_{p1}^{\,2}) \leq 1{,}0 \rightarrow$$

$$\rightarrow t_{p1} \geq \sqrt{[\,(6\, M_{Ed,\,max} / (a\, f_y\, \gamma_{M0})\,]}$$

t_{p1} means: $t_p + t_r$ $\quad t_p$

Recommended thickness of bottom plate in presence of rocker:

$$t_p \geq \max \{0{,}75\ t_f \quad ; \quad \sqrt{[\ (6\ M_{Ed,\ max} / (a\ f_y\ \gamma_{M0})\ ]} - t_r\ \}$$

$$t_p \geq \max \{0,\ 75 \cdot 26\ mm \quad ; \quad \sqrt{[\ (6 \cdot 11{,}244\ kNm / (0{,}218\ m \cdot 235\ MPa \cdot 1{,}0)]} - 0{,}04\ m\ \} =$$

$$= \max (20\ mm \quad ; \quad 36\ mm - 40\ mm) = \max (20\ mm \quad ; \quad -4\ mm) = 20\ mm$$

This thickness of bottom plate plate will be applicated in both cases; rocker on wall and rocker on column.

Cap plate - column weldings

Lecture #17, Example 3a
for
$M_{Ed} = 0 \quad ; \quad V_{Ed} = 0$

$$\sqrt{[(\sigma_\perp)^2 + 3(\tau_\parallel^2 + \tau_\perp^2)]} \leq f_u / (\beta_w \gamma_{M2})$$

$$\sigma_\perp \leq 0{,}9 f_u / \gamma_{M2}$$

$$\gamma_{M2} = 1{,}25$$

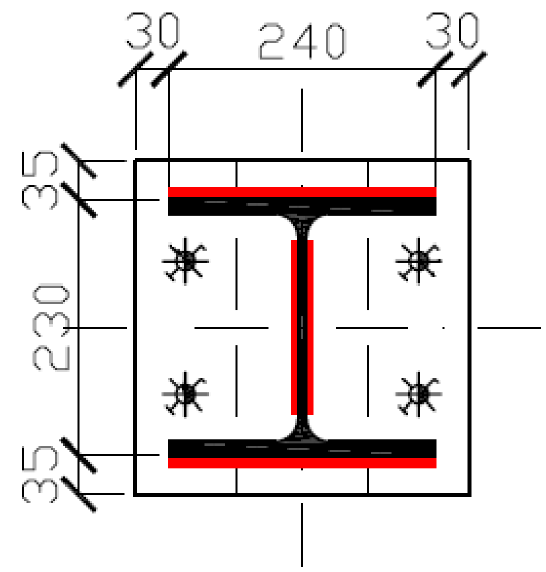


Photo: Author

N_{Ed} = 1 184,235 kN

f_u = S235 = 360 MPa

β_w = S235 = 0,8

Cap plate - flange:

t_1 = thickness of flange = 12 mm

t_2 = thickness of cap plate = 26 mm

$0{,}2\ t_2 \leq a_f \leq 0{,}7\ t_1 \rightarrow a_f = 8$ mm

l_f = length of weld = 240 mm

Cap plate - web:

t_1 = thickness of web = 7,5 mm

t_2 = thickness of cap plate = 26 mm

$0{,}2\ t_2 \leq a_w \leq 0{,}7\ t_1 \rightarrow a_w = 5$ mm

l_w = length of weld = 160 mm

$A = 2\ a_f\ l_f + 2\ a_f\ l_f = 54{,}4\ cm^2$

$\sigma\ (N_{Ed}) = N_{Ed}\ /\ A = 217{,}690\ MPa$

$\sigma_\perp = \tau_\perp = \sigma\ /\ \sqrt{2} = 153{,}930\ MPa$

$\tau_\parallel = 0{,}000\ MPa$

$\sqrt{[(\sigma_\perp)^2 + 3(\tau_\parallel{}^2 + \tau_\perp{}^2)]} = 307{,}860\ MPa$

$f_u\ /\ (\beta_w\ \gamma_{M2}) = 360{,}0\ MPa$

$307{,}860\ /\ 360{,}0 = 0{,}835$

condition satisfied

$\sigma_\perp = 153{,}930\ MPa$

$0{,}9\ f_u\ /\ \gamma_{M2} = 259{,}200\ MPa$

$153{,}930\ /\ 259{,}200 = 0{,}594$

condition satisfied

Cap plate-column compression

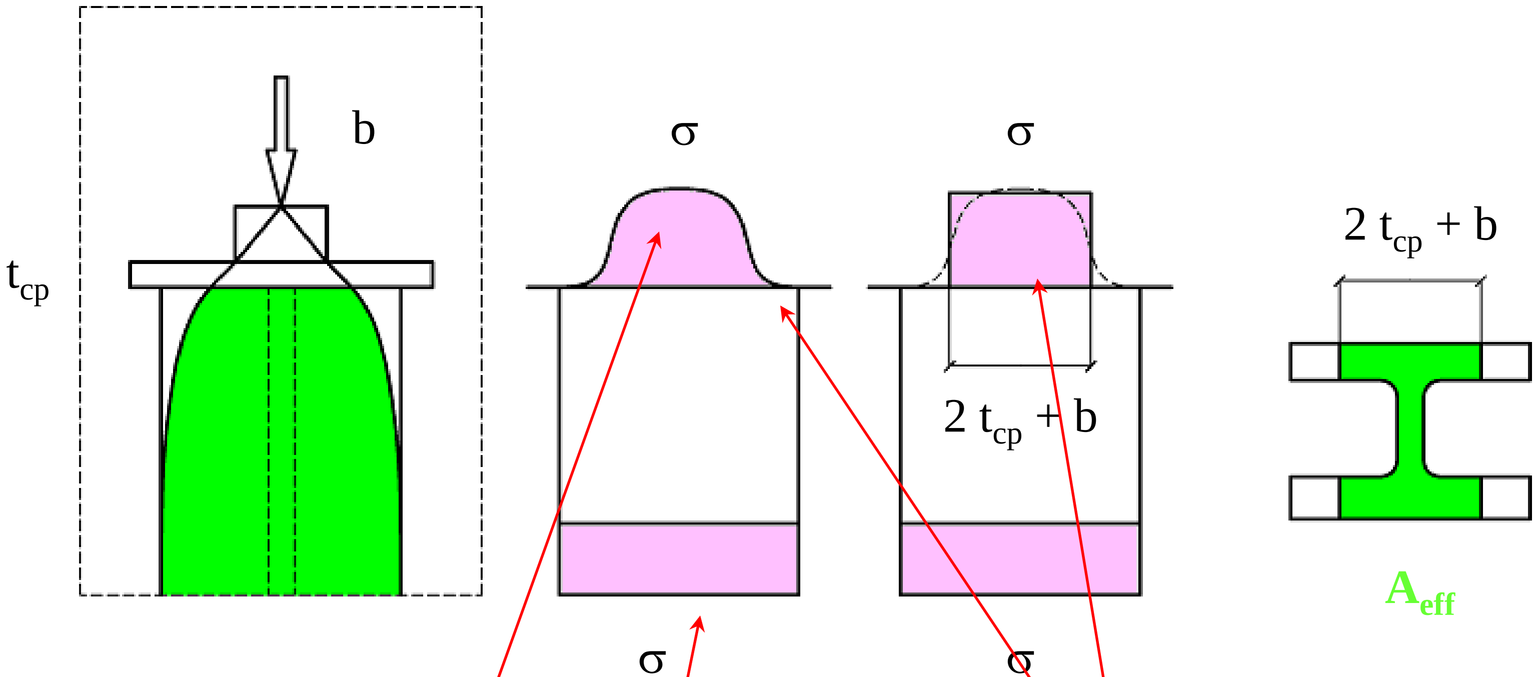


Photo: Author

Cap plate - wide thin member; stress between cap plate and column has non-linear character; we assume big constant value in central part and zero at ends of flange. Constant value of stress exists in distance from end of column. Linearisation of top stresse will be applicated

$$N_{Ed} / (A_{eff}\ f_y\ \gamma_{M0}) \leq 1{,}0$$

b = width of rocker = 120 mm

t_{cp} = thickness of cup plate = 20 mm

b_f = widht of column's flange = 240 mm

t_f = thickness of column's flange = 12 mm

$A_{HEA240} = 76{,}84\ cm^2$

$N_{Ed} = 1055\ kN$

$f_y = S235 = 235\ MPa$

$s = b + 2\ t_{cp} = 160\ mm$

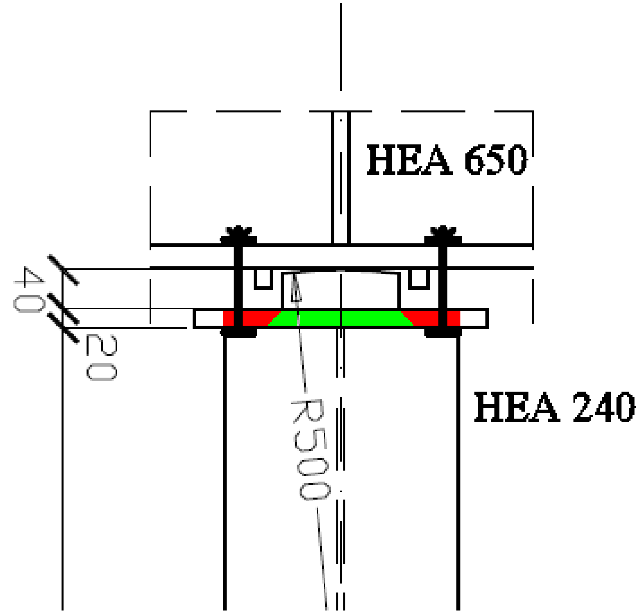
$x = b_f - s = 240 - 160\ mm = 80\ mm$

$A_{eff} = A_{HEA240} - 2 \cdot (x / 2) \cdot t_f =$

$= 76{,}84 - 8 \cdot 1{,}2 = 67{,}24\ cm^2$

$A_{eff}\ f_y\ \gamma_{M0} = 1\ 510{,}345\ kN$

$1\ 184{,}235 / 1\ 510{,}345 = 0{,}784 < 1{,}0$ OK



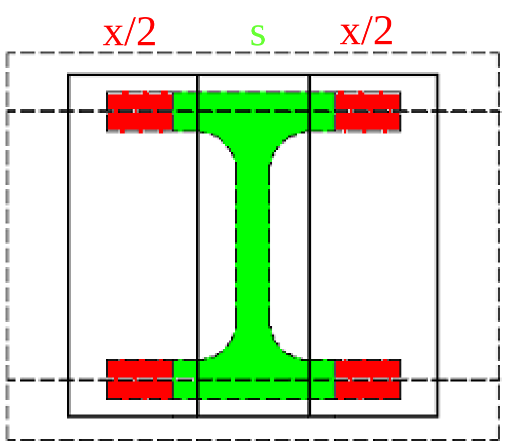


Photo: Author

V^{th} example of calculations – vertical stiffeners

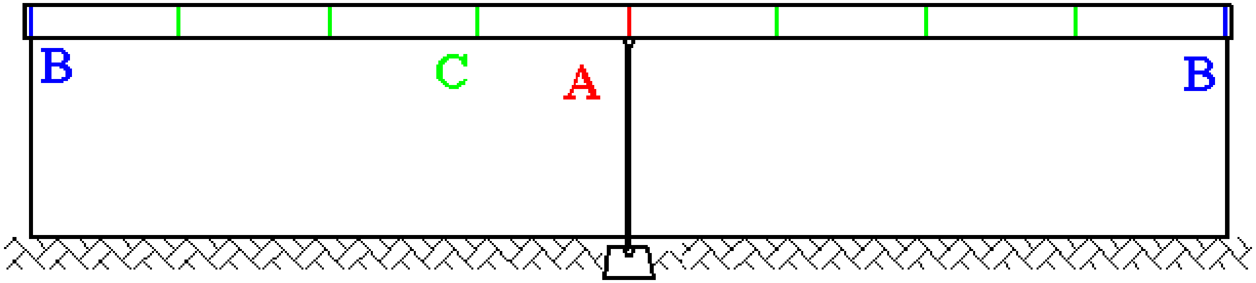


Photo: Author

New complex of data

Secondary beam: IPE 300

S 235 → ε = 1,0

Primary beam: HEA 650

A	Reaction from column	1049,9 kN
B	Reaction from wall	287,8 kN
C	2x force from secondary beam	2x36,4 = 72,8 kN

Conditions important for vertical stiffeners:

Independent of load		Load-dependent
1. Thickness of adjacent elements (#t / 33)		5. Contact stress (#t / 38)
2. Class of the cross-section (#t / 34)		6. Axial compression (#t / 39-49)
3. Prevent torsional buckling of stiffener (#t / 35)		7. Welds (#t / 50-55)
4. Rigid supports for web panel (#t / 36-37)		

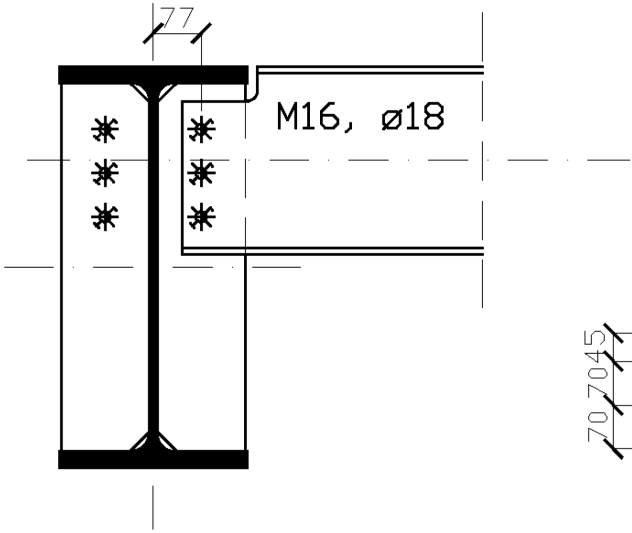
The same value for stiffeners A, B, C:

$t_{s\text{-}in}$ = max [(1) ; (2) ; (3) ; (4)]

(#t / 35)

Final thickness of stiffener depends on external action, but non smaller than $t_{s\text{-}in}$ (#t / 47)

1. Thickness of adjacent elements



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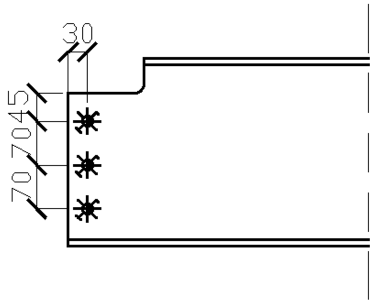


Vertical stiffener is used to connect secondary beam and primary beam. There are holes for bolts in both members (stiffener / web of secondary beam). Both elements should be analised because of bearing resistance. Assuming

thickness of stiffener > thickness of secondary beam web

we obtain a higher resistance of stiffener than resistance of web of secondary beam. We don't need to check resistance of stiffener.

$$\underline{t_{s\text{-}in,\,1} \geq t_{w\_sb} = 10{,}7\ \text{mm}}$$

2. Class of the cross-section

2.a. According to old Polish Standard PN-B 3200, stiffeners can't be IVth class of cross-section;

2.b. According to Eurocode EN 1993-1-5, stiffeners are calculated based on total geometry (as for Ist, IInd or IIIrd class of cross-section); not as for IVth where we must use effective geometry.

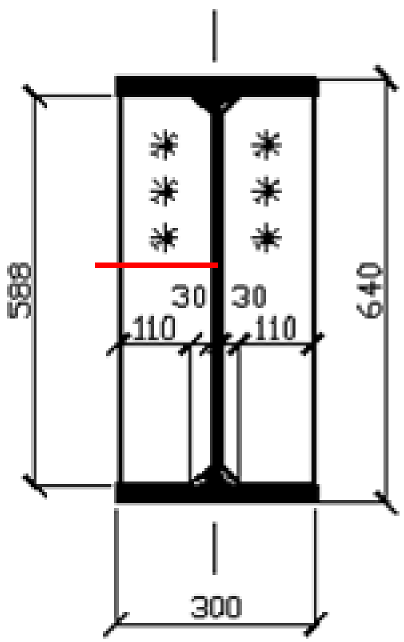
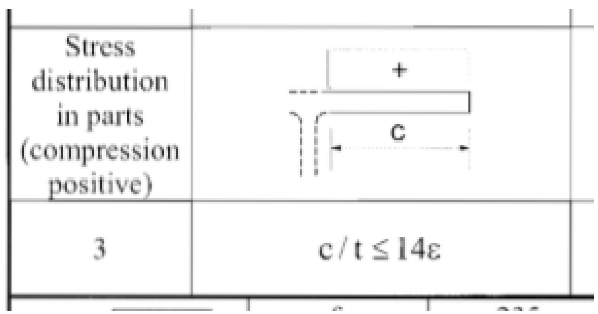


Photo: Author



Stress distribution in parts (compression positive)	
3	$c / t \leq 14\varepsilon$

Photo: EN 1993-1-1 tab. 5.2

Vertical stiffener under compression in horizontal cross-section looks like quasi-flange.

$b_s / t_s \leq 14\ \varepsilon$

$b_s = 140$ mm

$\underline{t_{s\text{-in},\,2} \geq 10{,}0\text{ mm}}$

3. Prevent torsional buckling of stiffener

EN 1993-1-5 (9.3)

$J_T / J_p \geq 5{,}3\ f_y / E$

Moment of inertia for one stiffener about edge fixed to the plate in horizontal cross-section (J_p) and torsional moment of inertia for one stiffener (J_T).

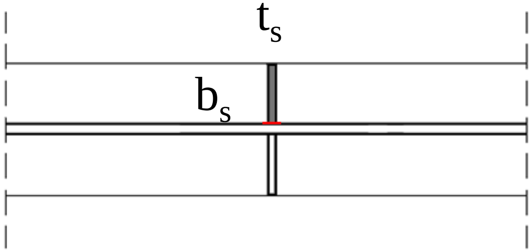


Photo: Author

$J_T = b_s\ t_s^3 / 3$

$J_p = b_s\ t_s\ (b_s / 2)^2 + b_s\ t_s^3 / 12 = b_s^3\ t_s / 4 + b_s\ t_s^3 / 12$

$5{,}3\ f_y / E = 0{,}006076$

$J_T / J_p = (b_s\ t_s^3 / 3) / (b_s^3\ t_s / 4 + b_s\ t_s^3 / 12) = b_s\ t_s\ (t_s^2 / 3) / [(b_s\ t_s / 4)\ (b_s^2 + t_s^2 / 3)\ ] =$

$= 4\ t_s^2 / (3\ b_s^2 + t_s^2)$

$4\ t_s^2 / (3\ b_s^2 + t_s^2) \geq 5{,}3\ f_y / E = 0{,}006076$

$4\ t_s^2 \geq 0{,}006076\ (3\ b_s^2 + t_s^2)$

$t_s^2\ (4 - 0{,}006076) \geq 3 \cdot 0{,}006076\ b_s^2$

$\underline{t_{s\text{-}in,\ 3} \geq 8{,}1\ \text{mm}}$

4. Rigid supports for web panel

EN 1993-1-5 9.3.3 (9.6)

$a / h_w \geq \sqrt{2} \rightarrow J_{st} \geq 1{,}50\, h_w^3\, t_w^3 / a^2$

$a / h_w < \sqrt{2} \rightarrow J_{st} \geq 0{,}75\, h_w\, t_w^3$

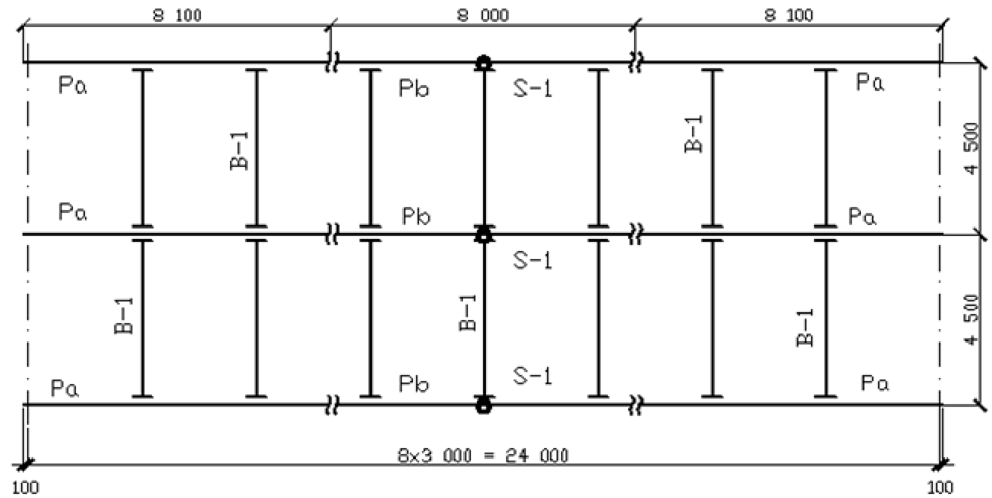
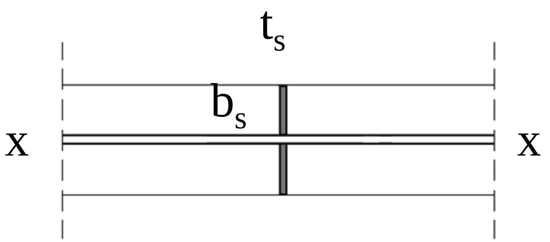


Photo: Author

Moment of inertia for pair of stiffeners about longitudinal axis x-x in horizontal direction

$J_{st} = 2\, [\, b_s^3\, t_s / 12 + b_s\, t_s\, (b_s + t_w)^2 / 4\,]$



a - distance between secondary beams = 3 000 mm

h_w - high of primary beam web (HEA 650) = 588 mm

t_w - thickness of primary beam web (HEA 650) = 13,5 mm

$a / h_w = 5{,}102 \geq \sqrt{2} \rightarrow 1{,}50\, h_w^3\, t_w^3 / a^2 = 8{,}336\ \text{cm}^4$

$J_{st} \geq 8{,}336\ \text{cm}^4 \rightarrow \underline{t_{s\text{-}in,\,4} \geq 0{,}1\ \text{mm}}$

$$t_{s\text{-}in} \geq \max[t_{s\text{-}in,\,1}\ ;\ t_{s\text{-}in,\,2}\ ;\ t_{s\text{-}in,\,3}\ ;\ t_{s\text{-}in,\,4}] = \max[10{,}7\text{ mm}\ ;\ 10{,}0\text{ mm}\ ;\ 8{,}1\text{ mm}\ ;\ 0{,}1\text{ mm}]$$

$$t_{s,\text{-}in} \geq 10{,}8\text{ mm}$$

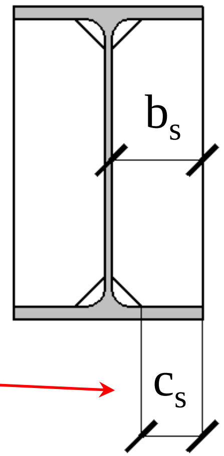
$$t_{s\text{-}in} = 11\text{ mm}$$

Conditions (1) – (4) are independen of loads; thickness $t_{s\text{-}in}$ is the same for each stiffeners.

For various stiffeners acts various forces. Is possible, than for big force, fatter stiffener is needed. It will be analised in second group of conditions: (5), (6), load-dependent.

5. Contact betwen stiffener and flange of primary beam

EN 1993-1-5 9.4 (2)

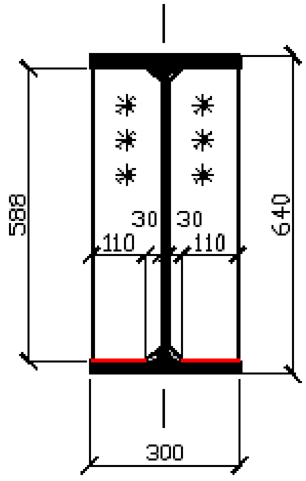


$$F_{s,\,Ed} / (2\, c_s\, t_s\, f_y\, \gamma_{M0}) \leq 1{,}0 \;\rightarrow\; t_s \geq F_{s,\,Ed} / (2\, c_s\, f_y\, \gamma_{M0})$$

c_s = 110 mm

	$F_{s,\,Ed}$	$t_{s,\,5}$
A	1049,9 kN	≥ 20, 3 mm
B	287,8 kN	≥ 5,6 mm
C	2x36,4 = 72,8 kN	≥ 1,4 mm

Photo: Author



6. Axial compression

EN 1993-1-5 9.2.1

EN 1993-1-5 9.4 (2)

Photo: lmsteelfab.com

- Stiffeners are treated as bar, compressed by axial force $F_{s1, Ed}$;
- We analyse flexural buckling about x-x axis;
- We take into consideration cross-section of stiffeners and cooperating part of web (┼ cross-section);
- Axial force $F_{s1, Ed}$ contains imperfections of stiffeners;
- Additionally we must analyse imperfections of web - represented by additional load q;
- We must analyse interaction between axial force $F_{s1, Ed}$, buckling about x-x and bending moment $M_{s, Ed}$ (q).

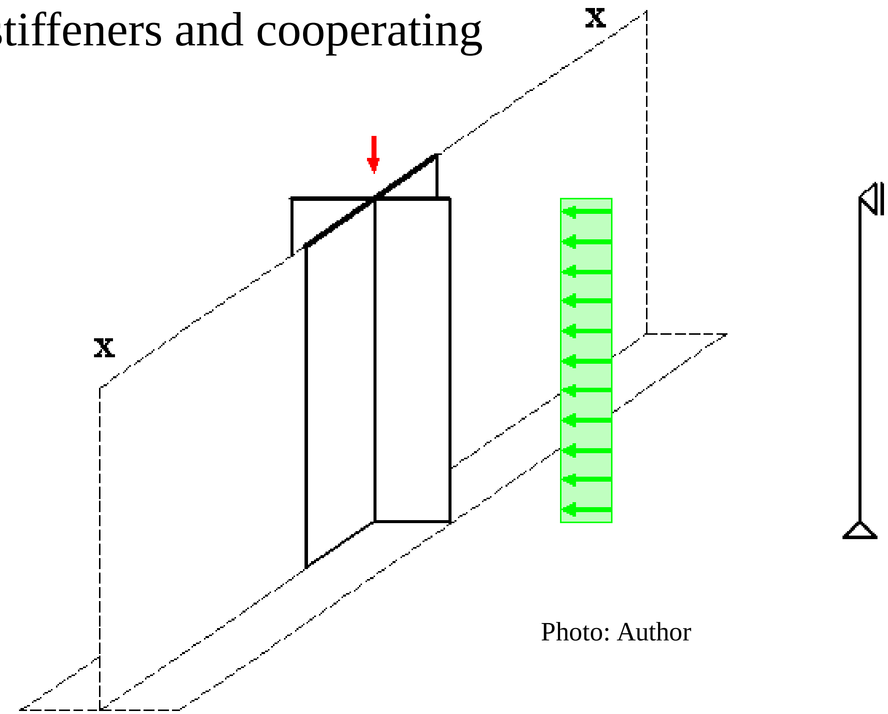


Photo: Author

Axial force: $F_{s1, Ed} = \max (F_{s, Ed} + \Delta N_{st} \; ; \; V^{*}_{Ed} + \Delta N_{st})$

The universal formula, to analyzed different cases:

Analyzed, when force acts direclty on the stiffener

Analyzed, when no forces act on the stiffener; stiffener is needen to prevent of web local buckling only.

Design project - violet part only

$$\Delta N_{st} = \sigma_m b^2 / \pi^2$$

$$\sigma_m = (\sigma_{cr,\,c} / \sigma_{cr,\,p}) (1 / a_1 + 1 / a_2) N_{Ed,\,eq} / b$$

$$\sigma_{cr,\,c} = \pi^2 E t_w{}^2 / [\ 12 (1 - \nu^2) a^2\ ] \approx 190\,000 (t_w / a)^2 \text{ [MPa]}$$

$$\sigma_{cr,\,p} = k_s (28{,}4\ \varepsilon)^2 f_y (t_w / b)^2 \approx 190\,000\ k_s (t_w / b)^2 \text{ [MPa]}$$

a, a_1, a_2 - distances between secondary beam; regular; irregular

$a = a_1 = a_2 = 3\ 000$ mm

$b = h_w$ - high of primary beam web (HEA 650) = 588 mm

t_w - thickness of primary beam web (HEA 650) = 13,5 mm

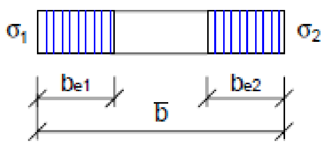
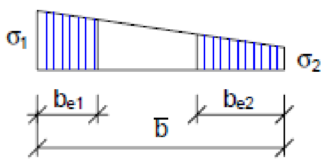
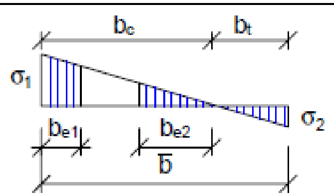
$N_{Ed,\,eq}$ - equivalent axial force in primary beam (→ #t / 41)

ν - Poisson ratio

k_σ - local buckling factor for primary beam (→ # t / 40)

Local buckling factor

EN 1993-1-5, tab 4.1

Stress distribution (compression positive)	Effective^p width b_{eff}
	$\underline{\psi = 1}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = 0{,}5\ b_{eff}$ $\quad$ $b_{e2} = 0{,}5\ b_{eff}$
	$\underline{1 > \psi \geq 0}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = \frac{2}{5-\psi} b_{eff}$ $\quad$ $b_{e2} = b_{eff} - b_{e1}$
	$\underline{\psi < 0}$: $b_{eff} = \rho\ b_c = \rho\ \bar{b} / (1-\psi)$ $b_{e1} = 0{,}4\ b_{eff}$ $\quad$ $b_{e2} = 0{,}6\ b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8{,}2 / (1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98\ (1 - \psi)^2$

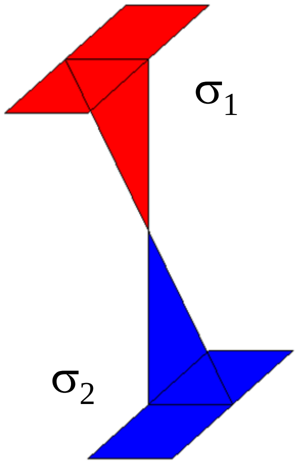


Photo: Author

Global stresses in primary beam web: bending moment only, no axian force

$$\sigma_1 = -\ \sigma_2 \rightarrow \psi = -1 \rightarrow k_\sigma = 23{,}9$$

Equivalent axial force

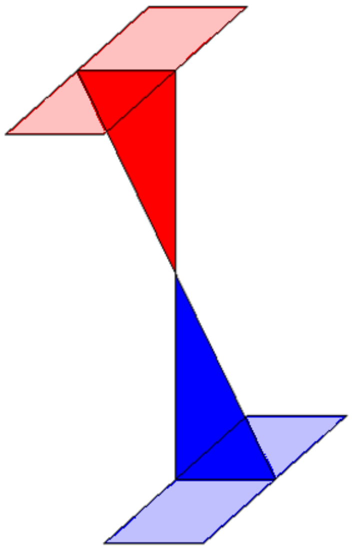


Photo: Author

$N_{Ed,\,eq}$ – effect of compression in web (dark red part only)

For global bending moment only

$N_{Ed,\,eq} = \sigma_{max} A/2$

A = area of web

Approximation:

$\sigma_{max} \approx f_y$

HEA 650:

$A = 13{,}5\text{ mm} \times 588\text{ mm} = 79{,}4\text{ cm}^2$

$N_{Ed,\,eq} = f_y A/2 = 932{,}95\text{ kN}$

$$a = a_1 = a_2 = 3\,000 \text{ mm}$$

$$h_w = 588 \text{ mm}$$

$$t_w = 13{,}5 \text{ mm}$$

$$k_s = 23{,}9$$

$$N_{Ed,\,eq} = 932{,}95 \text{ kN}$$

$$\sigma_{cr,\,c} = \pi^2 E\, t_w^{\,2} / [\ 12\,(1 - \nu^2)\, a^2\ ] = 190\,000\,(t_w / a)^2\ [\text{MPa}] = 3{,}848 \text{ MPa}$$

$$\sigma_{cr,\,p} = k_s\,(28{,}4\,\varepsilon)^2 f_y\,(t_w / b)^2 = 190\,000\, k_s\,(t_w / b)^2\ [\text{MPa}] = 2\,393{,}671 \text{ MPa}$$

$$\sigma_m = (\sigma_{cr,\,c} / \sigma_{cr,\,p})\,(1 / a_1 + 1 / a_2)\, N_{Ed,\,eq} / b = 1{,}700 \text{ kPa}$$

$$\Delta N_{st} = 0{,}060 \text{ kN}$$

Additional load q (effect of web imperfections):

$$q = \pi \sigma_m (w_0 + w_{el}) / 4$$

$$M_{s, Ed} = q\, l^2 / 8$$

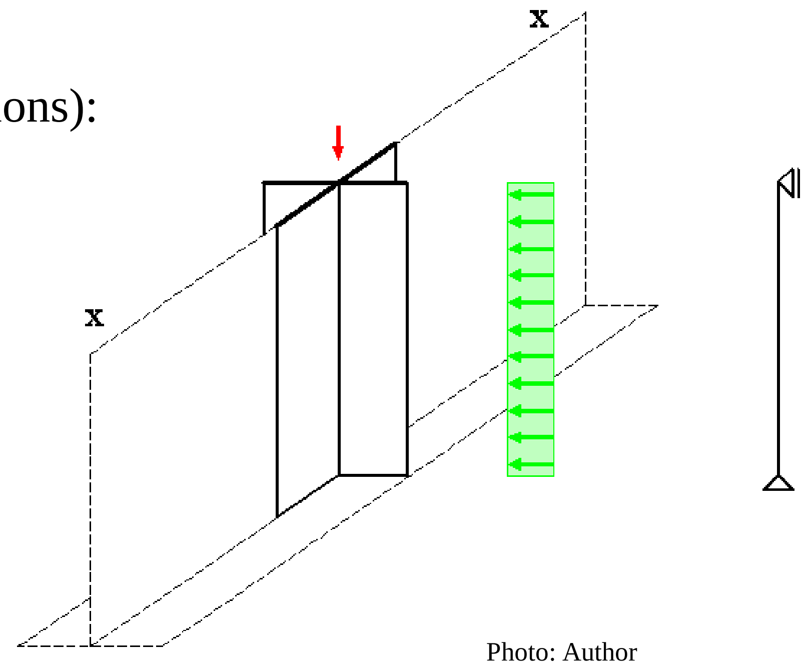


Photo: Author

Full formula for stiffener:

EN 1993-1-1 NA. 20

$$F_{s1, Ed} / (\chi_x N_{Rd}) + M_{s, Ed} / M_{Rd} \leq 1{,}0 - \Delta_{0, x}$$

Resistance:

$F_{s1,\,Ed} = F_{s,\,Ed} + \Delta N_{st}$

$M_{s,\,Ed} = q\, l^2 / 8$

$q = \pi\, \sigma_m\, (w_0 + w_{el}) / 4$

$w_0 = s / 300$

$s = \min\, (a_1 \;;\; a_2 \;;\; b)$

$w_{el} = b / 300$

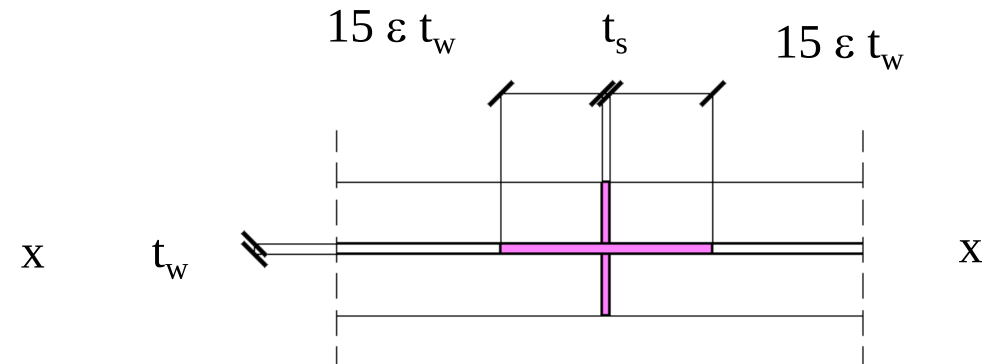


Photo: Author

N_{Rd} , M_{Rd} and χ_x as for violet cross-section (horizontal cross-section of pair of stiffeners + cooperated part of web).

M_{Rd} and χ_x are calculated for longitudinal axis x-x

Stability:

$\chi_x = \chi_x\, (c, ┼, l_{cr})$

$l_{cr} = \mu\, h_w$

$\mu = 0{,}75$

$b = h_w = 588$ mm

$b_s = 140$ mm

$a = a_1 = a_2 = 3\ 000$ mm

$\sigma_m = 1{,}700$ kPa

$s = \min (a_1\ ;\ a_2\ ;\ b) = 588$ mm

$w_0 = s / 300 = 1{,}96$ mm

$w_{el} = b / 300 = 1{,}96$ mm

$q = \pi\ \sigma_m\ (w_0 + w_{el}) / 4 = 0{,}005$ kN / m

$l = h_w$

$M_{s,\ Ed} = q\ l^2 / 8 = 0{,}0004$ kNm – neglegible, there will be analised buckling under axial force $F_{s1,\ Ed}$ only

Stiffener	$F_{s,\,Ed} + \Delta N_{st}$	$t_{s\text{-}in}$	A_{+}	$N_{Rd} = A_{+}\, f_y$
A	1049,96 kN	11 mm	$2 \cdot 15\, \varepsilon\, t_w +$ $+\, t_{s,\,1\text{-}4}\,(2\, b_s + t_w) =$ $= 72{,}785\ cm^2$	1710,446 kN
B	287,86 kN			
C	72,86 kN			

$J_{x,+} = (2\, b_s + t_w)^3\, t_{s,\,1\text{-}4} / 12 + 2 \cdot 15\, \varepsilon\, (t_w)^4 / 12 \approx (2\, b_s + t_w)^3\, t_{s,\,1\text{-}4} / 12 = 2\,317{,}585\ cm^4$

$i_{x,+} = \sqrt{(J_{x,+} / A_{+})} = 5{,}643\ cm$

$l_{cr} = h_w\, \mu = 44{,}1\ cm$

$\lambda_1 = 93{,}9\, \varepsilon = 93{,}9$

$\lambda = (l_{cr} / i)\,(1 / \lambda_1) = 0{,}083$

$\lambda < 0{,}2$ – buckling is not important, $\chi = 1{,}0$ no impact of lost of stability, resistance checked on #t / 36

Stiffener	$F_{s1, Ed}$	χN_{Rd}	E / R
A	1049,96 kN	1710,446 kN	0,614
B	287,86 kN		0,168
C	72,86 kN		0,043

Conclusions:

From **conditions 1-4**, 11 mm must be applied for each stiffeners;

From **condition 5**, $t_{s, A}$ > 20,3 mm, for rest 11 mm is enough.

From **condition 6**, 11 mm is enough for each stiffeners;

Final decision:

Stiffener	t_s
A	21 mm
B	11 mm
C	11 mm

Lecture #17, Example 1
for
$N_{Ed} = 0$

Welds between transverse stiffeners and web

Photo: steelconstruction.info

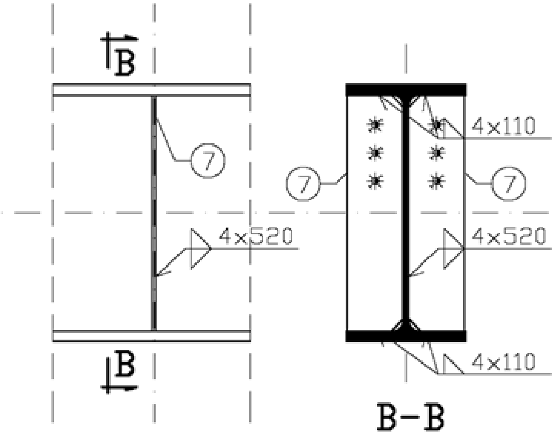


Photo: Author

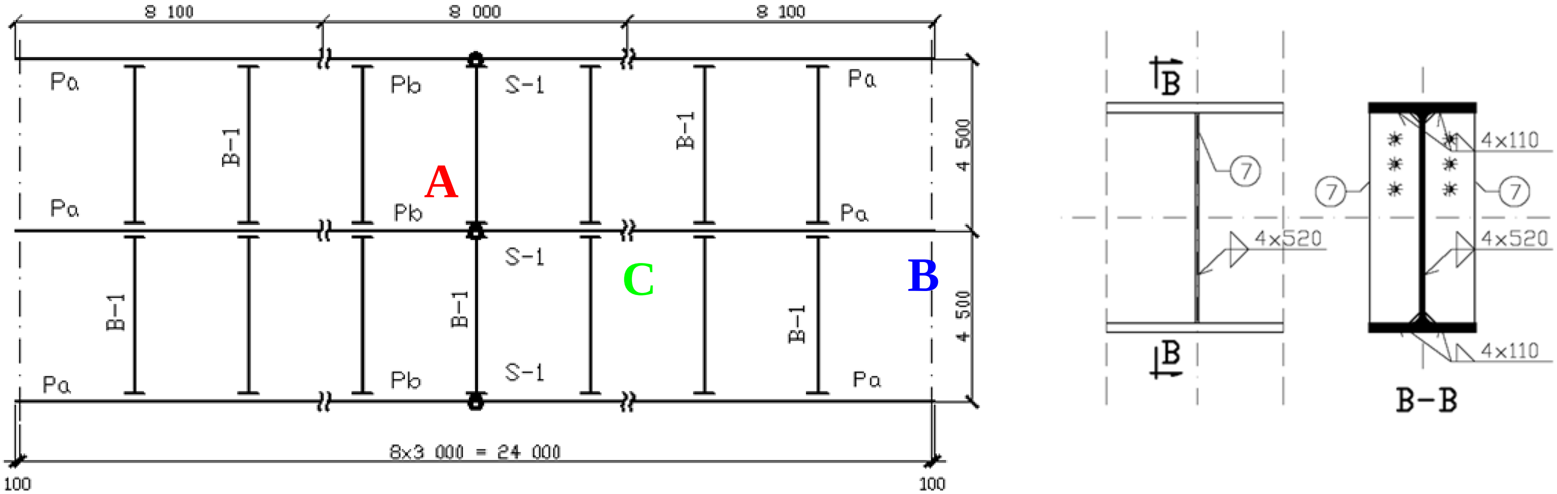


Photo: Author

A	Column - reaction	1049,9 kN
B	Wall - reaction	287,8 kN
C	2x force from secondary beam	2x36,4 = 72,8 kN

Position of center of gravity depends on thickness of welds. Thickness of weld a between two elements of thickness t_1 and t_2 could be taken ito consideration according to old Polish Standar:

$$0{,}2\ t_2 \leq a \leq 0{,}7\ t_1$$

$$t_2 \geq t_1$$

(PN-B 3200)

Thickness of stiffeners:

Stiffener	t_s
A	21 mm
B	11 mm
C	11 mm

Thickness of web (HEA 650) = 13,5 mm

Thickness of flange (HEA 650) = 26 mm

Stiffener	Stiffener - flange		Stiffener- web	
A	$t_2 = 26 \geq t_1 = 21$	$5 \leq a \leq 15$	$t_2 = 21 \geq t_1 = 13{,}5$	$4 \leq a \leq 9$
B	$t_2 = 26 \geq t_1 = 11$	$5 \leq a \leq 8$	$t_2 = 13{,}5 \geq t_1 = 11$	$3 \leq a \leq 8$
C				

For each welds for calculations will be taken into consideration a = 5 mm

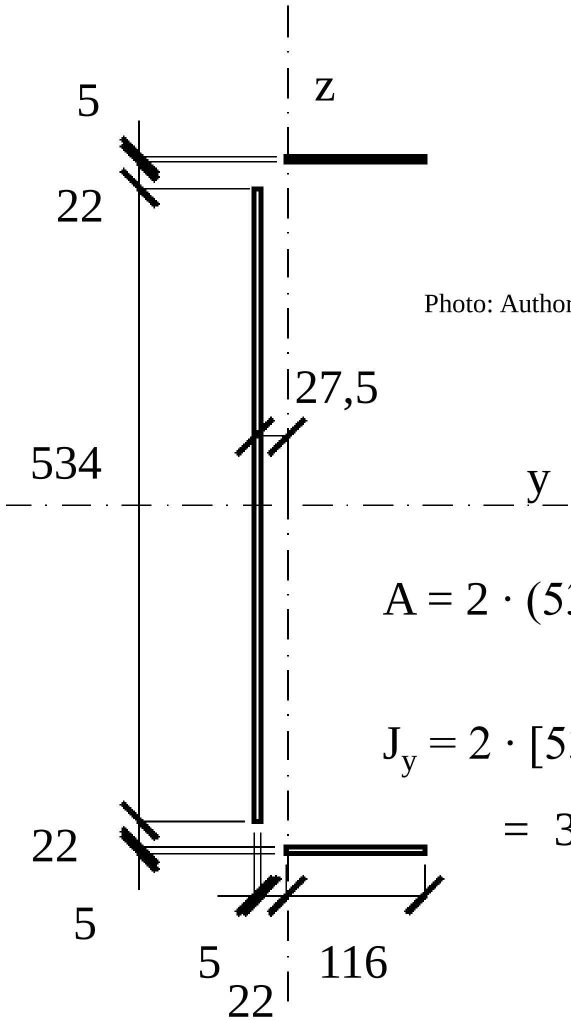
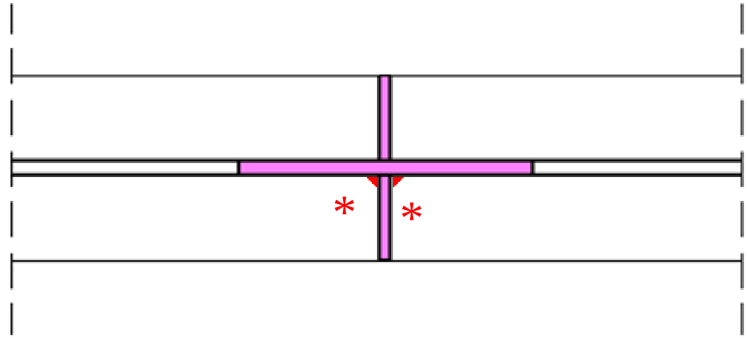


Photo: Author



Geometrical characteristics of welds are multiplied by 2 because of 2 times complex of welds (on left and right side of stiffener).

$$A = 2 \cdot (53{,}4 \cdot 0{,}5 + 2 \cdot 11{,}6 \cdot 0{,}5) = 76{,}6\ \text{cm}^2$$

$$J_y = 2 \cdot [53{,}4^3 \cdot 0{,}5 / 12 + 2 \cdot 11{,}6 \cdot 0{,}5 \cdot (53{,}4 / 2 + 2{,}2 + 0{,}5 / 2)^2] =$$

$$= 32\ 403\ \text{cm}^4$$

$$J_z = 2 \cdot [2 \cdot 11{,}6^3 \cdot 0{,}5 / 12 + 53{,}4 \cdot 0{,}5 \cdot (2{,}75 - 0{,}5/2)^2 + 2 \cdot 11{,}6 \cdot 0{,}5 \cdot (0{,}5 + 2{,}2 + 11{,}6/2 - 2{,}75)^2] =$$

$$= 1444\ \text{cm}^4$$

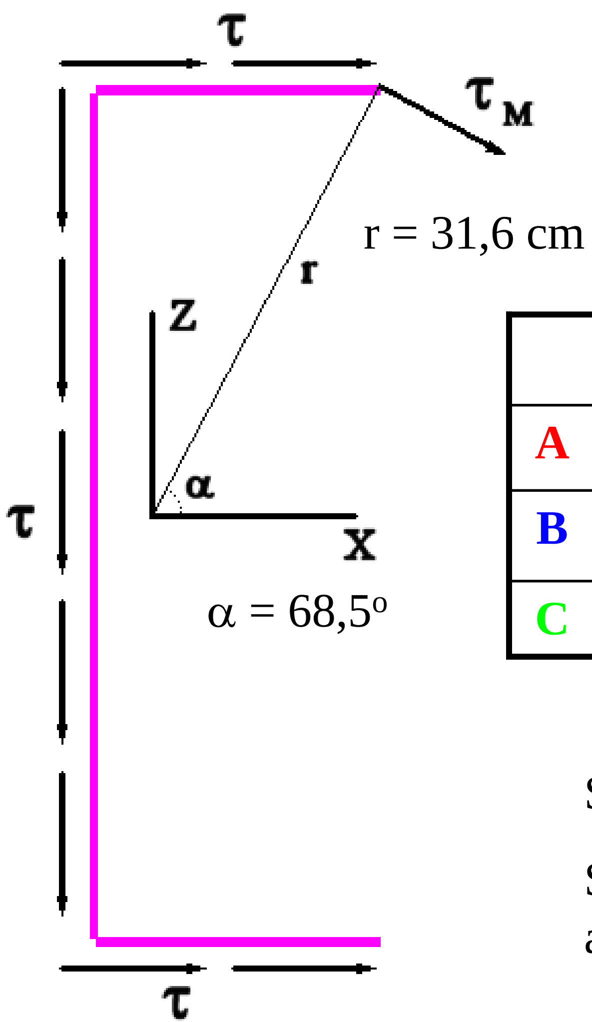


Photo: Author

$$\tau = V_{Ed} / A$$
$$\overline{\tau_M} = M\,\overline{r} / J_o$$
$$J_o = J_y + J_z$$

	V_{Ed}	M_{Ed}	τ	$\overline{\tau_M}$
A	524,95 kN	14,44 kNm	68,53 MPa	13,48 MPa
B	143,60 kN	3,95 kNm	18,75 MPa	3,69 MPa
C	36,40 kN	1,00 kNm	4,75 MPa	0,93 MPa

Stress $\tau(V_{Ed})$ is parallell to axis of welds ($= \tau_{\|}$)

Stress $\tau(M_{Ed})$ must be recalculated to $\tau_{\|}$ and $\tau_{\perp}$ by sin α and cos α

	$\tau_{\|}(V_{Ed})$	$\tau(M_{Ed})$	$\tau_{\|}(M_{Ed})$	$\tau_{\perp}(M_{Ed})$	$\tau_{\|} = \tau_{\|}(V_{Ed}) + \tau_{\|}(M_{Ed})$
A	68,53 MPa	13,48 MPa	12,54 MPa	4,94 MPa	81,07 MPa
B	18,75 MPa	3,69 MPa	3,43 Mpa	1,35 MPa	22,18 MPa
C	4,75 MPa	0,93 MPa	0,87 MPa	0,34 MPa	5,62 MPa

The biggest stresses are for stiffener A. This is enough to calculate weld for this one case.

$$\sqrt{[(\sigma_{\perp})^2 + 3(\tau_{\|}^2 + \tau_{\perp}^2)]} = 140{,}678 \text{ MPa} < f_u / (\beta_w \gamma_{M2}) = 360 \text{ MPa} \quad \text{OK}$$

$$\sigma_{\perp} = 0{,}000 \text{ MPa} < 0{,}9 f_u / \gamma_{M2} = 259{,}200 \text{ MPa} \quad \text{OK}$$

VI^th example of calculations - column base

Photo: j-p.com.ua

New complex of data:

N_{Ed} = 1 050,0 kN

Column HEA 240

S235

Concrete base C 30/37 f_{ck} = 30 MPa

Analysis of supporting joint concerns few important questions:

- Stiffness of joint (according assumption – hinge joint) → #t / 60
- Resistance of concrete onder base plate → #t / 61 - 62
- Welds column-base plate → #t / 63

One of two methods of calculation can be appicated: rough or accurate method.

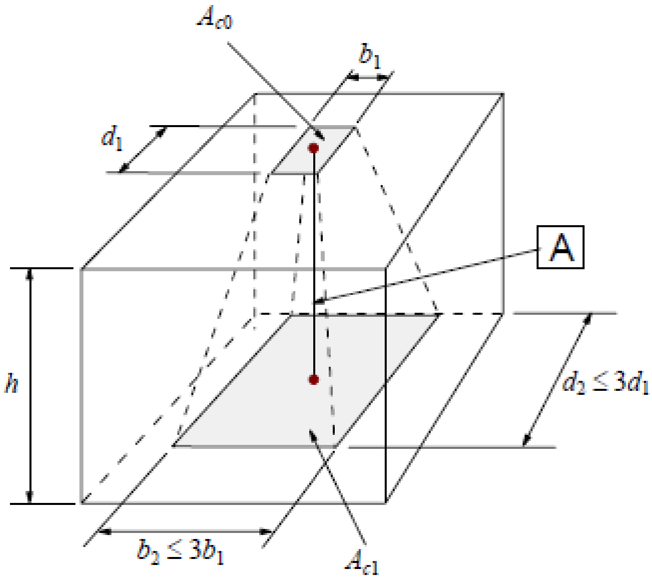


Photo: EN 1992-1-1 fig 6.29

$h \geq (b_2 - b_1)$ i $h \geq (d_2 - d_1)$

Accurate method based on analysis of stress distribution in concrete base block.

Rough analysis based on one fundamental assumption: horizontal area of concrete base block can’t be smaller than 2,25 area of steel base plate.

Rough analysis will be adopted into calculation.

Both methods are based on observation: distribution of stresses from steel base plate against concrete foundation is non-linear. The highest values occur under column shaft and in immediate vicinity, while they decrease further away. Linearization is assumed: a large constant stress value under shaft and in its vicinity, zero value in remaining area. Vicinity is described by distance c, depending on thickness of base plate t_p . Thickness of base plate t_p should be calculated for conndition: stresses can't exceed strength of concrete.

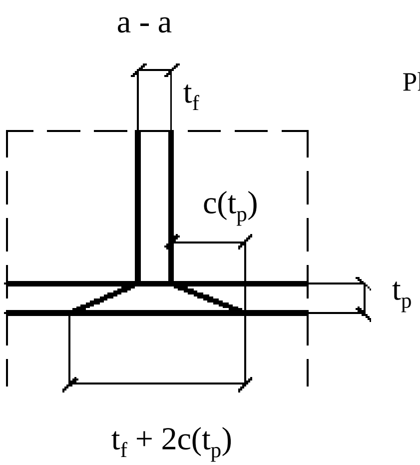
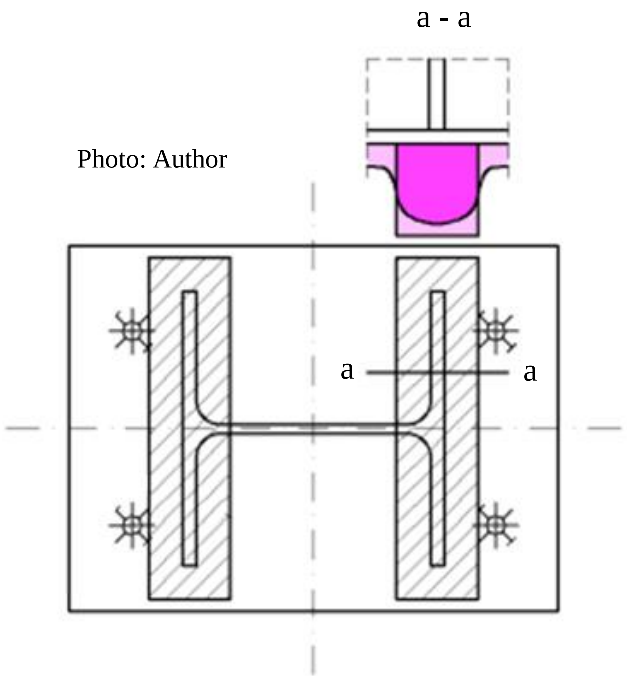


Photo: Author

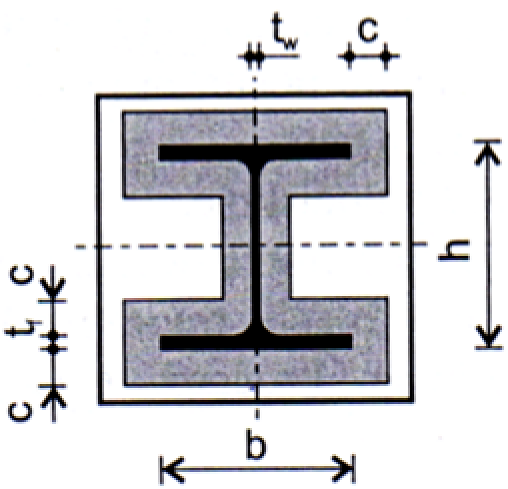
Photo: Author



Stiffness of column’s base

According to #15 / 79, column’s base with one cpuple of anchor bolts, located in strong axid of column’s cross-section is hinge support. This means, that real behaviou of support is the same as in initial assumption (hinge support).

Resistance of concrete block



Nośność podstawy słupa:

$$N_{Rd} = f_{cd}\left[2(b+2c)(t_f+2c)+(t_w+2c)(h-2c-2t_f)\right]$$

Maksymalny wysięg strefy docisku:

$$c = -\frac{X_2-\sqrt{X_2^2-4X_1X_3+4X_1N_{Rd}}}{2X_1}\,*$$

$$X_1 = 4f_{cd}$$

$$X_2 = 4bf_{cd}+2hf_{cd}-2t_wf_{cd}$$

$$X_3 = 2bt_ff_{cd}+ht_wf_{cd}-2t_ft_wf_{cd}$$

There is an error in source, it should be N_{Ed}, not N_{Rd}

c and N_{Rd} according to table:
"Konstrukcje stalowe, przykłady obliczeń według PN-EN 1993-1, część II, stropy i pomosty", praca zbiorowa pod redakcją A. Kozłowskiego, Oficyna Wydawnicza Politechniki Rzeszowskiej, Rzeszów 2009.

t_f (HEA 240) = 12 mm
t_w (HEA 240) = 7,5 mm
h (HEA 240) = 230 mm
b (HEA 240) = 240 mm

$f_{cd} = \alpha_{cc} f_{ck} / \gamma c = 30$ MPa $\cdot 1 / 1,5 = 20$ MPa

$X_1 = 4 f_{cd} = 80$ MPa
$X_2 = 4 b f_{cd} + 2 h f_{cd} – 2 t_w f_{cd} = 28,1$ MN/m
$X_3 = 2 b t_f f_{cd} + 2 h t_w f_{cd} – 2 t_w t_f f_{cd} = 0,181$ MN
$N_{Ed} = 1,050$ MN

$c = -\{X_2 - \sqrt{[X_2^2 – 4 X_1 X_3 + 4 X_1 N_{Ed}]}\} / (2 X_1) = 29$ mm
Thickness of base plate $t_p = c / \sqrt{[f_y / (3 f_{jd} \gamma_{M0})]} = 15$ mm

$N_{Rd} = f_{cd} [2(b + 2c)(t_f + 2c) + (t_w + 2c)(h - 2c – 2 t_f)] = 1087,443$ kN

1050 kN / 1087,443 kN = 0,966 < 1,0 OK

Base plate dimensions: non smaller than (c + b + c) x (c + h + c) =
= (29 + 240 + 29) x (29 + 230 + 29) = 298 x 288; it colud be taken as 300 mm x 290 mm.

Welds

Welds betweem shaft of column and base plate can be taken the same as between column and bottom plate of rocker (#t / 26 – 28)

Thank you for attention

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