Metal Structures

Design Project II

Floor girders – examples of calculation (part I)

Ist example of calculations – primary beam

Initial assumptions about cross-section

$h \approx L_{pb} / 20 \div L_{pb} / 25$

Steel S 355

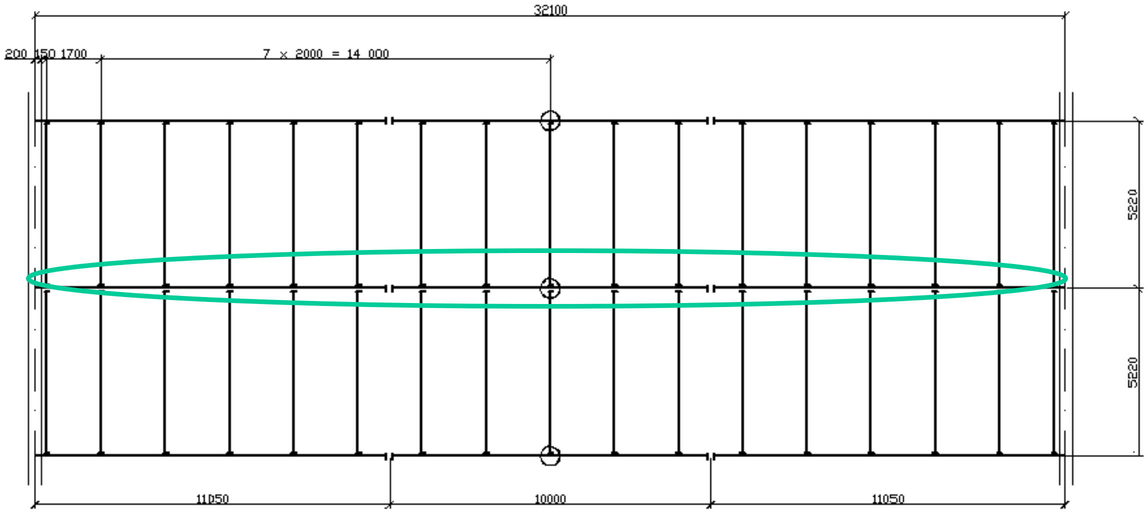
$L_{podciąg} = 16{,}05m$

$h = 642 \div 803$ mm

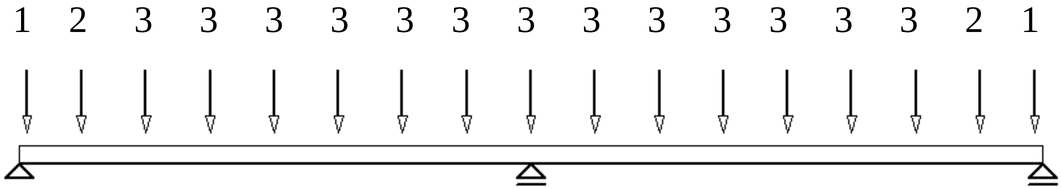
HEA 800

Ist class of cross-section

Photo: Author



Erection stage



$g_d = 2{,}967$ kN / m

	G_d [kN]	Q_d [kN]
1	21,602	0
2	37,745	0
3	40,503	0

Statics of srtucture

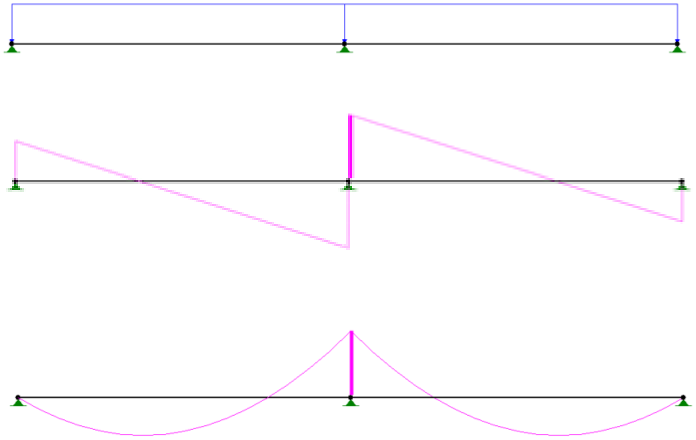


Photo: Author

Asymmetrical - no

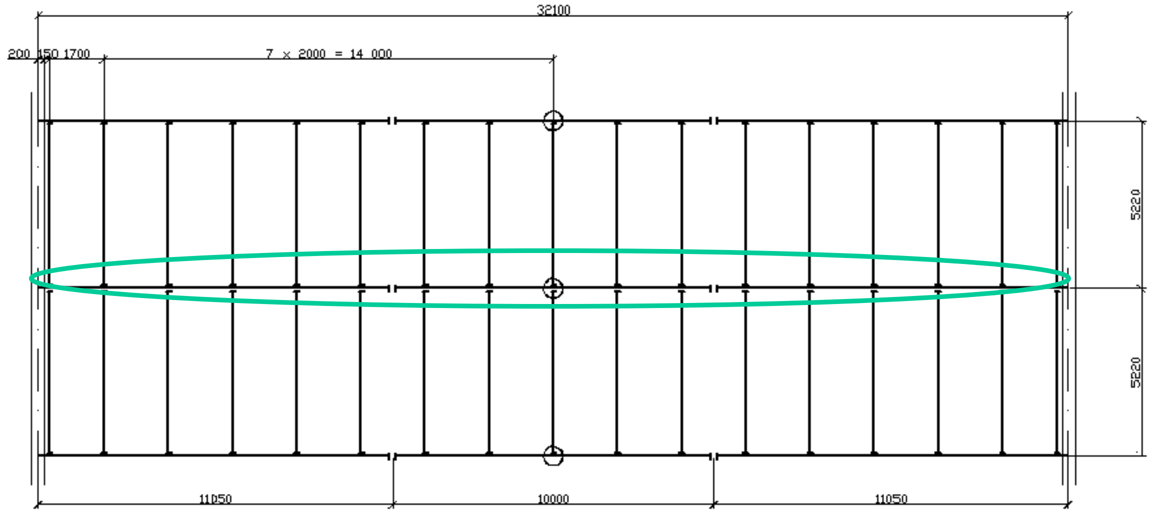
Symmetrical

	Bending moments [kNm]			Reactions [kN]			Shear forces [kN]	
	Left span	Central support	Right span	Left span	Central support	Right span	Central supports – left side	Central supports – right side
Asym								
Sym	374,690	**669,089**	374,690	125,063	**833,755**	125,063	**416,878**	416,878

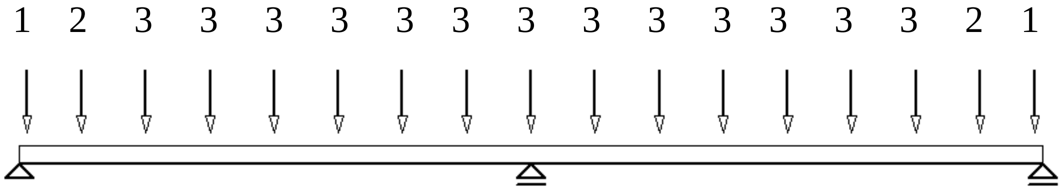
Ist example of calculations – primary beam

Steel S 355

Photo: Author



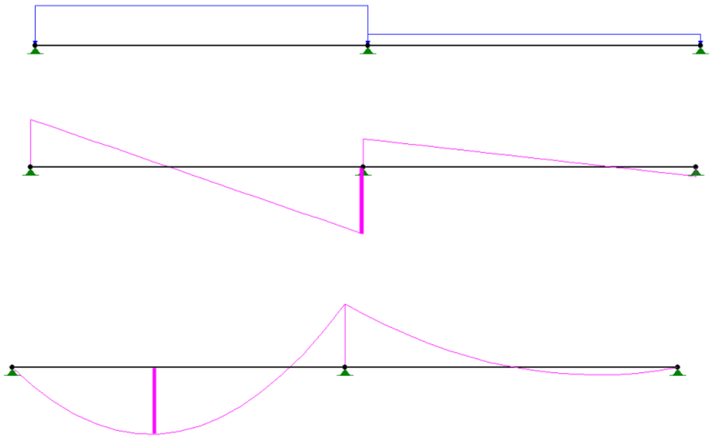
Exploitation stage



$g_d = 2{,}967 \text{ kN / m}$

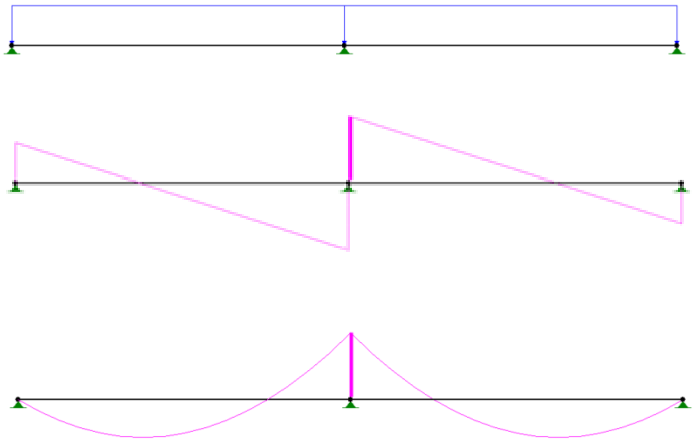
	G_d [kN]	Q_d [kN]
1	20,558	58,725
2	36,701	108,641
3	39,549	117,450

Statics of srtucture



Asymmetrical

Photo: Author



Symmetrical

	Bending moments [kNm]			Reactions [kN]			Shear forces[kN]	
	Left span	Central support	Right span	Left span	Central support	Right span	Central supports – left side	Central supports – right side
Asym	1 849,801	1 659,846	33,165	561,099	1 034,172	82,180	750,248	283,924
Sym	1 452,384	**2 593,542**	1 452,384	484,774	**3 231,828**	484,774	**1 615,914**	1 615,914

Bending moment only:

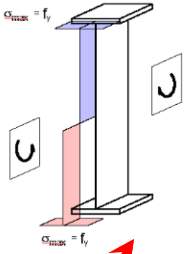


Photo: Author

$$M_{Ed} / M_{Rd} \leq 1{,}0$$

Class of cross-section	IVth	IIIrd	IInd	Ist
Distribution of σ across cross-section	Elastic		Plastic	
Effects	Local instability	Resistance of cross-section		
$M_{Ed} =$	From „normal" static calculations of structure			From special recalculation to new static scheme and new loads (**redistribution**)
$M_{Rd} =$	$W_{eff}\, f_y / \gamma_{M0}$	$W_{el}\, f_y / \gamma_{M0}$	$W_{pl}\, f_y / \gamma_{M0}$	

W_{eff} – IInd laboratory

W_{el} – tables for design

W_{pl} – Ist laboratory

Erection stage (small loads)

Exploitation stage (full loads)

Axial force and shear force

$N_{Ed} / N_{Rd} \leq 1{,}0$ $V_{Ed} / V_{Rd} \leq 1{,}0$

Class of cross-section	IVth	IIIrd	IInd	Ist
N_{Ed}	From „normal” static calculations of structure			
V_{Ed}	From „normal” static calculations of structure			
$N_{Rd,\,T} =$	$A\, f_y / \gamma_{M0}$			
$N_{Rd,\,C} =$	$A_{eff}\, f_y / \gamma_{M0}$	$A\, f_y / \gamma_{M0}$		
$V_{Rd} =$	$A_{v(4)}\, f_y / (\gamma_{M0}\, \sqrt{3})$	$A_v\, f_y / (\gamma_{M0}\, \sqrt{3})$		

There is no axial force in analysed case

A_{eff} – effective area

A – total area

A_v – active area for shear force

Simplification: due to the redistribution of bending moments, there will also be a slight redistribution of shear forces

Erection stage: shear force

EN 1993-1-1, (6.17), (6.18)

$V_{Ed} / V_{Rd} \leq 1,0$ $V_{Rd} = A_V f_y / (\sqrt{3}\, \gamma_{M0})$

Active area for shear force is defined in EN 1993-1-1 6.2.6.(3), but the most often, for I-beams, in stake into consideration safe estimation $h \cdot t_f$;
in analysed case = $79 \cdot 1,5 = 118,5\ cm^2$

$V_{Ed} = 416,878$ kN
$V_{Rd} = 2\,428,768$ kN

$V_{Ed} / V_{Rd} = 0,172 < 1,0$ 👍

Erection stage: interaction shear force – bending moment

Above the central support, we have a large bending moment and shear force simultaneously. In this situation, stress concentrations from bending and shear occur. We calculate this by applying a reduction in the bending resistance if the bending stress exceeds 50%. In analysed case is 17,2%, there is no need for reduction.

EN 1993-1-1 (6.12), (6.13), (6.29), (6.30)

Unreduced lresistance, Ist cross-section class:

$M_{Rd} = W_{y,\ pl}\ f_y\ /\ \gamma_{M0} = 3\ 088{,}145$ kNm

Checking resistance:

$M_{Ed} = 669{,}089$ kNm

$M_{Ed}\ /\ M_{Rd,\ red} = 0{,}217 > 1{,}0$ 👍

Global instability: lateral buckling

Photo: civildigital.com

Lateral buckling: occurs when bending about a strong axis; the member axis simultaneously bends and twists, without deforming adjacent cross-sections. Analyzed only for bending about a strong axis or bi-axial bending. Hazardous for bending elements (beams always; columns if they have at least one rigid node)

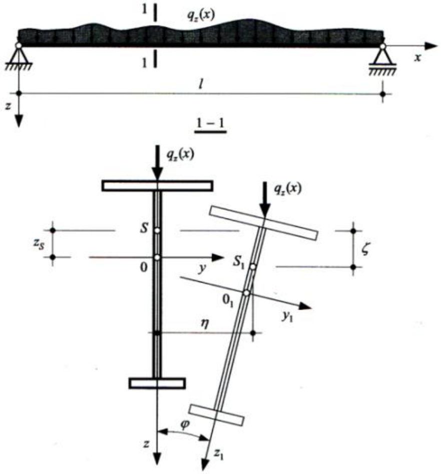


Photo: K. Rykaluk, Zagadnienia stateczności konstrukcji metalowych, DWE 2012

Phenomenon is described by complex of three derivative formulas:

$$E\, J_z\, \eta'''' + (M_y\, \varphi)'' = 0 \qquad [1]$$

$$E\, J_w\, \varphi'''' - [(2\, \beta_z\, M_y + G\, J_T)\, \varphi']' + q_z\, (e_z - z_s)\, \varphi + M_y\, \eta'' = 0 \qquad [2]$$

$$\beta_z = \{ \int_A [\, z\, (y^2 + z^2)\, dA] - z_s\} / (2\, J_y) \qquad [3]$$

There is no general analytical solution; there are only approximate solutions or solutions for limiting assumptions.

The simplest and easiest situation is a single-span beam supported by pins, with a constant bi-symmetric cross-section and a constant bending moment (many limiting assumptions).

$$N_{cr,\,z} = \pi^2 EJ_z / (\mu_z\, l_{0z})^2$$

$$N_{cr,\,T} = [\pi^2 EJ_w / (\mu_T\, l_{0T})^2 \; + \; GJ_t] / i_s^2$$

$$M_{cr} = i_s \sqrt{}\,(N_{cr,\,z}\, N_{cr,\,T})$$

The case of M = const is, however, very rare in structural analysis (one in a billion beams?), so the formula given in both the old Polish Standard B-3200 and Access Steel is **most often used** in calculations:

$$M_{cr} = \{\sqrt{}[\Xi^2 N_{cr,\,z}\, N_{cr,\,T}\, i_s^2 + (\Psi\, N_{cr,\,z})^2]\} - \Psi\, N_{cr,\,z}$$

Coefficients Ξ Ψ depend on the support conditions and the shape of the bending moment along the beam length and the geometrical characteristics of the cross-section; it is also taken into account whether the continuous load q is applied to the compression or tension flange. The specific values Ξ Ψ are specified slightly differently in PN, AS, and still other values can be found in various literature (approximations). $\Xi = 1$ and $\Psi = 0$ give a formulas like the one at the top of the slide.

First step is calculation of critical forces, the same as for flexural bucklning (as for compressive axial force)

$$N_{cr,\,z} = \pi^2\, EJ_z \,/\, (\mu_z\, l_{0z})^2$$

$$N_{cr,\,T} = [\pi^2\, EJ_w \,/\, (\mu_T\, l_{0T})^2 \;+\; GJ_t] \,/\, i_s^{\,2}$$

For both forces, we will only refer to the section of the span where the lower (unsecured) flange is compressed.

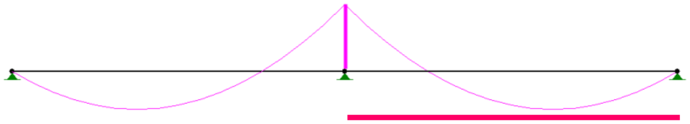


Photo: Author

Erection stage: no concrete plate, no protection against instability

In this stage, $l_{0z} = l_{0T}$ = total span = 16,050 m

The form of instability approximately corresponds to the behavior of a beam supported by a supporting beam.

$$\mu_z = \mu_T = 1{,}0$$

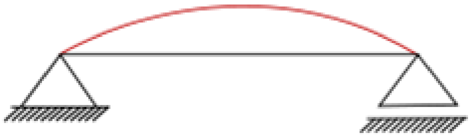


Photo: wikipedia

When analyzing lateral buckling, it is necessary to determine the relative position of three points in the cross-section:

- center of gravity;
- point of load application;
- shear center.

Load in poin	Effect
1	Bending, shear force, **torsion**
2	Bending, shear force
3	

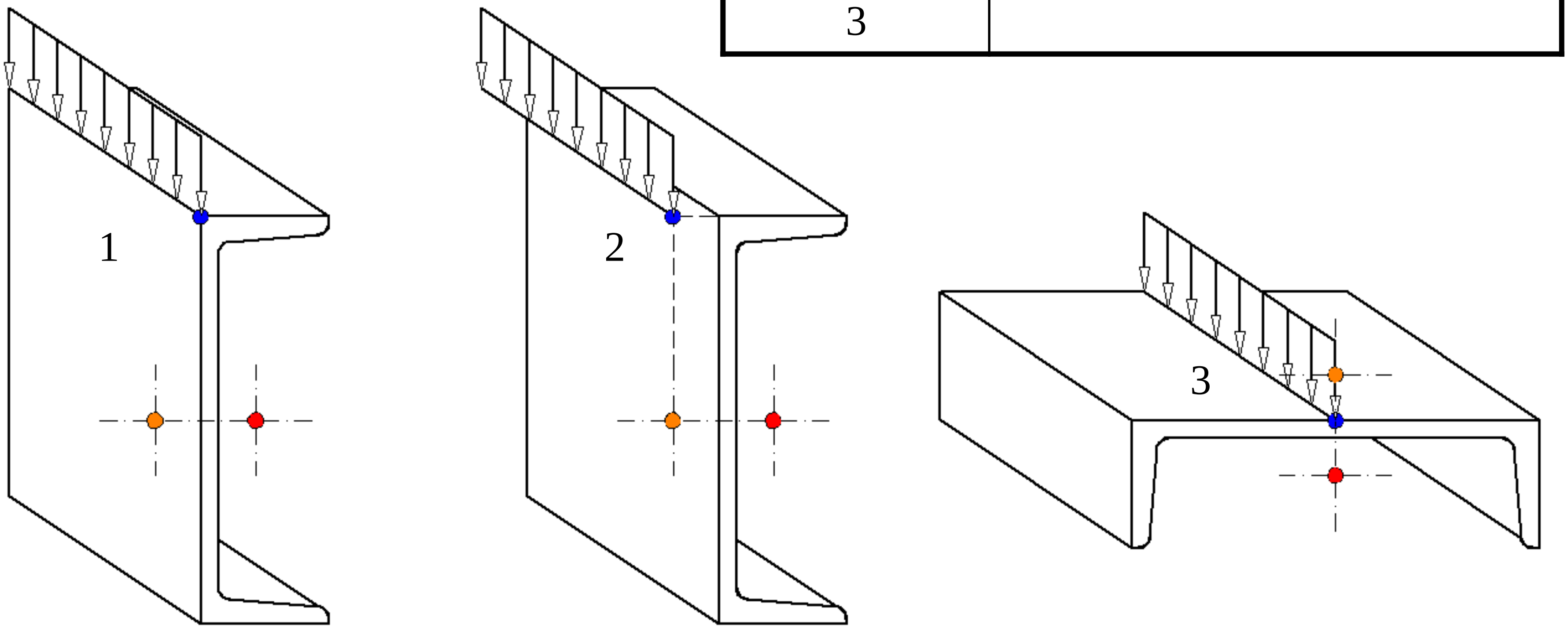
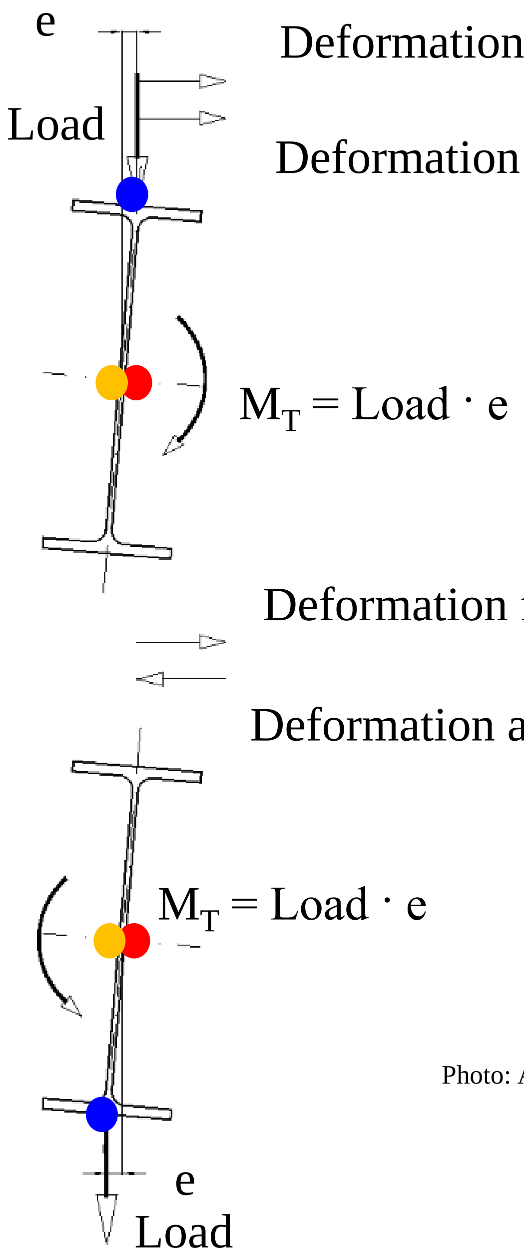


Photo: Author

If a cross-section has an axis of symmetry, the center of gravity and the shear center lie on this axis. For bisymmetric cross-sections (I-beams), these two points coincide. If the direction of the transverse load from point of load application does not pass through the shear center, then in addition to the classical bending and shearing effects, it will also cause torsion in the element:

$$m\ [kNm / m] = q\ [kN / m] \cdot \text{eccentricity}\ (sc - pla)\ [m]$$

Lateral buckling is a form of loss of stability involving simultaneous bending and rotation, so rotation a member with an eccentric load can accelerate buckling.



Deformation from initial imperfection

Deformation as the result of torsional moment

Initial imperfections make eccentricity e and torsional moment M_T as the secondary effect of load.

→ #5 / 77

Deformation from initial imperfection

Deformation as the result of torsional moment

Various points of load’s application makes various effects of deformations. These deformations from torsional moment can intensify or weaken the impact of initial imperfections. As a result, the cross-section may lose stability more easily (smaller M_{cr}) or more difficult (larger M_{cr})

Photo: Author

In the lateral buckling calculations, the distance between the shear center and the center of gravity, denoted z_s , will appear. This distance is measured perpendicular to directon of load. In analysed case: bi-symmetrical I-beam – this distance is equal 0.

In turn, the location of the load application point relative to the center of gravity is very important, as it differentiates the upper and lower flanges. Depending on the selected flange, the value of the distance z_g may be greater or less than 0.

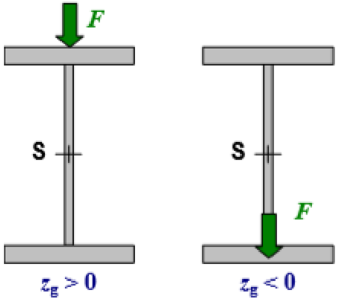


Photo: eurocodes.jrc.ec.europa.eu

Another important parameter is the asymmetry arm: a measure of how asymmetric a particular cross-section is about a specific axis. If it is an axis of symmetry, the asymmetry arm is zero. For bi-symmetric cross-sections, both arms equal zero.

$r_y = r_z = 0$

HEA 800:

$J_y = 303\ 400\ \text{cm}^4$

$J_z = 12\ 640\ \text{cm}^4$

$J_t = 596{,}9\ \text{cm}^4$

$J_w = 18\ 290\ 000\ \text{cm}^6$

$l_{0z} = l_{0T} = 16{,}050\ \text{m}$

$\mu_z = \mu_T = 1{,}0$

$z_s = 0{,}0\ \text{m}$

$i_s = \sqrt{}\,(i_y^2 + i_z^2 + z_s^2) = 0{,}333\ \text{m}$

$N_{cr,\,z} = \pi^2\, EJ_z / (\mu_z\, l_{0z})^2 = 1\ 016{,}988\ \text{kN}$

$N_{cr,\,T} = [\pi^2\, EJ_w / (\mu_T\, l_{0T})^2\ +\ GJ_t] / i_s^2 = 5\ 687{,}192\ \text{kN}$

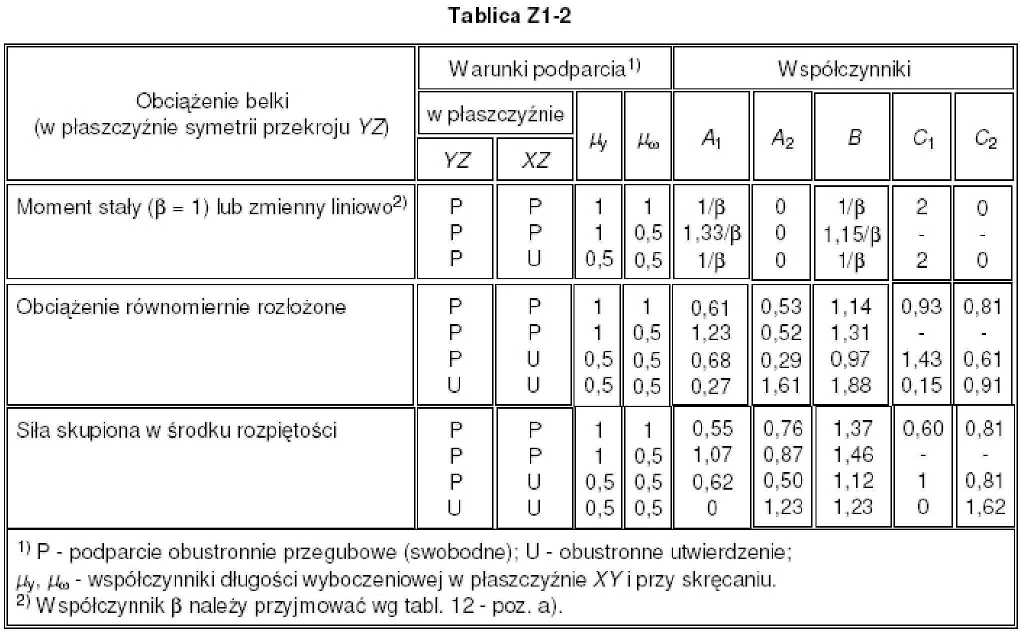
Rough estimate for the I-beam under consideration:

$M_{cr} = i_s \sqrt{}\,(N_{cr,\,z}\, N_{cr,\,T}) = 800{,}500\ \text{kNm}$

A more accurate estimate is possible using the formula

$$M_{cr} = \{\sqrt{[\Xi^2 N_{cr,\,z} N_{cr,\,T} i_s^2 + (\Psi N_{cr,\,z})^2]}\} - \Psi N_{cr,\,z}$$

But the problem is that the Ξ and Ψ coefficients, both in PN and AS, are given for beams with identical support at both ends. This doesn't quite fit the behavior of a two-span beam.



Tablica Z1-2

Obciążenie belki (w płaszczyźnie symetrii przekroju *YZ*)	Warunki podparcia[1]				Współczynniki				
	w płaszczyźnie		μ_y	μ_ω	A_1	A_2	B	C_1	C_2
	YZ	*XZ*							
Moment stały (β = 1) lub zmienny liniowo[2]	P P P	P P U	1 1 0,5	1 0,5 0,5	1/β 1,33/β 1/β	0 0 0	1/β 1,15/β 1/β	2 - 2	0 - 0
Obciążenie równomiernie rozłożone	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,61 1,23 0,68 0,27	0,53 0,52 0,29 1,61	1,14 1,31 0,97 1,88	0,93 - 1,43 0,15	0,81 - 0,61 0,91
Siła skupiona w środku rozpiętości	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,55 1,07 0,62 0	0,76 0,87 0,50 1,23	1,37 1,46 1,12 1,23	0,60 - 1 0	0,81 - 0,81 1,62

[1] P - podparcie obustronnie przegubowe (swobodne); U - obustronne utwierdzenie;
μ_y, μ_ω - współczynniki długości wyboczeniowej w płaszczyźnie *XY* i przy skręcaniu.
[2] Współczynnik β należy przyjmować wg tabl. 12 - poz. a).

Photo: PN B-3200

x → z ; y → x ; z → x

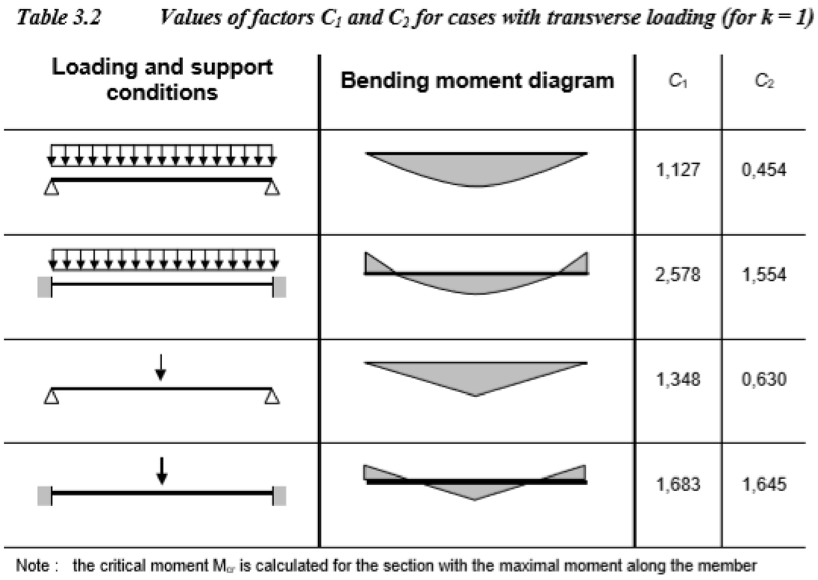


Table 3.2 Values of factors C_1 and C_2 for cases with transverse loading (for k = 1)

Loading and support conditions	**Bending moment diagram**	C_1	C_2
		1,127	0,454
		2,578	1,554
		1,348	0,630
		1,683	1,645

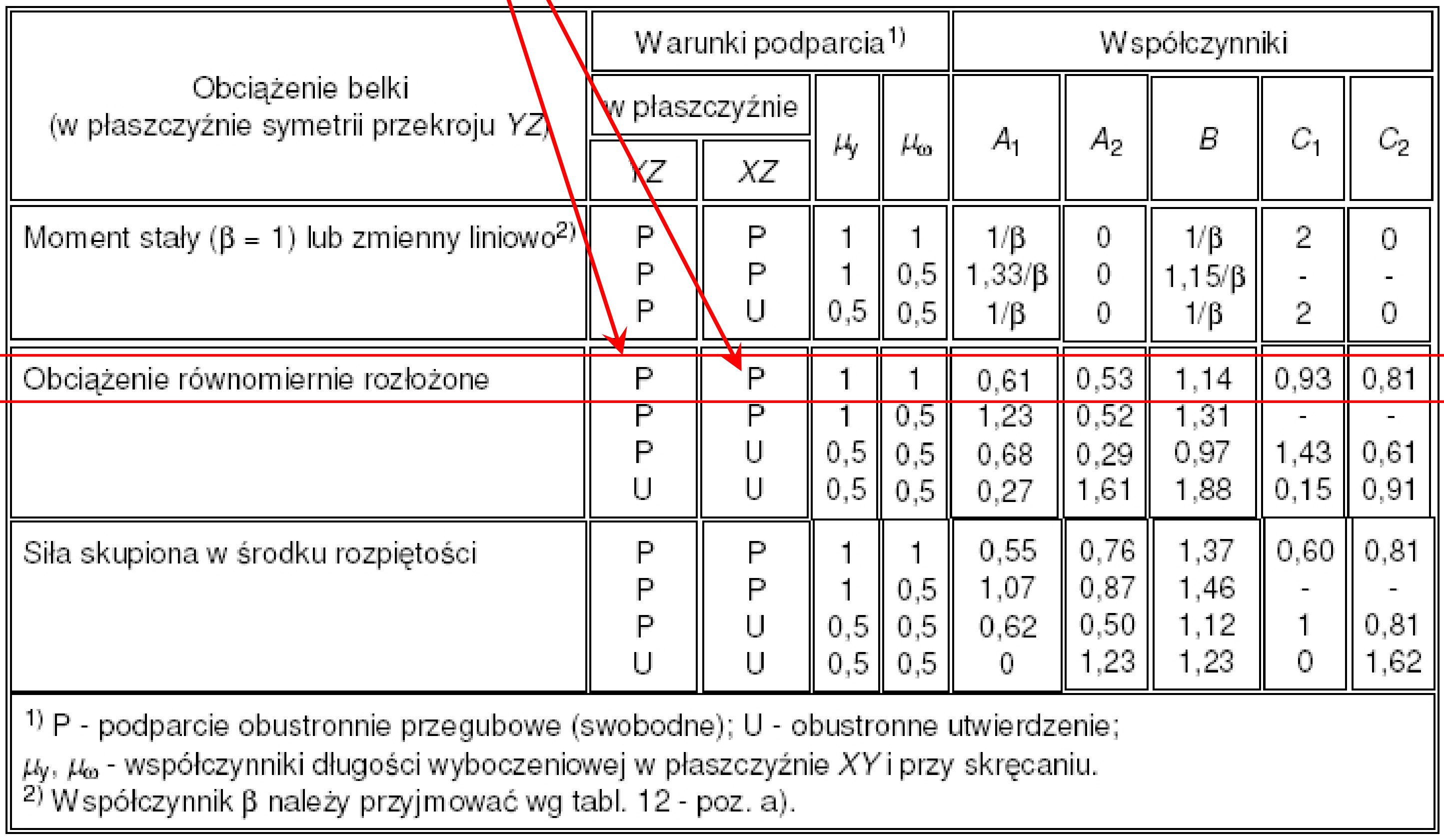
Note : the critical moment M_{cr} is calculated for the section with the maximal moment along the member

Photo: eurocodes.jrc.ec.europa.eu

Estimation according to PN: a more flexible way of supporting the beam is adopted (hinges at both ends P-P; estimation on the safe side)

Complex of multiple concentrated forces on a span can be approximated by a continuous load

Tablica Z1-2



Obciążenie belki (w płaszczyźnie symetrii przekroju YZ)	Warunki podparcia 1)				Współczynniki				
	w płaszczyźnie		μ_y	μ_ω	A_1	A_2	B	C_1	C_2
	YZ	XZ							
Moment stały (β = 1) lub zmienny liniowo 2)	P P P	P P U	1 1 0,5	1 0,5 0,5	1/β 1,33/β 1/β	0 0 0	1/β 1,15/β 1/β	2 - 2	0 - 0
Obciążenie równomiernie rozłożone	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,61 1,23 0,68 0,27	0,53 0,52 0,29 1,61	1,14 1,31 0,97 1,88	0,93 - 1,43 0,15	0,81 - 0,61 0,91
Siła skupiona w środku rozpiętości	P P P U	P P U U	1 1 0,5 0,5	1 0,5 0,5 0,5	0,55 1,07 0,62 0	0,76 0,87 0,50 1,23	1,37 1,46 1,12 1,23	0,60 - 1 0	0,81 - 0,81 1,62

1) P - podparcie obustronnie przegubowe (swobodne); U - obustronne utwierdzenie;
μ_y, μ_ω - współczynniki długości wyboczeniowej w płaszczyźnie XY i przy skręcaniu.
2) Współczynnik β należy przyjmować wg tabl. 12 - poz. a).

x → z ; y → x ; z → x

Photo: PN B-3200

Symbols according to PN:

$\Xi = B = 1{,}14$

$\Psi = A_0 = A_1\, b_y + A_2\, a_s$

$A_1 = 0{,}61$

$A_2 = 0{,}53$

$b_y = y_s – r_y / 2$

r_y – arm of assymetry; for bi-symmetrical cross-section = 0

y_s – distance shear center – gravity center; for bi-symmetrical cross-section = 0

a_s – distane gravity center – point of load application; in this case = 49,5 cm

$\Psi = A_0 = 0 + A_2\, a_s = 0{,}262$ m

$$M_{cr} = \{\sqrt{[\Xi^2\, N_{cr,\, z}\, N_{cr,\, T}\, i_s{}^2 + (\Psi\, N_{cr,\, z})^2]}\} - \Psi\, N_{cr,\, z}$$

$M_{cr} = 684{,}606$ kNm

Estimation according to AS: a more flexible way of supporting the beam is adopted (hinges at both ends; estimation on the safe side

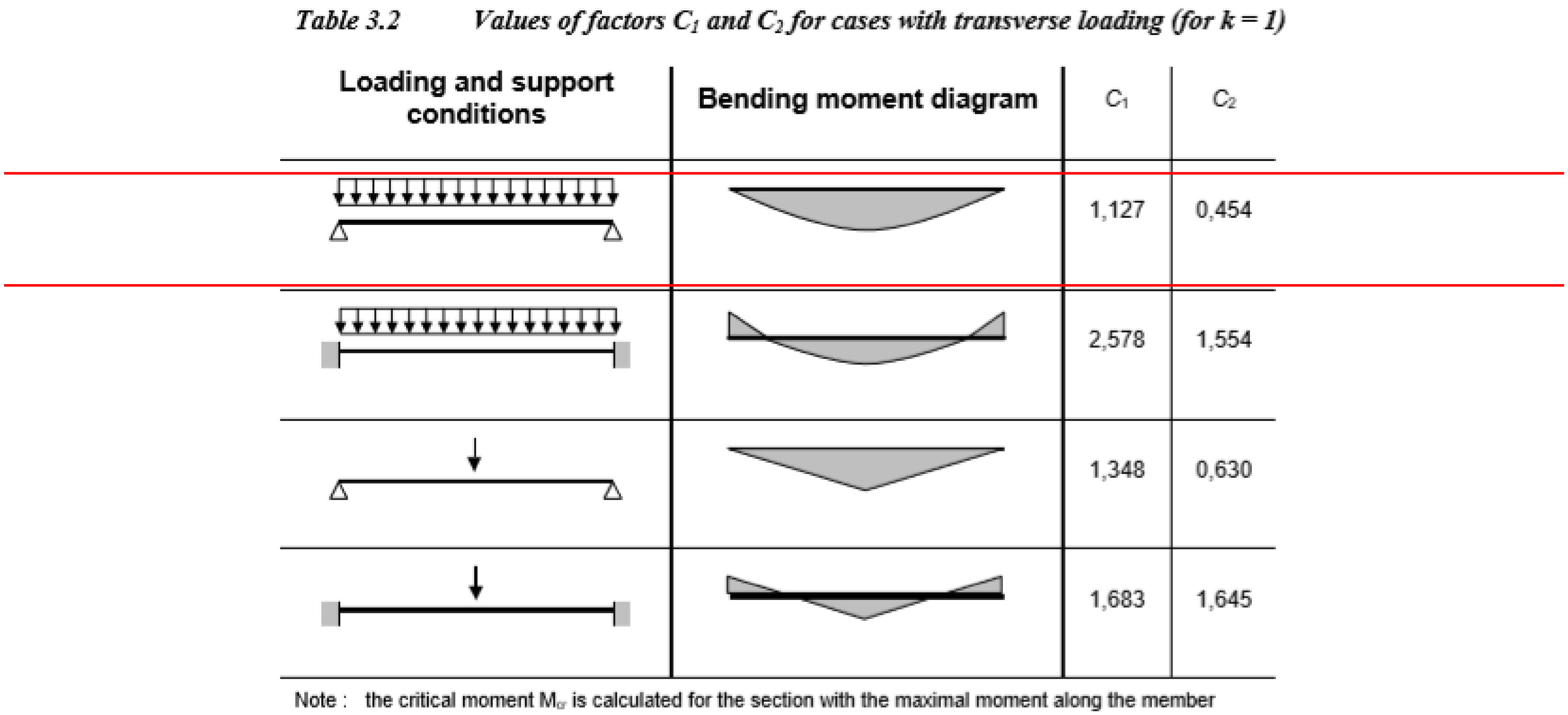


Table 3.2 Values of factors C_1 and C_2 for cases with transverse loading (for $k = 1$)

Loading and support conditions	Bending moment diagram	C_1	C_2
		1,127	0,454
		2,578	1,554
		1,348	0,630
		1,683	1,645

Note : the critical moment M_{cr} is calculated for the section with the maximal moment along the member

Photo: eurocodes.jrc.ec.europa.eu

Symbols according to AS:

$\Xi = C_1 = 1,127$

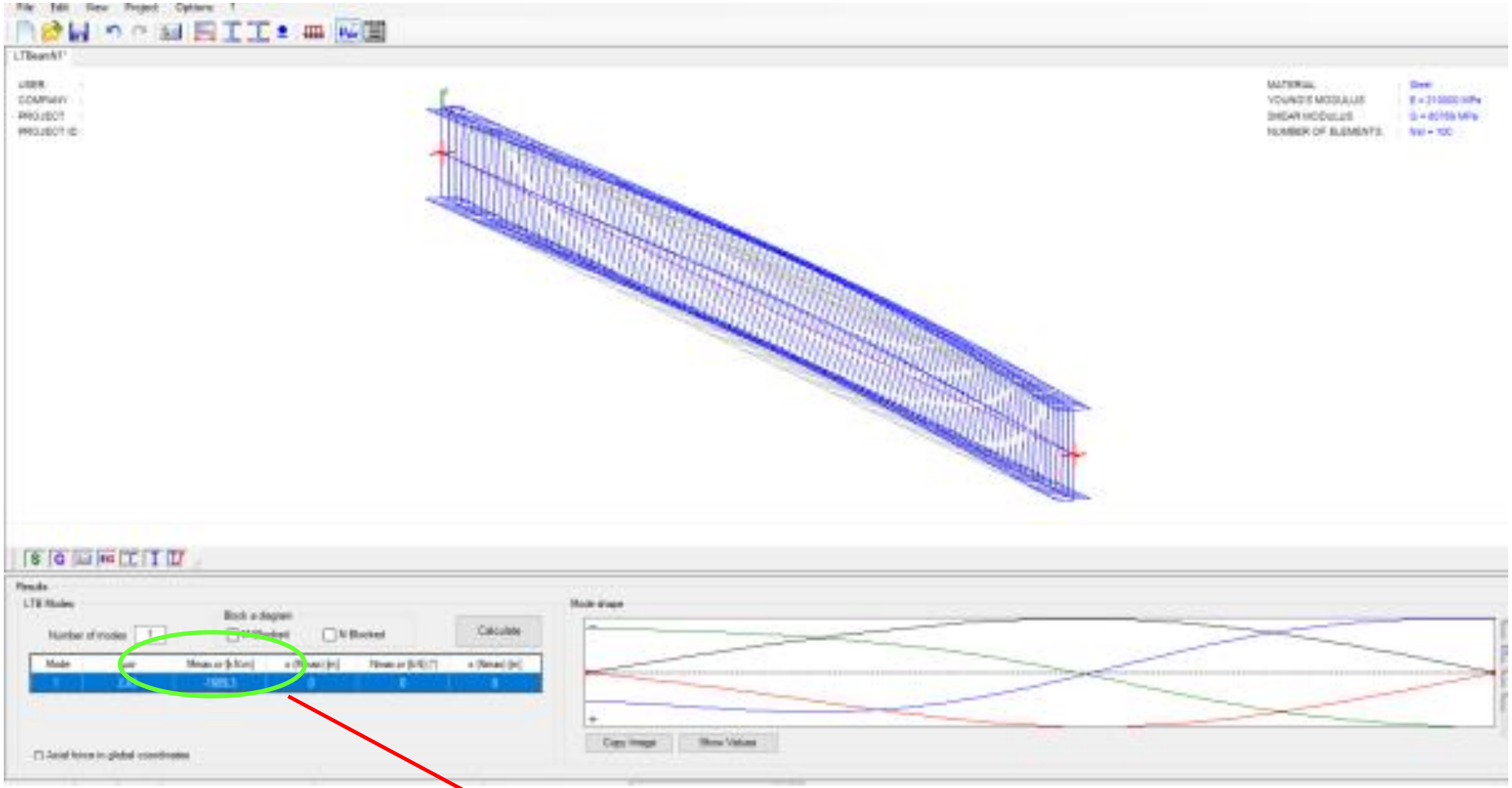
$\Psi = C_1 C_2 z_g = 1,127 \cdot 0,454 \cdot 0,495 \text{ m} = 0,235 \text{ m}$

$$M_{cr} = \{\sqrt{[\Xi^2 N_{cr,\,z} N_{cr,\,T} i_s^2 + (\Psi N_{cr,\,z})^2]}\} - \Psi N_{cr,\,z}$$

$M_{cr} = 666,698 \text{ kNm}$

Calculations using the LT-Beam program:

Photo: Author



M_{cr} = 1 809,3 kNm

Rougt estimate	PN	AS	LT-Beam
800,500 kNm	684,606 kNm	666,698 kNm	1 809,3 kNm

In Your project, consider one of these three methods.

For the next step of calculations, value from LT-Beam is assumed

$$M_{cr} = 1\ 809{,}3\ \text{kNm}$$

Lateral buckling

EN 1993-1-1   6.3.2

$\lambda_{LT} =$ $= \sqrt{(W_y\, f_y / M_{cr})}$	EJ = const	$\Phi_{LT} = [1 + \alpha_{LT} (\lambda_{LT} - 0{,}2) + \lambda_{LT}^2] / 2$ $\alpha_{LT} \rightarrow$ tab. 6.3, 6.4, EN 1993-1-1	$\chi_{LT} = \min\{$ $1/[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}]$; $1{,}0\}$
	Hot-roled and welded I-sections	$\Phi_{LT} =$ $= [1 + \alpha_{LT} (\lambda_{LT} - 0{,}4) + 0{,}75\, \lambda_{LT}^2] / 2$ $\alpha_{LT} \rightarrow$ tab. 6.3, 6.5, EN 1993-1-1	$\chi_{LT} = \min\{$ $1/[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \lambda_{LT}^2)}]$; $1/ \lambda_{LT}^2$; $1{,}0\}$

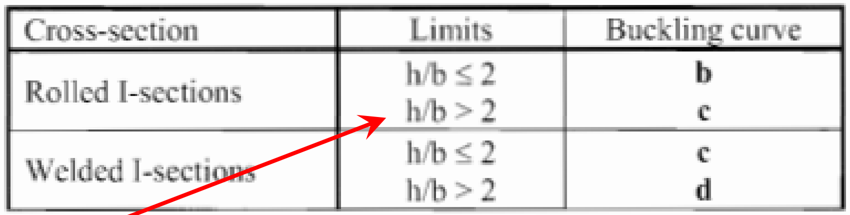
$\lambda_{LT} \leq 0{,}4 \rightarrow \chi_{LT} = 1{,}0$

The Eurocode is rather unclear about the two proposals (I-sections usually have EJ = const). This probably refers to the demarcation between I-sections and other elements (channels) of constant cross-section.

$\lambda_{LT} = \sqrt{}\,(W_y\, f_y / M_{cr}) = 1{,}707$

Imperfections: buckling curve c

(depth of cross-section / width of flange)



Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**b** **c**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**

Photo: EN 1993-1-1 tab. 6.5

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

Photo: EN 1993-1-1 tab. 6.3

$\alpha_{LT} = 0{,}49$

$\Phi_{LT} = [1 + \alpha_{LT}\,(\lambda_{LT} - 0{,}4) + 0{,}75\,\lambda_{LT}{}^2] / 2 = 1{,}926$

$\chi_{LT} = \min\{\ 1/[\Phi_{LT} + \sqrt{}\,(\Phi_{LT}{}^2 - \lambda_{LT}{}^2)]\ \ ;\ \ 1/\,\lambda_{LT}{}^2\ \ ;\ \ 1{,}0\ \} =$

$= \min\ (\ 0{,}355\ \ ;\ \ 0{,}582\ \ ;\ \ 1{,}0\ ) = 0{,}355$

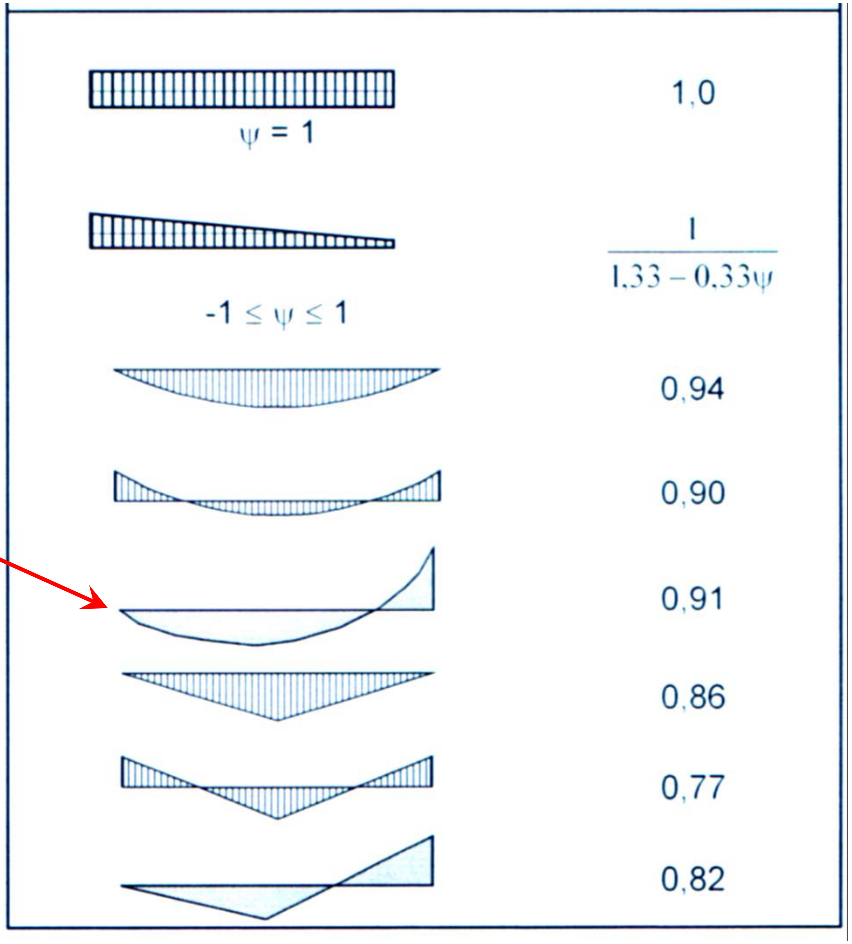
According to Eurocode EN 1993-1-1 6.3.2.2.(2), the distribution of bending moments along the length of the beam must also be taken into account.

k_c:

$$\chi_{LT,\, mod} = \min\left(\chi_{LT} / f \quad ; \quad 1{,}0\right)$$

$$f = \min\left\{1 - 0{,}5(1-k_c)[1 - 2(\lambda_{LT} - 0{,}8)^2] \quad ; \quad 1{,}0\right\}$$

In the case under consideration, the moment distribution looks like this:



$\psi = 1$	1,0
$-1 \le \psi \le 1$	$\frac{1}{1.33 - 0.33\psi}$
	0,94
	0,90
	0,91
	0,86
	0,77
	0,82

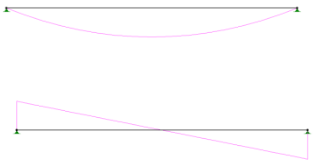
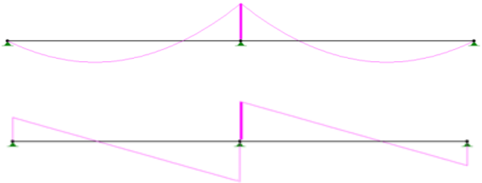
Photo: EN 1993-1-1 tab 6.6

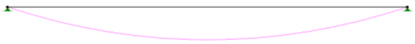
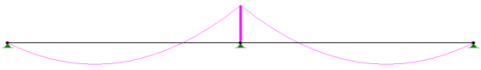
$$k_c = 0{,}91$$

$$f = \min\left(1{,}029 \quad ; \quad 1{,}0\right)$$

$$\chi_{LT,\, mod} = 0{,}355$$

$$M_{Ed} / (\chi_{LT,\, mod}\, M_{Rd}) = 0{,}611 < 1{,}0$$ 👍

Erection stage - resistance	Secondary beam	Primary beam
Static scheme	One-span beam, L = 5,220 m	Two-span,beam, L = 16,050 m
Supports	2 times hinge; bolted joint to vertical stiffeners on primary beam	3 times hinge; inside masonry wall and over column
Loads	q = conts	Gropup of forces in points; so dense that they can be roughly treated as q = conts
Distribution of cross-sectional forces	 Photo: Author	
Shear resistance	Yes	Yes
Interaction shear force – bendint moment	No (for max M, V = 0; for max V, M=0)	Yes
Bending resistance	Yes	Yes

Erection stage – lateral buckling	Secondary beam	Primary beam
Initian assumption – one-span beam	Yes	Two-span, but one-span will be taken into consideration as approximation
Supports	2 times hinge; bolted joint to vertical stiffeners on primary beam → no prevention for deplanation → one-span beam	3 times hinge; inside masonry wall and over column → no prevention for deplanation → situation estimated by one-span beam
Loads	q = conts	Gropup of forces in points; so dense that they can be roughly treated as q = conts
Parameters according to PN / AS	$A_1 = 0,61$; $A_2 = 0,53$; $B = 1,14$; $C_1 = 0,93$; $C_2 = 0,81$	
	$C_1 = 1,127$; $C_2 = 0,454$	
Distribution of bending moments; final recalculation of χ_{LT} (factor k_c)	 Photo: Author	

Exploitation stage: shear force

V_{Ed} = 1 615,914

V_{Rd} = 2 428,768 kN

V_{Ed} / V_{Rd} = 0,665 < 1,0 👍

But, at now, M_{Ed} is not from stati calculation, but from **redistribution of bending moments**

Redistribution of bending moments

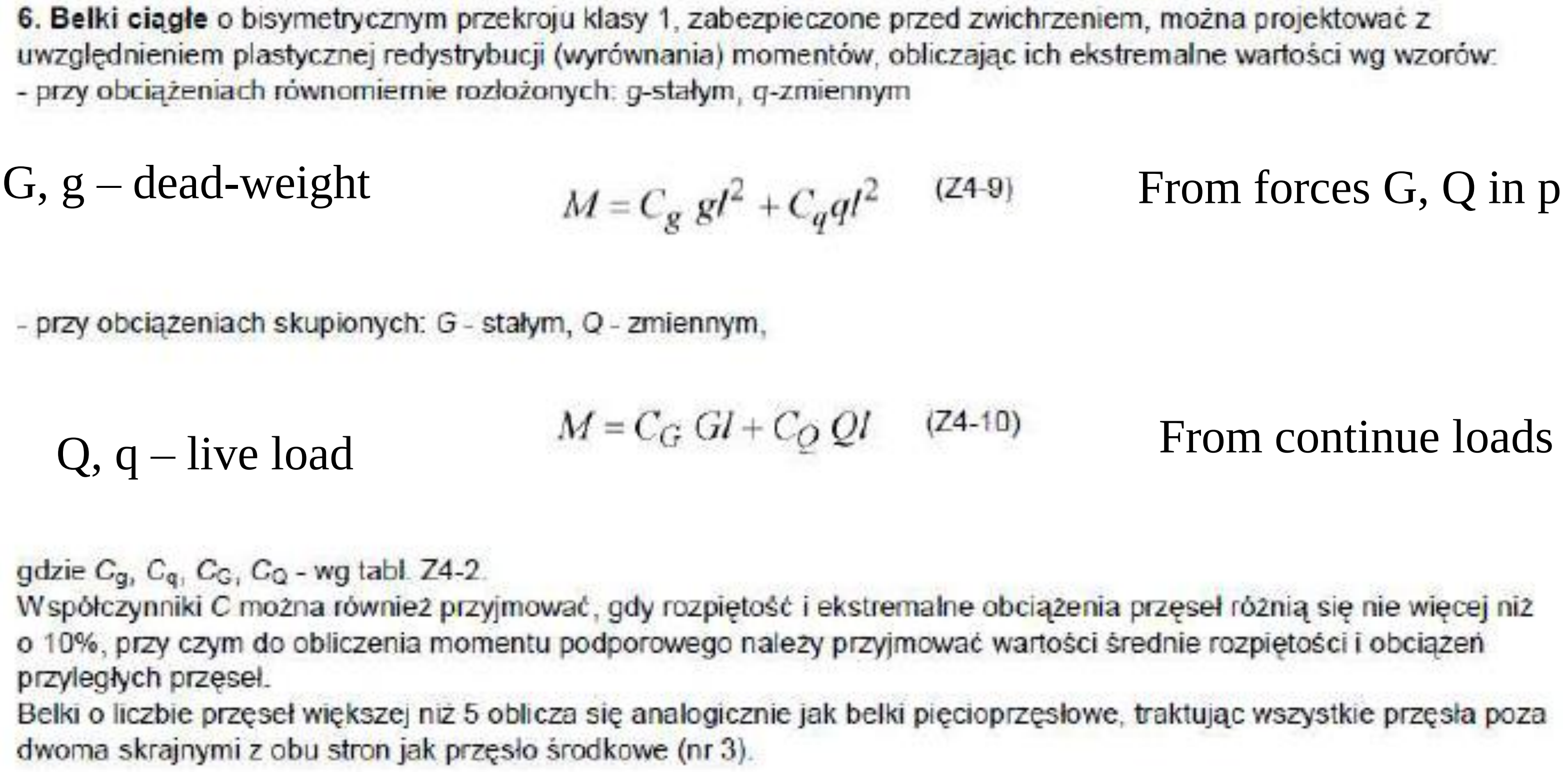
There are two methods of recalculations of bending moments as an effect of redistribution.

Method	Time	M_{Ed}	V_{Ed}	Accuracy
"Table"	👍	👍	👎	👎
"Graphical"	👎	👍	👍	👍

→ #11 / 78

→ #11 / 80

Table: PN B 3200



6. Belki ciągłe o bisymetrycznym przekroju klasy 1, zabezpieczone przed zwichrzeniem, można projektować z uwzględnieniem plastycznej redystrybucji (wyrównania) momentów, obliczając ich ekstremalne wartości wg wzorów:

- przy obciążeniach równomiernie rozłożonych: g-stałym, q-zmiennym

G, g – dead-weight

$$M = C_g\, gl^2 + C_q ql^2 \qquad \text{(Z4-9)}$$

From forces G, Q in points

- przy obciążeniach skupionych: G - stałym, Q - zmiennym,

Q, q – live load

$$M = C_G\, Gl + C_Q\, Ql \qquad \text{(Z4-10)}$$

From continue loads g, q

gdzie C_g, C_q, C_G, C_Q - wg tabl. Z4-2.

Współczynniki C można również przyjmować, gdy rozpiętość i ekstremalne obciążenia przęseł różnią się nie więcej niż o 10%, przy czym do obliczenia momentu podporowego należy przyjmować wartości średnie rozpiętości i obciążeń przyległych przęseł.

Belki o liczbie przęseł większej niż 5 oblicza się analogicznie jak belki pięcioprzęsłowe, traktując wszystkie przęsła poza dwoma skrajnymi z obu stron jak przęsło środkowe (nr 3).

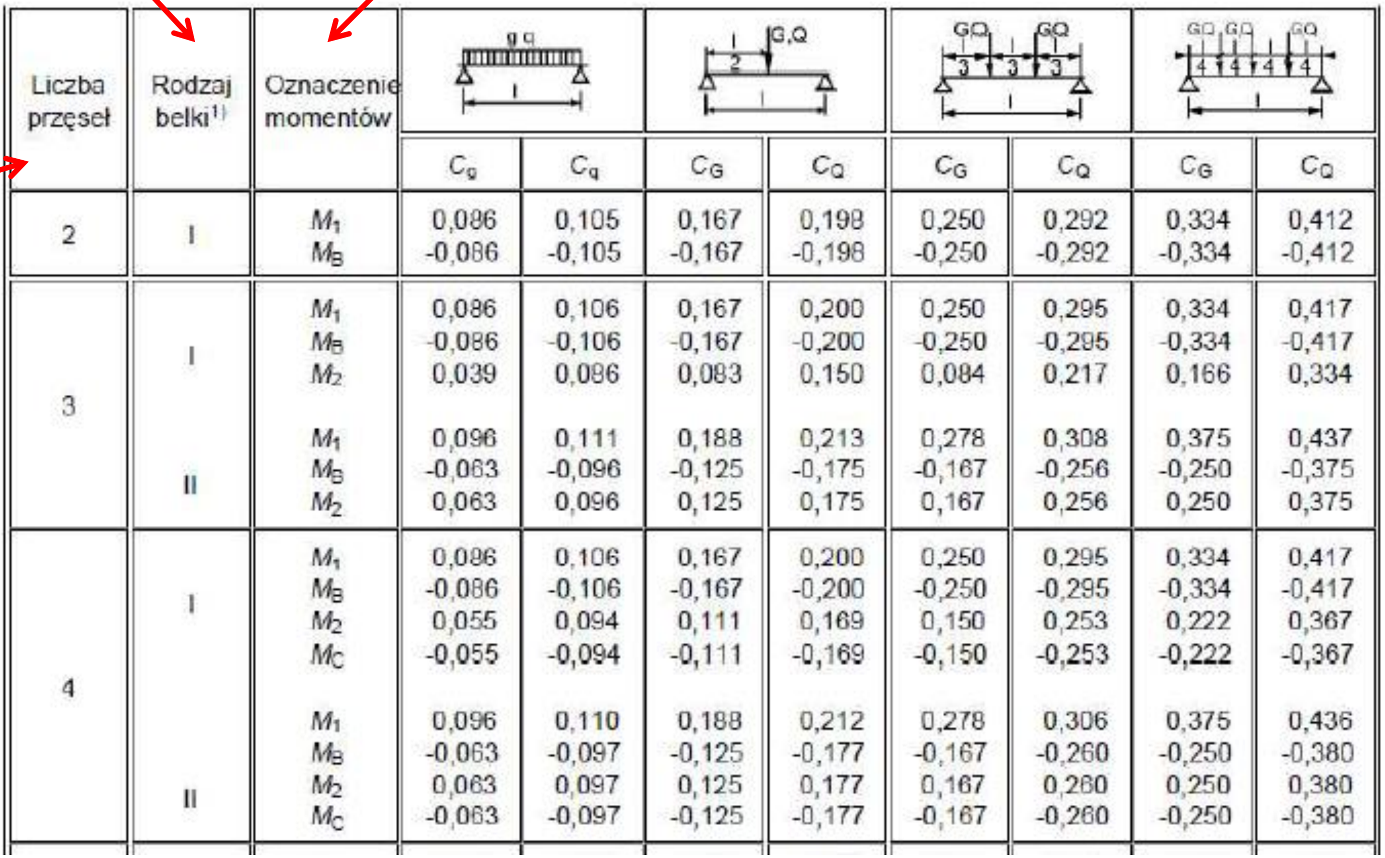
Moments (→ #11 / 82):

Stiffness distribution along beam (→ #11 / 82)

1, 2, 3 – in spans ; A, B, C – over supports

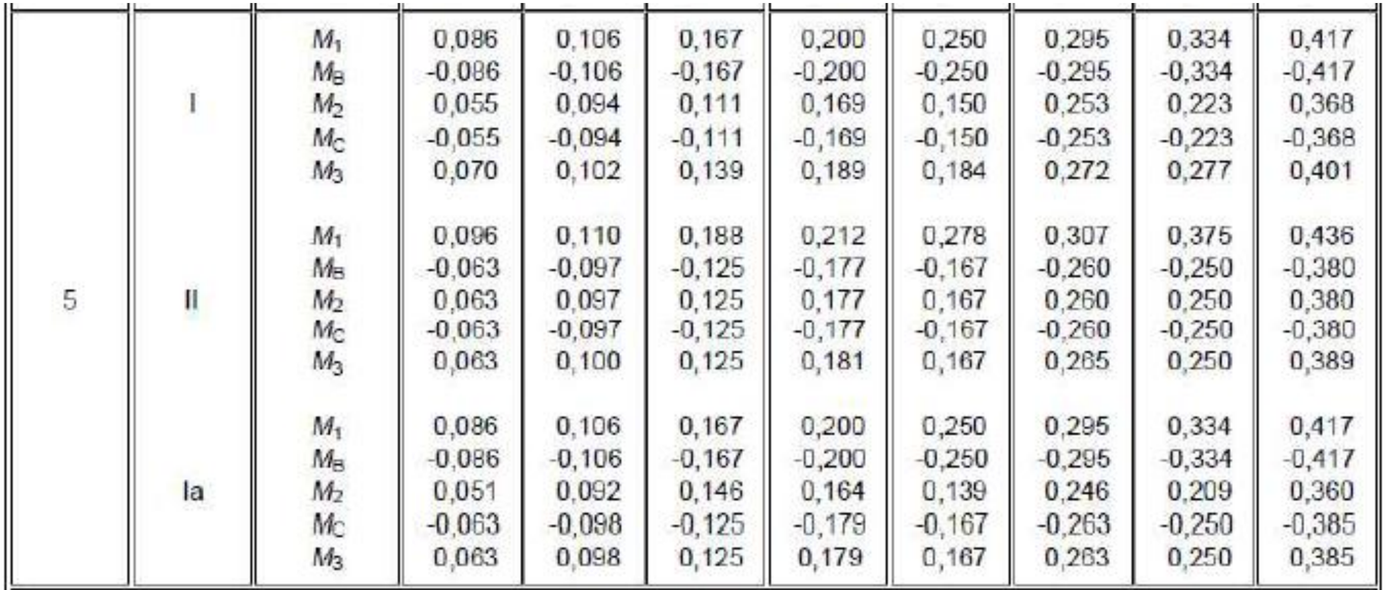
Type of action

Number of spans



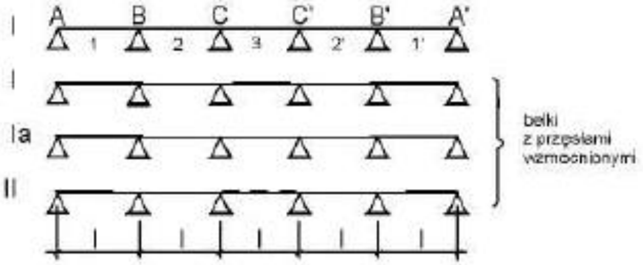
Liczba przęseł	Rodzaj belki¹⁾	Oznaczenie momentów	g q		G,Q		GQ GQ		GQ GQ GQ	
			C_g	C_q	C_G	C_Q	C_G	C_Q	C_G	C_Q
2	I	M_1	0,086	0,105	0,167	0,198	0,250	0,292	0,334	0,412
		M_B	-0,086	-0,105	-0,167	-0,198	-0,250	-0,292	-0,334	-0,412
3	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,039	0,086	0,083	0,150	0,084	0,217	0,166	0,334
	II	M_1	0,096	0,111	0,188	0,213	0,278	0,308	0,375	0,437
		M_B	-0,063	-0,096	-0,125	-0,175	-0,167	-0,256	-0,250	-0,375
		M_2	0,063	0,096	0,125	0,175	0,167	0,256	0,250	0,375
4	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,222	0,367
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,222	-0,367
	II	M_1	0,096	0,110	0,188	0,212	0,278	0,306	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380

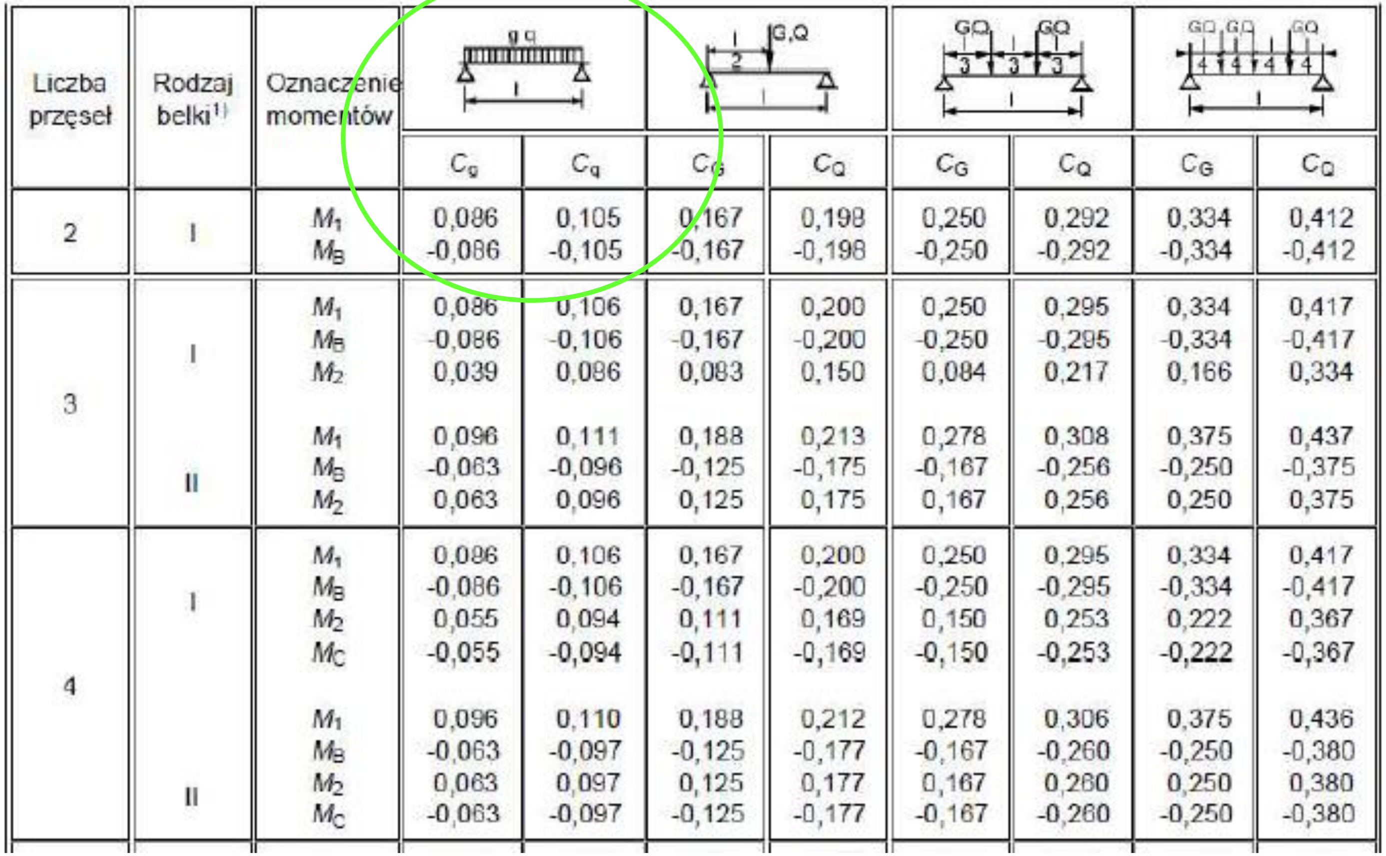
→ #11 / 81



5	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,223	0,368
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,223	-0,368
		M_3	0,070	0,102	0,139	0,189	0,184	0,272	0,277	0,401
	II	M_1	0,096	0,110	0,188	0,212	0,278	0,307	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_3	0,063	0,100	0,125	0,181	0,167	0,265	0,250	0,389
	Ia	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,051	0,092	0,146	0,164	0,139	0,246	0,209	0,360
		M_C	-0,063	-0,098	-0,125	-0,179	-0,167	-0,263	-0,250	-0,385
		M_3	0,063	0,098	0,125	0,179	0,167	0,263	0,250	0,385

1)





Liczba przęseł	Rodzaj belki$^{1)}$	Oznaczenie momentów								
			C_g	C_q	C_G	C_Q	C_G	C_Q	C_G	C_Q
2	I	M_1	0,086	0,105	0,167	0,198	0,250	0,292	0,334	0,412
		M_B	-0,086	-0,105	-0,167	-0,198	-0,250	-0,292	-0,334	-0,412
3	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,039	0,086	0,083	0,150	0,084	0,217	0,166	0,334
	II	M_1	0,096	0,111	0,188	0,213	0,278	0,308	0,375	0,437
		M_B	-0,063	-0,096	-0,125	-0,175	-0,167	-0,256	-0,250	-0,375
		M_2	0,063	0,096	0,125	0,175	0,167	0,256	0,250	0,375
4	I	M_1	0,086	0,106	0,167	0,200	0,250	0,295	0,334	0,417
		M_B	-0,086	-0,106	-0,167	-0,200	-0,250	-0,295	-0,334	-0,417
		M_2	0,055	0,094	0,111	0,169	0,150	0,253	0,222	0,367
		M_C	-0,055	-0,094	-0,111	-0,169	-0,150	-0,253	-0,222	-0,367
	II	M_1	0,096	0,110	0,188	0,212	0,278	0,306	0,375	0,436
		M_B	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380
		M_2	0,063	0,097	0,125	0,177	0,167	0,260	0,250	0,380
		M_C	-0,063	-0,097	-0,125	-0,177	-0,167	-0,260	-0,250	-0,380

Based on #t / 4: $g = 19{,}584$ kN / m $q = 57{,}993$ kN / m

$$\mathbf{M_1 = |M_B| = 0{,}086 \cdot 19{,}584\ [kN / m] \cdot (16{,}05\ [m])^2 + 0{,}105 \cdot 57{,}993\ [kN / m] \cdot (16{,}05\ [m])^2 =}$$

$$\mathbf{= 2\ 002{,}470\ kNm}$$

Exploitation stage, verification:

Redistribution:

$M_{\text{exploit, „normal”, asym, Ed, span}} < \mathbf{M_{\text{span , redistribution}}} = \mathbf{M_{\text{support, redistribution}}} < M_{\text{exploit, „normal”, sym, Ed, support}}$

1 452,384 < 2 002,470 < 2 593,542

OK, redistribution correctly

Erection stage: interaction shear force – bending moment

Above the central support, we have a large bending moment and shear force simultaneously. In this situation, stress concentrations from bending and shear occur. We calculate this by applying a reduction in the bending resistance if the bending stress exceeds 50%. In analysed case is 66,5%.

EN 1993-1-1 (6.12), (6.13), (6.29), (6.30)

Unreduced resistance, Ist cross-section class:

$M_{Rd} = W_{y,\,pl}\, f_y / \gamma_{M0} = 3\ 088{,}145$ kNm

Reductin:

$\rho = [(2\, V_{Ed} / V_{Rd}) - 1]^2 = 0{,}109$

$M_{Rd,\,red} = \min \{M_{Rd} \quad ; \quad [W_{y,\,pl} - \rho\, (A_w)^2 / (4\, t_w)]\, f_y / \gamma_{M0}\ \} = 3\ 022{,}115$ kNm

Sprawdzenie nośności na zginanie:

$M_{Ed} = 2\ 002{,}470$ kNm

$M_{Ed} / M_{Rd,\,red} = 0{,}663 < 1{,}0$ 👍

Lateral buckling: difference in relation to erection stage

Currently, the upper flange is stiffened by a reinforced concrete slab. The only loss of stability occurs in the section where the lower flange is compressed.

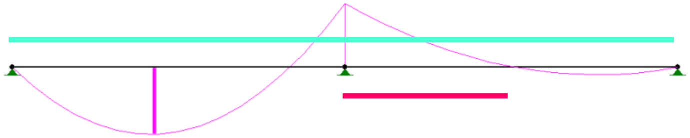


Photo: Author

The compression range of the lower flange is greatest for asymmetric loading.

In analysed case: $l_{0z} = l_{0T} = 7{,}658$ m

he adjacent sections: the second span and the section of the span that does not lose stability make it possible to estimate the buckling length coefficients as

$$\mu_z = \mu_T = 0{,}707$$

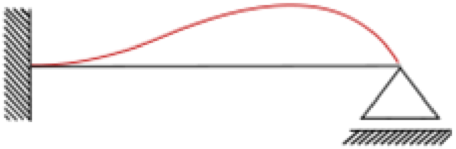


Photo: wikipedia

Rest calculation are the same as for previous case; only critical length is different.

M_{cr} (PN) = 2 769,308 kNm

M_{cr} (AS) = 2 857,686 kNm

For the next step of calculations, the minimum (PN ; AS) was assumed:

M_{cr} = 2 769,308 kNm

$\chi_{LT} = 0{,}732$

$f = 0{,}964$

$\chi_{LT,\ mod} = 0{,}767$

$M_{Ed} / (\chi_{LT,\ mod}\ M_{Rd}) = 0{,}864 < 1{,}0$ 👍

Deflections:

Two methods:

- Value fro mstatic calculations;
- Approximation value:

$\Delta = 0{,}50\ [5\ g\ L^4 / (384\ E\ J_y)] + 0{,}75\ [5\ q\ L^4 / (384\ E\ J_y)]$

where

Δ - deflection; g – dead-weight; q – live load; L – one-span length

Member	Accepted value
Main roof girder (truss or beam)	L / 250
Purlin	L / 200
Corrugated sheel - roofing	L / 150
Floor girder: primary beam secondary beam	 L / 350 L / 250
Door head or window head	L / 500

Various values for various part of structurews

EN 1993-1-1 N.A. 22

In analysed case:

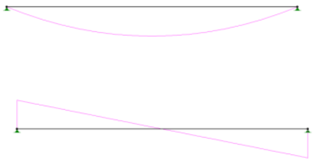
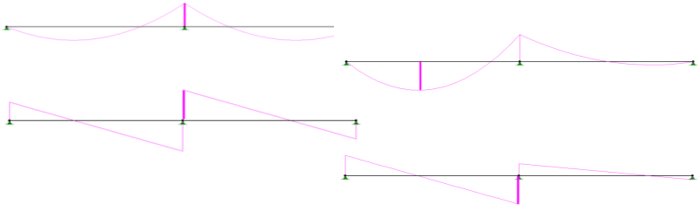
$\Delta = 0{,}039$ m

$\Delta_{dop} = 16{,}05 / 350 = 0{,}046$ m

$\Delta / \Delta_{dop} = 0{,}840 < 1{,}0$ 👍

Efforts

Stage	Shear force resistance	Interaction shear force – bending moment	Bending moment resistance	Lateral buckling	Deflection
I	0,172	≤ 0,50	0,217	0,611	
II	0,665	> 0,50	0,663	**0,864**	0,840

Exploitation stage - resistance	Secondary beam	Primary beam
Static scheme	One-span beam, L = 5,220 m	Two-span,beam, L = 16,050 m
Supports	2 times hinge; bolted joint to vertical stiffeners on primary beam	3 times hinge; inside masonry wall and over column
Loads	q = conts	Gropup of forces in points; so dense that they can be roughly treated as q = conts
Distribution of cross-sectional forces	 Photo: Author	
Shear resistance	Yes	Yes
Interaction shear force – bendint moment	No (for max M, V = 0; for max V, M=0)	Yes
Bending resistance	Yes	Yes

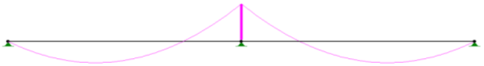
Exploitation stage – lateral buckling	Secondary beam	Primary beam
Initian assumption – one-span beam	No (flange under compression is fully protected by concrete plate)	Two-span, but one-span will be taken into consideration as approximation
Supports		3 times hinge; inside masonry wall and over column → no prevention for deplanation → situation estimated by one-span beam
Loads		Gropup of forces in points; so dense that they can be roughly treated as q = conts
Parameters according to PN / AS		A_1 = 0,61 ; A_2 = 0,53 ; B = 1,14 ; C_1 = 0,93 ; C_2 = 0,81
		C_1 = 1,127 ; C_2 = 0,454
Distribution of bending moments; final recalculation of χ_{LT} (factor k_c)		Photo: Author 

Photo: mscsteel.com

IInd example of calculations – hinge joint

Hinge joint, secondary beam - primary beam

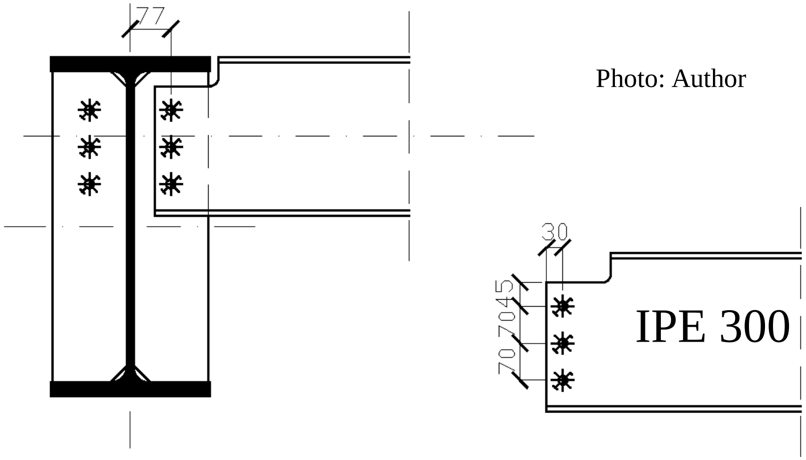


Photo: Author

(New complex of data):

Span of secondary beam L = 3,00 m

Vertical force = reaction at the end of secondary beam V_{Ed} = 90,50 kN

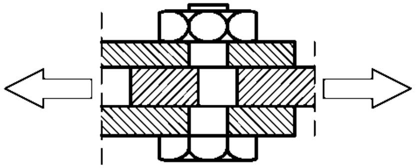
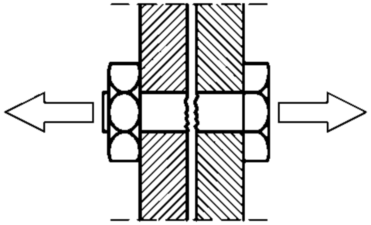
Secondary bending moment $M_{Ed} = 0{,}077 \cdot 90{,}50 = 6{,}969$ kNm

Analysis of bolted joint concerns few important questions:

- Initial analysis (category of joint, class of bolt, dimension, length, geometry) → #t / 46 - 51
- Stiffness of joint (according assumption – hinge joint) → #t / 52 - 58
- Distribution of extrenal actions between bolts → #t / 59 - 61
- Checking of resistances → #t / 62 - 88
- Stiffener → example Vth
- Conclusions → #t / 89

→ #15 / 13

Categories of bolted joints and loads

					
Categories of bolted joint	A	B	C	D	E
Types of loads	Static without changing the direction of the bending moments; aerodynamic	Static with changing the direction of the bending moments; aerodynamic	Dynamic	Static; aerodynamic	Dynamic
Types of bolts	„normal”	preloaded		„normal”	preloaded

Changing the direction of the bending moment:
various combinations of loads

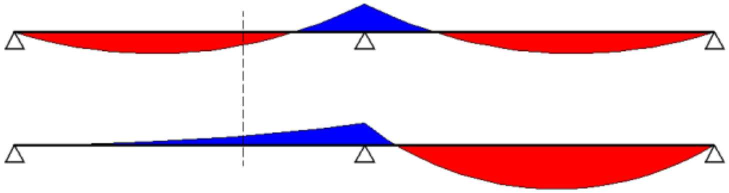


Photo: Author

Bolts:

Category	Class	Diameter
A, D	4.6, 4.8	d =16 mm
	5.6, 5.8, 6.8	d = 20 - 24 mm
	8.8, 10.9	d ≥ 24 mm
B	8.8, 10.9	d ≥ 24 mm
C, E		

Symbol:

M16 → d = 16mm

Class: X.Y

$X = f_{ub} / 100 \rightarrow f_{ub} = 100\ X$

$Y = 10\ f_{yb} / f_{ub} \rightarrow f_{yb} = 10\ X\ Y$

Class: 4.8

$f_{ub} = 400$ MPa

$f_{yb} = 320$ MPa

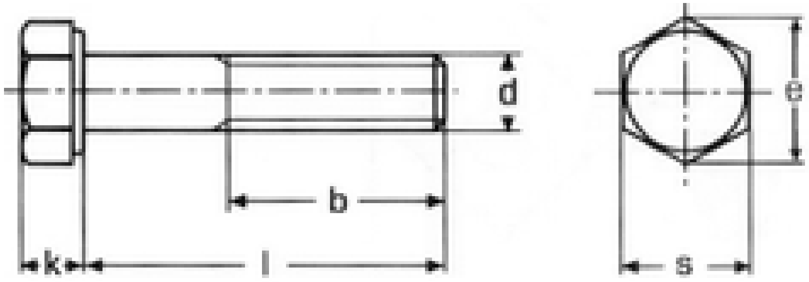


Photo: Author

Recommendation, diameter of bolts: function of thickness of elements

$1{,}5\ t_{min} \leq d \leq 2{,}5\ t_{min}$

A, D: $\Sigma t \leq 5d$

B, C, E: $\Sigma t \leq 8d$

$f_{ub} > f_u$

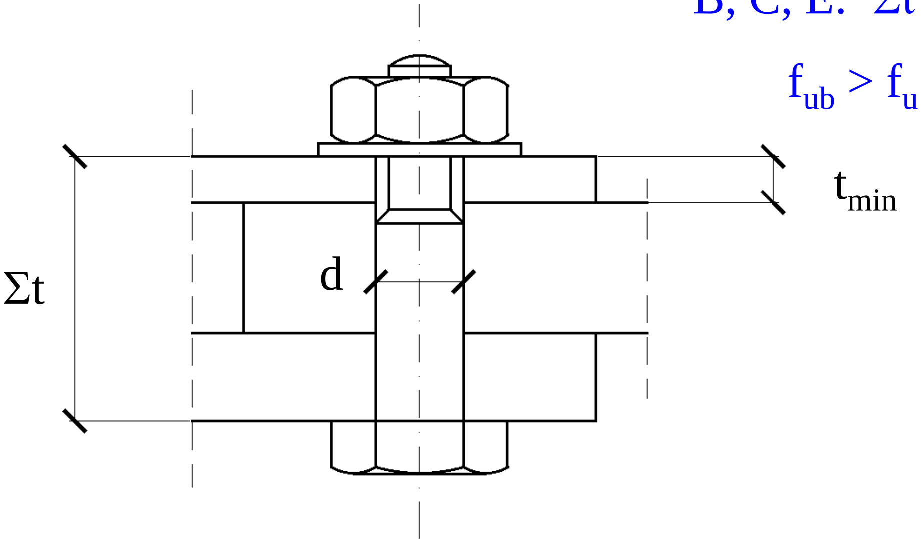


Photo: Author

Recommendation type of bolt for category A: type SB with a thread along part of shank length according to EN 4014

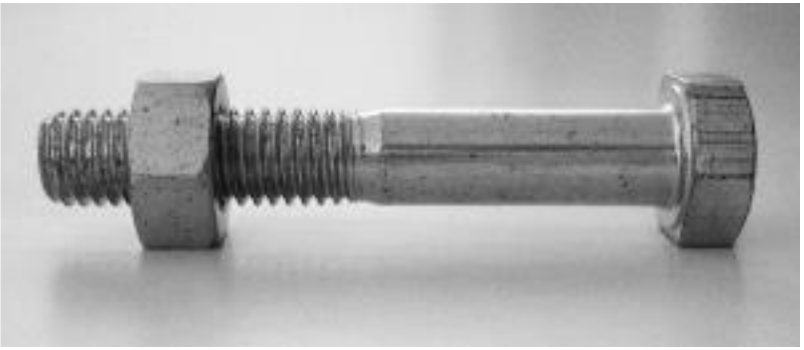


Photo: wikipedia

Assumptions: static action without change of direction of actions: joint category A, bolts M16, class 4.8

t_{min} = thickness of web IPE 300 = 7,1 mm

$$1{,}5\ t_{min} = 11\ \text{mm} \leq d = 16\ \text{mm} \leq 2{,}5\ t_{min} = 18\ \text{mm}$$

OK

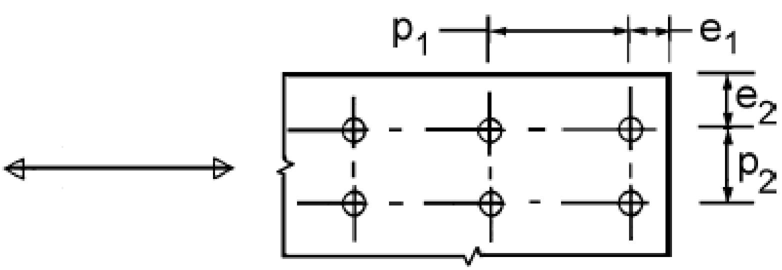
$$\text{A:}\ \ \Sigma t \leq 5d = 80\ \text{mm}$$

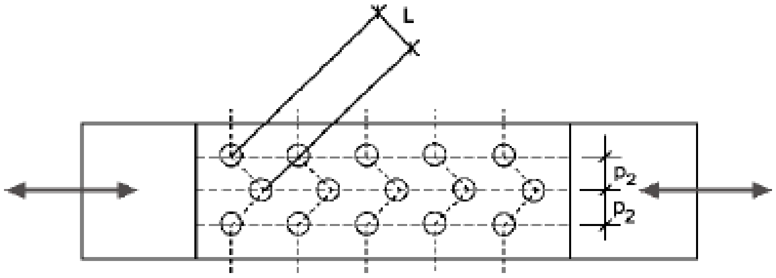
Max recommended thickness of all elements

OK

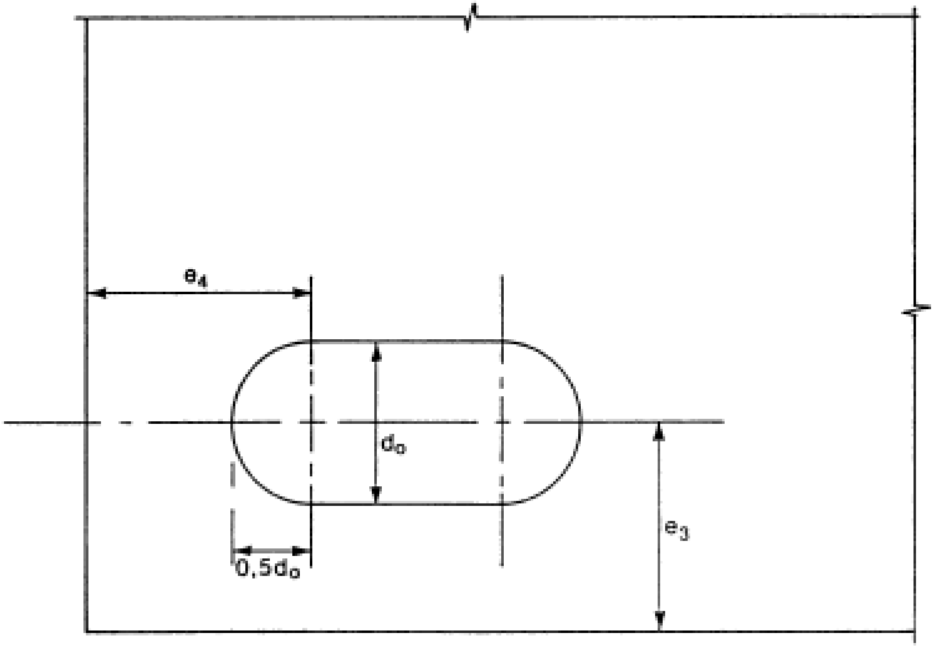
$$f_{ub} = 400\ \text{MPa} > f_u = (\text{S235}) = 360\ \text{MPa}$$

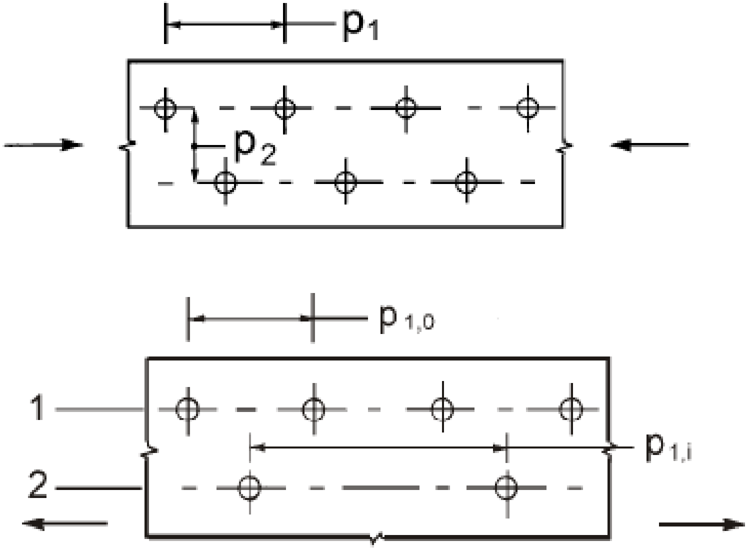
OK





Distances according to EN 1993-1-8 fig. 3.1

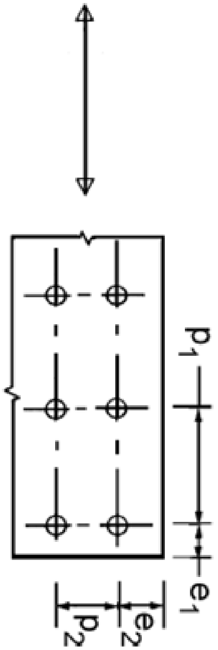




EN 1993-1-8 tab 3.3

Dimensions	Minimum	Maximum		
		„Normal” steels		Stainless steels
		Steel exposed to the weather / corrosion influences	Not exposed	
e_1	$1{,}2\ d_0$	$4\ t_{e,\ min} + 40$ mm		max($8\ t_{e,\ min}$; 125 mm)
e_2	$1{,}2\ d_0$	$4\ t_{e,\ min} + 40$ mm		max($8\ t_{e,\ min}$; 125 mm)
e_3	$1{,}5\ d_0$			
e_4	$1{,}5\ d_0$			
p_1	$2{,}2\ d_0$	min($14\ t_{e,\ min}$; 200 mm)	min($14\ t_{e,\ min}$; 200 mm)	min($14\ t_{min}$; 175 mm)
$p_{1,0}$		min($14\ t_{e,\ min}$; 200 mm)		
$p_{1,i}$		min($14\ t_{e,\ min}$; 200 mm)		
p_2	$2{,}4\ d_0$ ($1{,}2\ d_0$ and $L \geq 2{,}4\ d_0$)	min($14\ t_{e,\ min}$; 200 mm)	min($14\ t_{e,\ min}$; 200 mm)	min($14\ t_{min}$; 175 mm)

Stiffness of joint



EN 1993-1-8 fig. 3.1

- Structure uses slotted holes;
- For slotted holes, Eurocode only provides distances from edge;
- There are no guidelines regarding distance of oval slotted from each other;
- Distances for slotted holes are more restrictive than for round one;
- Design adopted guidelines for round holes.

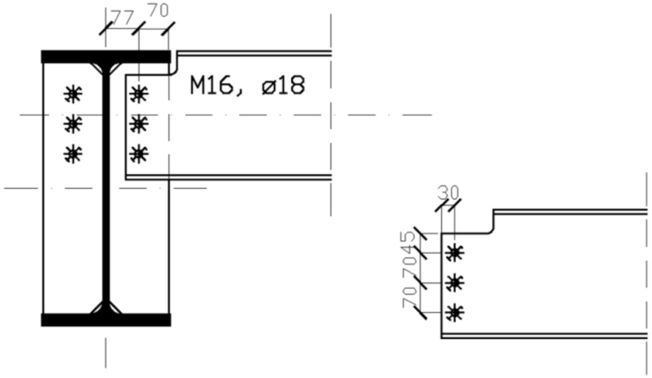


Photo: Author

Dimension	min [mm]	max [mm]	real [mm]	Conclusion
e_1 (e_3)	24	68	45	OK
e_4 (e_4)	24	68	30 ; 70	OK
p_1	35	99	70	OK
p_2	38	99		

e_2 – 30 for beam, 70 for stiffener (**stiffener will be fatter than web of beam, so for calculation is taken into consideration olny web of beam**)

According to results of experiments, we can assume, that there are always pinned joints, if:

→ #14 / 42

- web only is supported;
- for bolts are applied slotted holes.

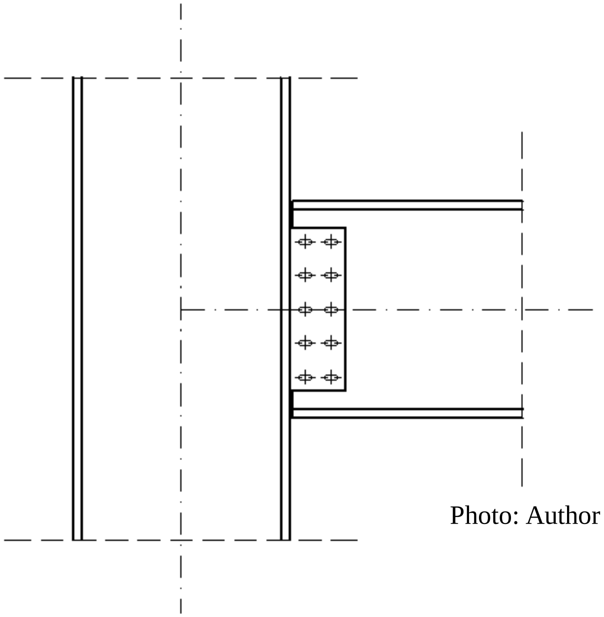


Photo: Author

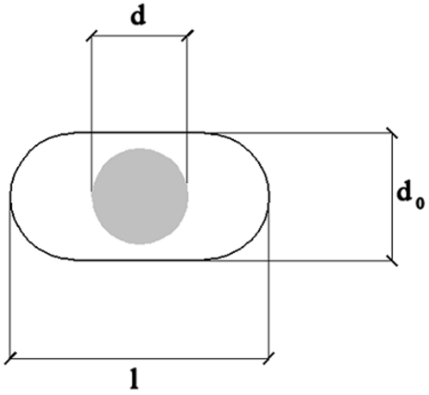


Photo: tekla-detailed-structural-fabrication.com

Differences between two diameters: d_0 - d = or l - d = [mm]

Bolts or pins:		M 12, M 14	M16, M 18, M 20, M 22	M 24	M 27, M 30 ...
Fit bolts		0			
Normal round holes		1 (0,5) 2	2 (1,5)		3 (2,5)
Oversize round holes		3	4	6	8
Short slotted holes	d_0	1 2	2		3
	l	4	6	8	10
Long slotted holes	d_0	1 2	2		3
	l	1,5 d			

EN 1090-2 tab 11

(mast, towers)

countersunk bolts

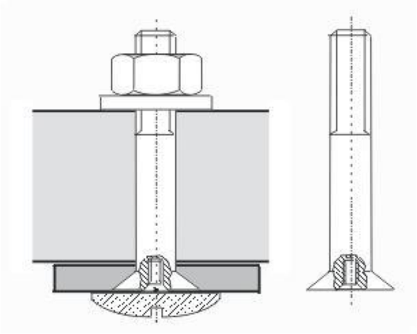


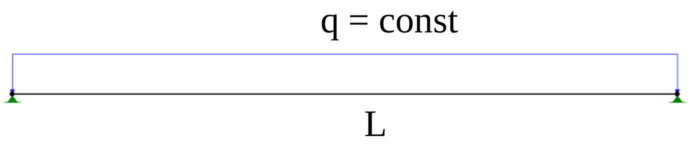
Photo: zeglarstwo.sail-ho.pl

Rivets - nominal hole diameter shall be specified individually in each design project

Due to application of slotted holes, there is clearance around bolts, between shanks and edges of holes. It allows for slight mutual rotation of connected elements and is important for good reproduction of behavior of hinge joint.

Is it sufficient to reproduce a perfectly hinged node?

General explanation of problem, **there is no need to repeat it in project**.



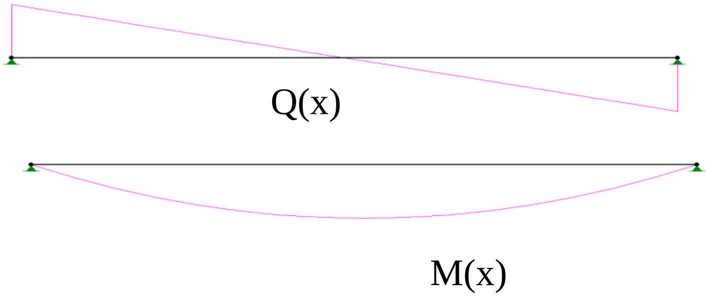


Photo: Author

Schwedler-Żurawski formulas:

$q(x) = -Q'(x) = -M''(x)$

$Q(x) = -q\,x + (q\,L / 2)$

$M(x) = -(q\,x^2 / 2) + (q\,L\,x / 2)$

Relations $M(x) \leftrightarrow w'(x) \leftrightarrow w(x)$:

$w''(x) = -M(x) / EJ$

$\underline{\phi(x) = w'(x) / (EJ) = [(q\,x^3 / 6) - (q\,L\,x^2 / 4) + (q\,L^3 / 24)] / (E\,J)}$

$w(x) = [(q\,x^4 / 24) - (q\,L\,x^3 / 12) + (q\,L^3\,x / 24)] / (E\,J)$

$w(x = L / 2) = w_{max} = 5\,q\,L^4 / (384\,E\,J)$

$V_{Ed} = Q(x = 0) = 90{,}50$ kN (→ #t / 44) ; L = 3,0 m (→ #t / 44) ; IPE 300 (→ #t / 44)

J (IPE 300) = 8 356 cm^4

$Q(x = 0) = q\, L / 2 = 90{,}50$ kN → q = 60,333 kN / m

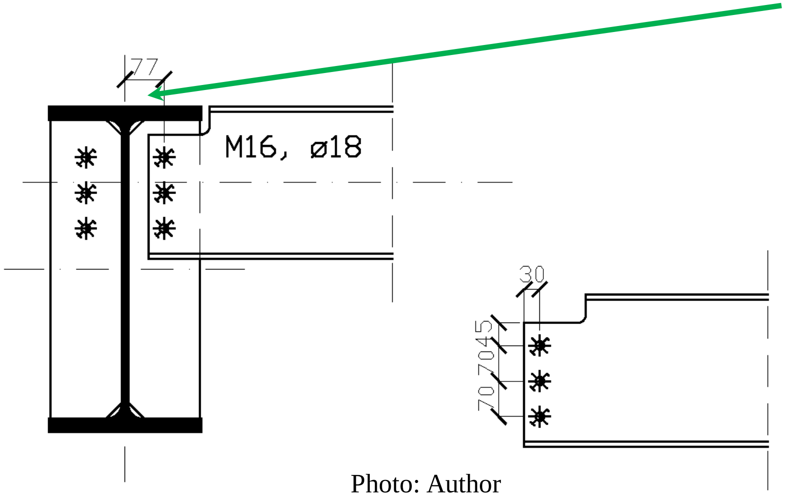


Photo: Author

Rotation angle in case of ideal hinge joit in axis of bolts: $\phi(x = 77 \text{ mm}) = 1{,}77^{o}$

Assumption: short slotted hole, M16

$d_0 = d + 2$ mm = 18 mm

l = d + 6 mm = 22 mm

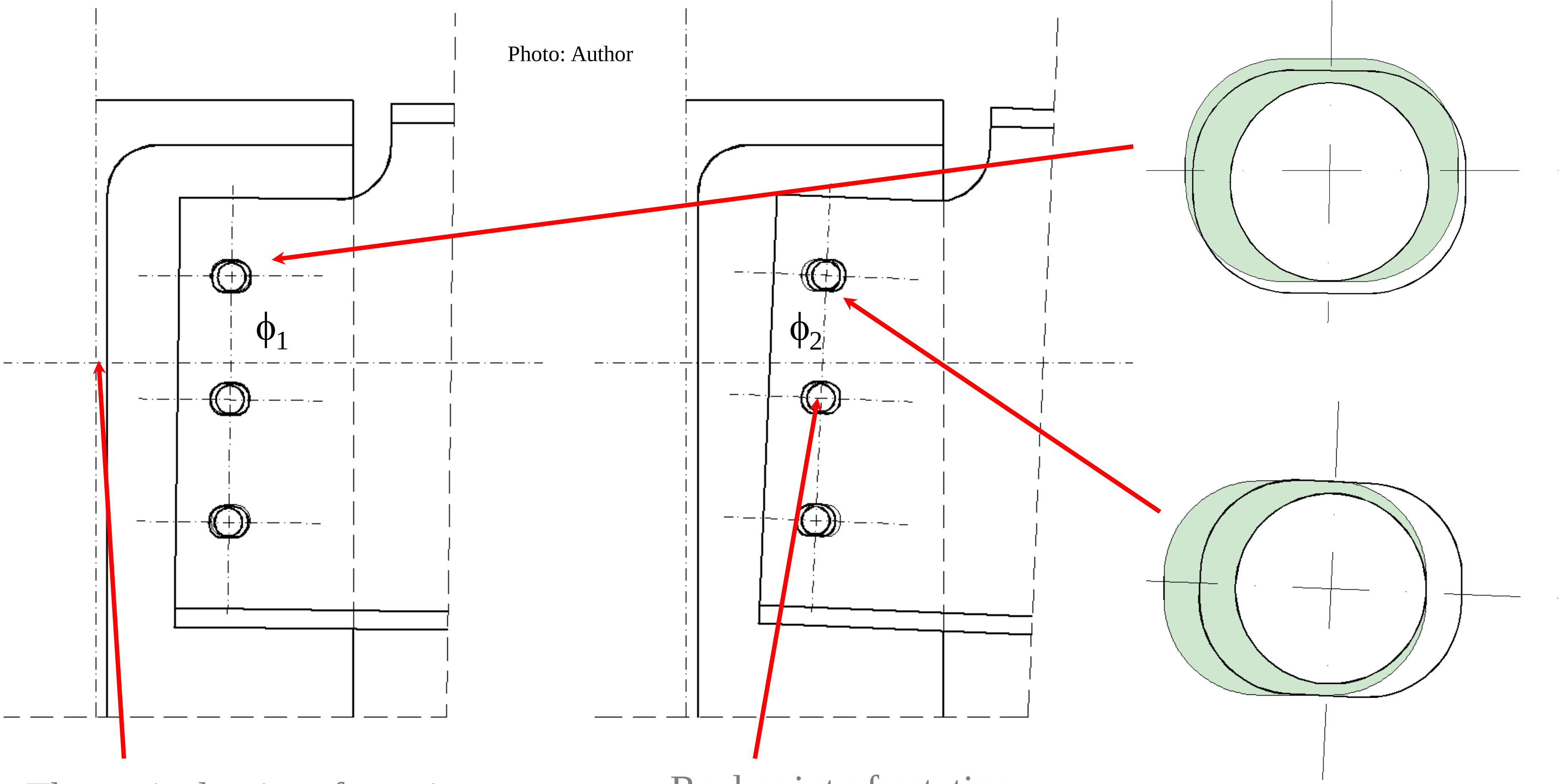


Photo: Author

Theoretical point of rotation

Real point of rotation

$\phi_1 \approx 0,7^o$

$\phi_2 \approx 2,5^o$

$\phi(x = 77\ \text{mm}) = 1,77\ ^o < \phi_2 \rightarrow$ no problem with rotation of beam thanks to slotted holes.

In ideal situation, thanks to clearances around bolt's shank, there is no contact between bolt and plate. Checking beraring resistance is not necessary.

In fact, due to imperfections, bolt may be right at edge of hole from the beginning and - due to rotation – immediatelly come into contact with plate. Checking beraring resistance is necessary just in case.

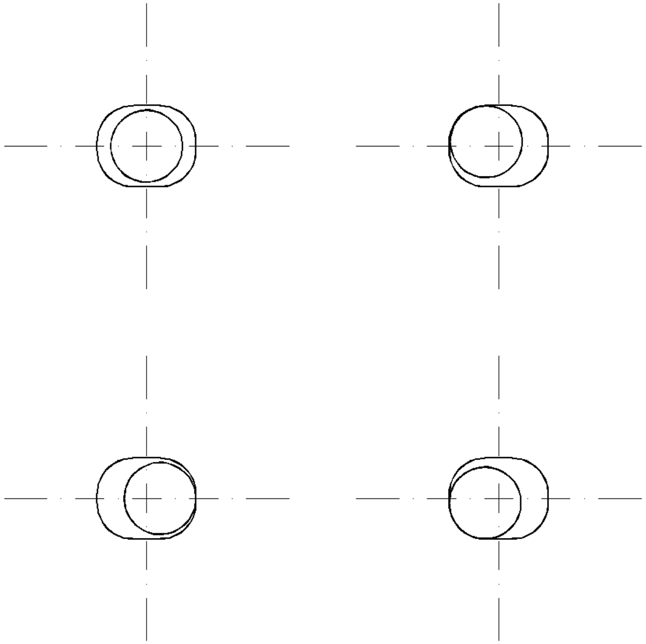
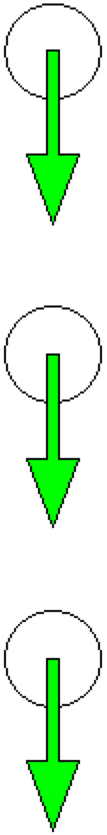


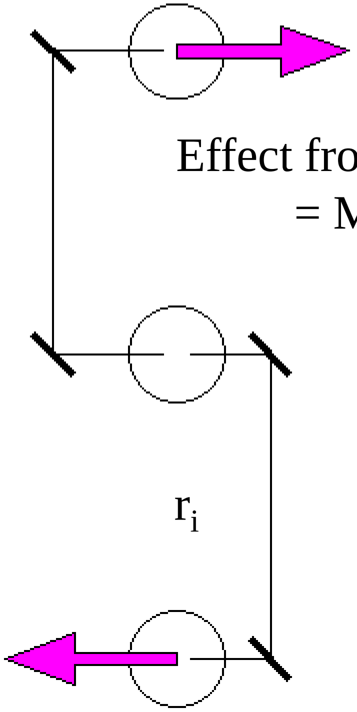
Photo: Author

Distribution of forces

Photo: Author

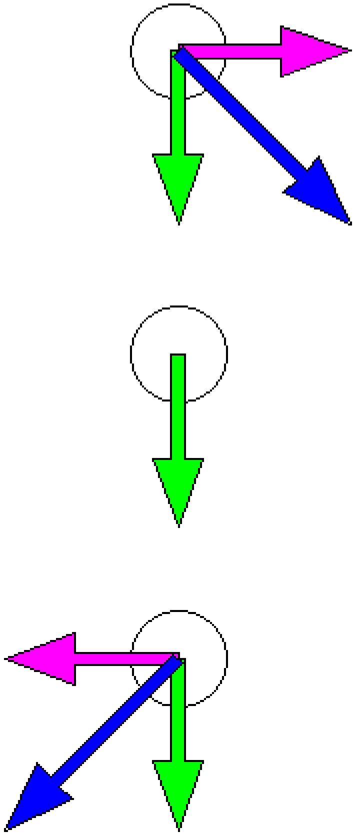
Effect from bending moment:
always perpendicular to arm r
between **center of bolts group** and
analised bolt.





Effect from bending moment =
$= M_{Ed} \cdot r_i / \Sigma\, [(r_i)^2]$

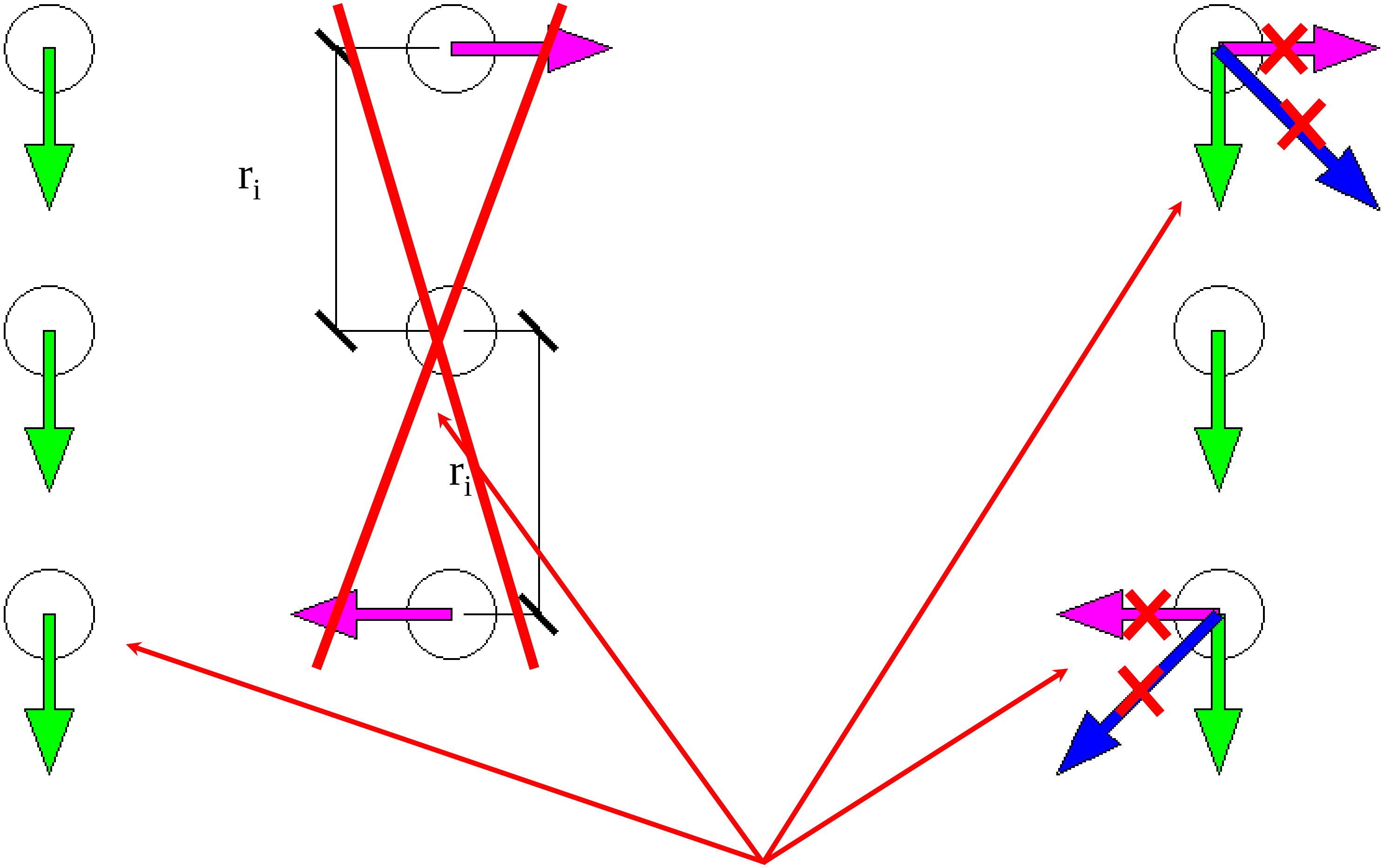
$\Sigma\, [(r_i)^2] \neq [\Sigma\, (r_i)]^2$



Resultants

Vertical force per one bolt =
= total force / number of bolts

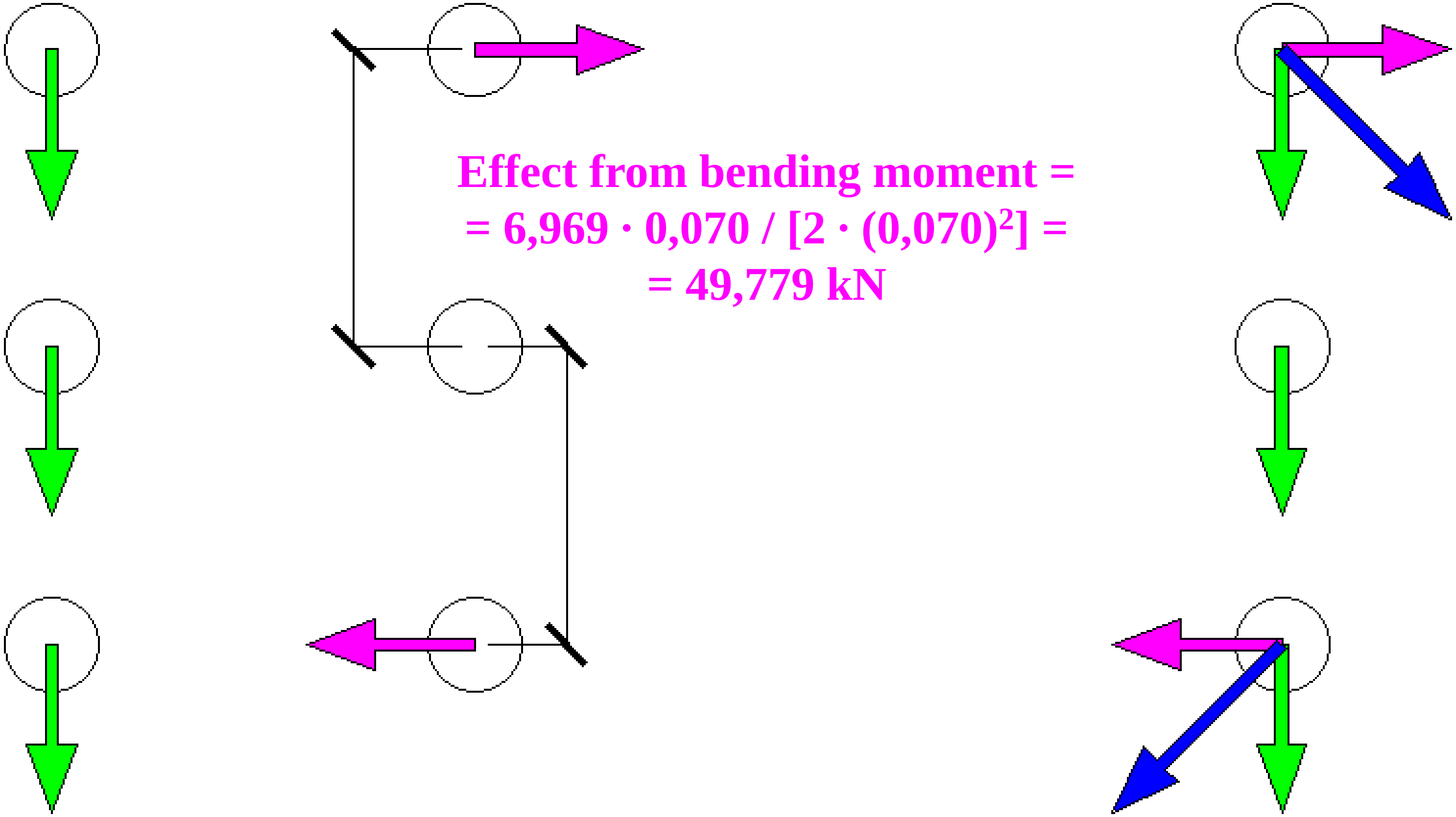
Secondary bending moment is very often neglected in calculation, because of its very small value. In example is presented full version; secondary bending moment is analysed.



Most often effect of analysis

Distribution of forces:

Photo: Author



Effect from bending moment =
$= 6{,}969 \cdot 0{,}070 / [2 \cdot (0{,}070)^2] =$
= 49,779 kN

Vertical force per one bolt =
= 90,50 / 3 = 30,167 kN

Resultant =
$= \sqrt{[(30{,}167)^2 + (49{,}779)^2]} =$
= 58,206 kN

Resistance of joint

Cathegory of joint A: three mechanism of destruction:

Shear resistance

(destruction of shank)

Photo: ceprofs.civil.tamu.edu

Bearing resistance

(local deformation of plate / web as effect of contact shank-plate)

Photo: ascelibrary.org

Block tearing

(total destruction of plate / web)

Photo: quora.com

Loads:

Shear resistance

Photo: ceprofs.civil.tamu.edu

Full value of force in one bolt

58,206 kN

Block tearing

Photo: quora.com

Total external vertical force

90,500 kN

Bearing resistance

Photo: ascelibrary.org

Both (vertical and horizontal) components in one bolt

30,167 kN **49,779 kN**

analyzed independently in both directions

Shear plane can goes through threated or unthreated part of bolt.

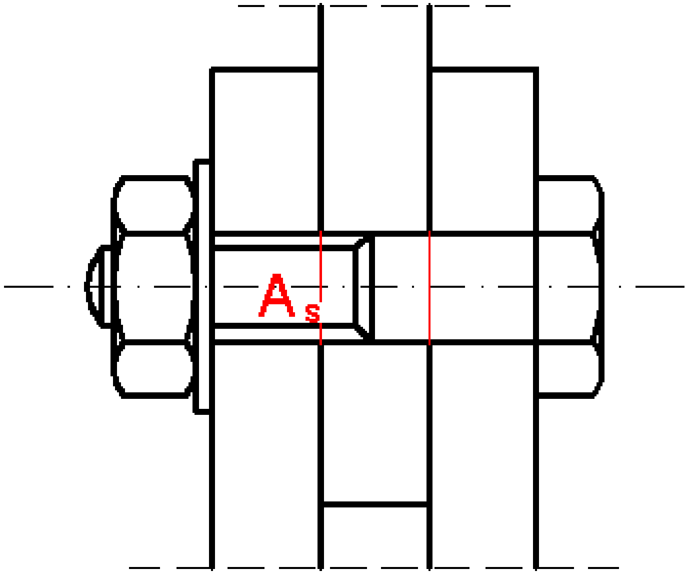


Photo: Author

It is necessary to check bolt length (length of thread along part of shank) now, and not - as in the previous project - only at the very end.

Recommendation for bolted joint category A: one washer under nut.

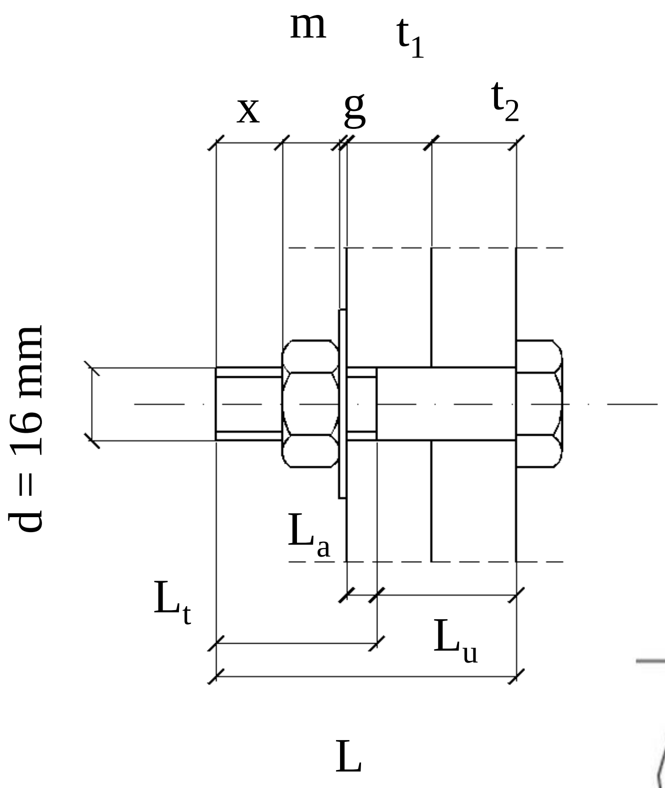


Photo: Author

t_1 – smaller one thickness; in this case: thickness of beam web, 7,1 mm

t_1 – thickness of stiffener; unknown for now, will be calculated in example Vth. As an estimate, we can assume 12 - 25 mm (different thicknesses in different places of beam). For this example of calculations, we will assume a maximum thickness of 25 mm.

L_a – recommended between 1P and 1/3 t_1 ($1/3 \cdot 7{,}1 = 2{,}5$ mm)

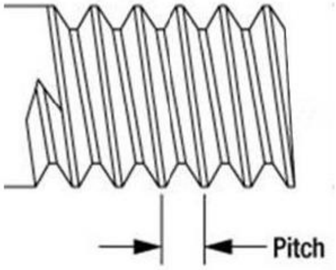


Photo: u-bolts-r-us.co.uk

EN ISO 4014:

Bolt	P [mm]
M16	2,0
M20	2,5
M24	3,0
M30	3,5

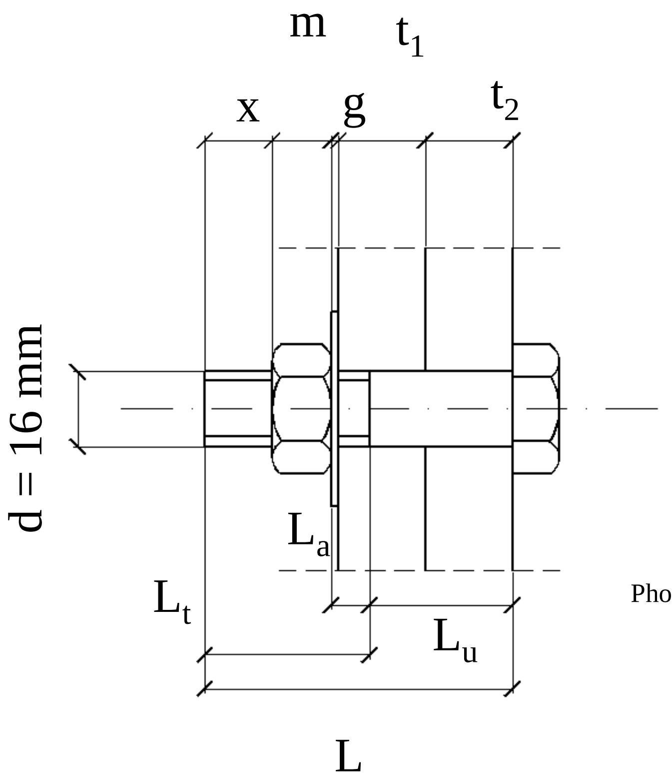


Photo: Author

$$d = 16 \geq x \geq 1P = 2$$

$$L = x + 14{,}8 + 4 + 7{,}1 + 25 = 53 – 76 \text{ mm}$$

$$t_2 + t_1 + g = 36 \geq L_u \geq t_2 + 2/3\ t_1 = 30 \text{ mm}$$

$$L_a \geq P$$

EN 14 399-5:

Bolt	g [mm]
M16	4,0
M20	4,0
M24	4,0
M30	5,0

EN ISO 4032:

Bolt	m [mm]
M16	14,8
M20	18,0
M24	21,5
M30	25,6

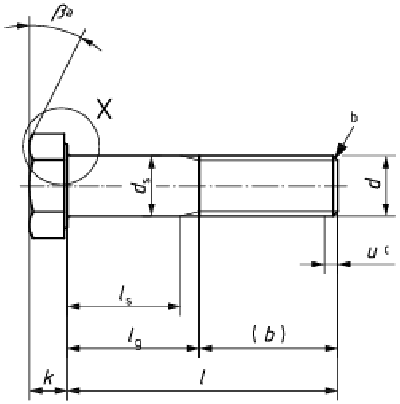


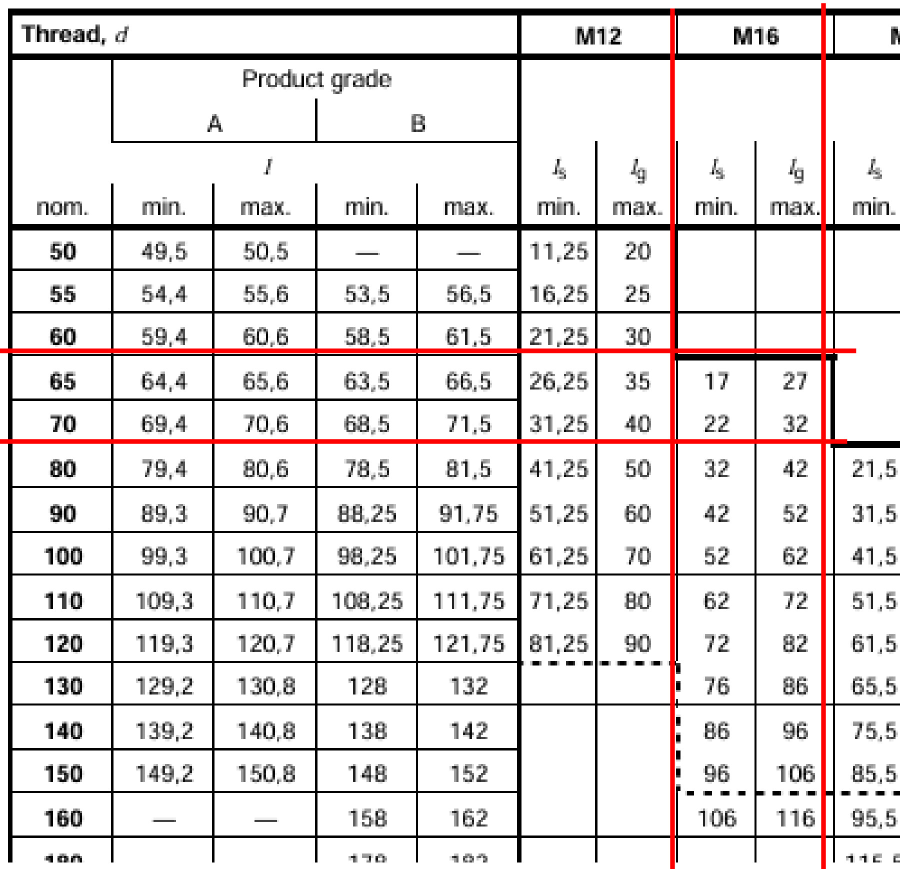
Photo: EN 4014 fig. 1

$$L = 53 – 76\text{ mm} \quad ; \quad l = 65\text{ mm} \quad \text{OK}$$

$$36 \geq L_u \geq 30\text{ mm} \quad ; \quad l_g = 27\text{ mm} \quad \text{too small}$$

$$b = l – l_g = 65 – 27 = 38$$

$$L_a = t_2 + t_1 + g – b = -2\text{ mm impossible}$$



Thread, d					M12		M16		M
	Product grade								
	A		B						
	l				l_s	l_g	l_s	l_g	l_s
nom.	min.	max.	min.	max.	min.	max.	min.	max.	min.
50	49,5	50,5	—	—	11,25	20			
55	54,4	55,6	53,5	56,5	16,25	25			
60	59,4	60,6	58,5	61,5	21,25	30			
65	64,4	65,6	63,5	66,5	26,25	35	17	27	
70	69,4	70,6	68,5	71,5	31,25	40	22	32	
80	79,4	80,6	78,5	81,5	41,25	50	32	42	21,5
90	89,3	90,7	88,25	91,75	51,25	60	42	52	31,5
100	99,3	100,7	98,25	101,75	61,25	70	52	62	41,5
110	109,3	110,7	108,25	111,75	71,25	80	62	72	51,5
120	119,3	120,7	118,25	121,75	81,25	90	72	82	61,5
130	129,2	130,8	128	132			76	86	65,5
140	139,2	140,8	138	142			86	96	75,5
150	149,2	150,8	148	152			96	106	85,5
160	—	—	158	162			106	116	95,5

Photo: EN 4014 tab. 1

$$L = 53 – 76\text{ mm} \quad ; \quad l = 70\text{ mm} \quad \text{OK}$$

$$36 \geq L_u \geq 30\text{ mm} \quad ; \quad l_g = 32\text{ mm} \quad \text{OK}$$

$$b = l – l_g = 70 – 32 = 38$$

$$L_a = t_2 + t_1 + g – b = -2\text{ mm impossible}$$

Recommended type of bolt (thread along part of shank length according to EN 14 399-4) does not satisfy geometric requirements. So, there will be adopted bolt with thread along entire shank length, type SB, according to EN 4017.

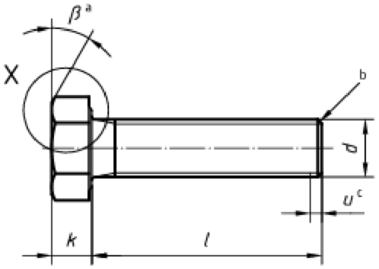
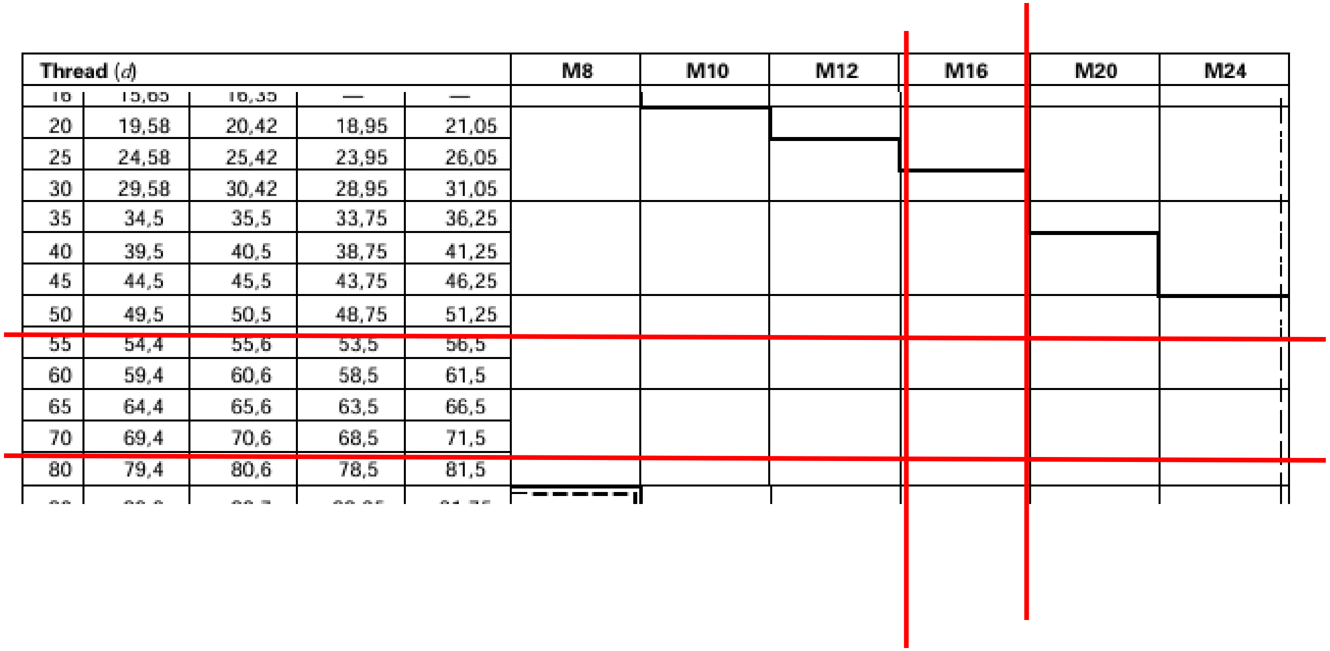


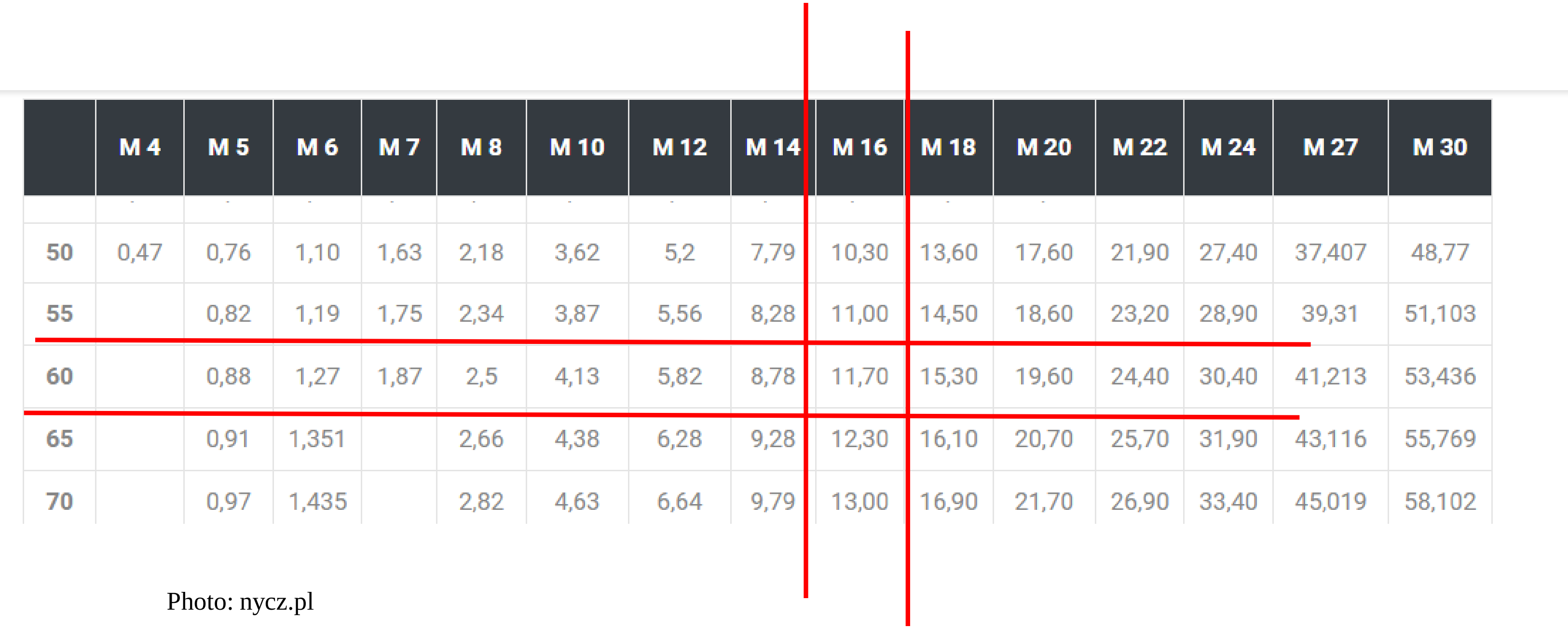
Photo: EN 4017 fig. 1



Thread (d)					M8	M10	M12	M16	M20	M24
16	15,65	16,35	—	—						
20	19,58	20,42	18,95	21,05						
25	24,58	25,42	23,95	26,05						
30	29,58	30,42	28,95	31,05						
35	34,5	35,5	33,75	36,25						
40	39,5	40,5	38,75	41,25						
45	44,5	45,5	43,75	46,25						
50	49,5	50,5	48,75	51,25						
55	54,4	55,6	53,5	56,5						
60	59,4	60,6	58,5	61,5						
65	64,4	65,6	63,5	66,5						
70	69,4	70,6	68,5	71,5						
80	79,4	80,6	78,5	81,5						

Photo: EN 4017 tab. 1

There is no problem with geommetrical requirements – but for sure shear plane goes through threated part of bolt. A_s = 1,61 cm^2



	M 4	M 5	M 6	M 7	M 8	M 10	M 12	M 14	M 16	M 18	M 20	M 22	M 24	M 27	M 30
50	0,47	0,76	1,10	1,63	2,18	3,62	5,2	7,79	10,30	13,60	17,60	21,90	27,40	37,407	48,77
55		0,82	1,19	1,75	2,34	3,87	5,56	8,28	11,00	14,50	18,60	23,20	28,90	39,31	51,103
60		0,88	1,27	1,87	2,5	4,13	5,82	8,78	11,70	15,30	19,60	24,40	30,40	41,213	53,436
65		0,91	1,351		2,66	4,38	6,28	9,28	12,30	16,10	20,70	25,70	31,90	43,116	55,769
70		0,97	1,435		2,82	4,63	6,64	9,79	13,00	16,90	21,70	26,90	33,40	45,019	58,102

Photo: nycz.pl

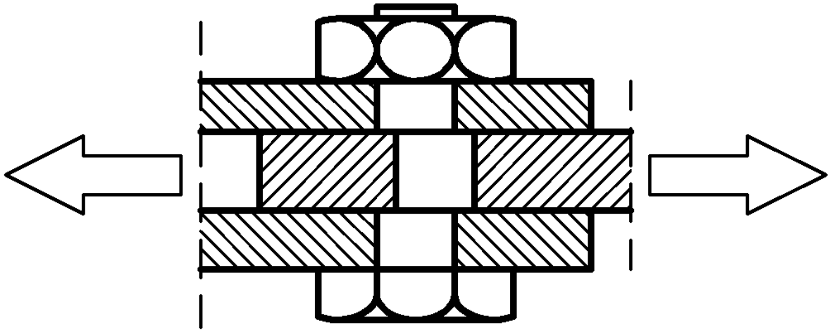
For example, for L = 60 mm, unit mass = 11,70 kg / 1000 pcs

Shear resistance: destruction of shank of bolt

Photo: Author

$F_{V,Rd} = n\ \alpha_v\ f_{ub}\ A_b / \gamma_{M2}$ EN 1993-1-8 tab 3.4

$\gamma_{M2} = 1{,}25$



A_b = area of unthreaded portion of bolt A, or threaded portion of bolt A_s

n - numer of shear planes (in analised case n = 1, between web of IPE 300 and vertical stiffener)

α_V - function of call of bolt:

$A_b = A_s$		$A_b = A$
4.6 5.6 8.8	4.8 5.8 6.8 10.9	
0,6	0,5	0,6

EN 1993-1-8 tab. 3.4

$F_{V,Rd} = n\ \alpha_v\ f_{ub}\ A_b\ /\ \gamma_{M2} = 1 \cdot 0{,}6 \cdot 400\ \mathrm{MPa} \cdot 1{,}61\ \mathrm{cm}^2\ /\ 1{,}25 = 24{,}730\ \mathrm{kN}$

Shear resistance

Photo: ceprofs.civil.tamu.edu

Total force in bolt **58,206 kN**

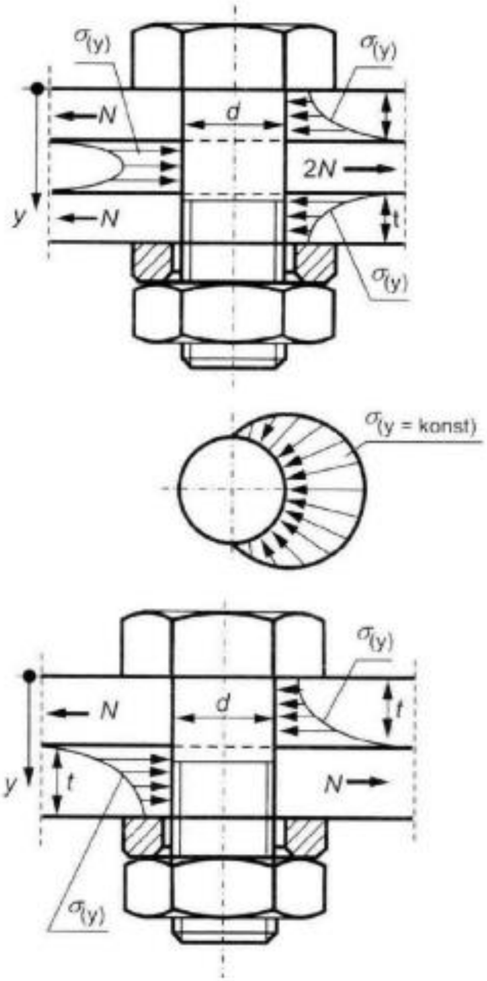
$R_{max}\ /\ F_{V,Rd} = 2{,}354 > 1{,}0$ wrong

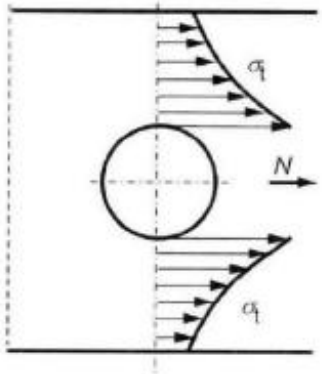
Conclusion: in joint should be more massive bolts or more than 3 bolts.

More thah 3 bolts → smaller external actions applicated to one bolt

More massive bolts → go to #t / 49, once again analysis of length of bolt and area; bigger resistance.

Bearing resistance





Deformation or destruction of plates as the effect of contact with shank of bolt.

Photo: A. Biegus, Projektowanie konstrukcji stalowych według Eurokodów, Politechnika Wrocławska

Local deformation or destruction - but no global destruction (block tearing)

Destruction could be parallel ‖ or perpendicular ⊥ to direction of force

Total destruction is problem of netto area or block tearing, not bearing resistance

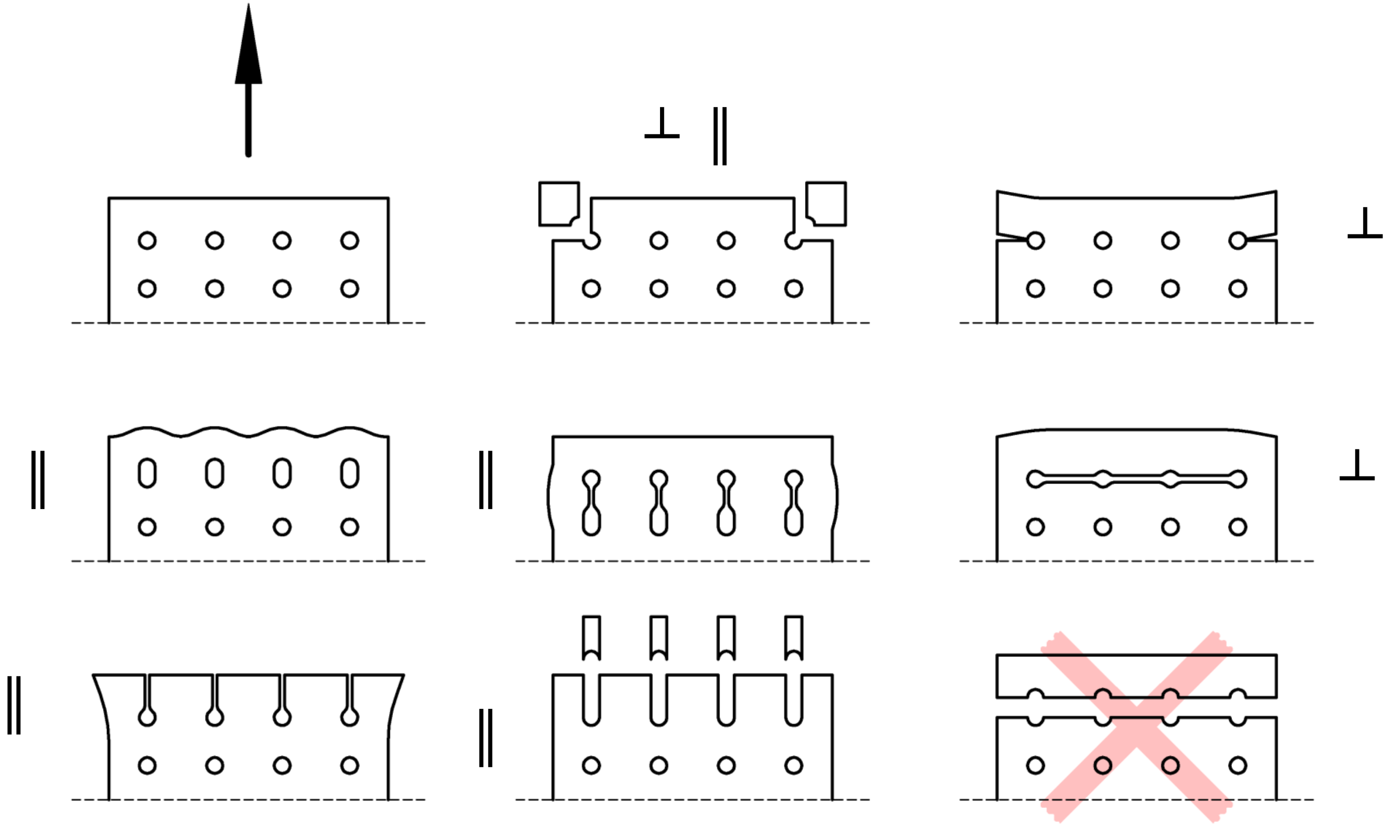


Photo: Author

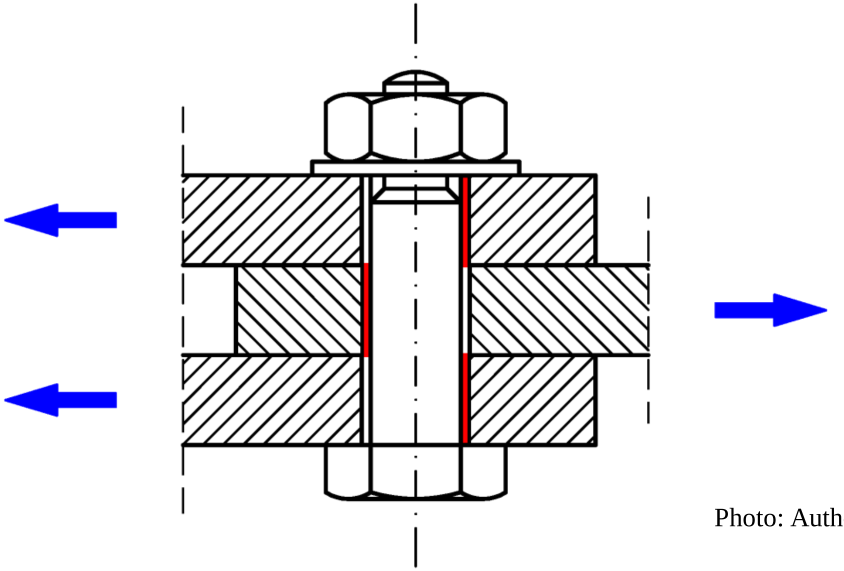


Photo: Author

$$F_{b,Rd} = \beta_b\, k_1\, \alpha_b\, f_u\, d\, t_{min} / \gamma_{M2}$$

EN 1993-1-8 tab 3.4, red part is given in bottom part of table.

β_b – parameter of shape of hole → #t / 75

k_1 – parameter for phenomenons in direction perpendicular ⊥ to force → #t / 76

α_b – parameter for phenomenons in direction parallel ‖ to force → #t / 75, 76

f_u – ultimate strength of plate

d – dimension of bolt

t_{min} – minimum total thickness of plate → #t / 75

$\gamma_{M2} = 1,25$

$$\alpha_b = \min(\alpha_d ;\ f_{ub} / f_u ;\ 1{,}0)$$

$$t_{min} = \min(\Sigma t_1 ;\ \Sigma t_2)$$

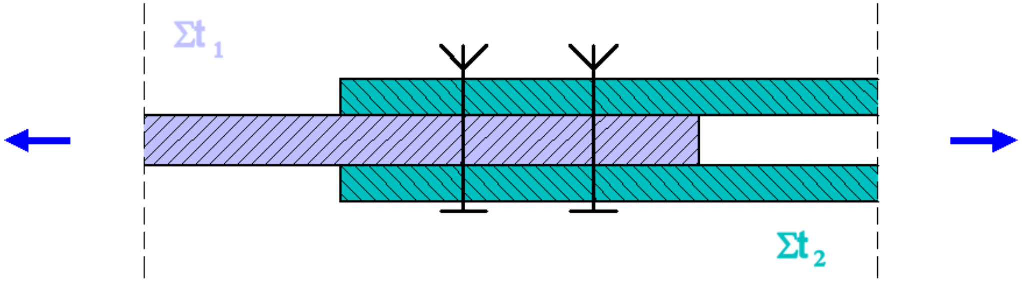


Photo: Author

Intentionally: no resistance, allowing free movement along hole's axis. This allows beam to rotate, as in an ideal hinge. After bolt has moved, it may possibly come into contact with plate. Then coefficient can be assumed as for an enlarged round hole 0,8 (to avoid damage of plate).

	β_b	
Fit bolts Normal round holes	1,0	
Oversized round holes	0,8	
Slotted holes	0,6	▬ ↑
		▬ ←

EN 1993-1-8 tab 3.4

k_1 – parameter for phenomenons in direction perpendicular to force $\perp$

α_b – parameter for phenomenons in direction parallel to force $\parallel$

$$\alpha_b = \min (\alpha_d \; ; \; f_{ub} / f_u \; ; \; 1{,}0)$$

EN 1993-1-8 tab 3.4

d_0 – diameter of hole

	k_1 $\perp$ Direction perpendicular to force
Inner	$\min (1{,}4\, p_2 / d_0 - 1{,}7 \; ; \; 2{,}5)$
Edge	$\min (2{,}8\, e_2 / d_0 - 1{,}7 \; ; \; 2{,}5)$

	α_d $\parallel$ Direction parallel to force
Inner	$p_1 / 3d_0 - 0{,}25$
End	$e_1 / 3d_0$

Index 1 and 2 in symbols e_1 e_2 p_1 p_2 - there are no horizontal H and vertical V directions, but always paralell (1) ‖ and perpendicular (2) ⊥ to direction of force:

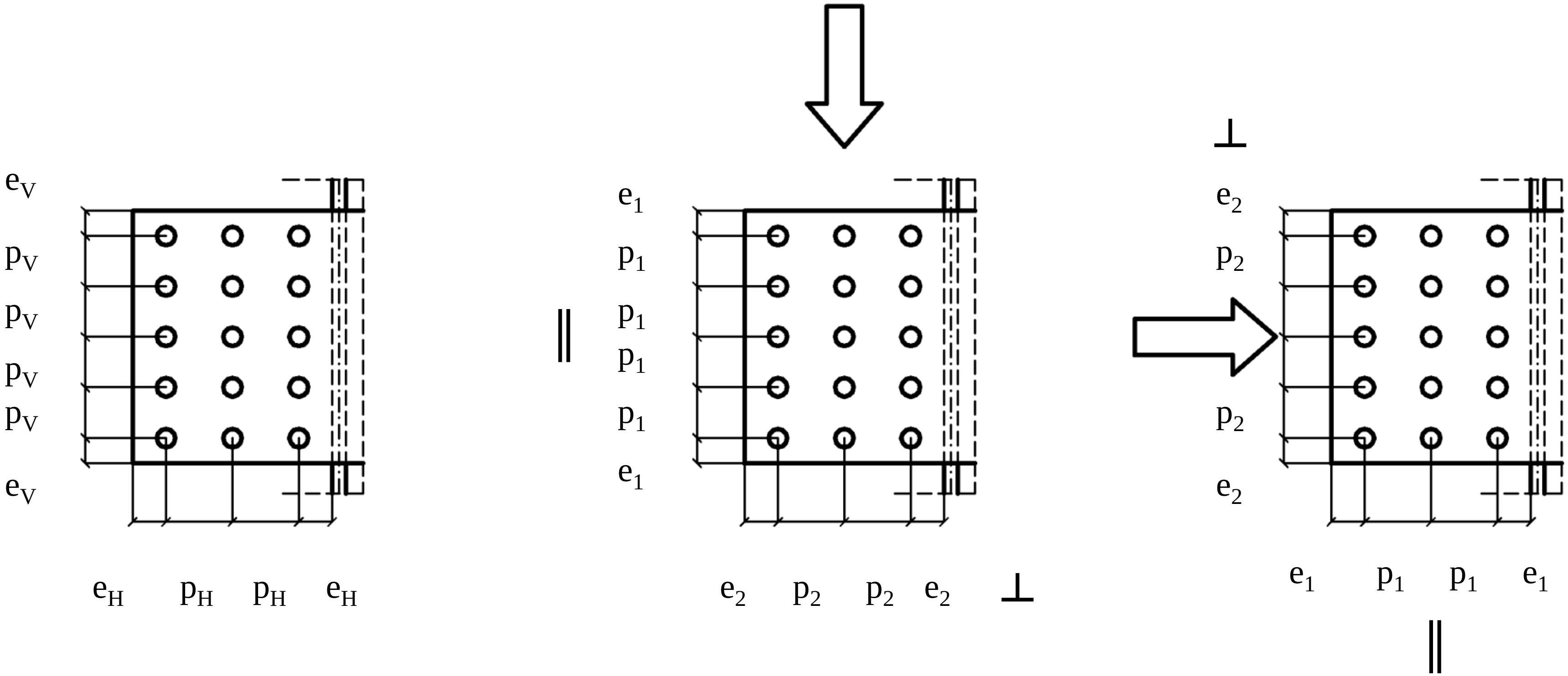


Photo: Author

	$k_1 \perp$	Notice
Inner	$\min(1{,}4\ p_2 / d_0 - 1{,}7\ ;\ 2{,}5)$	Neighboring bolts on **both sides** ⊥ **to force**
Edge	$\min(2{,}8\ e_2 / d_0 - 1{,}7\ ;\ 2{,}5)$	Neighboring bolts on **one side only**

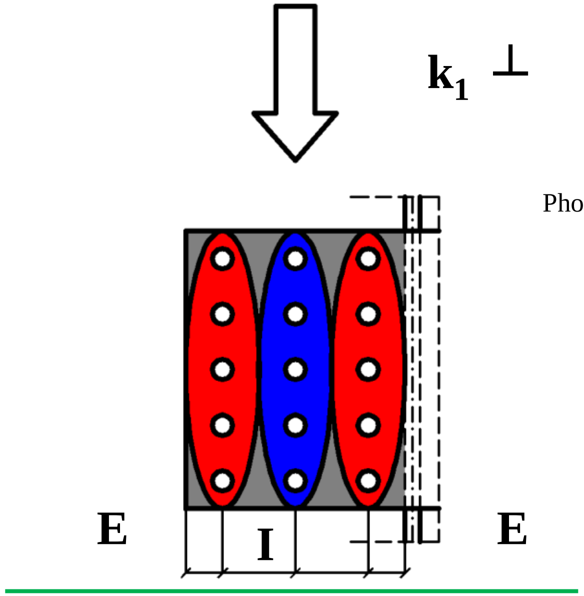
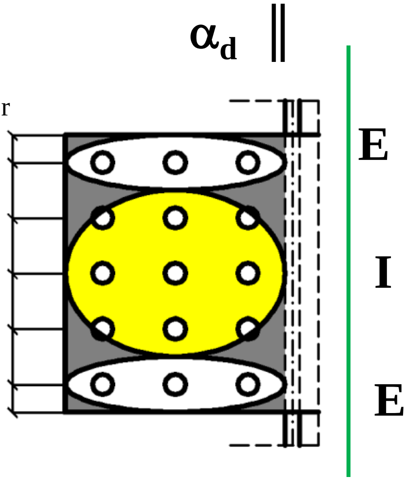
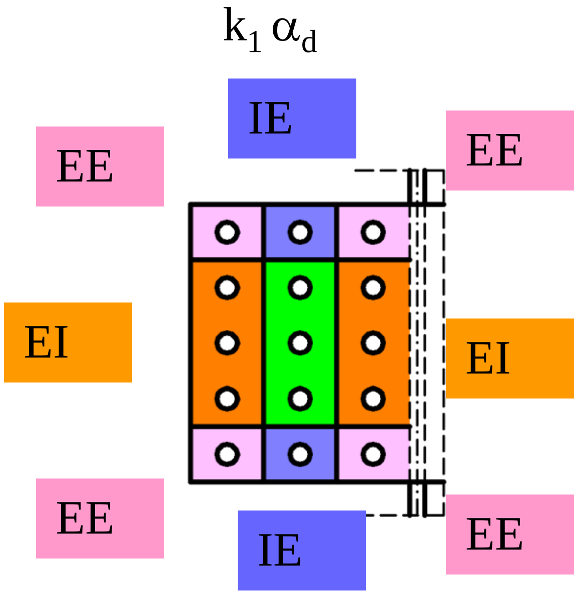


Photo: Author





	$\alpha_d \parallel$	Notice
Inner	$p_1 / 3d_0 - 0{,}25$	Neighboring bolts on **both sides** ∥ **to force**
End	$e_1 / 3d_0$	Neighboring bolts on **one side only**

	$k_1 \perp$	Notice
Inner	min(1,4 p_2 / d_0 - 1,7 ; 2,5)	Neighboring bolts on **both sides** ⊥ **to force**
Edge	min(2,8 e_2 / d_0 - 1,7 ; 2,5)	Neighboring bolts on **one side only**

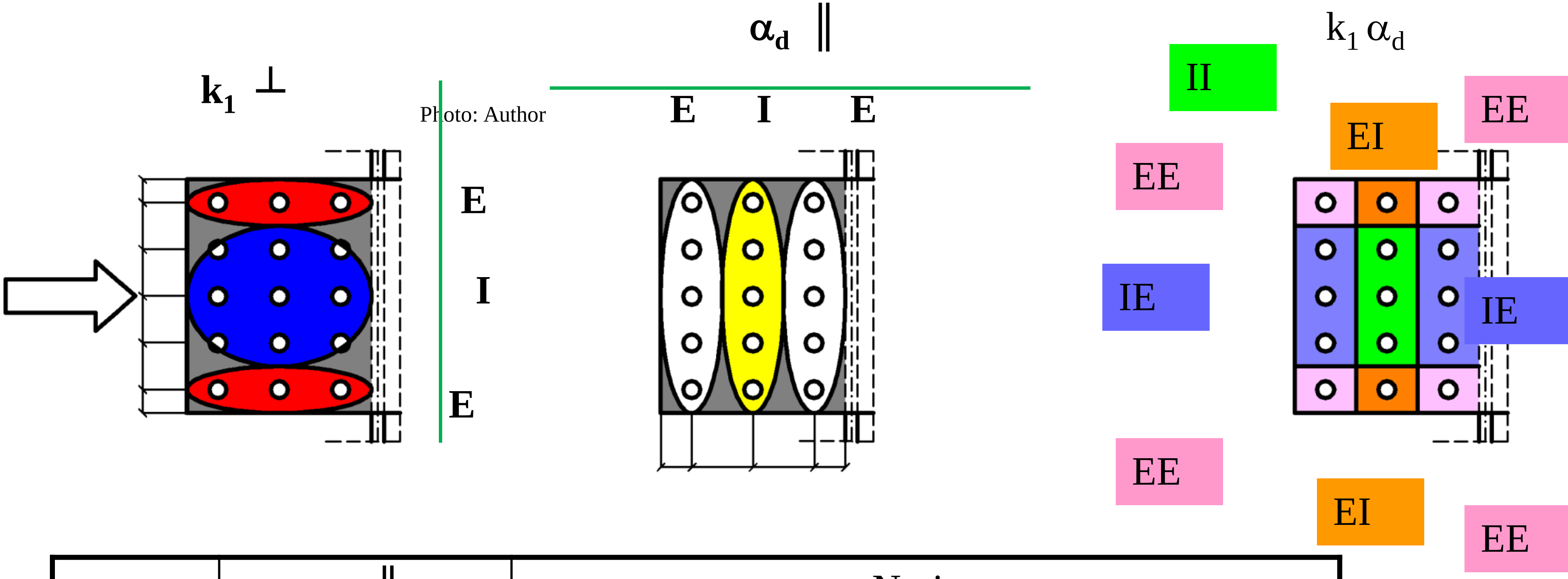
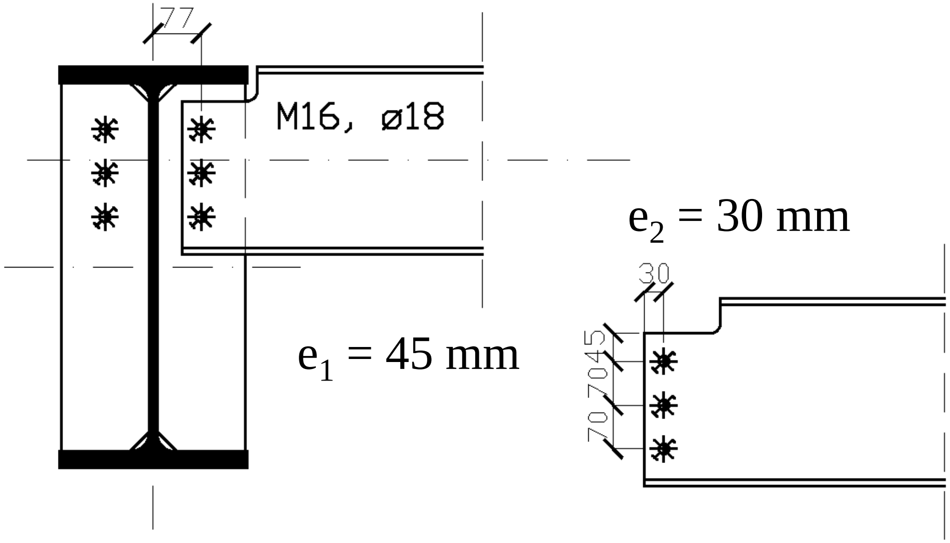


Photo: Author

	α_d ∥	Notice
Inner	p_1 / 3d_0 - 0,25	Neighboring bolts on **both sides** ∥ **to force**
End	e_1 / 3d_0	Neighboring bolts on **one side only**



e_2 = 30 mm

e_1 = 45 mm

p_1 = 70 mm

Photo: Author

No inner k_1 ⊥ α_d ∥

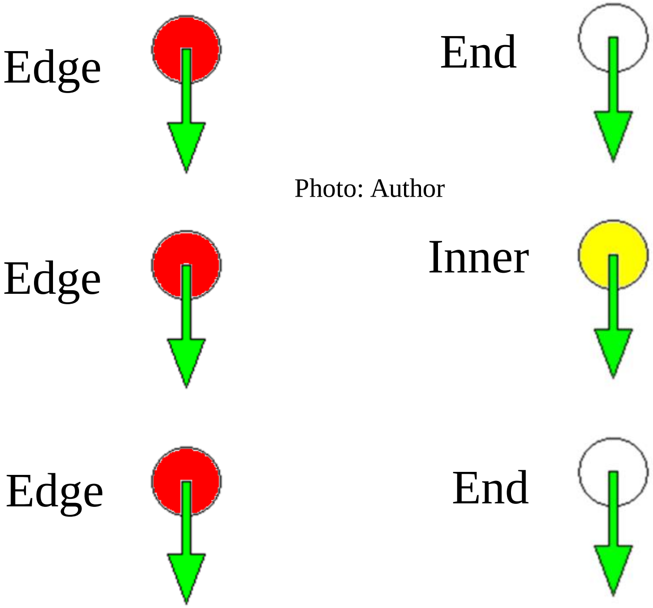


Photo: Author

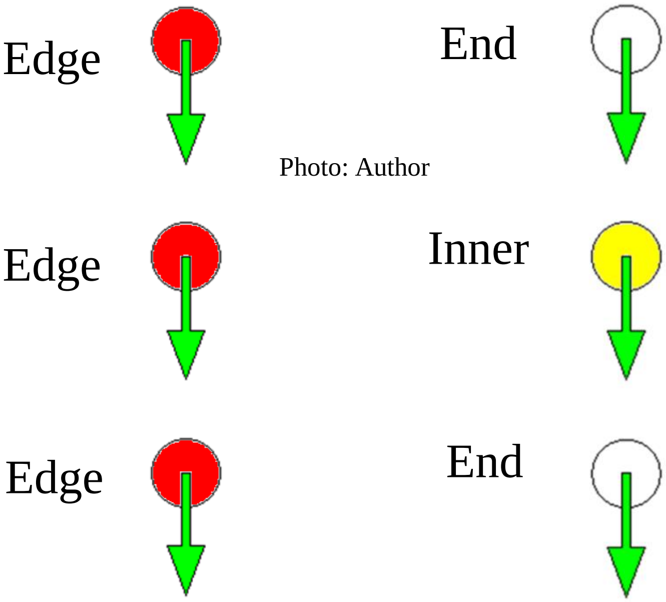
d_0 = (slotted hole: l) = 22 mm

d_0 = 18 mm

	k_1 ⊥ Direction perpendicular to force
Inner	min (1,4 p_2 / d_0 - 1,7 ; 2,5)
Edge	min (2,8 e_2 / d_0 - 1,7 ; 2,5)

	α_d ∥ Direction parallel to force
Inner	p_1 / $3d_0$ - 0,25
End	e_1 / $3d_0$

No inner k_1 ⊥ α_d ∥



Edge End

Photo: Author

Edge Inner

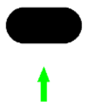
Edge End

$f_{ub} / f_u = 400 / 360 = 1,111$

	k_1 ⊥
Inner	
Edge	2,5

	α_d ∥	$\alpha_b = \min(\alpha_d ; \ f_{ub} / f_u ; 1,0)$ ∥
Inner	1,046	1,000
End	0,833	0,833

$$t_{min} = \min(\Sigma t_1 ; \Sigma t_2) = \min(7{,}1 \;\; ; \;\; 30{,}0) = 7{,}1 \text{ mm}$$



$\beta_b = 0{,}6$

Photo: Author

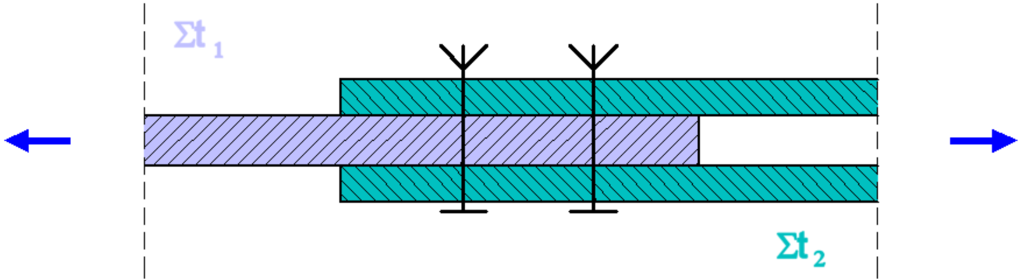
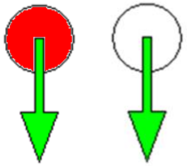
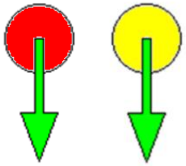


Photo: Author

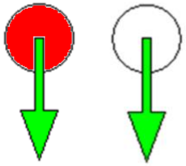
Resistance for vertical direction:



$$F_{b,V,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} = 0{,}6 \cdot 2{,}5 \cdot 0{,}833 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$
$$= 45{,}422 \text{ kN}$$



$$F_{b,V,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} = 0{,}6 \cdot 2{,}5 \cdot 1{,}0 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$
$$= 54{,}528 \text{ kN}$$



$$F_{b,V,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} = 0{,}6 \cdot 2{,}5 \cdot 0{,}833 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$
$$= 45{,}422 \text{ kN}$$

k_1 ⊥

No inner

$\alpha_d \parallel$

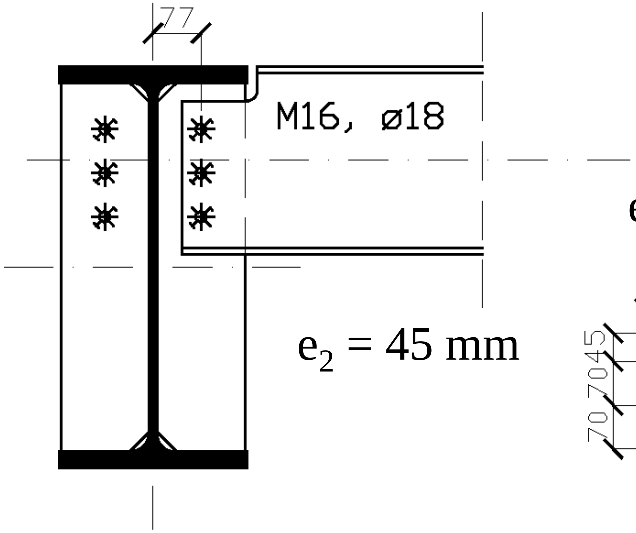
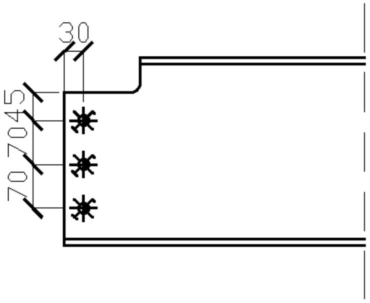


Photo: Author

e_1 = 30 mm

e_2 = 45 mm



p_2 = 70 mm

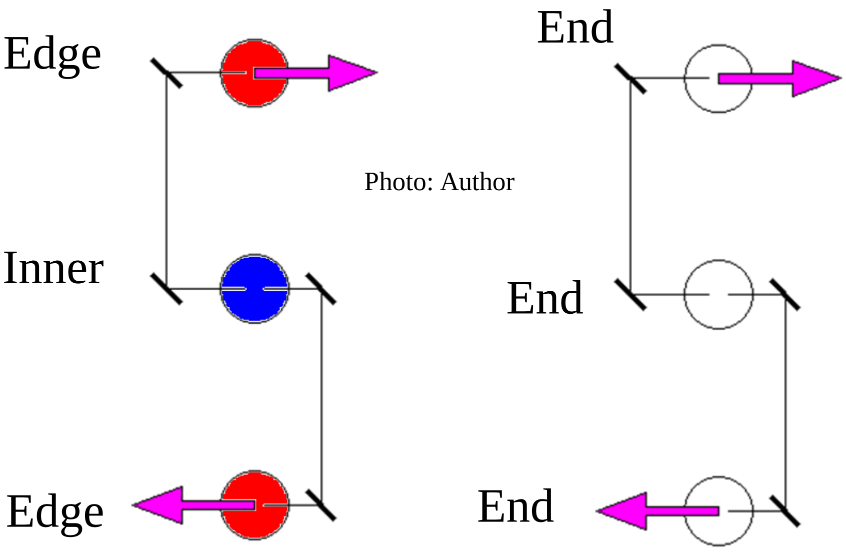


Photo: Author

d_0 = (slotted hole: l) = 22 mm

	k_1 ⊥ Direction perpendicular to force
Inner	min (1,4 p_2 / d_0 - 1,7 ; 2,5)
Edge	min (2,8 e_2 / d_0 - 1,7 ; 2,5)

	$\alpha_d \parallel$ Direction parallel to force
Inner	p_1 / 3d_0 - 0,25
End	e_1 / 3d_0

k_1 ⊥ **No inner** $\alpha_d \parallel$

	k_1 ⊥
Inner	2,5
Edge	2,5

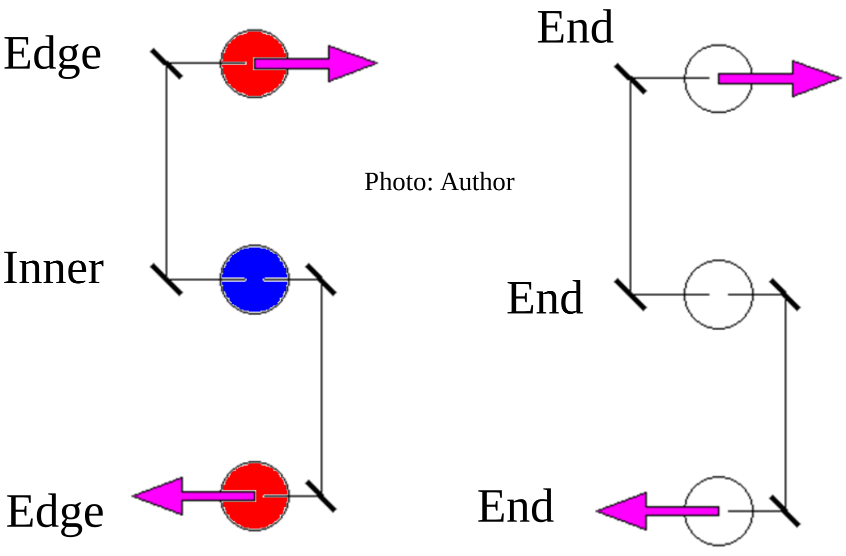


Photo: Author

$$f_{ub} / f_u = 400 / 360 = 1{,}111$$

	$\alpha_d \parallel$	$\alpha_b = \min(\alpha_d ;\ f_{ub} / f_u ;\ 1{,}0) \parallel$
Inner		
End	0,455	0,455

$$t_{min} = \min(\Sigma t_1 ; \Sigma t_2) = \min(7{,}1 \quad ; \quad 30{,}0) = 7{,}1 \text{ mm}$$

$$\beta_b = 0{,}8$$

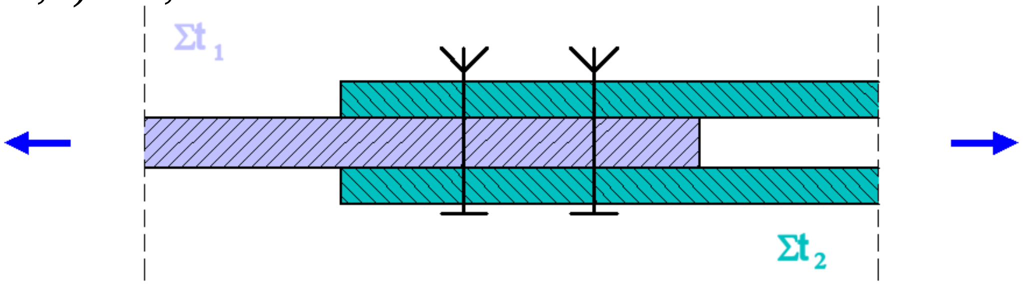


Photo: Author

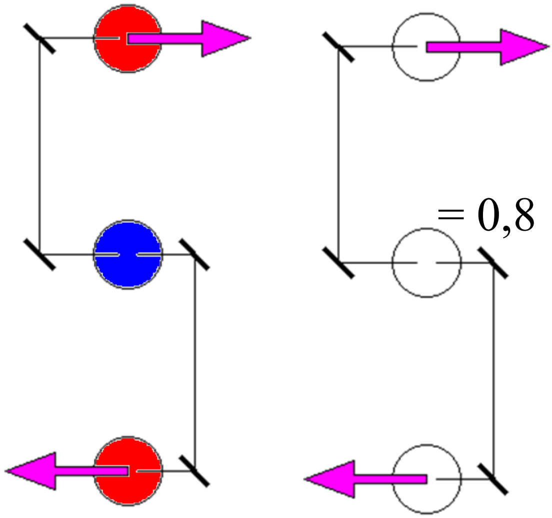
Resistance for horizontal direction:

$$F_{b,H,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} =$$

$$= 0{,}8 \cdot 2{,}5 \cdot 0{,}455 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$

$$= 34{,}459 \text{ kN}$$

Photo: Author



$$F_{b,H,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} =$$

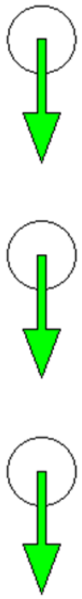
$$= 0{,}8 \cdot 2{,}5 \cdot 0{,}455 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$

$$= 34{,}459 \text{ kN}$$

$$F_{b,H,Rd} = \beta_b \, k_1 \, \alpha_b \, f_u \, d \, t_{min} / \gamma_{M2} =$$

$$= 0{,}8 \cdot 2{,}5 \cdot 0{,}455 \cdot 400 \text{ MPa} \cdot 16 \text{ mm} \cdot 7{,}1 \text{ mm} / 1{,}25 =$$

$$= 34{,}459 \text{ kN}$$



Vertical direction

Force [kN]	Resistance [kN]	E / R
30,167	45,422	0,664 < 1,0 OK
30,167	54,528	0,553 < 1,0 OK
30,167	45,422	0,664 < 1,0 OK

Horizontal direction

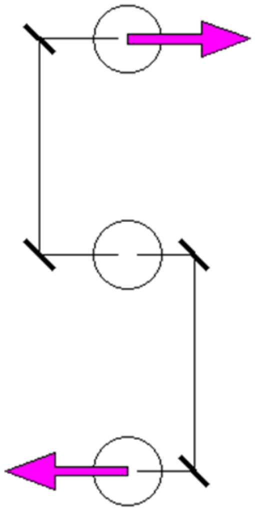


Photo: Author

Force [kN]	Resistance [kN]	E / R
49,779	34,459	1,445 > 1,0
0,000	34,459	0,000 < 1,0 OK
49,779	34,459	1,445 > 1,0

Conclusion: more than 3 bolts should be applicated.

More thah 3 bolts → smaller external actions applicated to one bolt

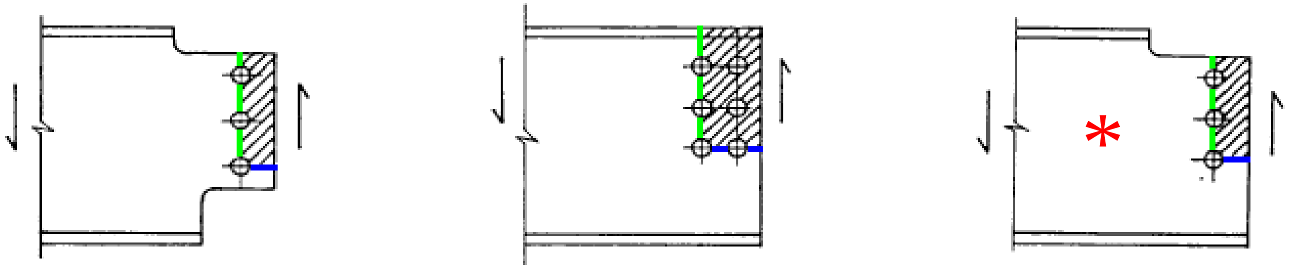
More massive bolts → change of d_0 and l, small impact for bearing resistance (but once again analysis of length of bolt)

Block tearing – total destruction of web

EN 1993-1-8 3.10.2

Shear

Tension



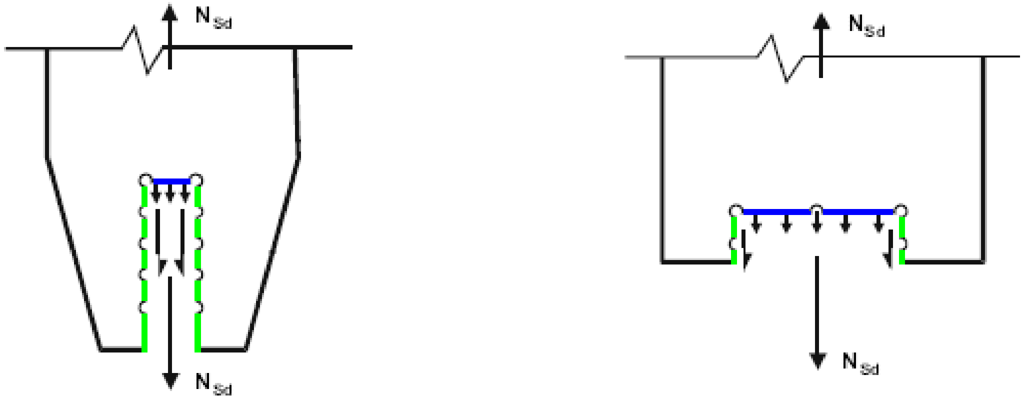
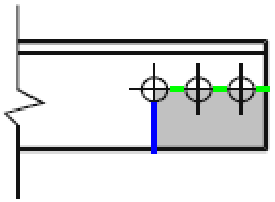


Photo: EN 1993-1-8 fig. 3.8



$$V_{eff,\,1,\,Rd} = \quad f_u\, A_{nt} / \gamma_{M2} + f_y\, A_{nv} / (\sqrt{3}\;\; \gamma_{M0}) \quad | \quad 0{,}5\, f_u\, A_{nt} / \gamma_{M2} + f_y\, A_{nv} / (\sqrt{3}\;\; \gamma_{M0})$$

$$\gamma_{M0} = 1{,}00; \qquad \gamma_{M2} = 1{,}25$$

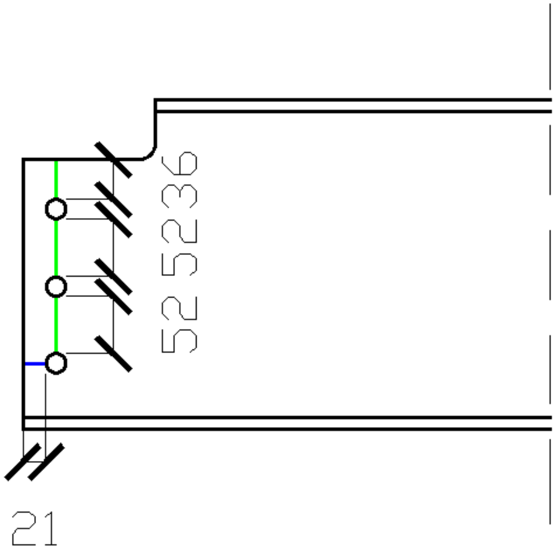


Photo: Author

Photo: quora.com

Total vertical force

90,50 kN

$A_{nv} = 0{,}71 \cdot (5{,}2 + 5{,}2 + 3{,}6) = 9{,}94 \text{ cm}^2$

$A_{nt} = 0{,}71 \cdot 2{,}1 = 1{,}49 \text{ cm}^2$

$V_{eff,\ 1,\ Rd} = 0{,}5\ f_u\ \mathbf{A_{nt}} / \gamma_{M2} + f_y\ \mathbf{A_{nv}} / (\surd 3\ \ \gamma_{M0}) =$

$= 0{,}5 \cdot 360 \text{ MPa} \cdot 1{,}49 \text{ cm}^2 / 1{,}25 +$

$+ 235 \text{ MPa} \cdot 9{,}94 \text{ cm}^2 / (\surd 3 \cdot 1{,}0) =$

$= 21{,}456 \text{ kN} + 134{,}863 \text{ kN} = 156{,}319 \text{ kN}$

$E / R = 90{,}50 / 156{,}319 = 0{,}579 < 1{,}0$ OK

Conclusions

- Slotted holes enable good imitation of hinge joint;
- Horizontal forces come from secondary bending moment are often neglected in calculation;
- In general case, bearing resistance must be analysed separately for horizontal and vertical direstions (various resistances because of various distances between bolts) for each bolts (various external loads because of distribution reactions from bending moment);

Thank you for attention

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